Operations Research Perspectives 9 (2022) 100259

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

A signed distance based ranking approach with unknown fuzzy priority vectors for medical diagnosis involving interval type-2 trapezoidal pythagorean fuzzy preference relations

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ARTICLE INFO

MSC:
06Dxx
06D72

Keywords:
"Interval type 2 trapezoidal pythagorean fuzzy preference relation" (IT2TrPFPR)
"Hybrid averaging" (HA) operation
"Ordered weighted averaging (OWA) operation"
"Signed-based distance closeness coefficient" (SDBCC)

ABSTRACT

In many of our real life problems, we often come across situations where there is no information about the priority weights which make it difficult to analyze the objects under consideration. Instead of employing simple fuzzy sets, "interval type-2 trapezoidal pythagorean fuzzy preference relations (IT2TrPFPRs)" can be used which have better representational power and ability to cope with uncertain situations. The approach discussed in this article is an effective tool for managing multiple criteria group decision-making situations with completely unknown priority weights modeled as IT2TrPFPRs. To aggregate the opinion of multiple decision-makers, a hybrid averaging operation based on weighted averaging and ordered weighted averaging (OWA) operations is employed for a collective decision environment. To calculate the fuzzy priority weight vectors in case of completely unknown environment, we construct a non-linear optimization model. An integrated optimization model based on a new signed distance-based closeness coefficients approach is employed to determine the priority ranking of alternatives. Feasibility of the proposed technique is discussed with an implementation of patient centered medicine system for choosing the appropriate treatment method. Moreover, a comparative investigation with previous approaches is conducted to demonstrate the effectiveness of the given approach.

1. Introduction

Decision-making information presented by group decision-makers is frequently vague, inaccurate and uncertain due to the deficiency of information, time hassle or insufficient concentration of decision-makers. Consequently, solutions related to Group decision-making (GDM) problems becomes difficult and challenging. The fuzzy set (FS) theory was initially proposed by Zadeh [1] in 1965 to signify the degree of elements belonging to a particular set. They were formerly proposed to cope up with the problems concerning with subjective uncertainties. Later on, type-2 fuzzy sets (T2FSs) [2–4], an extension of type-1 fuzzy sets (T1FSs) were introduced since they are better in measuring imprecise information and provide us a more suitable representation which is computationally feasible to cope with uncertain situations. But the secondary memberships in T2FSs often makes them very difficult to apply in certain practical situations. Therefore to handle this situation interval type-2 (IT2FSs) are vastly employed where the secondary membership grade is fixed to 1. IT2FSs are very effective in dealing with uncertain and ambiguous problems because of their representational power and less computational cost as compared to T2FSs. Therefore, in

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https://doi.org/10.1016/j.orp.2022.100259
Received 29 June 2022; Received in revised form 27 October 2022; Accepted 28 October 2022
Available online 13 November 2022

our approach we will also employ the approach of IT2FSs by Mendel [5] instead of T2FSs because of the complexity associated with them.

By considering both the membership and non-membership degree of an element in a fuzzy set, Atanassov [6,7] generalized the fuzzy sets to intuitionistic fuzzy sets (IFSs) which have widely been used to deal effectively with diverse decision making problems as they are more attuned to the philosophical aspect of considering positive and negative aspects of objects under consideration. IFSs were further extended by Yager [8] by presenting the idea of "Pythagorean fuzzy sets (PFSs)". By considering the additional aspect of non-membership along with the membership grades various authors [9–17] have effectively made use of IFSs and PFSs to deal with numerous decision making situations. Moreover, these fuzzy sets have also widely been employed for Covid-19 diagnosis by different authors [18–23].

Classical "preference relations" (PRs) [24] is one of the most commonly used tool to assess relative preference of different objects by expressing the degrees of preferences of one object on the other. In decision sciences a paradigmatic change has occurred with the fuzzification of the classical concept of preference relations by the introduction of "Fuzzy preference relations" (FPRs) [1] and the "intuitionistic fuzzy preference relations" (IFPRs) [25,26]. Here we introduce the concept of "interval type-2 trapezoidal preference relation" as an extension of IFPRs which will be combined with the idea of signed based distance proposed by Chen ([27–30] to formulate a new idea of ranking. The approach discussed in this article is an effective tool for managing multiple criteria group decision-making situations with completely unknown priority weights modeled as IT2TrPFPRs. To aggregate the opinion of multiple decision-makers, a hybrid averaging operation based on weighted averaging and ordered weighted averaging (OWA) operations is employed for a collective decision environment. To calculate the fuzzy priority weight vectors in case of completely unknown environment, we construct a non-linear optimization model. An integrated optimization model based on a new signed distance-based closeness coefficients approach is employed to determine the priority ranking of alternatives. Feasibility of the proposed technique is discussed with an implementation of patient centered medicine system for choosing the appropriate treatment method. Moreover, a comparative investigation with previous approaches is conducted to demonstrate the effectiveness of the given approach.

The article organizes as follows: Section 2 presents basic definitions and ideas of IT2TrFNs. Section 3 discusses the idea of the signed distance-based technique. Section 4 involves the introduction of IT2TrPFRs, the discussion of environment with completely unknown fuzzy priority weight vectors and the construction of non-linear optimization model using signed distance based closeness coefficient for ranking the alternatives. Section 5 summarizes our technique and is presented as an algorithm. Section 6 demonstrates the feasibility of the proposed technique and comparisons with the existing methods. Finally, we conclude our discussion in Section 7.

2. Preliminaries

In this section, we briefly discuss some definitions and ideas.

Definition 2.1 (*[31]*). A PFS $\overline{\mathrm{P}}$ on universal set $\mathfrak{W}$ is defined such as:

$$\bar{p} = \{\langle \sigma, \mu_{\bar{p}}(\sigma), \nu_{\bar{p}}(\sigma)\rangle \big| \mu^2_{\bar{p}}(\sigma) + \nu^2_{\bar{p}}(\sigma) \le 1, \mu_{\bar{p}}(\sigma), \nu_{\bar{p}}(\sigma) \in [0,1], \sigma \in \mathfrak{W}\} \quad (1)$$

where $\mu_{\bar{p}}(\sigma)$ and $\nu_{\bar{p}}(\sigma)$ represent the Pythagorean membership degree and Pythagorean non-membership degree of $\bar{p}$ at σ respectively.

Definition 2.2 (*[32,33]*). The indeterminacy degree of σ to $\bar{p}$ is defined such as:

$$\pi_{\bar{p}}(\sigma) = \sqrt{1 - \mu^2_{\bar{p}}(\sigma) - \nu^2_{\bar{p}}(\sigma)} \quad (2)$$

where $\pi_{\bar{p}}(\sigma) \in [0,1]$.

Definition 2.3. Let $\bar{p}(\sigma) = [\bar{p}^L(\sigma), \bar{p}^U(\sigma)]$ be IT2PFS on universal set $\mathfrak{W}$ where $\sigma \in \mathfrak{W}$ and $\bar{p}^L : \mathfrak{W} \to [0,1]$ and $\bar{p}^U : \mathfrak{W} \to [0,1]$ are type-1 Pythagorean fuzzy sets (T1PFSs) known as upper and lower Pythagorean fuzzy sets respectively with conditions $0 \le \bar{p}^L(\sigma) \le \bar{p}^U(\sigma) \le 1$. If $\bar{p} \in \sigma$ is convex and described on a enclosed interval then $\bar{p}$ is called IT2PFN on $\mathfrak{W}$.

Definition 2.4. The indeterminacy degree of IT2PFN define as:

$$\begin{aligned}\pi_{\bar{p}}(\sigma) =& [\pi^L_{\bar{p}}(\sigma), \pi^U_{\bar{p}}(\sigma)] \\ =& [\sqrt{1 - (\mu^L_{\bar{p}})^2(\sigma) - (\nu^L_{\bar{p}})^2(\sigma)}, \sqrt{1 - (\mu^U_{\bar{p}})^2(\sigma) - (\nu^U_{\bar{p}})^2(\sigma)}]\end{aligned} \quad (3)$$

where $\pi^L_{\bar{p}}(\sigma), \pi^U_{\bar{p}}(\sigma) \in [0,1]$.

Definition 2.5 (*[34]*). Let $\bar{p}^L = [\bar{p}_1{}^L, \bar{p}_2{}^L, \bar{p}_3{}^L, \bar{p}_4{}^L; \mu^L_{\bar{p}}, \nu^L_{\bar{p}}]$ and $\bar{p}^U = [\bar{p}_1{}^U, \bar{p}_2{}^U, \bar{p}_3{}^U, \bar{p}_4{}^U; \mu^U_{\bar{p}}, \nu^U_{\bar{p}}]$ be the lower and upper trapezoidal Pythagorean fuzzy number (TrPFN) on the universal set $\mathfrak{W}$ where $0 \le \bar{p}_1{}^L \le \bar{p}_2{}^L \le \bar{p}_3{}^L \le \bar{p}_4{}^L \le 1$, $0 \le \mu^L_{\bar{p}} \le \mu^U_{\bar{p}} \le 1$, $0 \le \nu^L_{\bar{p}} \le \nu^U_{\bar{p}} \le 1$ and $\bar{p}^L \subset \bar{p}^U$. The Pythagorean membership and non-membership function $\mu_{\bar{p}}$ and $\nu_{\bar{p}}$ is defined respectively:

$$\mu_{\bar{p}}(\sigma) = \begin{cases} \frac{(\sigma - \bar{p}_{11})\mu_{\bar{p}}}{\bar{p}_{12} - \bar{p}_{11}} & \bar{p}_{11} \le \sigma < \bar{p}_{12} \\ \mu_{\bar{p}} & \bar{p}_{12} \le \sigma \le \bar{p}_{13} \\ \frac{(\bar{p}_{14} - \sigma)\mu_{\bar{p}}}{\bar{p}_{14} - \bar{p}_{13}} & \bar{p}_{13} < \sigma \le \bar{p}_{14} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\nu_{\bar{p}}(\sigma) = \begin{cases} \frac{(\sigma - \bar{p}_{11})\nu_{\bar{p}}}{\bar{p}_{12} - \bar{p}_{11}} & \bar{p}_{11} \le \sigma < \bar{p}_{12} \\ \nu_{\bar{p}} & \bar{p}_{12} \le \sigma \le \bar{p}_{13} \\ \frac{(\bar{p}_{14} - \sigma)\nu_{\bar{p}}}{\bar{p}_{14} - \bar{p}_{13}} & \bar{p}_{13} < \sigma \le \bar{p}_{14} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where $\mu_{\bar{p}} = [\mu_{\bar{p}^L}(\sigma), \mu_{\bar{p}^U}(\sigma)]$ and $\nu_{\bar{p}} = [\nu_{\bar{p}^L}(\sigma), \nu_{\bar{p}^U}(\sigma)]$ are IT2PFNs.

The number $\bar{p}$ represented as $\bar{p} = [\bar{p}^L, \bar{p}^U] = ([\bar{p}_1{}^L, \bar{p}_2{}^L, \bar{p}_3{}^L, \bar{p}_4{}^L; \mu^L_{\bar{p}}, \nu^L_{\bar{p}}], [\bar{p}_1{}^U, \bar{p}_2{}^U, \bar{p}_3{}^U, \bar{p}_4{}^U; \mu^U_{\bar{p}}, \nu^U_{\bar{p}}])$ and is called IT2TrPFN.

