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EDITORIAL

SPECIAL ISSUE SECTION ON FRACTAL AI-BASED ANALYSES AND APPLICATIONS TO COMPLEX SYSTEMS: PART III

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Fractals, as a universal language, are often considered to be an abundant source of creativity, surprise, beauty and reality. Being regarded and employed as a powerful tool to communicate, interpret, describe and analyze complex ideas and complexity in nature and other imaginable systems, fractals can most of the time remind one of a story or a narrative element of ever-evolving scientific revolution in the productive realms. Mathematics translated from nature is reliant on fractals which have extensive and multidisciplinary applications evolving into many various disciplines.

Complex systems, composed of multiple interconnected interacting entities that manifest the subtle global scale dynamics, entail the understanding of complexity which has been through an evolution with cumulative nature where order is considered to be the unifying framework. Characterized by complexity and complex structures emerging in the dynamic and chaotic settings with unified repercussions, nonlinear science has various interacting components that bring about a pressing requirement to be supported by the novel advanced mathematical, analytical and computational techniques based on cutting-edge engineering and technologies to describe, cluster, regulate, categorize, classify, identify, approximate, synchronize, control, regulate, optimize and analyze inherent irregular, multilayer and unpredictable behaviors. Considering the profound complexity, proficiency in computational complexity provides and ensures a multifarious, sophisticated and integrative outlook to solve complicated challenging problems. Therefore, applicable theories and implementations need to be put into real life to identify the subtle properties of complex dynamic systems. Fractal systems and chaotic formations share some elements in common and can be regarded as an important component of complex phenomena. Furthermore, nonlinear phenomena, consisting of pattern formation, evolution, learning, bifurcation and adaptation, which must involve both inter- and transdisciplinary approaches including but not limited to modeling, experimental mathematics, rigorous mathematical methods, data analysis, nonlinear engineering models as well as experimental observations.

The solution of different problems in complex and nonlinear dynamic systems as well as related domains depends on more than one single variable, which requires the modeling and analysis to be conducted by related equations and systems including relationships with a function of two or more independent variables. Thus, complex-fractional models, fractional-order models, nonlinear fractional equations, nonlinear integral equations, soliton wave solutions, time fractional diffusion equations, different dimensional time-fractional diffusion evolution equations, fractal-fractional differential operators, fractional calculus (FC), complex order fractional derivatives, variable-order fractional, fractional derivatives, partial derivatives, variable order derivatives, ordinary differential equations (ODEs), partial differential equations (PDEs), local and non-local kernels, Mittag-Leffler kernel, integer and fractional-orders, hidden attractors, distributed-order integro-differential equations, fractional quantum, fractal geometry and complex geometry among many others are addressed oriented for innovative and applicable solutions to challenging problems of complexity and nonlinearity in complex dynamical processes. Such extensive schemes consist of spatial, temporal, size-related, hierarchical, speed-related and topological structures with diverse possible granularities of the particular system through differential equations, data analyses, simulations, analytical and numerical estimations, numerical analyses and algorithms, which include numerical algorithms, evolutionary algorithms, artificial neural networks (ANNs), wavelet ANNs, neural network clustering, biological networks dynamics, variable-order fractional network, variable-order fractional chaotic system, multi-agent variable-order fractional supply chain (SC) network, control of structures and systems, network dynamics control, controllability, optimal control, nonlinear controllers, multi-agent systems, deep learning, quantum computing approaches as well as quantum forecasting methods covering a wide-ranging spectrum.

The universe, with its fractal structure patterns, includes infinite array of elements interacting in complex systems, while manifesting adaptability, self-organization and sensitivity to the external environment. Fractal-based analysis, in this regard, relies on self-similarity and contractivity with iterative function systems. A fractal system being a nonlinear, complex and interactive system is capable of adapting to an uncertain environment in which FC reflects the actual state properties of the related systems having latent and unforeseeable variations, so it allows the integration and differentiation. Correspondingly, fractal artificial intelligence (Fractal AI) enables the derivation of novel, efficacious as well as productive mathematical and computational methods to make accurate and prompt critical decisions, extract hidden information from the dynamics of a complex system, quantizing processes and accelerating experiments by performing big data analyses for exploration in line with the paradigm of modern science. Within this framework, fractal, FC, fractional-order systems, fractional-order dynamical systems, controllability, quantum, deterministic analysis, stochastic analysis, fixed point theory, (non-) equilibrium points, wavelet-based applications, entropy-based applications, fractional nonlinear system, quantum fractals theory of chaos along with modeling, technical analysis, numerical simulation, data-driven identification and quantification, experimental validation as well as applications are some of the most widely used methods to solve the related multifaceted real-world problems ranging from epidemiology to diseases and addiction problems, economic uncertainty, finance, ecology, interpersonal relationships, SCs, computer viruses, fluid dynamics to heating, mechanical systems and structures, hydrodynamics, geophysical fluids, velocity, mass and energy, energy transfer, reaction-diffusion systems, among many others.

The main objective of this Special Issue is directed toward the new developments based on fractal, fractional calculus, multivariate fractional methods, quantum, Artificial Intelligence and so on in conjunction with their applications in complex systems, including the theoretical and numerical dimensions. Robust and alternative solutions for the problems of complex systems, thus, tend to have dynamic, nonlinear, differentiable, nondifferentiable, transient and chaotic properties. In view of that and to address the related challenges of complexity and nonlinearity, various papers on fractal, FC, quantum, wavelet, fuzzy adaptive techniques, complexity of computation, entropy-based as well as fractal AI-based analyses and applications to complex systems, among many other means, have been incorporated into this Special Issue following the completion of detailed peer-review processes and related procedures.

The key points and contributions of the accepted papers of the Part III of our Special Issue are specified in the following:

The study entitled "Numerical Solution of Distributed-Order Integro-Differential Equations" presents a numerical algorithm to obtain an approximate solution of distributed-order integro-differential equations. Expressed in the form of a polynomial with unknown coefficients and in place of differential and integral operators, matrices deduced from the shifted Legendre polynomials are employed by the authors who computed the numerical values of the polynomial coefficients by setting up a system of equations tallying with the number of unknowns achieved through Legendre–Gauss quadrature formula and the collocation technique. The applicability and validity of the method are demonstrated by the related illustrative examples.

The manuscript named "The Impact of Delay Strategies on the Dynamics of Coronavirus Pandemic Model with Nonlinear Incidence Rate" proposes the delayed susceptible–exposed–infected–recovered model with a nonlinear incidence rate to study the effective role of control strategies in the case of the novel corona virus. The authors discuss three types of equilibria for the model, which are trivial, coronavirus free and coronavirus existence with delay terms for their analysis. They use well-posed notations like the Lasalle invariance principle, Routh Hurwitz criterion and Lyapunov function to investigate the local and global stabilities. The manuscript, where the proposed model is based on the four components of subpopulations as susceptible–exposed–infected–recovered, also presents some useful replications on the subject matter.

