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Initial value problems spreadsheet solver using VBA for engineering education

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Article Info

Abstract

Keywords: Excel spreadsheet, Initial Value Problems (IVPs) spreadsheet solver, Runge-Kutta methods, VBA programming 2010 AMS: 65LXX, 65YXX, 68WXX,68UXX, 68NXX Received: 4 March 2018 Accepted: 22 April 2018 Available online: 30 June 2018

Spreadsheet solver using VBA programming has been designed for solving initial value problems (IVPs), analytically and numerically by all Runge-Kutta (RK) methods including also fifth order with calculation of true percent relative error for corresponding RK method. This solver is user-friendly especially for beginner users of Excel and VBA.

1. Introduction

IVPs arise in any field of science and engineering education such as mechanics, geotechnics, dynamics, chemical kinetics, optimization and stability, et cetera. There are computing approaches; exact solution method and numerical methods for solving these IVPs. Numerical methods are both applicable and practical in solving IVPs in many engineering problems because of the existence of complicated problems in engineering and limitations of exact solution method [1, 2]. Numerical methods yield approximate the solutions of the IVPs, particularly for the nonlinear IVPs.

This study mainly has focussed on numerical solutions followed by Euler and various Runge-Kutta methods for solving single IVPs. These methods progress the solution over step starting from some given initial condition at the initial starting point. To simplify the steps in solving IVPs by RK methods, a tool is used. This tool is a prevalent spreadsheet application, fundamentally called as Excel, also commonly used by professionals for diverse applications in business [3], engineering and science [4]-[6].

Numerical methods in science and engineering may also be implemented in by use of Excel and also VBA. Use of VBA in explicit form Visual Basic for Applications programming capability lurks in the background behind Excel handled in the texts like Lilley and Chapra [2, 7]. In addition to this, a series of studies in literature employed spreadsheet as a calculator or solver to focus on design of solver and calculator for polynomial interpolation [8, 9], solution for systems of linear and nonlinear equations [10, 11], computation of eigenvalues [12, 13], design of spreadsheet calculator for numerical differentiation [14]-[16], spreadsheet solver for solution of partial differential equations [17], a spreadsheet solution of system of initial value problems using fourth-order RK method [18], and fourth-order RK method by spreadsheet [19]. Only the works of Tay et al. [20, 21] include design of spreadsheet calculator for solving system of IVPs using fourth-order RK method and also solving IVPs using fourth-order RK method with use of VBA programming.

In this study, a spreadsheet solver is designed to solve both IVPs by all RK methods and also exact solution method in the spreadsheet environment based on VBA programming. Microsoft Excel 2010 and Microsoft Visual Basics for Applications 7.0 are used during this study. The generation of VBA programming includes three steps. The first step is to develop an user interface input form is designed to acquire the needed information such as initial conditions of independent and dependent variables for each RK method, step size and number of steps. Then a general VBA code for any IVPs is created behind the Solve button in user interface input form. The third step is to generate function files depending on the related IVP and its analytical solution. Once the SOLVE button in user interface input form is clicked, the complete numerical and analytical solutions of the IVP and corresponding true percent relative error will be computed automatically for each order of RK method.

Examples are presented from various fields of engineering to demonstrate the merits of this unconventional solver design which shields the tedious algorithmic implementation details from the user (such as students and educators) and greatly simplifies solving an IVP using RKSOLVER.

This spreadsheet solver is user-friendly such that users only require to enter initial conditions of independent and dependent variables for each RK method, step size and number of steps at the first step to compute the complete solution of the IVPs automatically without typing any commands in the spreadsheet cells. Here, complete solution of the IVPs means solutions from each order of RK method, exact solutions and also true percent relative errors in terms of comparison with each RK method and exact solutions. So users as educators have an oppurtunity to elucidate students the differences and similarities that exist between each order of RK method and also exact solutions at the same time and be able to comment on the solution of any engineering problem including IVPs correctly. There is no need to know the various derivations of RK methods and memorize the complicated formulations of RK methods. The solver is general and standard for any engineering problem. The main aim of this paper is to design a tool in other words spreadsheet solver which employes both numerical methods: RK methods with fifth order and also analytical methods giving exact solutions with automatically calculated true percent relative errors in solving IVPs at the same time. Therefore this solver is called as IVP spreadsheet solver.

2. Runge Kutta (RK) methods

This section is devoted to solving IVPs of the form given below:

$$\frac{dy}{dx} = f(x, y) \tag{2.1}$$

with the initial value $y(x_0) = y_0$ for the number of points *n* within the interval $x_0 \le x \le x_n$. Here *x* is the independent variable, *y* is the dependent variable, *f* is the function of derivation (in other words slope) and *h* is the fixed step size. *n*, the number of steps can be found as $(x_n - x_0)/h$ [1].