Definition 2.6. Let two IT2TrPFNs $\bar{p}_1 = ([\bar{p}_{11}{}^L, \bar{p}_{12}{}^L, \bar{p}_{13}{}^L, \bar{p}_{14}{}^L; \mu^L_{\bar{p}}, \nu^L_{\bar{p}}], [\bar{p}_{11}{}^U, \bar{p}_{12}{}^U, \bar{p}_{13}{}^U, \bar{p}_{14}{}^U; \mu^U_{\bar{p}}, \nu^U_{\bar{p}}])$ and $\bar{p}_2 = ([\bar{p}_{21}{}^L, \bar{p}_{22}{}^L, \bar{p}_{23}{}^L, \bar{p}_{24}{}^L; \mu^L_{\bar{p}}, \nu^L_{\bar{p}}], [\bar{p}_{21}{}^U, \bar{p}_{22}{}^U, \bar{p}_{23}{}^U, \bar{p}_{24}{}^U; \mu^U_{\bar{p}}, \nu^U_{\bar{p}}])$, and $\xi \ge 0$. The primary operations described on IT2TrPFNs are follows:

1. Addition:

$$\begin{aligned}\bar{p}_1 \oplus \bar{p}_2 =& \Big\langle \big[\, \bar{p}^L_{11} + \bar{p}^L_{21}, \bar{p}^L_{12} + \bar{p}^L_{22}, \bar{p}^L_{13} + \bar{p}^L_{23}, \bar{p}^L_{14} + \bar{p}^L_{24}; \\ & \sqrt{(\mu^L_1)^2 + (\mu^L_2)^2 - (\mu^L_1)^2(\mu^L_2)^2}, \nu^L_1 \nu^L_2 \,\big], \\ & \big[\, \bar{p}^U_{11} + \bar{p}^U_{21}, \bar{p}^U_{12} + \bar{p}^U_{22}, \bar{p}^U_{13} + \bar{p}^U_{23}, \bar{p}^U_{14} + \bar{p}^U_{24}; \\ & \sqrt{(\mu^U_1)^2 + (\mu^U_2)^2 - (\mu^U_1)^2(\mu^U_2)^2}, \nu^U_1 \nu^U_2 \,\big] \Big\rangle\end{aligned}$$

2. Multiplication:

$$\begin{aligned}\bar{p}_1 \otimes \bar{p}_2 =& \Big\langle \big[\, \bar{p}^L_{11}\bar{p}^L_{21}, \bar{p}^L_{12}\bar{p}^L_{22}, \bar{p}^L_{13}\bar{p}^L_{23}, \bar{p}^L_{14}\bar{p}^L_{24}; \\ & \mu^L_1 \mu^L_2, \sqrt{(\nu^L_1)^2 + (\nu^L_2)^2 - (\nu^L_1)^2(\nu^L_2)^2} \,\big], \\ & \big[\, \bar{p}^U_{11}\bar{p}^U_{21}, \bar{p}^U_{12}\bar{p}^U_{22}, \bar{p}^U_{13}\bar{p}^U_{23}, \bar{p}^U_{14}\bar{p}^U_{24}; \\ & \mu^U_1 \mu^U_2, \sqrt{(\nu^U_1)^2 + (\nu^U_2)^2 - (\nu^U_1)^2(\nu^U_2)^2} \,\big] \Big\rangle\end{aligned}$$

3. Multiplication by an ordinary number:

$$\begin{aligned}\xi \bar{p}_1 =& \Big\langle \big[\eta \bar{p}^L_{11}, \eta \bar{p}^L_{12}, \eta \bar{p}^L_{13}, \eta \bar{p}^L_{14}; \sqrt{1 - (1 - (\mu^L_1)^2)^\eta}, (\nu^L_1)^\eta\big], \\ & \big[\eta \bar{p}^U_{11}, \eta \bar{p}^U_{12}, \eta \bar{p}^U_{13}, \eta \bar{p}^U_{14};\end{aligned}$$

$\sqrt{1-(1-(\mu_1^U)^2)^\eta}, (\nu_1^U)^\eta] \rangle$

4. Exponential:

$$\overline{p}_1{}^\eta = \langle [\overline{p}_{11}{}^L\eta, \overline{p}_{12}{}^L\eta, \overline{p}_{13}{}^L\eta, \overline{p}_{14}{}^L\eta; (\mu_1^L)\eta, \sqrt{1-(1-(\nu_1^L)^2)\eta}],$$
$$[\overline{p}_{11}{}^U\eta, \overline{p}_{12}{}^U\eta, \overline{p}13^U\eta, \overline{p}_{14}{}^U\eta;$$
$$(\mu_1^U)\eta, \sqrt{1-(1-(\nu_1^U)^2)^\eta}] \rangle$$

Definition 2.7 (*[35]*). Let a preference matrix $\overline{P} = (\overline{p}_{ij})_{n\times n}$ where $\overline{p}_{ij} = (\overline{p}_{ij1}, \overline{p}_{ij2}, \overline{p}_{ij3}, \overline{p}_{ij4})$ is trapezoidal fuzzy numbers, and where $0 \le \overline{p}_{ij1} \le \overline{p}_{ij2} \le \overline{p}_{ij3} \le \overline{p}_{ij4} \le 1\ \forall\ i, j = 1, 2, \dots n$, then $\overline{P}$ is known as additive trapezoidal fuzzy preference relation.

$$\overline{p}_{ii1} = \overline{p}_{ii2} = \overline{p}_{ii3} = \overline{p}_{ii4} = 0.5, i, j = 1, 2, \dots n$$

$$\overline{p}_{ij1} + \overline{p}_{ii4} = \overline{p}_{ij2} + \overline{p}_{ij3} = \overline{p}_{ij3} + \overline{p}_{ii2} = \overline{p}_{ij4} + \overline{p}_{ii1} = 1, i, j = 1, 2, \dots n$$

Definition 2.8 (*[36]*). Let $\overline{P} = (\overline{p}_{ij})_{n\times n}$ be IT2TrPFPR where $\overline{P}_{ij} = [(\overline{p}_{1ij}{}^L, \overline{p}_{2ij}{}^L, \overline{p}_{3ij}{}^L, \overline{p}_{4ij}{}^L; \mu^L_{\overline{p}_{ij}}, \nu^L_{\overline{p}_{ij}}), (\overline{p}_{1ij}{}^U, \overline{p}_{2ij}{}^U, \overline{p}_{3ij}{}^U, \overline{p}_{4ij}{}^U; \mu^U_{\overline{p}_{ij}}, \nu^U_{\overline{p}_{ij}})]$ be IT2TrPFPR

$\overline{P}_{ij} = (\overline{p}_{ij1}, \overline{p}_{ij2}, \overline{p}_{ij3}, \overline{p}_{ij4})$ is IT2TrPFPNs, where $0 \le \overline{p}_{ij1} \le \overline{p}_{ij2} \le \overline{p}_{ij3} \le \overline{p}_{ij4} \le 1\ \forall\ i, j = 1, 2, \dots n$, then $\overline{P}$ is known as additive interval type 2 trapezoidal pythagorean fuzzy preference relation.

$$\overline{p}_{ii1} = \overline{p}_{ii2} = \overline{p}_{ii3} = \overline{p}_{ii4} = 0.5, i, j = 1, 2, \dots n$$

$$\overline{p}_{ij1} + \overline{p}_{ii4} = \overline{p}_{ij2} + \overline{p}_{ij3} = \overline{p}_{ij3} + \overline{p}_{ii2} = \overline{p}_{ij4} + \overline{p}_{ii1} = 1, i, j = 1, 2, \dots n$$

3. Signed distance based ranking approach with completely unknown weight environment involving OWA aggregation operator

Here we have proposed a new approach based on signed distances to find out the ranking of IT2TrPFPR using positive and negative values equally to characterize the ordering of IT2TrPFPRs. But the basic idea of TOPSIS is the same that the optimal choice should be at minimum distance from positive ideal solution and at maximum distance from the negative ideal solution. The respective reference values for the positive and negative ideal solutions are set as [(1, 1, 1, 1; 1, 1, 1),(1, 1, 1, 1; 1, 1, 1, 1)] and [(0, 0, 0, 0; 1, 1, 1),(0, 0, 0, 0; 1, 1, 1)] since the he attribute values modeled in the form of IT2TrNNs lie between zero and one. That is, they are placed respectively at $x = 1$ and $x = 0$ on the y-axis. The proposed scheme is different from typical distance measures approaches as this study involves signed-distance based approach associated with IT2TrPFPR.

Proposition 1. *Consider $\overline{p}$ be an IT2TrPFPR on the universal set $\mathfrak{W}$ and $\overline{p} = [\overline{p}^L, \overline{p}^U] = ([\overline{p}_1{}^L, \overline{p}_2{}^L, \overline{p}_3{}^L, \overline{p}_4{}^L; \mu^L_{\overline{p}}, \nu^L_{\overline{p}}], [\overline{p}_1{}^U, \overline{p}_2{}^U, \overline{p}_3{}^U, \overline{p}_4{}^U; \mu^U_{\overline{p}}, \nu^U_{\overline{p}}])$ where $0 \le \mu^L_{\overline{p}} \le \mu^U_{\overline{p}} \le 1$ and $0 \le \nu^L_{\overline{p}} \le \nu^U_{\overline{p}} \le 1$.*

The signed distances-based of $\overline{p}$ from $\overline{0}_1$ (at x = 0) or $\overline{1}_1$ (at x = 1) are as follows:

$$d(\overline{p}, \overline{0}_1) = \frac{1}{8}\Big[2(\overline{p}_1{}^L + \overline{p}_2{}^L + \overline{p}_3{}^L + \overline{p}_4{}^L) + 2(\overline{p}_1{}^U + \overline{p}_2{}^U + \overline{p}_3{}^U + \overline{p}_4{}^U) + 3(\overline{p}_2{}^U + \overline{p}_3{}^U - \overline{p}_1{}^U - \overline{p}_4{}^U)\left(\frac{\mu^L_{\overline{p}}}{\mu^U_{\overline{p}}} - \frac{\nu^L_{\overline{p}}}{\nu^U_{\overline{p}}}\right)\Big] \quad (6)$$

$$d(\overline{p}, \overline{1}_1) = \frac{1}{8}\Big[2(\overline{p}_1{}^L + \overline{p}_2{}^L + \overline{p}_3{}^L + \overline{p}_4{}^L) + 2(\overline{p}_1{}^U + \overline{p}_2{}^U + \overline{p}_3{}^U + \overline{p}_4{}^U) + 3(\overline{p}_2{}^U + \overline{p}_3{}^U - \overline{p}_1{}^U - \overline{p}_4{}^U)\left(\frac{\mu^L_{\overline{p}}}{\mu^U_{\overline{p}}} - \frac{\nu^L_{\overline{p}}}{\nu^U_{\overline{p}}}\right) - 16\Big] \quad (7)$$

Proof. The derivation is analogous to previously derived signed based distance formulas by Chen [28,29] in Appendix D and B respectively.

Example 1. Let $\overline{P}$ be an IT2TrPFPR on the universal set $\mathfrak{W}$ and $\overline{p} = [\overline{p}^L, \overline{p}^U] = [(0.3, 0.5, 0.6, 0.8; 0.3, 0.2), (0.2, 0.4, 0.7, 0.9; 0.2, 0.1)]$ be "interval type-2 trapezoidal pythagorean fuzzy number"(IT2TPFN) The signed distance $\overline{p}$ from $\overline{0}_1$ (at x = 0) or $\overline{1}_1$ (at x = 1) are calculated as

$$d(\overline{P}, \overline{0}_1) = \frac{1}{8}\Big[2(\overline{p}_1{}^L + \overline{p}_2{}^L + \overline{p}_3{}^L + \overline{p}_4{}^L) + 2(\overline{p}_1{}^U + \overline{p}_2{}^U + \overline{p}_3{}^U + \overline{p}_4{}^U) + 3(\overline{p}_2{}^U + \overline{p}_3{}^U - \overline{p}_1{}^U - \overline{p}_4{}^U)\left(\frac{\mu^L_{\overline{p}}}{\mu^U_{\overline{p}}} - \frac{\nu^L_{\overline{p}}}{\nu^U_{\overline{p}}}\right)\Big]$$
$$= \frac{1}{8}\Big[2(0.3 + 0.5 + 0.6 + 0.8) + 2(0.2 + 0.4 + 0.7 + 0.9) + 3(0.4 + 0.7 - 0.2 - 0.9)(\frac{0.3}{0.2} - \frac{0.2}{0.1})\Big]$$
$$= 1.1$$

And the signed distance $d(\overline{P}^1_{12}, \overline{1}_1)$ using Eq. (7) are calculated as:

$$d(\overline{P}, \overline{1}_1) = \frac{1}{8}\Big[2(\overline{p}_1{}^L + \overline{p}_2{}^L + \overline{p}_3{}^L + \overline{p}_4{}^L) + 2(\overline{p}_1{}^U + \overline{p}_2{}^U + \overline{p}_3{}^U + \overline{p}_4{}^U) + 3(\overline{p}_2{}^U + \overline{p}_3{}^U - \overline{p}_1{}^U - \overline{p}_4{}^U)\left(\frac{\mu^L_{\overline{p}}}{\mu^U_{\overline{p}}} - \frac{\nu^L_{\overline{p}}}{\nu^U_{\overline{p}} - 16}\right)\Big]$$
$$= \frac{1}{8}\Big[2(0.3 + 0.5 + 0.6 + 0.8) + 2(0.2 + 0.4 + 0.7 + 0.9) + 3(0.4 + 0.7 - 0.2 - 0.9)(\frac{0.3}{0.2} - \frac{0.2}{0.1}) - 16\Big]$$
$$- 0.9000$$

Property 1. *Consider $\overline{P}$ be an IT2TrPFPR on the universal set $\mathfrak{W}$*

$$\overline{p} = [\overline{p}^L, \overline{p}^U] = ([\overline{p}_1{}^L, \overline{p}_2{}^L, \overline{p}_3{}^L, \overline{p}_4{}^L; \mu^L_{\overline{p}}, \nu^L_{\overline{p}}], [\overline{p}_1{}^U, \overline{p}_2{}^U, \overline{p}_3{}^U, \overline{p}_4{}^U; \mu^U_{\overline{p}}, \nu^U_{\overline{p}}]).$$

(i) $d(\overline{p}, \overline{0}_1) - d(\overline{p}, \overline{1}_1) = 2.$

(ii) *$\overline{p}$ is located at $\overline{1}_1$ (i.e. $\overline{p}_1{}^L = \overline{p}_2{}^L = \overline{p}_3{}^L = \overline{p}_4{}^L = 1$) $\Longleftrightarrow d(\overline{p}, \overline{1}_1) = \overline{p}_1{}^U = \overline{p}_2{}^U = \overline{p}_3{}^U = \overline{p}_4{}^U = 0$ and $d(\overline{p}, \overline{0}_1) = 2$.*