In another paper bearing the title "Investigating Fractal-Fractional Mathematical Model of Tuberculosis (TB) under Fractal-Fractional Caputo Operator", the authors, who investigated the fractional mathematical model of tuberculosis, discuss a new operator under fractal-fractional Caputo derivative. Along with the Ulam–Hyers approach, the methods of basic theorems of fractal-FC and the iterative numerical techniques of fractional Adam–Bashforth method are employed for approximate solution. The authors consider different values for fractional-order α and fractal dimension β and compare the results with integer order for the real data. The application of the fractal-FC technique demonstrates the related benefit to know about the real-world problem and the new operator in question is said to generate better results than ordinary integer order, which is one of the significant contributions of the paper.

The authors of the study entitled "Explainability of Neural Network Clustering in Interpreting the COVID-19 Emergency Data" show the importance of clinical emergence and the problematic features regarding the data of the reporting patients. The study demonstrates the importance of self-organizing maps for the interpretation of hospital data, particularly for the COVID-19 pandemic. The authors conclude that the correct interpretation of data can result in trustworthy forecasting models. The step-by-step modeling approach put forth in the study aims to contribute positively to the possible upcoming waves of COVID-19 and cases emerging from other variants of SARS-2 such as the Delta variant. The identification of important variables with the aid of mappings and networks is provided as contributions of the study.

The paper having the title of "Analysis of a Nonlinear Dynamical Model of Hepatitis B Disease" provides contributions on mathematical epidemiology which is significant for the understanding of the dynamics of the infectious diseases. The focus of the paper is the mathematical model of hepatitis B with fractional-order derivative. The authors derive the analysis of the required solution by the application of fixed theory approach and further get the derivation of the Ulam–Hyers stability (UHS) techniques by perturbing the model proposed. They utilize a modified Euler method, namely Taylor's series method to achieve the iterative series solution of the proposed system related to hepatitis. The adequate and significant numerical results are presented along with their comparison with integer order to validate and show the importance of the fractional operators in the study. It is suggested that it is possible to reveal the transmission of hepatitis B through the use of the fractional-order method, which is important in terms of providing positive theoretical support for the prevention and treatment of the disease.

In the manuscript entitled "Global Analysis of Different Compartments in a Giving-Up Smoking Model", the authors analyze different common kinds of smoking phases. They formulate the form of a system related to the ODEs in line with the existing literature. They investigate their model for possible equilibria as well as positivity and boundedness of the solution. Exhibiting six equilibrium points of the model, the authors also investigate the stability analysis of each fixed point. Linearization approach is employed to perform the local stability analysis of each equilibrium point, which shows that the related equilibria are locally asymptotically stable under certain conditions. The fixed points for global analysis are checked with the help of Lyapunov stability theory and stability criteria are obtained accordingly. Local sensitivity analysis is also performed in the study, where the theoretical results are verified by a scheme developed through the use of nonstandard finite difference (NSFD) method. Simulations point the high degree of accuracy supporting the analytical findings.

"On an Extension of the Operator with Mittag-Leffler Kernel" is a manuscript which deals with the challenging task of dealing with nonsingular kernels. This challenge is due to their restrictions at origin. The authors of the manuscript suggest an extension of the fractional operator involving the Mittag-Leffler kernel which admits integrable singular kernel at the origin. The authors report new solutions of the related differential equations along with some related perspectives from the modeling viewpoint. The results of the study show the importance of the Caputo-like-type derivatives, while also demonstrating that the solutions of several fractional equations with the MABC-derivative which are not solvable with the Atangana–Baleanu–Caputo (ABC) derivative.

"On the Analysis of Fractal-Fractional Order Model of Middle Eastern Respiration Syndrome Corona Virus (Mers-Cov) Under Caputo Operator" is another manuscript which establishes the dynamical behavior of MERS-CoV via a sense of Caputo fractal-fractional order system. As a novel approach of fractional-operator, the authors of the manuscript apply fractal-fractional Riemann–Liouville derivative to the model in question. The existence and the uniqueness of the solution pertaining to the considered model is verified by the approach based on the fixed point theory. The UHS technique from nonlinear functional analysis is used for the investigation of the local and global stability of the developed system. To establish the numerical solution of the related problem, the authors use the fractal-fractional type Adams–Bashforth iterative method and simulate the model proposed by considering different fractal dimensions and fractional order converging to the integer-order. By concluding that all the compartmental quantities show convergence and stability in fractal-fractional form, the study demonstrates that fractal-fractional techniques can be used as powerful tools for the investigation of global dynamics related to the diseases.

In the manuscript named "Dynamical Behavior of the Long Waves in the Nonlinear Dispersive Media through Analytical and Numerical Investigation", a well-known mathematical model's analytical wave solutions [modified Benjamin–Bona–Mahony (BBM) equation] are studied. The authors apply six different recent analytical and semi-analytical schemes to the model considered to construct abundant analytical as well as semi-analytical solutions. The variety of the solutions, as a significant contribution of the study, has the aim of investigating the accuracy of the analytical techniques through the calculation of absolute error between analytical and semi-analytical solutions, revealing the corresponding match between them. The features of the Hamiltonian system are used for the stability characterization of the analytical results in the manuscript whose originality is discussed as compared with other previous studies published.

In another study named "A Quantum Fractals Theory of the Pions Chaos and Their Futuristic Applications with Granular Sources", higher-order correlations measured in lead collisions are examined besides the coherence-chaotic characteristics of the particles creation region. The authors of the study examine the pions excretion from a dispersed collection of fluid droplets with coherence snippets which produce quantum interferences to explore the nature of fluid and chaos fraction characteristics in collisions of heavy nuclei. The contribution of the study is concerned with its analysis of hybrid fluid with different correlation functions through the number of droplets. The applicability aspect of the novel methodology is stated in relation with the various engineering applications.

In the following manuscript entitled "Qualitative Analysis of SEIRS Endemic Model Both from PDEs and ODEs Perspective", the authors propose an age-structured susceptible–infected–exposed–recovered (SEIRS) endemic model with the analysis that involves the utilization of the PDEs tools. The model's well-posed quality is demonstrated by developing an abstract Cauchy problem from the proposed system. The authors considered the age-free parameters and the problem is converted into an ODEs model which is investigated for disease-free and endemic equilibria and the global stability of each equilibrium is presented. The manuscript also presents a number of simulations to explain the central theorem as well as the use of a numerical scheme along with the prediction of the problem's dynamics through the sample curves.

The authors of the study "Revisiting Fejér-Hermite-Hadamard Type Inequalities in Fractal Domain and Applications" have established some bounds for the difference between left and central parts regarding the three integral identities coupled with Raina's function. The difference between the center and right parts in FHH inequality is also handled. The authors have also developed various generalizations for random variables, cumulative distribution functions and special function theory as the applications of local fractional integrals. The consequences as established by the study provide contributions to the inequality theory, FC and probability theory from the application perspective so that the other associated classes of functions can be established. It is stated that it is promising to comprise further bounds of other type of variants which involve local fractional techniques based on the related methodologies presented by the study.