1) First-Order RK Method

Euler's Method:

 $y_{i+1} = y_i + hk_1 \tag{2.2}$

where $k_1 = f(x, y)$

2) Second-Order RK Methods

a) Heun's Method:

$$y_{i+1} = y_i + h(\frac{k_1 + k_2}{2}) \tag{2.3}$$

where $k_2 = f(x_i + h, y_i + hk_1)$

b) Midpoint (Improved Polygon) Method:

$$y_{i+1} = y_i + hk_2 \tag{2.4}$$

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2})$

c) Ralston's Method:

$$y_{i+1} = y_i + \left(\frac{k_1 + 2k_2}{3}\right)h\tag{2.5}$$

where $k_2 = f(x_i + \frac{3h}{4}, y_i + \frac{3hk_1}{4})$

3) Third-Order RK Method

$$y_{i+1} = y_i + \left(\frac{k_1 + 4k_2 + k_3}{6}\right)h\tag{2.6}$$

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2}), \quad k_3 = f(x_i + h, y_i - k_1 h + 2k_2 h)$

J

4) Fourth-Order RK Method

$$y_{i+1} = y_i + \left(\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}\right)h$$
(2.7)

Function $f(x, y0, h)$
f = y0 / 0.2254
End Function

Table 1: Function module for stress-strain relationship IVP

Function fexact(x, y0, h, i)
fexact = Exp((h * i) / 0.2254)
End Function

Table 2: Function module for exact solution of stress-strain relationship

where $k_2 = f(x_i + \frac{h}{2}, y_i + \frac{k_1 h}{2}), \quad k_3 = f(x_i + \frac{h}{2}, y_i + \frac{k_2 h}{2}), \quad k_4 = f(x_i + h, y_i + k_3 h)$

5) Fifth-Order RK Method

$$y_{i+1} = y_i + \left(\frac{7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6}{90}\right)h$$
(2.8)

where $k_2 = f(x_i + \frac{h}{4}, y_i + \frac{k_1 h}{4}), \quad k_3 = f(x_i + \frac{h}{4}, y_i + \frac{k_1 h}{8} + \frac{k_2 h}{8}), \quad k_4 = f(x_i + \frac{h}{2}, y_i - \frac{k_2 h}{2} + k_3 h), \quad k_5 = f(x_i + \frac{3h}{4}, y_i + \frac{3k_1 h}{16} + \frac{9k_4 h}{16}),$ and $k_6 = f(x_i + h, y_i - \frac{3k_1 h}{7} + \frac{2k_2 h}{7} + \frac{12k_3 h}{7} - \frac{12k_4 h}{7} + \frac{8k_5 h}{7})$

It should be noted that k's are recurrence relationships. In other words, k_1 appears in the equation for k_2 which appears in the equation for k_3 and so on. Since each k is a functional evaluation, this recurrence makes RK methods efficient for computations [1].

In this work, fifth-order RK method yields the superior results in terms of less error than the other order of RK methods. As the order of RK method increases, convergence to the exact results also increases in terms of less errors.

3. Numerical examples

Numerical examples are presented from various engineering applications.

1) Geotechnical Engineering

To mIVPl the the behavior of soil under the effect of load, it is required to formulate the stress and strain relationship and this is achieved by the following IVP:

$$\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{c_C} \tag{3.1}$$

The exact solution for equation (3.1) is

$$\sigma = e^{\frac{\varepsilon}{c_C}} \tag{3.2}$$

where σ is the stress, ε is the strain of soil and c_C is the compression index and it is 0.2254 for this soil type. Initial conditions are, ε_0 is 0 for independent variable and σ_0 is 1 kPa for dependent variable. Final ε is 1.2 and step size (h) is 0.1. This means that number of steps (n) is 12. At first, for each numerical example, function modules are prepared for both IVP and exact solution of it respectively. These modules change from example to example. The functions for IVP and exact solution are illustrated in the following tables.

Here x is the independent variable, y0 is the initial dependent variable, i is the counter of steps.

Then equations (2.2) to (2.8) are applied to obtain the solutions by each order of RK method respectively. Besides exact solution of the IVP with true percent relative error for each RK method are also incorporated in the computations.

Finally IVP spreadsheet solver is applied which is discussed in the next section to obtain the complete solutions.