(iii) *$\overline{p}$ is at $\overline{0}_1$ (i.e. $\overline{p}_1{}^L = \overline{p}_2{}^L = \overline{p}_3{}^L = \overline{p}_4{}^L = \overline{p}_1{}^U = \overline{p}_2{}^U = \overline{p}_3{}^U = \overline{p}_4{}^U = 0$) $\Longleftrightarrow d(\overline{p}, \overline{0}_1) = 0$ and $d(\overline{p}, \overline{1}_1) = -2$.*

Property 2. *Let $\overline{p}$, $\overline{F}$, $\overline{S}$ and $\overline{X}$ be four IT2TrPFPR defined on universal set $\mathfrak{W}$ where*

$$\overline{p} = [(\overline{p}_1{}^L, \overline{p}_2{}^L, \overline{p}_3{}^L, \overline{p}_4{}^L; \mu^L_{\overline{p}}, \nu^L_{\overline{p}}), (\overline{p}_1{}^U, \overline{p}_2{}^U, \overline{p}_3{}^U, \overline{p}_4{}^U; \mu^U_{\overline{p}}, \nu^U_{\overline{p}})],$$

$$\overline{F} = [(\overline{f}_1{}^L, \overline{f}_2{}^L, \overline{f}_3{}^L, \overline{f}_4{}^L; \mu^L_{\overline{f}}, \nu^L_{\overline{f}}), (\overline{f}_1{}^U, \overline{f}_2{}^U, \overline{f}_3{}^U, \overline{f}_4{}^U; \mu^U_{\overline{f}}, \nu^U_{\overline{f}})],$$

$\overline{S} = [(1, 1, 1, 1; 1, 1), (1, 1, 1, 1; 1, 1)]$ *and* $\overline{X} = [(0, 0, 0, 0; 1, 1), (0, 0, 0, 0; 1, 1)].$

(i) *$\overline{p}$ is closer to $\overline{S}$ than $\overline{F} \Longleftrightarrow d(\overline{p}, \overline{1}_1) > d(\overline{F}, \overline{1}_1)$.*

(ii) *$\overline{p}$ is farther from $\overline{X}$ than $\overline{F} \Longleftrightarrow d(\overline{p}, \overline{0}_1) > d(\overline{F}, \overline{0}_1)$.*

From Property 1, it is follows that the signed distances -based $d(\overline{p}, \overline{0}_1) \in [0, 2]$ and $d(\overline{p}, \overline{1}_1) \in [-2, 0]$. Additionally regulation of trichotomy holds for signed distances based because $d(\overline{p}, \overline{0}_1)$ and $d(\overline{F}, \overline{0}_1)$ are real numbers that satisfying linear ordering i.e. The following conditions hold: $d(\overline{p}, \overline{0}_1) > d(\overline{F}, \overline{0}_1)$, $d(\overline{p}, \overline{0}_1) = d(\overline{F}, \overline{0}_1)$ or either $d(\overline{p}, \overline{0}_1) < d(\overline{F}, \overline{0}_1)$. Similarly $d(\overline{p}, \overline{1}_1)$ and $d(\overline{F}, \overline{1}_1)$ also satisfies. Therefore, signed base distance can be used to rank IT2TrPFPR.

4. Signed distance based group decision making approach with IT2TrPFPR AND incomplete weights

Consider a GDM problem. Let $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_S\}$ be set of DMs and $V = \{V_1, V_2, \dots, V_m\}$ be the alternatives set. This segment analyzes a decision-making situation with IT2TrPFPR with incomplete information. For aggregate the common ranking of DMs HA approach is offered in the form of collective choice. Further optimization model based on SDBCC is investigating the alternative weights. Lastly, this approach is summarized via offering an algorithm to solve the GDM problem primarily based on IT2TrPFPR.

4.1. Collective decision primarily based on HA technique

Multiple GDM may have their own opinions. There is one of a sort of aggregation operator observed for aggregating the records. such that HA operation use signed distance-based OWA operation [26] to accumulate IT2TrPFPR statistics to form a collective decision making. It represents the choice of DMs and committed common judgment of OWA operation, reordering the arguments in ascending order and weighting them.

Definition 4.1. Let $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_S\}$ be the sets of DMs and weight vectors are $\Upsilon = (\Upsilon_1, \Upsilon_2, \dots, \Upsilon_{\mathfrak{R}})$ where $\Upsilon_\tau \geq 0$ for $\tau = 1, 2, \dots, \mathfrak{R}$ and $\sum_{\tau=1}^{\mathfrak{R}} \Upsilon_\tau = 1$.

Let $\overline{P}_{ij}^{\tau} = [(\overline{p}_{1ij}^{\tau L}, \overline{p}_{2ij}^{\tau L}, \overline{p}_{3ij}^{\tau L}, \overline{p}_{4ij}^{\tau L}; \mu^{\tau L}_{\overline{P}_{ij}}, \nu^{\tau L}_{\overline{P}_{ij}}), (\overline{p}_{1ij}^{\tau U}, \overline{p}_{2ij}^{\tau U}, \overline{p}_{3ij}^{\tau U}, \overline{p}_{4ij}^{\tau U}; \mu^{\tau U}_{\overline{P}_{ij}}, \nu^{\tau U}_{\overline{P}_{ij}})]$ IT2TrPFPR expressed by DMs where $\lambda_\tau \in \lambda$. Let $\tau = 1, 2, \dots, \mathfrak{R}$ is defined as:

$$\begin{aligned}\dot{P}^{\tau}_{ij} &= WA(\overline{P}^{1}_{ij}, \overline{P}^{2}_{ij}, \dots, \overline{P}^{\mathfrak{R}}_{ij}) = (\Upsilon_1 \cdot \overline{P}^{1}_{ij}) \oplus (\Upsilon_2 \cdot \overline{P}^{2}_{ij}) \oplus \cdots \oplus (\Upsilon_{\mathfrak{R}} \cdot \overline{P}^{\mathfrak{R}}_{ij}) \\ &= \Bigg[\Bigg(\sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau U}_{1ij}), \sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau U}_{2ij}), \sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau U}_{3ij}), \\ &\sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau U}_{4ij}); \min_\tau(\mu^{\tau U}_{\overline{P}_{ij}}, \nu^{\tau U}_{\overline{P}_{ij}}) \Bigg), \\ &\Bigg(\sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau L}_{1ij}), \sum_{s=1}^{S} (\Upsilon_\tau \times \overline{p}^{\tau L}_{2ij}), \sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau L}_{3ij}), \\ &\sum_{\tau=1}^{\mathfrak{R}} (\Upsilon_\tau \times \overline{p}^{\tau L}_{4ij}); \min_\tau(\mu^{\tau L}_{\overline{P}_{ij}}, \nu^{\tau L}_{\overline{P}_{ij}}) \Bigg)\Bigg]\end{aligned} \tag{8}$$

Apply WA of V_i then we obtain: $\dot{P}^{\tau}_{ij} = [(\dot{P}^{s\tau L}_{1ij}, \dot{P}^{s\tau L}_{2ij}, \dot{P}^{\tau L}_{3ij}, \dot{P}^{\tau L}_{4ij}; \mu^{\tau L}_{\dot{P}_{ij}}, \nu^{\tau L}_{\dot{P}_{ij}}), (\dot{P}^{\tau U}_{1ij}, \dot{P}^{\tau U}_{2ij}, \dot{P}^{\tau U}_{3ij}, \dot{P}^{\tau U}_{4ij}; \mu^{\tau U}_{\dot{P}_{ij}}, \nu^{\tau U}_{\dot{P}_{ij}})]$ where $0 \leq \dot{P}^{L}_{1ij} \leq \dot{P}^{L}_{2ij} \leq \dot{P}^{L}_{3ij} \leq \dot{P}^{L}_{4ij} \leq 1$, $0 \leq \dot{P}^{U}_{1ij} \leq \dot{P}^{U}_{2ij} \leq \dot{P}^{U}_{3ij} \leq \dot{P}^{U}_{4ij} \leq 1$, $0 \leq \mu^{U}_{\dot{P}_{ij}} \leq \mu^{L}_{\dot{P}_{ij}} \leq 1$, $0 \leq \nu^{U}_{\dot{P}_{ij}} \leq \nu^{L}_{\dot{P}_{ij}} \leq 1$ and $\dot{P}^{U}_{ij} \subset \dot{P}^{L}_{ij}$.

4.1.1. Aggregation using OWA operation based on signed distances

OWA operation is the regularly used aggregation method. [26,29]. Signed-based distance OWA operation is used to accumulate weighted ratings and build a collective decision matrix by using HA operation. The proposed OWA operation takes into reordering arguments in ascending order and subsequently weighting them. At the begin of Proposition 1, the signed distance of IT2TrPFPR. $\overline{P} = [\overline{P}^{L}, \overline{P}^{U}] = [(\overline{p}_1^{L}, \overline{p}_2^{L}, \overline{p}_3^{L}, \overline{p}_4^{L}; \mu^{L}_{\overline{p}}, \nu^{L}_{\overline{p}}), (\overline{p}_1^{U}, \overline{p}_2^{U}, \overline{p}_3^{U}, \overline{p}_4^{U}; \mu^{U}_{\overline{p}}, \nu^{U}_{\overline{p}})]$ from $\overline{0}_1$ is calculated in this way:

$$\begin{aligned}d(\overline{P}, \overline{0}_1) = \frac{1}{8}\Bigg[& 2(\overline{p}_1^{L} + \overline{p}_2^{L} + \overline{p}_3^{L} + \overline{p}_4^{L}) + 2(\overline{p}_1^{U} + \overline{p}_2^{U} + \overline{p}_3^{U} + \overline{p}_4^{U}) \\ & + 3(\overline{p}_2^{U} + \overline{p}_3^{U} - \overline{p}_1^{U} - \overline{p}_4^{U}) \\ & \left(\frac{\mu^{L}_{\overline{p}}}{\mu^{U}_{\overline{p}}} - \frac{\nu^{L}_{\overline{p}}}{\nu^{U}_{\overline{p}}} \right) \Bigg]\end{aligned}$$

One of the foremost be counted in OWA operation is to determine the associated weights. Weighting vectors derived by using the normal distribution method [26], due to the fact this technique can reduce the effect of unfair arguments on the consequences of averaging by way of assigning low weights to outliers, inclusive of unduly high or unduly low arguments, and consequently makes the outcomes greater affordable utilizing virtue of group consensus critiques. It is expressed as follows:

$$\zeta_\tau = \frac{e^{-\frac{(\tau - \phi_\tau)^2}{2 \cdot \psi_\tau^2}}}{\sum_{\tau=1}^{M} e^{-\frac{(\tau - \phi_\tau)^2}{2 \cdot \phi_\tau^2}}}, \quad \tau = 1, 2, \dots, M. \tag{9}$$

where ϕ_τ represents the Mean and ψ_τ ($\psi_\tau \geq 0$) be the standard deviation of $1, 2, \dots, M$. such that:

$$\phi_\tau = \frac{1}{M} \cdot \frac{M(1+M)}{2} = \frac{1+M}{2} \tag{10}$$

$$\psi_\tau = \sqrt{\frac{1}{M} \sum_{\tau=1}^{M} (\tau - \phi_\tau)^2} \tag{11}$$

Definition 4.2. Let $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_\tau\}$ be the set of DMs and an IT2TrPFPR, $\overline{P}_{ij}^{\tau} = [(\overline{p}_{1ij}^{\tau L}, \overline{p}_{2ij}^{\tau L}, \overline{p}_{3ij}^{\tau L}, \overline{p}_{4ij}^{\tau L}; \mu^{\tau L}_{\overline{p}_{ij}}, \nu^{\tau L}_{\overline{p}_{ij}}), (\overline{p}_{1ij}^{\tau U}, \overline{p}_{2ij}^{\tau U}, \overline{p}_{3ij}^{\tau U}, \overline{p}_{4ij}^{\tau U}; \mu^{\tau U}_{\overline{p}_{ij}}, \nu^{\tau U}_{\overline{p}_{ij}})]$ that represent the alternatives ratings corresponding to λ_j by the DMs $\lambda_\tau \in \lambda$. The signed-based distance OWA operation with their corresponding weight vectors ($\zeta = \zeta_1, \zeta_2, \dots, \zeta_{\mathfrak{R}}$) is defined such as:

$$\begin{aligned}\dot{P}_{ij} &= OWA(\overline{p}^{1}_{ij}, \overline{p}^{2}_{ij}, \dots, \overline{p}^{\mathfrak{R}}_{ij}) = (\zeta_1 \cdot \overline{p}^{\vartheta(1)}_{ij}) \oplus (\zeta_2 \cdot \overline{p}^{\vartheta(2)}_{ij}) \oplus \cdots \oplus (\zeta_{\mathfrak{R}} \cdot \overline{p}^{\vartheta(\mathfrak{R})}_{ij}) \\ &= \Bigg[\Bigg(\sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)L}_{1ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)L}_{2ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)L}_{3ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)L}_{4ij}); \\ &\min_\tau(\mu^{\vartheta(\tau)L}_{\overline{p}_{ij}}, \nu^{\vartheta(\tau)L}_{\overline{p}_{ij}}) \Bigg), \\ &\Bigg(\sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)U}_{1ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)U}_{2ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)U}_{3ij}), \sum_{\tau=1}^{\mathfrak{R}} (\zeta_\tau \times \overline{p}^{\vartheta(\tau)U}_{4ij}); \\ &\min_\tau(\mu^{\vartheta(\tau)U}_{\overline{p}_{ij}}, \nu^{\vartheta(\tau)U}_{\overline{p}_{ij}}) \Bigg)\Bigg]\end{aligned} \tag{12}$$

where $(\vartheta(1), \vartheta(2), \dots, \vartheta(M))$ be permutation of $(1, 2, \dots M)$.