The study which is entitled "New Generalization Involving Convex Functions via $\hbar$-Discrete AB-Fractional Sums and Their Use I Fractional Difference Equations" aims at exploring the novel variants for convex functions via discrete AB-fractional operator within the frame of time scale calculus $\hbar - Z$ with $0 < \alpha < 1$ and $0 < \hbar \leq 1$. The authors of the study provide the new discrete fractional inequalities that have $\hbar$-discrete generalized Mittag-Leffler function in the kernel, generating some known results with the possibility of utilization as handy tools to investigate qualitative and quantitative properties of solutions of certain classes of difference equations. The investigation of the study demonstrates $\hbar$-discrete AB-fractional operators by use of time scale calculus. The results obtained generalize and extend a number of well-known results as computed by Liu. The study also addresses the boundedness and uniqueness of solutions to certain partial finite difference and sum-difference equations as set up through the application of the outcome acquired.

The manuscript named "Dynamics of Visceral Leishmania Epidemic Model with Non-Singular Kernel" takes into consideration a dangerous form of leishmaniasis, namely visceral leishmaniasis, which has the highest rate of fatality. The related analysis of the study is conducted using ABC operator with a convex incidence rate. By using the fixed point theorems, the authors establish the existence and uniqueness for the solutions of the fractional leishmania epidemic model. In order to find the numerical solution, Newton polynomial and Adams–Bashforth methods are used. The numerical method is demonstrated to be accurate in both classical and fractional differential equation cases.

In the study "Caputo Based Model for Increasing Strains of Corona-Virus: Theoretical Analysis and Experimental Design", the authors address COVID-19 and point out the importance of delving deeper into the complexities of this elusive virus. The authors of the study propose a Caputo-based model for increasing COVID-19 strains. The memory effect and hereditary properties of the fractional variant for the model enable a full comprehension of the dynamics of the model's features. In addition, the authors discuss the existence of unique solution using the fixed point theorem and Arzelà Ascoli principle besides the stability analysis of the model by means of UHS and generalized Ulam–Hyer stability (GUHS). The best-suited parameters and the optimized Caputo fractional-order parameter α are obtained through the running of the simulations for both the models. The authors state that the proposed model can comprehend the dynamical behavior of the virus better compared to the integer order version. Various numerical simulations are also carried out using an efficient numerical scheme so that more light can be shed on the characteristics of the model.

In another paper, entitled "Traveling Wave Solutions to a Mathematical Model of Fractional order $(2+1)$-Dimensional Breaking Soliton Equation", the authors aim at considering a significant mathematical model of fractional-order $(2+1)$-dimensional breaking soliton (Calogero) equation with solution by the Khater method. The derivatives are stated to be in local fractional derivative sense. The authors of the paper utilize the fractional transformation equation in order to convert the proposed nonlinear fractional-order differential equation into nonlinear ODE. For the construction of the closed form traveling wave solutions of the fractional differential equation in question, the Khater method is employed. Furthermore, the authors construct many of the new exact solutions, demonstrating the convenience, power and ease of the Khater method for the solution of the nonlinear fractional differential equation that arises in mathematical physics. The broader applicability of the method is also provided as a contribution of the study since the established solutions proved reliability and decreasing of the size computational work give this method a broader applicability.

The work named "Cyclocopula Technique to Study the Relationship between Two Cyclostationary Time Series with Fractional Brownian Motion Errors" is concerned with the detection of relationship between two time series, which is stated to be important in different scientific fields. The authors of the research introduce a new copula-based method (cyclocopula) for the detection of the relationship between two cylostationary time series with fractional Brownian motion (fBm) errors. The authors study the performance of their proposed method by employing numerous simulated datasets. The applicable aspects of the approach introduced is examined in the real world problems and the numerical as well as applied studies validate the performance of the related introduced technique.

Another manuscript is entitled "The Comparative Report on Dynamical Analysis about Fractional Nonlinear Drinfeld-Sokolov-Wilson System" in which the authors have used the new extended direct algebraic technique to achieve the new exact solitary wave solutions of the nonlinear fractional couple Drinfeld–Sokolov–Wilson system. Fractional theory of calculus is used as the perspective to develop the system and thus, Riemann–Liouville fractional derivative (RLFD), β-fractional differential operator, and Atangana–Baleanu fractional derivative (ABFD) have been utilized. The related approach is stated to have compatibility and effectiveness as a scientific approach to examine the various space-time fractional models in fields like physics and engineering regarding the real life issues.

The authors of the study "Analysis of Hidden Attractors of Non-Equilibrium Fractal Fractional Chaotic System with One Signum Function" have explored a chaotic system containing only one signum function through the Caputo fractal-fractional operator. The existence theory has been developed by employing the fixed point result of Leray–Schauder to prove that the considered chaotic system has at least one solution. According to Banach's fixed point theorem, the chaotic system proposed is said to have a unique solution. Furthermore, the authors show that the suggested chaotic system is Ulam–Hyers (UH) stable under the novel operator of power law kernel using nonlinear functional analysis. For the evaluation of the numerical outcomes of the model considered, the Adams–Bashforth technique is used. Eventually, the authors reveal the complex structure of numerical solutions for different fractional-order and fractal dimension values.

"Analysis of a Fractional Order Smoking Models with Relapse and Harmonic Mean Type Incidence Rate under ABC Operator" is a paper which investigates the newly constructed fractional-order smoking models with relapse and harmonic mean type incidence rate under the fractional operator ABC. The authors establish the existence results and uniqueness of the solution with the aid of the fixed point theory approach, and they prove the stability of the model with the help of Ulam–Hyers approach by considering a small perturbation in the problem. Furthermore, the established scheme is simulated against the available data for some smoking communities in which various types of potential, moderate, and quit smokers live. By using different fractional orders for simulation, the results obtained show that fractional order converges to integer order as order is increased. In addition to that finding, it is also presented that it is possible to control smoking through means of education, employment and awareness regarding its harm put forth by doctors as well as the related dosage of some drugs.

Another research paper is entitled "On Analytical and Numerical Simulation for the Ultra-Short Pulses Mathematical Model in Optical Fibers" and it is on the investigation of the solitons and breathers wave solutions of the generalized nonlinear Schrödinger equation of third-order by the new generalized computational [generalized Khater (GK)] technique. By employing these related solutions, the authors of the study have constructed the initial and the boundary conditions so that they can apply a new numerical scheme known by the trigonometric quintic spline, which is a mathematical model recognized for ultra-short pulses in fields like engineering and applied sciences particularly in the case of optical fibers. The paper has obtained many distinct exact traveling and solitary wave solutions which are illustrated by sketches explaining the novel properties of the fractional models considered. Besides this, the novelty and accuracy of the solutions obtained by the study are also explained and verified based on their effectiveness and power.