2) Mechanical Engineering

To determine the change in velocity in other words acceleration of a free-falling body to the forces acting on it with considering the air resistance, the following IVP is used:

$$\frac{dv}{dt} = g - \frac{c}{m}v \tag{3.3}$$

The exact solution for equation (3.3), which also gives velocity of the object, is

$$v(t) = \frac{gm}{c} (1 - e^{(-\frac{c}{m})t})$$
(3.4)

where *v* is the velocity (dependent variable *y*), *t* is the time in seconds (indepedent variable *x*), *g* is the gravitational constant, 9.8 m/s², m is the mass of the object, 68.1 kg and c is the drag coefficient, 12.5 kg/s. Initial conditions are, t_0 is 0 s and v_0 is 0 m/s [1]. Final value of time is 5 s and step size (h) is 0.5. This means that number of steps (n) for computation is 10.

At first, for this example, function modules are written for both IVP and exact solution of it respectively. These functions are illustrated in Table 3 and Table 4 respectively.

Here x is the independent variable corresponding to time, y0 is the initial dependent variable corresponding to velocity.

Function f(x, y0, h)	
f = 9.8 - ((12.5 / 68.1) * y0)	
End Function	

Table 3: Function module for exact solution yielding velocity

```
Function fexact(x, y0, h,i)
fexact = ((9.8 * 68.1) / 12.5) * (1 - Exp((-12.5 / 68.1) * (h * i)))
End Function
```

Table 4: Function module for exact solution yielding velocity

Like geotechnical engineering example, equations (2.2) to (2.8) are employed to find the solutions by each order of RK method respectively. Besides exact solution of the IVP with true percent relative error for each RK method are also inserted in the computations. Finally IVP spreadsheet solver is used which is mentioned in the next section to obtain the complete solutions.

3) Chemical Engineering: Mixture Problem

The mixture problem related to a tank containing 1000 L of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same time. In this problem, it is required to determine the amount of salt after t minutes in this tank. For this reason, the following IVP is employed:

$$\frac{dA}{dt} = \frac{-A}{100} \tag{3.5}$$

A(t) is the amount of salt after t minutes in tank, also the dependent variable is obtained by the following exact solution:

$$A(t) = 15e^{\left(\frac{-t}{100}\right)} \tag{3.6}$$

Initial conditions are, t_0 is 0 min and A_0 is 15 kg. Final value of time is 0.96 min and step size (h) is 0.02. Number of steps (n) for computation is 49.

At first, function modules are formed for both IVP and exact solution of the problem respectively. These functions are displayed in Table 5 and Table 6 respectively.

Here x is the independent variable corresponding to time, y0 is the initial dependent variable corresponding to amount of salt after t minutes in the tank.

Then, equations (2.2) to (2.8) are used to determine the solutions by writing codes for each order of RK method respectively. These codes are standard and valid for any scince and engineering problem including IVP. So there is no need to write cIVP for various problems. Besides exact solution of the IVP with true percent relative error for each RK method are also included in the computations. True percent relative error is in the following form:

$$\varepsilon_T = \left| \frac{ExactResult - ApproximateResult}{ExactResult} \right| \times 100 \tag{3.7}$$

Where Exact Result in other words true result represents the solution obtained by analytically. Approximate Result corresponds with the corresponding solution obtained by numerical methods, any order of RK methods.

Finally IVP spreadsheet solver is employed which is argued in the next section to obtain the complete solutions.

4. IVP spreadsheet solver

Using this IVP spreadsheet solver leads to a macro named RKSOLVER which solves the whole IVP at once completely.

The general procedure for obtaining complete solution of an IVP is composed of some steps. These steps are standard and applicable for any type of IVP.

The first step is to design an user interface input form (userform) called as UserForm4 to enable users to enter required data for solving an IVP completely. The standard form of UserForm4 for any problem is illustrated in Figure 4.1.

The second step is to generate a new tab name as IVP Solver with RKSOLVER macro including codes for solving IVP by both numerically (by each order of RK method) and analytically (gives exact solution). RKSOLVER also provides user to compute true percent relative error for each RK method.

Figure 4.2 illustrates the standard IVP Solver tab with RKSOLVER button. One more variation is to add a button assigned RKSOLVER macro in the spreadsheet. So user is able to run the macro simply by clicking this button. It is sufficient to start the complete solution procedure of IVPs.

Function f(x, y0, h)
f = -y0 / 100
End Function

Function fexact(x, y0, h, i)
fexact = 15 * Exp(-(h * i) / 100)
End Function

 Table 6: Function module for exact solution of the problem

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Figure 4.1: The standard userform for all examples

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Figure 4.2: The standard IVP Solver tab with RKSOLVER button

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Figure 4.3: The standard blank spreadsheet image with k's (recurrence relationships) titles

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Figure 4.4: The standard blank spreadsheet image with RK results, exact results and error titles

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Figure 4.5: Userform for geotechnical engineering example

Then the only thing is to specify sufficient place in spreadsheet cells to make macro fill them with solutions for any IVP examples. For this reason, the titles for k's, RK results, exact results and error titles are written as is the case with Figure 4.3 and Figure 4.4 respectively.