OWA consideration of V_i on λ_j are follows: $\dot{P}_{ij} = [\dot{P}^{L}_{ij}, \dot{P}^{U}_{ij}] = [(\dot{P}^{L}_{1ij}, \dot{P}^{L}_{2ij}, \dot{P}^{L}_{3ij}, \dot{P}^{L}_{4ij}; \mu^{L}_{\dot{P}_{ij}}, \nu^{L}_{\dot{P}_{ij}}), (\dot{P}^{U}_{1ij}, \dot{P}^{U}_{2ij}, \dot{P}^{U}_{3ij}, \dot{P}^{U}_{4ij}; \mu^{U}_{\dot{P}_{ij}}, \nu^{U}_{\dot{P}_{ij}})]$ where $0 \leq \dot{P}^{L}_{1ij} \leq \dot{P}^{L}_{2ij} \leq \dot{P}^{L}_{3ij} \leq \dot{P}^{L}_{4ij} \leq 1$, $0 \leq \dot{P}^{U}_{1ij} \leq \dot{P}^{U}_{2ij} \leq \dot{P}^{U}_{3ij} \leq \dot{P}^{U}_{4ij} \leq 1$, $0 \leq \mu^{L}_{\dot{P}_{ij}} \leq \mu^{U}_{\dot{P}_{ij}} \leq 1$, $0 \leq \nu^{L}_{\dot{P}_{ij}} \leq \nu^{U}_{\dot{P}_{ij}} \leq 1$ and $\dot{P}^{L}_{ij} \subset \dot{P}^{U}_{ij}$.

Example 2. Let $\overline{P}$ be an IT2TrPFPR on the universal set $\mathfrak{W}$ and P^1, P^2, P^3 be three judgment matrices. Find the sign distances $d(\overline{P}^{1}_{12}, \overline{0}_1)$, $d(\overline{P}^{2}_{12}, \overline{0}_1)$ and $d(\overline{P}^{3}_{12}, \overline{0}_1)$ respectively.

$$\begin{aligned}d(\overline{P}^{1}_{12}, \overline{0}_1) = \frac{1}{8}\Bigg[& 2(\overline{p}_{112}{}^{1}L + \overline{p}_{212}{}^{1}L + \overline{p}_{312}{}^{1}L + \overline{p}_{412}{}^{1}L) \\ & + 2(\overline{p}_{112}{}^{1}U + \overline{p}_{212}{}^{1}U + \overline{p}_{312}{}^{1}U + \overline{p}_{412}{}^{1}U) + \\ & 3(\overline{p}_{212}{}^{1}U + \overline{p}_{312}{}^{1}U - \overline{p}_{112}{}^{1}U - \overline{p}_{412}{}^{1}U) \\ & \left(\frac{\mu^{1}_{\overline{p}}L}{\mu^{1}_{\overline{p}}U} - \frac{\nu^{1}_{\overline{p}}L}{\nu^{1}_{\overline{p}}U} \right) \Bigg] \\ = \frac{1}{8}\Bigg[& 2(0.3 + 0.5 + 0.6 + 0.8) + 2(0.2 + 0.4 + 0.7 + 0.9) \\ & + 3(0.4 + 0.7 - 0.2 - 0.9)(\frac{0.3}{0.2} - \frac{0.2}{0.1}) \Bigg] \\ = 1.1 &\end{aligned}$$

$$d(\overline{P}^{2}_{12}, \overline{0}_1) = \frac{1}{8}\Bigg[2(\overline{p}_{112}{}^{2}L + \overline{p}_{212}{}^{2}L + \overline{p}_{312}{}^{2}L + \overline{p}_{412}{}^{2}L)$$

$$+2(\overline{p}_{112}{}^2U+\overline{p}_{212}{}^2U+\overline{p}_{312}{}^2U+\overline{p}_{412}{}^2U)+3(\overline{p}_{212}{}^2U+\overline{p}_{312}{}^2U-\overline{p}_{112}{}^2U-\overline{p}_{412}{}^2U)\left(\frac{\mu_{p}^2L}{\mu_{p}^2U}-\frac{\nu_{p}^2L}{\nu_{p}^2U}\right)\Bigg]$$

$$=\frac{1}{8}\Bigg[2(0.6+0.7+0.8+0.9)+2(0.4+0.6+0.7+0.8)+3(0.6+0.7-0.4-0.8)\left(\frac{0.6}{0.5}-\frac{0.5}{0.4}\right)\Bigg]$$

$$=1.3481$$

$$d(\overline{P}^3_{12},\overline{0}_1)=\frac{1}{8}\Bigg[2(\overline{p}_{112}{}^3L+\overline{p}_{212}{}^3L+\overline{p}_{312}{}^3L+\overline{p}_{412}{}^3L)+2(\overline{p}_{112}{}^3U+\overline{p}_{212}{}^3U+\overline{p}_{312}{}^3U+\overline{p}_{412}{}^3U)+3(\overline{p}_{212}{}^3U+\overline{p}_{312}{}^3U-\overline{p}_{112}{}^3U-\overline{p}_{412}{}^3U)\left(\frac{\mu_{p}^3L}{\mu_{p}^3U}-\frac{\nu_{p}^3L}{\nu_{p}^3U}\right)\Bigg]$$

$$=\frac{1}{8}\Bigg[2(0.2+0.3+0.3+0.4)+2(0.1+0.2+0.2+0.5)+3(0.2+0.2-0.1-0.5)\left(\frac{0.8}{0.2}-\frac{0.2}{0.1}\right)\Bigg]$$

$$=0.4$$

As $d(P^1_{12},\overline{0}_1)=1.1$, $d(P^2_{12},\overline{0}_1)=1.3481$ and $d(P^3_{12},\overline{0}_1)=0.4$ As $d(P^3_{12},\overline{0}_1)<d(P^1_{12},\overline{0}_1)<d(P^2_{12},\overline{0}_1)$ so $\vartheta(1)=3$, $\vartheta(2)=1$, $\vartheta(3)=2$. Hence $P^{\vartheta(1)}_{12}=P^3_{12}$, $P^{\vartheta(2)}_{12}=P^1_{12}$, $P^{\vartheta(3)}_{12}=P^2_{12}$.

$$\dot{P}_{12}=\text{OWA}(\overline{p}^1_{12},\overline{p}^2_{12},\dots,\overline{p}^{\mathfrak{R}}_{12})=(\zeta_1\cdot\overline{p}^{\vartheta(1)}_{12})\oplus(\zeta_2\cdot\overline{p}^{\vartheta(2)}_{12})\oplus\cdots\oplus(\zeta_{\mathfrak{R}}\cdot\overline{p}^{\vartheta(\mathfrak{R})}_{12})$$

$$=\Bigg[\Bigg(\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})L}_{112}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})L}_{212}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})L}_{312}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})L}_{412});\min_{\mathfrak{r}}(\mu^{\vartheta(\mathfrak{r})L}_{\overline{p}_{12}},\nu^{\vartheta(\mathfrak{r})L}_{\overline{p}_{12}})\Bigg),$$
$$\Bigg(\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})U}_{112}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})U}_{212}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})U}_{312}),\sum_{\mathfrak{r}=1}^{\mathfrak{R}}(\zeta_{\mathfrak{r}}\times\overline{p}^{\vartheta(\mathfrak{r})U}_{412});\min_{\mathfrak{r}}(\mu^{\vartheta(\mathfrak{r})U}_{\overline{p}_{12}},\nu^{\vartheta(\mathfrak{r})U}_{\overline{p}_{12}})\Bigg)\Bigg]$$

$$=[(0.2429\times0.2+0.5142\times0.3+0.2429\times0.6,0.2429\times0.3+0.5142\times0.5+0.2429\times0.7,0.2429\times0.3+0.5142\times0.6+0.2429\times0.8,0.2429\times0.4+0.5142\times0.8+0.2429\times0.8);\min((0.8,0.3,0.6),(0.2,0.2,0.5))(0.2429\times0.1+0.5142\times0.2+0.2429\times0.4,0.2429\times0.2+0.5142\times0.4+0.2429\times0.6,0.2429\times0.2+0.5142\times0.7+0.2429\times0.7,0.2429\times0.5+0.5142\times0.9+0.2429\times0.8);\min((0.2,0.2,0.5),(0.1,0.1,0.4))]$$

$$\dot{P}_{12}=[(0.34858,0.5,0.57571,0.70284;0.3,0.2)\ (0.22429,0.4,0.57855,0.77855;0.2,0.1)]$$

Collective decision matrix $\dot{E}$ as follows:

$$\dot{E}=\begin{bmatrix}[\dot{P}^L_{11},\dot{P}^U_{11}] & [\dot{P}^L_{12},\dot{P}^U_{12}] & \cdots & [\dot{P}^L_{1n},\dot{P}^U_{1n}]\\ [\dot{P}^L_{21},\dot{P}^U_{21}] & [\dot{P}^L_{22},\dot{P}^U_{22}] & \cdots & [\dot{P}^L_{2n},\dot{P}^U_{2n}]\\ \vdots & \vdots & \dots & \vdots\\ [\dot{P}^L_{n1},\dot{P}^U_{n1}] & [\dot{P}^L_{n2},\dot{P}^U_{n2}] & \cdots & [\dot{P}^L_{nm},\dot{P}^U_{nm}]\end{bmatrix} \tag{13}$$

4.2. Normalization of collective matrix

Normalized the collective information to demisable and inadmissible values. The normalized process are defined as:

Let $\dot{P}^*_j=\max_i\ \dot{P}^U_{1ij}$ (for $x_{\underline{j}}\in X_b$) and $\dot{P}^-_j=\min_i\ \dot{P}^L_{4ij}$ (for $x_{\underline{j}}\in X_c$). P_{ij} is acquired as:

$$P_{ij}=[P^L_{ij},P^U_{ij}]=[(p^L_{1ij},p^L_{2ij},p^L_{3ij},p^L_{4ij};\mu^L_{P_{ij}},\nu^L_{P_{ij}}),(p^U_{1ij},p^U_{2ij},p^U_{3ij},p^U_{4ij};\mu^U_{P_{ij}},\nu^U_{P_{ij}})]$$
$$=\begin{cases}\Bigg[\left(\frac{\dot{P}^L_{1ij}}{\dot{P}^*_j},\frac{\dot{P}^L_{2ij}}{\dot{P}^*_j},\frac{\dot{P}^L_{3ij}}{\dot{P}^*_j},\frac{\dot{P}^L_{4ij}}{\dot{P}^*_j};\mu^L_{\dot{P}_{ij}},\nu^L_{\dot{P}_{ij}}\right),\left(\frac{\dot{P}^U_{1ij}}{\dot{P}^*_j},\frac{\dot{P}^U_{2ij}}{\dot{P}^*_j},\frac{\dot{P}^U_{3ij}}{\dot{P}^*_j},\frac{\dot{P}^U_{4ij}}{\dot{P}^*_j};\mu^U_{\dot{P}_{ij}},\nu^U_{\dot{P}_{ij}}\right)\Bigg] & \text{if } x_{\underline{j}}\in X_b\\ \Bigg[\left(\frac{\dot{P}^-_j}{\dot{P}^L_{4ij}},\frac{\dot{P}^-_j}{\dot{P}^L_{3ij}},\frac{\dot{P}^-_j}{\dot{P}^L_{2ij}},\frac{\dot{P}^-_j}{\dot{P}^L_{1ij}};\mu^L_{\dot{P}_{ij}},\nu^L_{\dot{P}_{ij}}\right),\left(\frac{\dot{P}^-_j}{\dot{P}^U_{4ij}},\frac{\dot{P}^-_j}{\dot{P}^U_{3ij}},\frac{\dot{P}^-_j}{\dot{P}^U_{2ij}},\frac{\dot{P}^-_j}{\dot{P}^U_{1ij}};\mu^U_{\dot{P}_{ij}},\nu^U_{\dot{P}_{ij}}\right)\Bigg] & \text{if } x_{\underline{j}}\in X_c\end{cases} \tag{14}$$

Normalized collective matrix E is as follows:

$$E=\begin{bmatrix}[P^L_{11},P^U_{11}] & [P^L_{12},P^U_{12}] & \cdots & [P^L_{1n},P^U_{1n}]\\ [P^L_{21},P^U_{21}] & [P^L_{22},P^U_{22}] & \cdots & [P^L_{2n},P^U_{2n}]\\ \vdots & \vdots & \ddots & \vdots\\ [P^L_{m1},P^U_{m1}] & [P^L_{m2},P^U_{m2}] & \cdots & [P^L_{mn},P^U_{mn}]\end{bmatrix} \tag{15}$$