The subsequent research paper is entitled "Numerical Investigation of the Nonlinear Fractional Ostrovsky Equation" and investigates the numerical solutions of the nonlinear fractional Ostrovsky equation through five recent numerical schemes which are Adomian decomposition, El Kalla, Cubic B-Spline, extended Cubic B-Spline and exponential Cubic B-Spline. The authors have investigated the obtained computational solutions via the generalized Jacobi elliptical functional and modified Khater methods. The accuracy of the numerical solutions obtained is examined through the calculation of the absolute error between the exact and numerical solutions, and the matching between them are illustrated by the related sketches given in the paper accordingly.

The authors of the paper entitled "Exact Soliton Solutions for Conformable Fractional Six Wave Interaction Equations" consider conformable fractional time derivative of order $\alpha \in (0, 1]$ in view of the Lax-pair of nonlinear operators in order to derive a fractional nonlinear evolution system of PDEs, which is called the Fractional-Six-Wave-Interaction-Equations as derived in terms of one temporal plus one and two spatial dimensions. Consisting of linear combinations of the hyperbolic functions with complex coefficients, an ansatz is utilized for obtaining an infinite set of exact soliton solutions for the related system. The authors also introduce a number of numerical examples to demonstrate the effective aspect of the ansatz method in terms of obtaining the exact solutions for similar systems of nonlinear evolution equations, which represent models for many physical problems in interacting waves theory.

The next work entitled "Fractional Mathematical Modeling to the Spread of Polio with the Role of Vaccination under Non-Singular Kernel" deals with the fractional mathematical model for the spread of polio in a community with variable size structure including the role of vaccination. The authors of the work have extended the model with the help of the Atangana–Baleanu in the sense of the Caputo (ABC) fractional operator. The positivity and boundedness of solution (positively invariant region) have been presented for the ABC-fractional model of polio. The fixed point theory has been adopted to examine the existing results and uniqueness of the solution for the related problem. The Ulam–Hyers stability scheme has also been used to investigate the stability result for the considered model and the fractional Adams–Bashforth technique has been employed to obtain numerical simulation. The plotting of the comparisons at different available rates of infection and exposition as well as graphical presentations have been included in the work for the understanding of the dynamics of the model at various fractional orders.

The other paper is entitled "A Hybrid Approach for Synchronizing between Two Reaction-Diffusion Systems of Integer- and Fractional-order Applied on Certain Chemical Models" in which a synchronization problem for spatiotemporal partial differential systems is addressed and researched in a subjectivist framework. Based on the Lyapunov direct method and some proposed nonlinear controllers, the authors of the paper have established a new scheme that are established to attain a full synchronization between two reaction-diffusion systems of integer- and fractional-order. They particularly address a novel vector-valued control law which derived analytically to achieve the desired synchronization between two chemical models, which are the Lengyel–Epstein and Gray–Scott models. Further numerical simulations have been performed in two-dimensional (2D) and three-dimensional (3D) configurations in order to validate the theoretical results obtained. The study concludes that the synchronization error converges over time to zero by the implementation of the proposed control laws.

"Use of Evolutionary Algorithms in a Fractional Framework to Prevent the Spread of Coronavirus" is a study which aims at designing the optimal management policies for the novel coronavirus disease 2019 (COVID-19). The authors have designed optimal policies based on multi-objective evolutionary algorithms to control the fractional-order model of the COVID-19 outbreak. The study consists of the presentation of a fractional-order model of the disease dynamic and considering of the impacts of the fractional derivative's value on the modeling and forecasting of the disease spread. The proposal of a multi-objective optimization problem has been done by taking into account rate of communication, transition of symptomatic infected class to the quarantined one as well as the release of quarantined uninfected individuals. The numerical results of the study validate that by solving the proposed multi-objective problem and the obtained policies, it is possible for the governments to take strategies that prevent the disease outbreak and mitigate destructive effects on the economy.

The work entitled "Numerical Computing to Solve the Nonlinear Corneal System of Eye Surgery Using the Capability of Morlet Wavelet Artificial Neural Networks" presents a novel heuristic computing technique in order to solve bioinformatics problem for the corneal shape model of eye surgery by the use of the Morlet wavelet ANN which is optimized by the global search schemes. Different cases based on nonlinear second-order differential equations governing the corneal model have been solved effectively for measuring the performance of the design network configuration. The authors have also implemented Adams method to compare the presented outcomes of the stochastic solver, showing the value of the related present scheme as based on accuracy and convergence with negligible values of absolute error in the range 10^{-7} to 10^{-8}. The algorithm complexity over the statistical operators using the mean value time, generations and count of function throughout the process has been addressed to optimize the design network variable that specifies the smooth accomplishment for all cases of the model.

Another study is named "Skewed Auto-Regressive Process with Exogenous Input Variables: An Application in the Administered Vaccine Doses on Covid-19 Spread" in which the authors focus on the prevalence of COVID-19 along with vaccination in the United States. The daily total infected cases of COVID–19 with total vaccinated cases as exogenous input are considered and modeled by utilizing light-/heavy-tailed auto-regressive with exogenous input model based on the innovations that belong to the flexible class of the two-piece scale mixtures of normal family. The authors put forth the conclusion that the prediction of COVID–19 spread is affected by the rate of vaccine injection. In addition, the presence of exogenous input variables in time series models increases the accuracy of modeling and leads to better and closer approximations in some issues that include predictions. Furthermore, the authors have considered expectation-maximization type algorithm to find the maximum likelihood estimations of the model parameters, including modeling and predicting the infected numbers of COVID-19 in the presence of the vaccinated cases in the United States.

The article entitled "Computational Study of Fractional Order Vector Borne Diseases Model" deals with the solution of a nonlinear fractional-order model to analyze the dynamical behavior of vector-borne diseases within the frame of Caputo fractional derivative. The proposed mathematical model advances the existing integer-order model on transmission and cure of vector-borne diseases. The authors have proven the existence and uniqueness of the solutions of the fractional-order model using the Banach contraction principle. In addition, the local asymptomatic stability for the obtained disease-free equilibrium point and global stability for the proposed model in the sense of Ulam–Hyers stability criteria have also been investigated. Corrector-Predictor algorithm has been used for the numerical solution of the projected model. Finally, the authors have performed numerical simulations for the different values of fractional-order derivative, making a comparison with the results of the integer-order derivative to illustrate the theoretical results obtained.