Example 3. By Table 1, it is obvious that $\dot{P}^*_1=0.8757,\dot{P}^-_2=0.2$, $\dot{P}^+_3=0.9$ and $\dot{P}^-_4=0.82713$. Then the normalized collective matrix D can be constructed in Table 2. The transformed outcomes P_{21} and P_{12} as follows:

$$P_{21}=\Bigg[\left(\frac{\dot{P}^L_{211}}{\dot{P}^*_1},\frac{\dot{P}^L_{221}}{\dot{P}^*_1},\frac{\dot{P}^L_{321}}{\dot{P}^*_1},\frac{\dot{P}^L_{421}}{\dot{P}^*_1};\mu^L_{\dot{P}_{21}},\nu^L_{\dot{P}_{21}}\right),\left(\frac{\dot{P}^U_{211}}{\dot{P}^*_1},\frac{\dot{P}^U_{221}}{\dot{P}^*_1},\frac{\dot{P}^U_{321}}{\dot{P}^*_1},\frac{\dot{P}^U_{421}}{\dot{P}^*_1};\mu^U_{\dot{P}_{21}},\nu^U_{\dot{P}_{21}}\right)\Bigg]$$
$$=\Bigg[\left(\frac{0.29716}{0.87571},\frac{0.42429}{0.87571},\frac{0.5}{0.87571},\frac{0.65142}{0.87571};0.2,0.3\right),\left(\frac{0.22145}{0.87571},\frac{0.42145}{0.87571},\frac{0.6}{0.87571},\frac{0.77571}{0.87571};0.5,0.5\right)\Bigg]$$
$$=[(0.3393,0.4845,0.5710,0.7439;0.2,0.3)\times(0.2529,0.4813,0.6857,0.7993;0.1,0.2)]$$

$$P_{12}=\Bigg[\left(\frac{\dot{P}^-_2}{\dot{P}^L_{412}},\frac{\dot{P}^-_2}{\dot{P}^L_{312}},\frac{\dot{P}^-_2}{\dot{P}^L_{212}},\frac{\dot{P}^-_2}{\dot{P}^L_{112}};\mu^L_{\dot{P}_{12}},\nu^L_{\dot{P}_{12}}\right),\left(\frac{\dot{P}^-_2}{\dot{P}^U_{412}},\frac{\dot{P}^-_2}{\dot{P}^U_{312}},\frac{\dot{P}^-_2}{\dot{P}^U_{212}},\frac{\dot{P}^-_2}{\dot{P}^U_{112}};\mu^U_{\dot{P}_{12}},\nu^U_{\dot{P}_{12}}\right)\Bigg]$$
$$=\Bigg[\left(\frac{0.2}{0.70284},\frac{0.2}{0.57571},\frac{0.2}{0.5},\frac{0.2}{0.34858};0.3,0.2\right),\left(\frac{0.2}{0.77855},\frac{0.2}{0.57855},\frac{0.2}{0.4},\frac{0.2}{0.22429};0.2,0.9\right)\Bigg]$$
$$=[(0.2846,0.3474,0.400,0.5738;0.3,0.2)\times(0.2569,0.3457,0.5000,0.8917;0.2,0.9)]$$

4.3. Completely unknown preference structure involving interval type-2 trapezoidal pythagorean fuzzy preference relations

In many of the real life problems, it is often not possible to judge the preference of criteria or factors responsible for a specific phenomena due to some uncertainty or complexities associated with it. However, it may be possible to provide the relative ordering of the factors partially like weak, strict or difference ordering or interval or ratio bounds. In GDM process, decision-makers may possibly put across few preference

Table 1
Aggregated values of alternatives in $\dot{E}$.

	$\dot{P}_{ij}^{L}$						$\dot{P}_{ij}^{U}$					
	$\dot{P}_{1ij}^{L}$	$\dot{P}_{2ij}^{L}$	$\dot{P}_{3ij}^{L}$	$\dot{P}_{4ij}^{L}$	$\mu_{\dot{P}_{ij}}^{L}$	$v_{\dot{P}_{ij}}^{L}$	$\dot{P}_{1ij}^{U}$	$\dot{P}_{2ij}^{U}$	$\dot{P}_{3ij}^{U}$	$\dot{P}_{4ij}^{U}$	$\mu_{\dot{P}_{ij}}^{U}$	$v_{\dot{P}_{ij}}^{U}$
$\dot{P}_{11}$	0.500	0.500	0.500	0.500	0.5	0.5	0.500	0.500	0.500	0.500	0.5	0.5
$\dot{P}_{12}$	0.3486	0.500	0.5757	0.7028	0.3	0.2	0.2243	0.400	0.5786	0.7786	0.2	0.1
$\dot{P}_{13}$	0.1486	0.2729	0.4486	0.6514	0.4	0.3	0.1243	0.2486	0.500	0.700	0.3	0.2
$\dot{P}_{14}$	0.3757	0.500	0.600	0.7271	0.3	0.2	0.2514	0.400	0.5514	0.8028	0.2	0.1
$\dot{P}_{21}$	0.2972	0.4243	0.500	0.6514	0.2	0.3	0.2215	0.4215	0.600	0.7757	0.1	0.2
$\dot{P}_{22}$	0.500	0.500	0.500	0.500	0.5	0.5	0.500	0.500	0.500	0.500	0.5	0.5
$\dot{P}_{23}$	0.2786	0.4271	0.5514	0.700	0.5	0.3	0.1514	0.2757	0.4215	0.800	0.3	0.2
$\dot{P}_{24}$	0.2243	0.4514	0.5514	0.700	0.3	0.2	0.1243	0.3243	0.5757	0.7757	0.2	0.3
$\dot{P}_{31}$	0.3486	0.5514	0.7271	0.8514	0.3	0.4	0.300	0.500	0.7514	0.8757	0.2	0.3
$\dot{P}_{32}$	0.300	0.4486	0.600	0.7757	0.3	0.5	0.200	0.4700	0.7243	0.8757	0.2	0.3
$\dot{P}_{33}$	0.500	0.500	0.500	0.500	0.5	0.5	0.500	0.500	0.500	0.500	0.5	0.5
$\dot{P}_{34}$	0.200	0.300	0.4757	0.7028	0.2	0.3	0.100	0.2729	0.4243	0.8271	0.3	0.1
$\dot{P}_{41}$	0.2729	0.400	0.500	0.6243	0.2	0.3	0.1972	0.4486	0.600	0.7486	0.1	0.2
$\dot{P}_{42}$	0.2729	0.4757	0.5757	0.7486	0.2	0.6	0.2729	0.4972	0.6729	0.8486	0.3	0.4
$\dot{P}_{43}$	0.2972	0.5243	0.700	0.800	0.5	0.2	0.1972	0.5757	0.7757	0.900	0.3	0.2
$\dot{P}_{44}$	0.500	0.500	0.500	0.500	0.5	0.5	0.500	0.500	0.500	0.500	0.5	0.5

Table 2
Normalized collective decision matrix E using HA operation.

	P_{ij}^{L}						P_{ij}^{U}					
	p_{1ij}^{L}	p_{2ij}^{L}	p_{3ij}^{L}	p_{4ij}^{L}	$\mu_{P_{ij}}^{L}$	$v_{P_{ij}}^{L}$	p_{1ij}^{U}	p_{2ij}^{U}	p_{3ij}^{U}	p_{4ij}^{U}	$\mu_{P_{ij}}^{U}$	$v_{P_{ij}}^{U}$
P_{11}	0.5710	0.5710	0.5710	0.5710	0.5	0.5	0.5710	0.5710	0.5710	0.5710	0.5	0.5
P_{12}	0.2846	0.3474	0.4000	0.5738	0.3	0.2	0.2569	0.3457	0.5000	0.8917	0.2	0.9
P_{13}	0.165	0.3032	0.4984	0.7238	0.4	0.3	0.1381	0.2762	0.5556	0.7778	0.3	0.2
P_{14}	0.4680	0.6228	0.7473	0.9057	0.3	0.2	0.3132	0.4982	0.6868	1.0000	0.2	0.1
P_{21}	0.3393	0.4845	0.5710	0.7439	0.2	0.3	0.2529	0.4813	0.6857	0.7994	0.1	0.2
P_{22}	0.4000	0.4000	0.4000	0.4000	0.5	0.5	0.4000	0.4000	0.4000	0.4000	0.5	0.5
P_{23}	0.3039	0.4746	0.6127	0.7778	0.5	0.3	0.1682	0.3063	0.4683	0.8889	0.3	0.2
P_{24}	0.2712	0.5458	0.6667	0.8463	0.3	0.2	0.1503	0.3921	0.6960	0.9378	0.2	0.3
P_{31}	0.3981	O.6297	0.8303	0.9723	0.3	0.4	0.3426	0.5710	0.8581	1.0000	0.2	0.3
P_{32}	0.2578	0.3333	0.4459	0.6667	0.3	0.5	0.2284	0.2761	0.4255	1.0000	0.2	0.3
P_{33}	0.5556	0.5556	0.5556	0.5556	0.5	0.5	0.5556	0.5556	0.5556	0.5556	0.5	0.5
P_{34}	0.2481	0.3627	0.5751	0.8497	0.2	0.3	0.1209	0.3299	0.5130	1.0000	0.3	0.1
P_{41}	0.3116	0.4568	0.5710	0.7129	0.2	0.3	0.2251	0.5122	0.6852	0.8548	0.1	0.2
P_{42}	0.2672	0.3474	0.4204	0.7329	0.2	0.6	0.2357	0.2972	0.4023	0.7329	0.3	0.4
P_{43}	0.3302	0.5825	0.7778	0.8889	0.5	0.2	0.2191	0.6397	0.8619	1.0000	0.3	0.2
P_{44}	0.6045	0.6045	0.6045	0.6045	0.5	0.5	0.6045	0.6045	0.6045	0.6045	0.5	0.5

conditions on criterion weights with respect to their understanding, past practice or subjective verdicts. Generally such information about criterion weights is partial [37] or completely unknown. The incomplete information about criterion weights can be commonly constructed by numerous fundamental ranking forms [37–39]. But here we are considering the situation where the fuzzy priority weight vectors are completely unknown. For this we present the following optimization model to determine these weights based on given IT2FPRs.

4.4. Non-linear optimization model for determining the unknown fuzzy priority vectors

In this subsection, at the perception of SDB, we accomplished a non-linear model to find the values of fuzzy priority weights of E under the partial information. The constructed non-linear model as follows:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{4}(p_{ijk} - 0.5 - log_{(81)}w_{ik} + log_{(81)}w_{j(5-k)})^2$$

$$[M2]\ s.t \begin{cases} 0 \le \sum_{i=1}^{n} w_{i1} \le 1, \\ \sum_{i=1}^{n} w_{i4} \ge 1, \\ 0 \le w_{i1} \le w_{i2} \le w_{i3} \le w_{i4} \le 1, \\ i = 1,2,3,\dots n \end{cases} \tag{16}$$

Using model M1, we get the fuzzy priority weight vectors $\varpi = w_{ij}$ of E.

For comparative significance value of alternatives the normalized weighted value of $\bar{\bar{P}}_{ij}$ is calculated.

$$\overline{\overline{P}}_{ij} = [(w_{ij}.\overline{\overline{p}}^{L}_{1ij}, w_{ij}.\overline{\overline{P}}^{L}_{2ij}, w_{ij}.\overline{\overline{P}}^{L}_{3ij}, w_{ij}.\overline{\overline{P}}^{L}_{4ij}; \mu^{L}_{\overline{\overline{P}}_{ij}}, v^{L}_{\overline{\overline{P}}_{ij}}), (w_{ij}.\overline{\overline{P}}^{U}_{1ij}, w_{ij}.\overline{\overline{P}}^{U}_{2ij}, \overline{\overline{P}}^{U}_{3ij}, w_{ij}.\overline{\overline{P}}^{U}_{4ij}; \mu^{U}_{\overline{\overline{P}}_{ij}}, v^{U}_{\overline{\overline{P}}_{ij}})]$$

we can also denote as:

$$\overline{\overline{P}}_{ij} = [\overline{\overline{P}}^{L}_{ij}, \overline{\overline{P}}^{U}_{ij}] = [(\overline{\overline{P}}^{L}_{1ij}, \overline{\overline{P}}^{L}_{2ij}, \overline{\overline{P}}^{L}_{3ij}, \overline{\overline{P}}^{L}_{4ij}; \mu^{L}_{\overline{\overline{P}}_{ij}}, v^{L}_{\overline{\overline{P}}_{ij}}), (\overline{\overline{P}}^{U}_{1ij}, \overline{\overline{P}}^{U}_{2ij}, \overline{\overline{P}}^{U}_{3ij}, \overline{\overline{P}}^{U}_{4ij}; \mu^{U}_{\overline{\overline{P}}_{ij}}, v^{U}_{\overline{\overline{P}}_{ij}})] \tag{17}$$

further, the normalized weighted matrix can be followed as:

$$\overline{E}_{\varpi} = \begin{bmatrix} [\overline{\overline{P}}^{L}_{11}, \overline{\overline{P}}^{U}_{11}] & [\overline{\overline{P}}^{L}_{12}, \overline{\overline{P}}^{U}_{12}] & \cdots & [\overline{\overline{P}}^{L}_{1n}, \overline{\overline{P}}^{U}_{1n}] \\ [\overline{\overline{P}}^{L}_{21}, \overline{\overline{P}}^{U}_{21}] & [\overline{\overline{P}}^{L}_{22}, \overline{\overline{P}}^{U}_{22}] & \cdots & [\overline{\overline{P}}^{L}_{2n}, \overline{\overline{P}}^{U}_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\overline{\overline{P}}^{L}_{m1}, \overline{\overline{P}}^{U}_{m1}] & [\overline{\overline{P}}^{L}_{m2}, \overline{\overline{P}}^{U}_{m2}] & \cdots & [\overline{\overline{P}}^{L}_{mn}, \overline{\overline{P}}^{U}_{mn}] \end{bmatrix} \tag{18}$$