Another paper entitled "Modeling the Dynamics of Stochastic Norovirus Epidemic Model with Time-Delay" studies the norovirus (NoV) epidemic model with random perturbations and a time delay. For this aim, the existence and uniqueness of the global positive solution have been obtained, and then the sufficient condition for the extinction of the disease have been derived. The authors have also established the appropriate Lyapunov function and discussed the existence of a stationary distribution. The analytical results acquired have been illustrated by presenting some numerical simulations. The authors have demonstrated that the stochastic SEIR model is an endeavor to understand the epidemiological aspects of NoV. It is also indicated that when ambient cross-immunity and noise are addressed in NoV epidemic models, the model can provide new insights into epidemiological problems.

"Numerical Analysis of Newly Developed Fractal-Fractional Model of Casson Fluid with Exponential Memory" is a paper which focuses its analysis on the examination of the Casson fluid under the influence of heat generation and magnetic field. Accordingly, the authors have considered the flow of the Casson fluid in between two vertical parallel plates. They have developed the linear coupled governing equation in terms of classical PDEs and generalized by using the operator of the fractal-fractional derivative with an exponential kernel. The numerical solution of the proposed problem has been revealed using the finite-difference technique presented by Crank–Nicolson which, as the finite difference scheme, has the benefit of being unconditionally stable and applicable directly to the PDEs without any transformation to ODEs. This technique, in sense of exponential memory, has been revealed to be unreported in the literature for such a proposed problem. The analysis results of the study show that the fractal-fractional order model has a large memory effect compared to that of the fractional-order and classical model owing to the fractal-order parameter.

Another study in Part III of our Special Issue is entitled "Modeling the Transmission Phenomena of COVID-19 Infection with the Effect of Vaccination via Non-Integer Derivative under Real Statistic" and it is concerned with the presentation of a mathematical model for the transmission of COVID-19 with vaccination effects. The authors of the study present the basic properties of FC to inspect the model. They further calculate the equilibria of the model and determine the reproduction number R_0. Local asymptotic stability conditions for the disease-free have also been obtained. The nonlinear least-square procedure has been utilized to parameterize the model from actual cases reported in Pakistan. The existence of a unique solution is proven by fixed theory and the numerical results presented address the simulation of the virus transmission. The results have been compared with those of the Caputo derivative in the study from which significant changes have been noticed by lowering the order of fractional derivative.

"Optical Soliton Wave Solutions of the Fractional Complex Paraxial Wave Dynamical Model Along with Kerr Media" addresses the optical wave solutions of the fractional complex paraxial wave dynamical (FPWD) model with Kerr media. The model in question investigates the frequency of the beam obeys, revealing nondispersive and nondiffractive spatio-temporal localized wave envelopes promulgating in optical Kerr media. The authors of the study also employ three of the recent computational schemes for the investigation of the considered model's abundant wave solutions which are subsequently used to evaluate the initial and boundary conditions. The authors' contribution is on the point of the investigation of the considered model in 2D, 3D and density plots of the real, imaginary and absolute value of the solutions.

The next work in our Special Issue is a research article named "Mathematical and Numerical Analysis of SIQR Epidemic Model of Measles Disease Dynamics". The article addresses measles which was responsible for 73% of child deaths in 2018. The work provides a description of the physical solution of the SIQR model for measles spread and the efficacy of solutions is compared based on three different numerical techniques. The preference of NSFD scheme over the rest is established. The support of the numerical results is also established by a strong classical analysis of the model in which the existence of a solution-vector in explicit subsets of the function spaces is guaranteed, leading to optimization of fixed-point methods. The provision of an accurate and reliable numerical solution of a measles epidemic model with the help of a NSFD method is a noteworthy contribution of the paper whose proposed strategy preserves all-important properties controlled by the measles epidemic model showing the method efficacy and validity for large step size.

The subsequent study named "A Nonlocal Modeling for Solving Time Fractional Diffusion Equation Arising in Fluid Mechanics" primarily investigates new techniques to obtain numerical solutions of time-fractional diffusion equations. The authors represent the fractional derivative term as represented in the Lagrange operational sense. After describing the temporal direction of the considered model using the Legendere orthogonal polynomials, nonlocal peridynamic differential operator (PDDO) is applied to achieve a full discretization approach. The stability and convergence of the scheme are investigated numerically, which verify the validity of the method proposed. Numerical results obtained by the study demonstrate the simplicity and accuracy of the presented method.

"Video-based Table Tennis Tracking and Trajectory Prediction Using Spatial-Temporal CNNs based on Deep Learning" is a study concerned with sports tracking, table tennis in particular, which shows that anything could happen and involves the significant difficulty related to computer-aided systems which must address how to record and analyze many game events. Fractal AI plays an essential role in dealing with complex structures, allowing effective solutions. The study proposes structured output convolutional neural network (CNN) based on deep learning approaches and a trajectory prediction model based on long-short term memory (LSTM) and mixture density networks (MDN) for object tracking. Compared with existing systems relying on a traditional method based on physical analysis and a nonmachine learning tracking algorithm, which can be complex and inflexible, the study provides a more reasonable motion model design considering the characteristics of table tennis, which is a significant novelty and contribution brought by the study.

Entitled "Double Diffusive Magneto-Free-Convection Flow of a Maxwell Fluid Over a Vertical Plate: Special Functions Based Analysis using Local and Non-Local Kernels to Heat and Mass Flux subject to Exponential Heating", the research paper aims at analyzing the general equations of double diffusive magneto-free convection in a rate-type fluid presented in a nondimensional form. Thermal transport phenomenon is discussed in the presence of constant concentration coupled with first-order chemical reaction with exponential heating. An innovative definition in power law (CF) and Mittag-Leffler (ABC) kernels form the time fractional operators, which are implemented to hypothecate the constitutive mass, heat and momentum equations. The authors obtain their results based on special functions using Laplace transformation technique to tackle the nondimensional equations for velocity, mass and energy. The paper discusses the considerable contributions of mass, thermal and mechanical components on the dynamics of fluid independently and provides the finding of an interesting property regarding the behavior of the fluid velocity when the movement is observed in the magnetic intensity along with the plate as a contribution.

The following study in our Special Issue is named "New Results for Parabolic Equation on the Sphere with Caputo-Fabrizio Operator", which studies the initial value problem for parabolic problem associated with the Caputo–Fabrizio derivative. The problem is addressed in two cases, namely linear inhomogeneous case and nonlinearity source term. While for the linear case, the authors derive the convergence result of the mild solution when the fractional-order $\alpha \to 1^-$ under some various assumptions on the initial datum, for the nonlinear problem, they show that the existence and uniqueness of the mild solution using Banach fixed point theory. As another contribution, the paper proves the convergence result of the mild solution when the fractional order $\alpha \to 1^-$.

The paper entitled "Appraisal of Analytical Solutions for (2+1)-Dimensional Nonlinear Chiral Schrödinger Equation" investigates the solitary wave solutions of the $(2+1)$-dimensional nonlinear Chiral Schrödinger ((2+1)-DCNLS) equation by employing the modern extended direct algebraic approach. The nonlinear model's different categories of solitary waves solutions are also found through the study. The division of semi-dark solitons, singular dark-pitch solitons, single solitons of type 1 along with 2, intermixed hyperbolical, trigonometrical, and rational solitons are developed and evaluated via software tools for numerical calculations as the important contributions of the paper which effectively depicts 3D- and 2D-contour output-1 plots to provide further details regarding the physical characteristics of the obtained solutions. The research fundamentally aims at using the novel extended direct algebraic method to assess some revolutionary solitons with their physical features to the needed equation.

The manuscript named "Mathematical Analysis of a Delayed Malware Propagation Model on Mobile Wireless Sensor Network" is on the investigation of a delayed malware propagation model on mobile wireless sensor network, which incorporates nonlinear incidence rate, logistic growth rate and recovery rate. The manuscript also provides the analysis of local asymptotic stability of the endemic equilibrium and existence of Hopf bifurcation at crucial value of the time delay. The authors provide the investigation of global exponential stability specifically via linear matrix inequality and an example in order to emphasize the effectiveness of findings in the paper numerically and graphically. The manuscript corroborates the finding that it is possible to model real phenomena with the aid of fractional-order systems owing to the fact that such systems inherently exhibit memory effects.

The next paper in our Special Issue bears the title of "A Review of the Miniaturization of Microstrip Patch Antenna Based on Fractal Shapes" focuses on wireless communication which has become ubiquitous in different sectors including military and commercial systems. The paper provides the overview of the subject matter, listing the various most common fractal approaches to microstrip patch antenna (MPA) miniaturization proposed in the literature, by underlining different kinds of fractal geometries extensively used recently by specifically focusing on the option of miniaturization of microstrip antennae based on the theories of Sierpinski and Minkowski which are considered to be the ideal ones.

Another paper, entitled "Forecasting Stock Market Crashes via Real-Time Recession Probabilities: A Quantum Computing Approach" shows the importance of fast and precise prediction of stock market crashes, providing a comparison of quantum forecast methods. The new prediction model of stock market crashes is established via real-time recession probabilities with the power to accurately estimate future global stock market downturn scenarios. The 104-country sample used allows the sample compositions to consider the regional diversity of the alert warning indicators. The authors employ several alternative techniques on the sample under study, with Quantum Boltzmann Machines, so that they can obtain a robust model. The results demonstrate very good prediction results owing to their capability to remember features and develop long-term dependencies from time series and sequential data. The model proposed offers contributions related to the large policy implications for an appropriate macroeconomic policy response to downside risks, which is applicable for achieving financial stability at the international level.

The manuscript named "Deterministic and Stochastic Analysis of a COVID-19 Spread Model" addresses the global dynamics of deterministic-stochastic COVID-19 mathematical model with quarantine class and in a way to incorporate a preventive vaccination. The authors use the Lyapnov functions for the global stability of disease free equilibrium point and graph theoretic for the construction of Lyapnov function for positive equilibrium point. The stability of model is discussed with respect to the reproductive number. In the study where real data were used, the spread is said to follow two major patterns which include deterministic trend and randomness, and thus, deterministic and stochastic approaches are employed to investigate the dynamic aspects and two behaviors stated. The main point of the work is that deterministic aspect helps with the capturing of the spread behavior following normal distribution, while the model with stochastic one helps with the capturing of the spread dynamic with randomness. Considering fractional and fractal-fractional differential operators, the method is said to allow the considering of three consecutive points and leading to a good approximation.

The next paper entitled "Effect of Occasional Heroin Consumers on the Spread of Heroin Addiction" is a research that focuses on showing the global behavior of each equilibrium of a heroin epidemic model with distributed delays. The authors obtain that the global behavior of the considered system is entirely governed by the value of the basic reproduction number $\Re_0$. When $\Re_0$ is less than 1, they establish the global stability of the drug-free equilibrium employing the Lyapunov direct method. On the other hand, when $\Re_0$ is greater than 1, it is found that the addiction persists and the drug equilibrium is globally asymptotically stable, which is also shown by the Lyapunov direct method. Another contribution of the research is controlling the heroin epidemic spread by adjusting the density of occasional heroin users. In addition, the measure is stated to be more effective compared to the treatment measure. The authors draw the dedication that controlling the occasional heroin consumers is the best strategy to reduce the spread of heroin addiction, which could eventually bring about the stopping of the heroin consumption spread, which is a serious health concern.

The following work named "Global Stability Analysis and Hopf Bifurcation of a Delayed Reversion Class Smoking Model" is also concerned with another addiction, that is to say, smoking which is considered to be a huge threat to global public health. The authors propose a delayed smoking model with reversion class in their paper and analyze the local stability of the smoking equilibrium and Hopf bifurcation by taking the time delay as the bifurcation parameter. They prove the global exponential stability through the employment of linear matrix inequality method. The authors conclude that in order to effectively control the spread of smoking behavior, the chain smokers should give up smoking as soon as possible. Furthermore, the chain smokers preparing to quit smoking should have strong willpower to avoid abandoning the plan and to avoid relapse as well. The theoretical results obtained by the study are validated both numerically and graphically.

"Pricing European Two Asset Option Using the Spectral Method with Second Kind Chebyshev Polynomials" is a study which employs a space-fractional Black–Scholes equation to price the two-asset European options that asset classes, which fulfill two distinct Lévy mechanisms using a spectral technique based on the Chebyshev basis. The solution is concerned with a collocation method that is based on the Chebyshev orthogonal basis. Moreover, the authors approximate the derivative operator based on the derivatives in the called model. As a contribution, the numerical examples show the accuracy and convergence order of the numerical procedure with numerical outcomes illustrating the accuracy of the scheme handled in the study.

"Dynamics of Love Affair of Romeo and Juliet through Modern Mathematical Tools: A Critical Analysis via Fractal-Fractional Differential Operator" is another paper of our Special Issue. The authors aim to express the feelings between Romeo and Juliet via mathematical tools. The love of Romeo and Juliet is shown as a coupled system of ODEs. The fractal fractional differential operator with the Mittag-Leffler function further generalizes the classical differential equations. Some theoretical analysis is also performed for the considered problem. The authors show the impact of the fractional-order parameter and fractal dimension parameter through the feelings of both individuals. Furthermore, the impact of various physical parameters on the love or hate of Romeo and Juliet is displayed and discussed in detail through the paper which also displays an interpersonal couple relationship in that for a stable relation among two lovers, financial stability and toleration of expenditure between both individuals plays an important role.