Example 4. By Table 2 the weighted normalized value of P_{ij} can be obtained by using (16)

$$\overline{\overline{P}}_{ij} = w_{ij}.\overline{\overline{P}}_{ij} = [(w_{ij}{}^{L}.\overline{\overline{p}}^{L}_{1ij}, w_{ij}{}^{L}.\overline{\overline{P}}^{L}_{2ij}, w_{ij}{}^{L}.\overline{\overline{P}}^{L}_{3ij}, w_{ij}{}^{L}.\overline{\overline{P}}^{L}_{4ij}; \mu^{L}_{\overline{\overline{P}}_{ij}}, v^{L}_{\overline{\overline{P}}_{ij}}),$$

$(w_{ij}{}^{U}.\overline{\overline{P}}^{U}_{1ij}, w_{ij}{}^{U}.\overline{\overline{P}}^{U}_{2ij}, \overline{\overline{P}}^{U}_{3ij}, w_{ij}{}^{U}.\overline{\overline{P}}^{U}_{4ij}; \mu^{U}_{\overline{\overline{P}}_{ij}}, \nu^{U}_{\overline{\overline{P}}_{ij}})]$

$\overline{\overline{P}}_{12}$

$w_{12}.\overline{\overline{P}}_{12} = [(w_{12}{}^{L}.\overline{\overline{p}}^{L}_{112}, w_{12}{}^{L}.\overline{\overline{P}}^{L}_{212}, w_{12}{}^{L}.\overline{\overline{P}}^{L}_{312}, w_{12}{}^{L}.\overline{\overline{P}}^{L}_{412}; \mu^{L}_{\overline{\overline{P}}_{12}}, \nu^{L}_{\overline{\overline{P}}_{12}}),$

$(w_{12}{}^{U}.\overline{\overline{P}}^{U}_{121}, w_{12}{}^{U}.\overline{\overline{P}}^{U}_{212}, \overline{\overline{P}}^{U}_{312}, w_{12}{}^{U}.\overline{\overline{P}}^{U}_{412}; \mu^{U}_{\overline{\overline{P}}_{12}}, \nu^{U}_{\overline{\overline{P}}_{12}})]$

$$\overline{\overline{P}}_{12} = [(0.1438397 \times 0.2846, 0.1438397 \times 0.3474, 0.1438397 \times 0.4000, 0.1438397 \times 0.5738; 0.3, 0.2)$$
$$(0.2733753 \times 0.2569, 0.2733753 \times 0.3457, 0.2733753 \times 0.5000, 0.2733753 \times 0.8917; 0.2, 0.9)]$$

$$\overline{\overline{P}}_{12} = [(0.0409, 0.0500, 0.0575, 0.0825; 0.3, 0.5) \times (0.0702, 0.0945, 0.1367, 0.2438; 0.5, 0.5)]$$

In IT2TrPFPR, we utilized the concept of signed distances that calculates the distance of alternatives from "positive-ideal solution"(PIS) and "negative-ideal solution"(NIS). At the end, correspondent to each alternative $V_i \in V$ we calculate the SDBCC CC_i. Values of normalized weighted rating are between 0 and 1, P^* used to represent the PIS and P^- for NIS are defined as:

$$P^* = \{\langle x_{\underline{j}}, [(1,1,1,1:1,1),(1,1,1,1;1,1)]\rangle \big| x_{\underline{j}} \in X\} \tag{19}$$

$$P^- = \{\langle x_{\underline{j}}, [(0,0,0,0:1,1),(0,0,0,0;1,1)]\rangle \big| x_{\underline{j}} \in X\} \tag{20}$$

Let $P^*_j = [(1,1,1,1;1,1),(1,1,1,1;1,1)]$ and $P^-_j = [(0,0,0,0;1,1),(0,0,0,0;1,1)]$ $\forall x_{\underline{j}} \in X$. According to Property 1, we know that $d(P^*_j, \overline{1}_1) = 0, d(P^*_j, \overline{0}_1) = 2, d(P^-_j, \overline{1}_1) = -2, d(P^-_j, \overline{0}_1) = 0$. For every alternative V_i, it is noticeable that the signed distances from P_{ij} to P^*_j and P_{ij} to P^-_j can be calculated with respect to $d(P_{ij}, \overline{1}_1)$ and $d(P_{ij}, \overline{0}_1)$. It follows such that:

$$d(\overline{\overline{P}}_{ij}, P^*_j) = d(\tilde{P}_{ij}, \overline{1}_1) \tag{21}$$

$$d(\overline{P}_{ij}, P^-_j) = d(\tilde{\tilde{P}}_{ij}, \overline{0}_1) \tag{22}$$

$$\begin{aligned} W(\hat{P}_{ij}, P^*_j) &= d(\overline{P}_{ij}, P^*_j) \\ &= \frac{1}{8}\Big[2(\overline{p}_{1ij}{}^{L} + \overline{p}_{2ij}{}^{L} + \overline{p}_{3ij}{}^{L} + \overline{p}_{4ij}{}^{L}) \\ &+ 2(\overline{p}_{1ij}{}^{U} + \overline{p}_{2ij}{}^{U} + \overline{p}_{3ij}{}^{U} + \overline{p}_{4ij}{}^{U}) \\ &+ 3(\overline{p}_{2ij}{}^{U} + \overline{p}_{3ij}{}^{U} - \overline{p}_{1ij}{}^{U} - \overline{p}_{4ij}{}^{U})\Big(\frac{\mu^{L}_{\overline{p}}}{\mu^{U}_{\overline{p}}} - \frac{\nu^{L}_{\overline{p}}}{\nu^{U}_{\overline{p}}}\Big) - 16\Big] \end{aligned} \tag{23}$$

$$\begin{aligned} W(\hat{P}_{ij}, P^-_j) &= d(\overline{P}_{ij}, P^-_j) \\ &= \frac{1}{8}\Big[2(\overline{p}_{1ij}{}^{L} + \overline{p}_{2ij}{}^{L} + \overline{p}_{3ij}{}^{L} + \overline{p}_{4ij}{}^{L}) \\ &+ 2(\overline{p}_{1ij}{}^{U} + \overline{p}_{2ij}{}^{U} + \overline{p}_{3ij}{}^{U} + \overline{p}_{4ij}{}^{U}) \\ &+ 3(\overline{p}_{2ij}{}^{U} + \overline{p}_{3ij}{}^{U} - \overline{p}_{1ij}{}^{U} - \overline{p}_{4ij}{}^{U})\Big(\frac{\mu^{L}_{\overline{p}}}{\mu^{U}_{\overline{p}}} - \frac{\nu^{L}_{\overline{p}}}{\nu^{U}_{\overline{p}}}\Big)\Big] \end{aligned} \tag{24}$$

where $W(\hat{P}_{ij}, P^*_j)$ and $W(\hat{P}_{ij}, P^-_j)$ is the weighted signed distances.

The ideal solutions and the closeness of alternatives L_i can be deduced by using weighted signed- distances. Afterward, the average signed-based distances $(\frac{1}{n}) \cdot \sum_{j=1}^{n}(W\hat{P}_{ij}, P^*_j)$ and $(\frac{1}{n}) \cdot \sum_{j=1}^{n} W(\bar{P}_{ij}, P^-_j)$ are calculated to order the SBDCC of every alternative V_i.

Let CC^m_i for $i = 1, 2, \ldots, \mathfrak{r}$ are use to represent the SBDCC of each V_i expressed as:

$$CC^m_i = \frac{\frac{1}{n}\sum_{j=1}^{n} W(\hat{P}_{ij}, P^-_j)}{\frac{1}{n}\sum_{j=1}^{n} W(\hat{P}_{ij}, P^-_j) - \frac{1}{n}\sum_{j=1}^{n} W(\hat{P}_{ij}, P^*_j)} \tag{25}$$

It is necessary, that $0 \le CC^{\mathfrak{r}}_i \le 1$. The DMs approved one alternative with the shortest SDBCC. Ranking of SDBCC $CC^{\mathfrak{r}}_i$ alternatives expressed in ascending order.

5. Proposed signed distance based ranking approach with unknown fuzzy priority vectors

Suppose a GDM problem, $(\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{\mathfrak{R}}\})$ sets of DMs and $V_i \in V$ be the set of alternatives. The decision makers compare every alternative with every other alternative.

IT2TrPFPR framework under partial data is summarized as follows:

Step 1: Appeal the DMs to utilize the IT2TrPFPR $P = (P_{ij})_{n\times n}$ for the evaluation of one alternatives with every other alternative.

Step 2: According to normal distribution method the OWA operation can be derived by using (9)–(11), to find the weighting vector $(\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_{\mathfrak{R}}))$ in the OWA operation can be derived.

step3: By using (6) the signed distance $d(P^{\mathfrak{r}}_{ij}, 0_1)$ from $P^{\mathfrak{r}}_{ij}$ to 0_1 can be calculated and weighted ratings can be reordered in ascending order of signed distances.

3.1: Using HA operation in (12), HA $\dot{P}^{\mathfrak{r}}_{ij}$ for group consensus opinions can be calculated for each $V_i \in V$ and $x_{\underline{j}} \in X$.

3.2: Build a collective decision matrix $\dot{E}$ as in (13).

Step 4: Construct normalized collective matrix E as in (15) using (14).

Step 5: Construct an integrated optimization model [M1] to obtain the fuzzy priority weight vector ϖ.

Step 5.1: Using (4.4) to find the weighted normalized matrix, afterward find the sign distance from $\overline{\overline{P}}_{ij}$ to P^*_j and $\overline{\overline{P}}_{ij}$ to P^-_j can be calculated using (23) and (24) respectively.

Step 5.2: Further calculate the SDBCC CC^m_i.

Step 6: Rank $\mathfrak{r}$ alternatives in ascending order of SDBCC $CC^{\mathfrak{R}}_i$ to conclude best choice.

The approach based on the above algorithm is an effective tool for managing multiple criteria group decision-making situations with completely unknown priority weights modeled as IT2TrPFPRs. To aggregate the opinion of multiple decision-makers, a hybrid averaging operation based on weighted averaging and ordered weighted averaging (OWA) operations is employed for a collective decision environment. To calculate the fuzzy priority weight vectors in case of completely unknown environment, we construct a non-linear optimization model. An integrated optimization model based on a new signed distance-based closeness coefficients approach is employed to determine the priority ranking of alternatives.

The obtained results will be notably unusual from those earlier used techniques such as aggregation method based on WA operation and classical additive weighting approach in number of aspects. The aggregation technique based on WA operation weights only comparative significance of the decision-makers whereas the projected aggregation technique with HA approach weights both the IT2TrPFN ratings and its ordered position. Specifically, the presented approach not only considers the significance of decision-makers but also the conformity of individual judgments. Consequently the outcomes obtained by presented technique should be more exact than the previously used techniques. On the contrary, the classical weighting approach is only valid to those GDM problems where information about the criterion weights is completely known. However, the proposed model can be applied to the GDM problems with completely unknown information. Therefore, the comparative study shows the potential of the proposed technique in practical applications. The only disadvantage of the proposed technique seems to be the added computational cost resulting by considering both aspects of IT2TrPFNN ratings and its ordered position.

6. Patient centered medicine based medical diagnosis involving signed distance based ranking approach with interval type-2 trapezoidal pythagorean fuzzy preference relations

In this part, we show an instance of scientific decision-making to demonstrate the possibility of a preference scheme that practically performs our proposed method. The proposed method is established through a comparative investigation with different techniques.

In such situations, the attending medical doctor is not capable of autonomously bringing out critical fitness-care judgments for the affected patients. For this cause, the affected person-focused health-care organism implements a group decision-making scheme that deems the proficient judgments of the complete scientific panel and defines the alternatives available for the restoration or remedy of ailment and attempts to maximize the probably health-care blessings with possible results. The next practical illustration engages a patient-targeted clinical problem to illustrate the feasibility of the predicted effective signed-based distance closeness coefficient technique inside the framework of IT2TrPFPR. There are severe diseases that are regularly misdiagnosed owing to the resemblance in their symptoms so it is important to perceive the symptoms as quickly as viable. If you sense you is probably in for a prognosis of typhoid, keep in mind you could have some other condition. There are numerous illnesses that proportion similar symptoms to typhoid. Assume that a patient is admitted in the health center for the analysis of his unknown ailment having the symptoms of weakness, stomach pain, headache, diarrhea and constipation, cough, thirst and hunger. The feasible chances are the patient is suffering from this sort of 3 illnesses inclusive of typhoid, multiple sclerosis or glaucoma. For the attending doctor it is difficult to pin down the precise cure for the disease and he cannot independently make selections about the fitness of the patient so for that purpose a committee of three professional decision makers $\lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ and $V = \{V_1, V_2, V_3, V_4\}$ be the set of alternatives. The DMs compare each alternative and give their judgment matrixes with IT2TrPFPR P^1, P^2, P^3, respectively, Where $P = (P_{ij})_{4\times4}$, shown as follows: These three decision-makers considered several alternative comprising possibilities of therapy $V = \{V_1, V_2, V_3, V_4\}$ and chose the best alternative. Following steps shows the numerical example of proposed algorithm for solving GDM problem of a medical decision-making.