The next study is named "Dynamical Investigation and Distributed Consensus Tracking Control of a Variable-Order Fractional Supply Chain Network Using a Multi-Agent Neural Network-Based Control Method" which addresses SCs as complex nonlinear systems in the presence of different types of uncertainties which include supply-demand and delivery uncertainties. The authors of the study propose a variable-order fractional SC network and investigate the dynamic of the system by the Lyapunov exponent and bifurcation diagram. They demonstrate that a minor change in the system's fractional-derivative may affect its behavior dramatically. The distributed consensus tracking of the multi-agent network is also examined by the study through whose end a control technique based on the sliding concept and Chebyshev neural network estimator is offered. The stability aspect of the study is demonstrated through the employment of the Lyapunov stability theorem and Barbalat's lemma. The related numerical results demonstrate the proposed controller's excellent performance for distributed consensus tracking of multi-agent SC network as a significant contribution.

Entitled "Computational Analysis of Fractional Order Imperfect Testing Infection Disease Model", the next paper in the Special Issue is concerned with both the theoretical and numerical aspects of fractional-order imperfect testing infectious disease (ITID) model in sense of AB-fractional derivative coupled with Caputo operator. The existence of the solution is obtained by the utilization of fixed point theorem. The authors study the dynamics of imperfect testing and diagnostics of infectious diseases model by replacing the integer order derivative in sense of fractional order (FO) Atangana–Baleanu operator coupled with Caputo operator. They also use the fixed point theorems for the verification of existence results and Picard's stability technique utilized for the stability study of the related model. By performing the numerical computations for the fractional-order ITID model, the authors illustrate the results graphically.

The subsequent study of the Special Issue is named "The Numerical Meshless Approach for Solving the 2-D Time Nonlinear Multi-Term Fractional Cable Equation in Complex Geometries" that examines the 2D time nonlinear multi-term fractional cable equation. By adopting a valid meshfree technique, the nonlinear multi-term time-fractional cable equations (NM-TTFCEs) consisting of government equations and their boundary conditions are transformed into the boundary value problems. The numerical results ensure the accuracy and high efficiency of the proposed method which has aspects concerning several computational domains. The improved singular boundary method (ISBM) and the dual reciprocity method (DRM) implemented are considered to be a potential alternative to existing meshless strong form approaches in solving multi-term fractional equation problems with complex geometries.

"New Fractional Order Stability Analysis of Sir Model for the Transmission of Buruli Disease: A Biomedical Application" is a paper in which a new computing technique is investigated for the cause of *Mycobacterium ulcerans* (MU) with the help of differential equations through the SIR model. The authors aim at finding the cause between the ulcer and the environment for the spread of Buruli ulcer (BU), and thus, they apply the fractional-order SIR model to discuss in detail the BU disease. The mathematical modeling of the presented scheme with the help of stability techniques is accomplished and the behavior of the scheme is accessed by the evaluation of the different parameters of interest. The attained numerical results are observed for the illustration of the benefit of introducing a fractional model. The authors reflect that the presented numerical scheme maintains all the required control measuring features of the corresponding dynamical system and decreases the ancillary cost when studied over a long time. Through a novel and unique approach, a significant observation is noted and recommended for other epidemiological fractional order models as a contribution.

The theoretical study entitled "Heat Transfer of MHD Oldroyd-B Fluid with Ramped Wall Velocity and Temperature in View of Local and Non-Local Differential Operators" focuses on the examination of the convective flow of Oldroyd-B fluid with ramped wall velocity and temperature. To depict the fluid flow, the coupled PDEs are settled using the Caputo (C) and Caputo Fabrizio (CF) differential time derivatives. The mathematical analysis of the fractionalized models of fluid flow is conducted by Laplace transform (LT). The authors also explore the complexity of temperature and velocity profile by the numerical inversion algorithms Stehfest and Tzou. As a result, the significant impact of the fractional parameter (memory effects) and other parameters on the dynamics of the fluid flow is demonstrated.

"Analysis of Fractional Order Diarrhea Model Using Fractal Fractional Operator" is another paper in which the authors construct a scheme of fractional-order mathematical model for the population infected by diarrhea using the four compartments S, I, T and R. The fractal-fractional derivative operator (FFO) with generalized Mittag-Leffler kernel is employed in order to obtain the solution of the proposed system. The system is analyzed qualitatively as well as verified as a nonnegative unique solution. The existence and uniqueness results of the fractional-order model under Atangana–Baleanu fractal-fractional operator are proven by fixed point theory. The authors make an error analysis for the proposed fractional-order model and carry out the simulation for the derived fractional-order scheme for the purpose of checking the effectiveness of the results which will be of help, as a contribution in public health, regarding the prevention and control of such a type of epidemic in the society.

The next paper in our Special Issue has the following title "Predictive Control of a Variable-Order Fractional Chaotic Ecological System" is one which for the first time considers ecological processes by variable-order fractional model in the literature. The authors, accordingly, propose to extend a predator-prey mathematical model with variable-order fractional derivatives whose underlying assumption in the proposed model lies in considering values of fractional derivatives as time-varying functions rather than constant parameters. The feature of dynamic features of the system are investigated followed by the examination of the proposed system's control. For this purpose, a nonlinear model predictive control is proposed for the variable-order fractional system. The necessary optimality and sufficient conditions for solving the nonlinear optimal control problem in the form of FC with variable-order output-derivative are also formulated while delineating the controller's design procedure. Consequently, numerical simulations are conducted to demonstrate the effectiveness and performance of the developed control technique concerning the variable-order fractional predator-prey model.

The article entitled "Application of q-Shehu Transform on q-Fractional Kinetic Equation Involving the Generalized Hyper-Bessel Function" introduces the q-Shehu transform and defines the generalized hyper-Bessel function. The objective of the article is to obtain the solutions of the q-fractional kinetic equations in terms of the established generalized hyper-Bessel function by applying the established q-Shehu transform. The special cases of the main results are also provided in the article which concludes by providing the numerical values and the graphical representations of the related solutions.

The next study entitled "A Numerical Scheme for the Generalized ABC Fractional Derivative Based on Lagrange Interpolation Polynomial" describes a numerical and analytical investigation of a Hepatitis C virus (HCV) transmission concept in the context of fractional order. The model is an extension of the classical model to a fractional order. The authors discuss existence, uniqueness, Hyers–Ulam type stability and numerical results are in the related work. The Lagrange's interpolation polynomial technique is used for the numerical outcomes which demonstrate that the proposed method has a high level of precision and a low computing cost, as a contribution. It is also observed that the numerical results for the fractional-order model also contain the dynamics of the previous integer-order model as a special case. As a final step, the authors implement the numerical solutions for the fractional-order HCV model to demonstrate the results in a graphical way.

"Regularization of Cauchy Problem for 2-D Time-Fractional Diffusion Evolution Equations" is a work where the authors focus on an initial value problem for a class of 2D time-fractional diffusion evolution equations with Riemann–Liouville fractional derivative. Three new results are obtained as contributions in the paper. First of all, the existence and ill-posedness result (in the sense of Hadamard) in three cases consisting of homogeneous, inhomogeneous and nonlinear problems are considered. Second, by using the Fourier truncation method, we show that regularized problems are well-posed. Finally, some demonstrated examples are presented to test the proposed method.