Step 1:

Construct three IT2TrPFPR matrices $P = (P^k_{ij})_{n\times n}$.

$$P^{(1)} = \begin{bmatrix} [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.3,0.5,0.6,0.8;0.3,0.2),(0.2,0.4,0.7,0.9;0.2,0.1)] \\ [(0.1,0.2,0.4,0.7;0.6,0.5),(0.1,0.2,0.5,0.7;0.5,0.4)] \\ [(0.4,0.6,0.7,0.8;0.8,0.3),(0.3,0.6,0.7,0.8;0.3,0.1)] \\ \\ [(0.2,0.4,0.5,0.7;0.2,0.3),(0.1,0.3,0.6,0.8;0.1,0.2)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.2,0.4,0.6,0.7;0.6,0.5),(0.1,0.3,0.7,0.8;0.7,0.4)] \\ [(0.3,0.4,0.5,0.8;0.8,0.2),(0.2,0.4,0.6,0.8;0.6,0.4)] \\ \\ [(0.3,0.6,0.8,0.9;0.5,0.6),(0.3,0.5,0.8,0.9;0.4,0.5)] \\ [(0.3,0.4,0.6,0.8;0.5,0.6),(0.2,0.3,0.7,0.9;0.4,0.7)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.1,0.2,0.3,0.4;0.2,0.6),(0.1,0.2,0.3,0.6;0.7,0.4)] \\ \\ [(0.2,0.3,0.4,0.6;0.3,0.8),(0.2,0.3,0.4,0.7;0.1,0.3)] \\ [(0.2,0.5,0.6,0.7;0.2,0.8),(0.2,0.4,0.6,0.8;0.4,0.6)] \\ [(0.6,0.7,0.8,0.9;0.6,0.2),(0.4,0.7,0.8,0.9;0.4,0.7)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \end{bmatrix}$$

$$P^{(2)} = \begin{bmatrix} [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.6,0.7,0.8,0.8;0.6,0.5),(0.4,0.6,0.7,0.8;0.5,0.4)] \\ [(0.3,0.5,0.6,0.7;0.4,0.3),(0.2,0.3,0.5,0.8;0.3,0.2)] \\ [(0.4,0.5,0.6,0.8;0.3,0.2),(0.3,0.4,0.6,0.9;0.2,0.1)] \\ \\ [(0.2,0.2,0.3,0.4;0.5,0.6),(0.2,0.3,0.4,0.6;0.4,0.3)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.1,0.3,0.4,0.7;0.5,0.4),(0.1,0.2,0.4,0.8;0.4,0.3)] \\ [(0.2,0.4,0.5,0.6;0.8,0.5),(0.1,0.3,0.5,0.7;0.5,0.4)] \\ \\ [(0.3,0.4,0.5,0.7;0.3,0.4),(0.2,0.5,0.7,0.8;0.2,0.3)] \\ [(0.3,0.6,0.7,0.9;0.4,0.5),(0.2,0.6,0.8,0.9;0.3,0.4)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.2,0.3,0.5,0.8;0.8,0.5),(0.1,0.2,0.4,0.9;0.5,0.4)] \\ \\ [(0.2,0.4,0.5,0.6;0.2,0.3),(0.1,0.4,0.6,0.7;0.1,0.2)] \\ [(0.4,0.5,0.6,0.8;0.5,0.8),(0.3,0.5,0.7,0.9;0.4,0.5)] \\ [(0.2,0.5,0.7,0.8;0.5,0.8),(0.1,0.6,0.8,0.9;0.4,0.5)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \end{bmatrix}$$

$$P^{(3)} = \begin{bmatrix} [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.2,0.3,0.3,0.4;0.8,0.2),(0.1,0.2,0.2,0.5;0.2,0.1)] \\ [(0.1,0.2,0.4,0.5;0.7,0.3),(0.1,0.3,0.5,0.6;0.3,0.2)] \\ [(0.3,0.4,0.5,0.5;0.6,0.4),(0.1,0.2,0.3,0.6;0.4,0.3)] \\ \\ [(0.6,0.7,0.7,0.8;0.2,0.8),(0.5,0.8,0.8,0.9;0.1,0.2)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.4,0.5,0.6,0.7;0.6,0.3),(0.2,0.3,0.3,0.8;0.3,0.2)] \\ [(0.2,0.5,0.6,0.7;0.3,0.7),(0.1,0.3,0.6,0.8;0.2,0.3)] \\ \\ [(0.5,0.6,0.8,0.9;0.3,0.7),(0.4,0.5,0.7,0.9;0.2,0.3)] \\ [(0.3,0.4,0.5,0.6;0.3,0.6),(0.2,0.7,0.7,0.8;0.2,0.3)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \\ [(0.3,0.4,0.6,0.8;0.4,0.3),(0.1,0.5,0.6,0.9;0.3,0.1)] \\ \\ [(0.5,0.5,0.6,0.7;0.4,0.6),(0.4,0.7,0.8,0.9;0.3,0.4)] \\ [(0.3,0.4,0.5,0.8;0.7,0.3),(0.2,0.4,0.7,0.9;0.3,0.2)] \\ [(0.2,0.4,0.6,0.7;0.3,0.4),(0.1,0.4,0.5,0.9;0.1,0.3)] \\ [(0.5,0.5,0.5,0.5;0.5,0.5),(0.5,0.5,0.5,0.5;0.5,0.5)] \end{bmatrix}$$

Step 2:

Now we derive OWA weighting vector $\zeta = (\zeta_1, \zeta_2, \zeta_3) = (0.2429, 0.5142, 0.2429)$.

Step 3:

Further we find the signed distance. For this, we use $d(P^1_{12}, \bar{0}_1) = 1.1$, $d(P^2_{12}, \bar{0}_1) = 1.3481$ and $d(P^3_{12}, \bar{0}_1) = 0.4$. As $d(P^3_{12}, \bar{0}_1) < d(P^1_{12}, \bar{0}_1) < d(P^2_{12}, \bar{0}_1)$ so $\vartheta(1) = 3$, $\vartheta(2) = 1$, $\vartheta(3) = 2$. Hence $P^{\vartheta(1)}_{12} = P^1_{12}$, $P^{\vartheta(1)}_{12} = P^1_{12}$, $P^{\vartheta(1)}_{12} = P^1_{12}$.

Step 3.1:

By utilizing HA operation Table 1 shows the sum up of ratings $\dot{P}^{\tau}_{ij}$ of $V_i \in V$.

Step 3.2:

Collective matrix $\dot{E}$ can be created.

Step 4:

By Table 1, it is obvious that $\dot{P}^*_1 = 0.8757, \dot{P}^-_2 = 0.2$, $\dot{P}^+_3 = 0.9$ and $\dot{P}^-_4 = 0.82713$. Then the normalized collective matrix $\dot{D}$ can be constructed in Table 2.

Step 5:

By using (M1) find the fuzzy priority vector of E. Where

$$\varpi_{ij} = [w^L_{ij}, w^U_{ij}] = (w^L_{1ij}, w^L_{2ij}, w^L_{3ij}, w^L_{4ij}), (w^U_{1ij}, w^U_{2ij}, w^U_{3ij}, w^U_{4ij})$$

Table 3
Weighted normalized collective decision matrix $\overline{\overline{E}}_{\varpi}$ using (4.4)

	$\overline{\overline{P}}^L_{ij}$						$\overline{\overline{P}}^U_{ij}$					
	$\overline{\overline{P}}^L_{1ij}$	$\overline{\overline{P}}^L_{2ij}$	$\overline{\overline{P}}^L_{3ij}$	$\overline{\overline{P}}^L_{4ij}$	$\mu^L_{\overline{\overline{P}}_{ij}}$	$\nu^L_{\overline{\overline{P}}_{ij}}$	$\overline{\overline{P}}^U_{1ij}$	$\overline{\overline{P}}^U_{2ij}$	$\overline{\overline{P}}^U_{3ij}$	$\overline{\overline{P}}^U_{4ij}$	$\mu^U_{\overline{\overline{P}}_{ij}}$	$\nu^U_{\overline{\overline{P}}_{ij}}$
$\overline{\overline{P}}_{11}$	0.0821	0.0821	0.0821	0.0821	0.5	0.5	0.1561	0.1561	0.1561	0.1561	0.5	0.5
$\overline{\overline{P}}_{12}$	0.0409	0.0500	0.0575	0.0825	0.3	0.2	0.0702	0.0945	0.1367	0.2438	0.2	0.1
$\overline{\overline{P}}_{13}$	0.0526	0.0967	0.1590	0.2309	0.4	0.3	0.0602	0.1204	0.2422	0.3391	0.3	0.2
$\overline{\overline{P}}_{14}$	0.3315	0.4412	0.5294	0.6416	0.3	0.2	0.2083	0.3314	0.4568	0.6651	0.2	0.1
$\overline{\overline{P}}_{21}$	0.1249	0.1783	0.2101	0.2738	0.2	0.3	0.0632	0.1203	0.1714	0.1998	0.1	0.2
$\overline{\overline{P}}_{22}$	0.2414	0.2414	0.2414	0.2414	0.5	0.5	0.2027	0.2027	0.2027	0.2027	0.5	0.5
$\overline{\overline{P}}_{23}$	0.2094	0.3270	0.4222	0.5359	0.5	0.3	0.1044	0.1902	0.2908	0.5519	0.3	0.2
$\overline{\overline{P}}_{24}$	0.2541	0.5114	0.6246	0.7929	0.3	0.2	0.1061	0.2767	0.4912	0.6618	0.2	0.3
$\overline{\overline{P}}_{31}$	0.0647	0.1023	0.1349	0.1580	0.3	0.4	0.0921	0.1534	0.2306	0.2687	0.2	0.3
$\overline{\overline{P}}_{32}$	0.0562	0.0726	0.0972	0.1453	0.3	0.5	0.0851	0.1029	0.1585	0.0373	0.2	0.3
$\overline{\overline{P}}_{33}$	0.2943	0.2943	0.2943	0.2943	0.5	0.5	0.2976	0.2976	0.2976	0.2976	0.5	0.5
$\overline{\overline{P}}_{34}$	0.2481	0.3627	0.5751	0.8497	0.2	0.3	0.1018	0.2777	0.4319	0.8419	0.3	0.1
$\overline{\overline{P}}_{41}$	0.0237	0.0348	0.0435	0.0543	0.2	0.3	0.0468	0.1065	0.1425	0.1778	0.1	0.2
$\overline{\overline{P}}_{42}$	0.0560	0.0728	0.0881	0.1536	0.2	0.6	0.1037	0.1308	0.1770	0.3225	0.3	0.4
$\overline{\overline{P}}_{43}$	0.1376	0.2428	0.3242	0.3705	0.5	0.2	0.1262	0.3685	0.4965	0.5761	0.3	0.2
$\overline{\overline{P}}_{44}$	0.5090	0.5090	0.5090	0.5090	0.5	0.5	0.4421	0.4421	0.4421	0.4421	0.5	0.5

$$\varpi_{ij} = [w^L_{ij}, w^U_{ij}] = \begin{bmatrix} [(0.1438376), (0.2733753)] \\ [(0.1438397), (0.2733753)] \\ [(0.3190571), (0.4359204)] \\ [(0.7298309), (0.6852155)] \\ \\ [(0.3680370), (0.2499342)] \\ [(0.6033931), (0.5066862)] \\ [(0.6890366), (0.6208660)] \\ [(0.9369136), (0.7056906)] \\ \\ [(0.1624929), (0.2686871)] \\ [(0.2178836), (0.3725696)] \\ [(0.5297458), (0.5355691)] \\ [(0.9999953), (0.8419122)] \\ \\ [(0.07614026), (0.2080033)] \\ [(0.2095657), (0.4400704)] \\ [(0.4168528), (0.5760887)] \\ [(0.8420989), (0.7313801)] \end{bmatrix}$$

Step 5.1:

Table 3 shows the Weighted Normalize collective decision matrix $\overline{\overline{E}}_{\varpi}$ can be constructed by Using (4.4).

Step 5.2:

Further we calculate the sign distance from $\overline{\overline{P}}_{ij}$ to P^*_j and $\overline{\overline{P}}_{ij}$ to P^-_j using (23) and (24) respectively and afterward SDBCC CC^m_i corresponding to each alternative $V_i \in V$. The computing of sign distance from $\overline{\overline{P}}_{ij}$ to P^*_j and $\overline{\overline{P}}_{ij}$ to P^-_j is expressed in Tables 4 and 5

Step 6:

We obtain the SDBCC CC^{τ}_i corresponding to every alternative V_i. The SDBCC $CC^{\tau}_1 = 0.2044$, $CC^{\tau}_2 = 0.2950$, $CC^{\tau}_3 = 0.2596$ and $CC^{\tau}_4 = 0.2665$. Then we have the ranking of alternative is as follows: $V_1 > V_3 > V_4 < V_2$. Which means that V_1 is the best alternative.