"Chaotic Behaviour in Fractional Helmholtz and Kelvin-Helmholtz Instability Problems with Riesz Operator" is the subsequent study that introduces some important dissipative problems which are recent and of intermittent interest. The classical dynamics of Helmholtz and Kelvin-Helmholtz instability equations are modeled with the Riesz operator which incorporates the left- and right-side of the Riemann–Liouville noninteger order operators to mimic naturally the physical patterns of the related models arising in hydrodynamics and geophysical fluids. The Laplace and Fourier transform techniques are utilized in the study to approximate the Riesz fractional operator in a spatial direction. The behaviors of the Helmholtz and Kelvin–Helmholtz equations are also observed for some values of fractional power in the regimes $0 < \alpha \leq 1$ and $0 < \alpha \leq 2$, using different boundary conditions on a square domain in 1D, 2D and 3D (spatial-dimensions). Numerical results of the study reveal impressive phenomena which emerge due to the variations in the initial and source functions, as well as fractional parameter α, for subdiffusive and superdiffusive scenarios.

The paper named "A High Order Numerical Scheme for Fractal-Fractional Laser System with Chaos Study" is primarily concerned with the investigation of the dynamical behavior of a fractional and fractal-fractional order laser chaotic model. The authors consider the model initially with Caputo's fractional derivative. An iterative technique based on the LT and its inverse aspect is employed to attain a particular solution for the fractional model. The existence of a unique solution is also examined for the fractional laser chaotic model. The authors subsequently address the laser chaotic model with a fractal-fractional approach in the Caputo sense. The approximate results are achieved for the proposed fractional and FF models by the employment of the Atangana–Seda numerical methods. The study puts forth that the presence of fractal order provides a richer diversity in graphical simulation which can be regarded as one important benefit of fractal-fractional derivatives over fractional derivatives.

The next article included in our Special Issue is entitled "A New Fractal Fractional Modelling of the Computer Viruses System" models the fractal-fractional system of the computer virus problem using the Atangana–Baleanu operator. The authors also present the existence and uniqueness of the results through applying the Schauder fixed point and Banach fixed theorems. By using the Atangana–Toufik technique, they obtain the approximate solutions by choosing various values of orders. Different values of fractal-fractional orders along with different amounts of initial conditions are chosen in order to examine the performance of the suggested numerical method on the new fractal-fractional system. Graphs in different dimensions presented in the study exhibit the solutions in a clear way.

The paper entitled "Generation and Robustness of Quantum Memory Assisted Entropic Uncertainty and Uncertainty Induced Non-Locality of Two Nitrogen-Vacancy Centers Coupled by Open Two Nanocavities" investigates the dynamics of quantum memory-assisted entropic uncertainty, uncertainty-induced nonlocality and log-negativity entanglement of two nitrogen-vacancy centers (NVC). The authors address the analyzing of the two nitrogen-vacancy centered qubits coupled with two coherent nanocavities in particular. They consider two different configurations of NVC qubits. They demonstrate that coherent nanocavities can successfully be employed to generate nonlocality and entanglement in the two NVC qubits while suppressing the associated entropic uncertainty. The NVC-qubits coupled with two coherent nanocavities promote entanglement sudden birth and death phenomena. The study provides contributions with regard to the quantum mechanical protocols that require robust practical generation of entanglement and nonlocality as well as the related information preservation for the increased efficiency of quantum devices.

The research paper named "Quantum Statistical Perspective to Examine the Source Chaos Fraction through Boson Femtoscopy" introduces an evolving source to describe chaotic information inside quantum entanglement and discusses the impact of the coherence order. They also investigate the role of source size parameter and particle number in the dynamics of the temperature profiles with the function of momentum variation. The paper further evaluates the influences of the modeling factors on the correlations with their normalized correlator in the perspective of source coherence and reveals that the orientation of the source has an impact on the diffusivity of the fluid under consideration. Graphical representations according to the geometry of the expanding source illustrate the preliminary findings of the paper which demonstrates that the consequences revealed can lead to an increase or decrease in the genuine correlations due to the distributions of velocity and temperature. The study provides contributions by showing the pivotal role played by Bose–Einstein correlations to understand the source better.

The authors of the study entitled "Case Study on Total Controllability and Optimal Control of Hilfer Neutral Non-Instantaneous Fractional Derivative" aim at discussing the controllability results of the Hilfer neutral noninstantaneous impulsive fractional integro-differential equations (HNNIIFIE). They identify effective strategies with HNNIIFIE regarding its total controllability and optimal control results. The authors also discuss the total controllability results by using the set-contraction theory and Kuratowski measure of noncompactness. They derive the results on optimal control by employing the appropriate conditions. The analysis shows the existence of an optimal pair for their desired system using set-contraction theory and Kuratowski measure of noncompactness. The illustration provided in the study depicts the validation of the outcomes.

The last study of our Special Issue entitled "Global Attractivity, Asymptotic Stability and Blow-Up Points for Nonlinear Functional-Integral Equations' Solutions and Applications in Banach Space $BC(R_+)$ with Computational Complexity" provides the basic definitions and results about Banach spaces along with the proof as well as their applications. The existence results of the equations are generally obtained based on the fundamental methods by which the fixed-point theorems are frequently applied. The aim of the study is to examine the nonlinear functional-integral equations in the Banach space $BC(R_+)$ using the measure of noncompactness. Moreover, the proof of the existence of global attractors and asymptotic stabilities, having at least one solution pertaining to $BC(R_+)$ is studied. The last stage includes the definitions of global attractivity and asymptotical stability for equations on Banach space $BC(R_+)$ while proving the existence of solutions and global attractivity. The related example of nonlinear integral equation as addressed in this study has been examined in terms of its blow-up point and asymptotic stability with computational complexity for the first time. The study enables a thorough analysis of computational complexity of the related nonlinear functional-integral equations. The applicable approach is significant in terms of the accurate transformation of the complex problems toward optimized solutions through the generation of advanced mathematical models in nonlinear and dynamic settings which are mainly characterized by complexity.

We are of the opinion and anticipation that our Special Issue will open up a new frontier so that novel and robust solutions can be reflected on and provided by scientists and researchers working on mathematics, physics, applied sciences, engineering, information theory, bioengineering, medicine, biomedicine, biomathematics, bionanotechnology, biology, public health, chemistry, ecology, finance, social sciences and so forth in order to address challenges concerning complex systems. Our Special Issue has its subsequent continuation as Part IV for further research and investigation.

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AUTHOR CONTRIBUTIONS

Yeliz Karaca is the lead guest editor for Part III of this Special Issue, and along with the editorial note for Part III, she wrote the editorial notes for Parts I and II. Yeliz Karaca, Dumitru Baleanu, Majaz Moonis, Yu-Dong Zhang and Osvaldo Gervasi are the guest editors of the Special Issue. All the guest editors have approved the submission of this editorial.