Table 4
Signed distances corresponding to each alternative.

Alternatives	$d(\tilde{\tilde{P}}_{ij}, \tilde{1}_1)$
$\tilde{\tilde{P}}_{11}$	−1.7618
$\tilde{\tilde{P}}_{12}$	−1.8456
$\tilde{\tilde{P}}_{13}$	−1.6747
$\tilde{\tilde{P}}_{14}$	−1.0827
$\tilde{\tilde{P}}_{21}$	−1.6592
$\tilde{\tilde{P}}_{22}$	−1.5559
$\tilde{\tilde{P}}_{23}$	−1.3530
$\tilde{\tilde{P}}_{24}$	−1.0703
$\tilde{\tilde{P}}_{31}$	−1.6974
$\tilde{\tilde{P}}_{32}$	−1.9449
$\tilde{\tilde{P}}_{33}$	−1.4081
$\tilde{\tilde{P}}_{34}$	−0.8729
$\tilde{\tilde{P}}_{41}$	−1.8334
$\tilde{\tilde{P}}_{42}$	−1.6869
$\tilde{\tilde{P}}_{43}$	−1.2987
$\tilde{\tilde{P}}_{44}$	−0.0489

Table 5
"Signed distances" corresponding to every alternative.

Alternatives	$d(\tilde{\tilde{P}}_{ij}, \tilde{0}_1)$
$\tilde{\tilde{P}}_{11}$	0.2382
$\tilde{\tilde{P}}_{12}$	0.1544
$\tilde{\tilde{P}}_{13}$	0.3253
$\tilde{\tilde{P}}_{14}$	0.9173
$\tilde{\tilde{P}}_{21}$	0.3409
$\tilde{\tilde{P}}_{22}$	0.441
$\tilde{\tilde{P}}_{23}$	0.6470
$\tilde{\tilde{P}}_{24}$	0.9297
$\tilde{\tilde{P}}_{31}$	0.3026
$\tilde{\tilde{P}}_{32}$	0.0551
$\tilde{\tilde{P}}_{33}$	0.5919
$\tilde{\tilde{P}}_{34}$	1.1271
$\tilde{\tilde{P}}_{41}$	0.1666
$\tilde{\tilde{P}}_{42}$	0.3131
$\tilde{\tilde{P}}_{43}$	0.7013
$\tilde{\tilde{P}}_{44}$	0.9511

6.1. Comparison and discussion

Comparative analysis was conducted to test the proposed method with some other existing approaches. Two comparisons have done with the distinct procedure, WA aggregation, and non-linear optimization model. In first comparative analysis, we have compared our proposed method with a well-known TOPSIS method. The fundamental perception of TOPSIS is that the chosen alternative should have shortest distance from PIS and farthest distance from NIS. Consider weighted

Table 6
Collective weighted normalized matrix $\overline{\overline{E}}_{\varpi}$ using (4.4)

	$\overline{\overline{P}}^L_{ij}$						$\overline{\overline{P}}^U_{ij}$					
	$\overline{\overline{P}}^L_{1ij}$	$\overline{\overline{P}}^L_{2ij}$	$\overline{\overline{P}}^L_{3ij}$	$\overline{\overline{P}}^L_{4ij}$	$\mu^L_{\overline{\overline{P}}_{ij}}$	$\nu^L_{\overline{\overline{P}}_{ij}}$	$\overline{\overline{P}}^U_{1ij}$	$\overline{\overline{P}}^U_{2ij}$	$\overline{\overline{P}}^U_{3ij}$	$\overline{\overline{P}}^U_{4ij}$	$\mu^U_{\overline{\overline{P}}_{ij}}$	$\nu^U_{\overline{\overline{P}}_{ij}}$
$\overline{\overline{P}}_{11}$	0.5721	0.5721	0.5721	0.5721	0.5	0.5	0.5721	0.5721	0.5721	0.5721	0.5	0.5
$\overline{\overline{P}}_{12}$	0.3185	0.3824	0.4292	0.5970	0.3	0.2	0.2849	0.4124	0.5464	0.9857	0.2	0.1
$\overline{\overline{P}}_{13}$	0.1689	0.3089	0.5022	0.6822	0.4	0.3	0.1400	0.2989	0.5556	0.7589	0.3	0.2
$\overline{\overline{P}}_{14}$	0.4424	0.6047	0.7286	0.8315	0.3	0.2	0.2652	0.4659	0.6221	0.9170	0.2	0.1
$\overline{\overline{P}}_{21}$	0.4610	0.5911	0.6617	0.8240	0.2	0.3	0.3693	0.6382	0.7856	0.9802	0.1	0.2
$\overline{\overline{P}}_{22}$	0.4000	0.4000	0.4000	0.4000	0.5	0.5	0.4000	0.4000	0.4000	0.4000	0.5	0.5
$\overline{\overline{P}}_{23}$	0.2889	0.4633	0.6089	0.7778	0.5	0.3	0.1589	0.3044	0.5000	0.8889	0.3	0.2
$\overline{\overline{P}}_{24}$	0.2862	0.5489	0.6729	0.8736	0.3	0.2	0.1623	0.4102	0.7113	0.9591	0.2	0.3
$\overline{\overline{P}}_{31}$	0.4416	0.6270	0.8261	0.9703	0.3	0.4	0.3627	0.5721	0.8364	1.0000	0.2	0.3
$\overline{\overline{P}}_{32}$	0.2703	0.3431	0.4425	0.6667	0.3	0.5	0.2334	0.2755	0.3636	1.0000	0.2	0.3
$\overline{\overline{P}}_{33}$	0.5556	0.5556	0.5556	0.5556	0.5	0.5	0.5556	0.5556	0.5556	0.5556	0.5	0.5
$\overline{\overline{P}}_{34}$	0.2627	0.3866	0.5960	0.8377	0.2	0.3	0.1239	0.4077	0.5638	1.0000	0.3	0.1
$\overline{\overline{P}}_{41}$	0.3764	0.4714	0.5858	0.7357	0.2	0.3	0.2975	0.5998	0.7140	0.8993	0.1	0.2
$\overline{\overline{P}}_{42}$	0.2601	0.3591	0.4376	0.8780	0.2	0.3	0.2301	0.2990	0.4695	0.8850	0.3	0.2
$\overline{\overline{P}}_{43}$	0.3600	0.5767	0.7644	0.8756	0.3	0.2	0.2144	0.6056	0.7456	1.0000	0.1	0.3
$\overline{\overline{P}}_{44}$	0.6196	0.6196	0.6196	0.6196	0.5	0.5	0.6196	0.6196	0.6196	0.6196	0.5	0.5

normalized matrix IT2TrPFPR $P^{\omega}_{\underline{i}}$ of V_i are as follows:

$$\begin{aligned}\mathrm{P}^{\omega}_{i} = \{ \langle\, \mathrm{x}_{\underline{j}}, [\, (\omega_j \mathrm{g}^U_{1ij}, \omega_j \mathrm{g}^U_{2ij}, \omega_j \mathrm{g}^U_{3ij}, \omega_j \mathrm{g}^U_{4ij}; \mu^U_{\mathrm{G}_{ij}}, \nu^U_{\mathrm{G}_{ij}}), \\ (\omega_j \mathrm{g}^L_{1ij}, \omega_j \mathrm{g}^L_{2ij}, \omega_j \mathrm{g}^L_{3ij}, \omega_j \mathrm{g}^L_{4ij}; \mu^L_{\mathrm{G}_{ij}}, \nu^L_{\mathrm{G}_{ij}}) \,]\, \rangle \,\big|\, \mathrm{x}_{\underline{j}} \in \mathrm{X} \,\}\end{aligned} \tag{26}$$

PIS and NIS are as follows:

$$\mathrm{P}^{\omega *} = \{ \langle \mathrm{x}_{\underline{j}}, [(\omega_j, \omega_j, \omega_j, \omega_j \,:\, 1, 1), (\omega_j, \omega_j, \omega_j, \omega_j; 1, 1)] \rangle \big| \mathrm{x}_{\underline{j}} \in \mathrm{X} \} \tag{27}$$

$$\mathrm{P}^{\omega} = \{ \langle \mathrm{x}_{\underline{j}}, [(0,0,0,0 \,:\, 1,1), (0,0,0,0; 1,1)] \rangle \big| \mathrm{x}_{\underline{j}} \in \mathrm{X} \} \tag{28}$$

$h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega *})$ and $h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega})$ distance of each normalized weighted IT2TrPFPR.

$$\begin{aligned} h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega *}) = \Bigg[\frac{1}{8} \sum_{j=1}^{n} \omega^2_{\underline{j}} \, \big((1-\mathrm{p}^U_{1ij})^2 + (1-\mathrm{p}^U_{2ij})^2 + (1-\mathrm{p}^U_{3ij})^2 + (1-\mathrm{p}^U_{4ij})^2 \\ + (1-\mathrm{p}^L_{1ij})^2 + (1-\mathrm{p}^L_{2ij})^2 + (1-\mathrm{p}^L_{3ij})^2 + (1-\mathrm{p}^L_{4ij})^2 \big) \Bigg]^{\frac{1}{2}} \end{aligned} \tag{29}$$

$$\begin{aligned} h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega}) = \Bigg[\frac{1}{8} \sum_{j=1}^{n} \omega^2_{\underline{j}} \, \big((\mathrm{p}^U_{1ij})^2 + (\mathrm{p}^U_{2ij})^2 + (\mathrm{p}^U_{3ij})^2 + (\mathrm{p}^U_{4ij})^2 \\ (\mathrm{p}^L_{1ij})^2 + (\mathrm{p}^L_{2ij})^2 + (\mathrm{p}^L_{3ij})^2 + (\mathrm{p}^L_{4ij})^2 \big) \Bigg]^{\frac{1}{2}} \end{aligned} \tag{30}$$

Further closeness coefficient $CC^{\mathbf{G}}_i (0 \le CC^{\mathbf{G}}_i \le 1)$ of alternative V_i are as follows:

$$CC^{\mathbf{G}}_i = \frac{h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega -})}{h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega *}) + h^{\mathbf{G}}(\mathrm{P}^{\omega}_{\underline{i}}, \mathrm{P}^{\omega -})} \tag{31}$$

by using (31) we cumulate the $CC^{\mathbf{G}}_i$ are follows $CC^{\mathbf{G}}_1 = 0.2201$, $CC^{\mathbf{G}}_2 = 0.3074$, $CC^{\mathbf{G}}_3 = 0.2686$ and $CC^{\mathbf{G}}_4 = 0.2565$. Then the alternative ranking is: $\mathrm{V}_1 > \mathrm{V}_4 > \mathrm{V}_3 > \mathrm{V}_2$. Which means V_1 is the beast alternative. Although the calculation of (29) and (31) is much heavy then our proposed method. Moreover our technique give realistic results than TOPSIS although it give same ranking alternative.

In the second comparative evaluation, we have taken into account the comparative significance of multiple decision-makers to establish a collective decision matrix using WA operation for aggregation process. The normalized decision matrix using WA operation is shown in Table 6.

The computed SDBCC are $\overline{\overline{\overline{CC_1}}} = 0.5357$, $\overline{\overline{\overline{CC_2}}} = 0.5351$, $\overline{\overline{\overline{CC_3}}} = 0.5624$ and $\overline{\overline{\overline{CC_4}}} = 0.6045$. Which means $\mathrm{V}_2 > \mathrm{V}_1 > \mathrm{V}_3 > \mathrm{V}_4$ so according to this method V_2 is the best choice. In comparative studies, weights vectors produced by using the WA method and those introduced by this study are the same but the decision rules and procedure in the paper are remarkably different from previous methods. Moreover, the proposed method with HA operation weights presents the PRs and their ordered position. So that the outcomes of the proposed technique should be realistic and authentic.

7. Conclusions

We have proposed a method which will work as an effective approach for managing uncertain situations where the fuzzy priority weight vectors are completely unknown and the preference information is in the form of interval type-2 trapezoidal pythagorean fuzzy preference relations. A hybrid averaging operation based on weighted averaging and ordered weighted averaging (OWA) operations has been employed for a collective decision environment to aggregate the multiple opinions. The completely unknown fuzzy priority weight vectors were determined using an optimization model. The main idea of the proposed GDM method is to choose the alternative with the smallest SDBCC. Finally, an application of medical decision-making has been illustrated to demonstrate the practicability of proposed approach. The proposed technique is constructive because of its flexibility regarding to the completely unknown information and can be extended in future research.

The proposed technique with HA approach weights both the IT2TrPFN ratings and its ordered position. Specifically, the presented approach not only considers the significance of decision-makers but also the conformity of individual judgments. Consequently the outcomes obtained by presented technique are more feasible than the previously used techniques.

CRediT authorship contribution statement

Muhammad Touqeer: Conceptualization, Methodology, Software. **Sadaf Shaheen:** Data curation, Writing – original draft. **Tahira Jabeen:** Data curation, Writing – original draft. **Saleh Al Sulaie:** Data curation, Reviewing original draft. **Dumitru Baleanu:** Data curation, Reviewing original draft. **Ali Ahmadian:** Supervision, Validation, Reviewing original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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