The working procedure for IVP solver namely RKSOLVER is described for each numerical examples (geotechnical engineering, mechanical engineering and chemical engineering). The steps for geotechnical engineering example are illustrated in the Figure 4.5- Figure 4.11.

The first step is to call userform by clicking run in the toolbar or simply clicking RKSOLVER button. The image of this userform for geotechnical engineering example is given in Figure 4.5. This userform is standard for any IVP example.

Due to the fact that initial conditions are different for all IVPs, the filled userform is distinctive for all problems. As is the case with geotechnical engineering example. Userform is filled with initial conditions of the problem in Figure 4.6. Then by clicking SOLVE button in UserForm4; k's, numerical solutions obtained form all RK methods, exact solutions (true solutions) and true percent relative errors can be obtained and displayed as the spreadsheet images in Figure 4.7 to Figure 4.11 respectively.

To Figure 4.10 and Figure 4.11, fifth-order RK method gives the best solution in terms of the least error and best convergence to exact solutions.

Similarly for mechanical engineering, userform is invoked by clicking RKSOLVER in Figure 4.12. Then this form is filled with necessary data as it is shown in Figure 4.13.

By clicking the SOLVE button in userform, computations are performed and given in the spreadsheet images of Figure 4.14 to Figure 4.18. To Figure 4.17 and Figure 4.18, the worst solution is obtained by Euler's method while fifth-order RK method is the best one with the least error and best convergence to the exact solution.

For mixture problem, userform is called by clicking RKSOLVER button in spreadsheet. Figure 4.19 illustrates this process.

Then this userform is filled by entering initial conditions as given in Figure 4.20. Clicking the SOLVE button in userform leads to complete solution of the problem. These solutions are displayed in Figure 4.21 to Figure 4.25.

To Figure 4.24 and Figure 4.25, all RK methods give quite well solutions with convergence to exact results in terms of less errors.

5. Conclusion

An IVP solver with use of RK methods including also the highest order; fifth order has been generated by VBA for the first time in literature. Emphasis was on all types of RK methods usable simultaneously and the solver generated applicable to IVPs for science and engineering problems.

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Figure 4.6: Filled userform for geotechnical engineering example

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7		0.4	19.27084	31.98343	28.80048	30.18418	27.92855	42.01913	31.61722	28.40868	39.25563	32.19948491	35.42978	40.7482
8		0.5	27.82046	47.14615	43.47453	44.80645	42.92275	64.29676	48.13482	43.76028	59.84624	49.73011234	54.5618	63.1125
9		0.6	40.16316	69.25174	65.50021	66.28005	65.881	98.03436	73.0507	67.30723	90.93029	76.67334187	83.90165	97.5584
LO		0.7	57.98178	101.4283	98.51241	97.75227	100.9975	149.0128	110.5568	103.383	137.7531	118.0289297	128.8446	150.534
11		0.8	83.70573	148.2025	147.9243	143.7986	154.6603	225.8868	166.9091	158.5952	208.1455	181.4298683	197.6162	231.896
12		0.9	120.8423	216.1221	221.7897	211.0657	236.5922	341.5963	251.4339	243.0114	313.7849	278.5197644	302.7478	356.700
L3		1	174.4546	314.6556	332.0807	309.2025	361.5829	515.4661	378.0198	371.9605	472.0657	427.0466658	463.3177	547.919
14		1.1	251.8524	457.4918	496.5791	452.2083	552.1159	776.3237	567.3289	568.7676	708.8726	654.0442822	708.3533	840.583
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Figure 4.7: Computation results for k's for geotechnical engineering example

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0	Р	Q	R	S	Т	U
yi+1Euler	yi+1Heun	yi+1Midpoint	yi+1Ralston's	yi+1Third Order	yi+1Fourth Order	yi+1Fifth Orde
1.443656	1.542071	1.542070924	1.542070924	1.556625079	1.558239338	1.55839448
2.084142	2.346466	2.36829718	2.353743026	2.404118656	2.411820362	2.42233410
3.008783	3.534409	3.624117018	3.563235792	3.69055243	3.712120001	3.75650291
4.343647	5.281982	5.528087280	5.358200416	5.638098147	5.686496553	5.81332352
6.270731	7.844696	8.40813576	8.012840169	8.579629901	8.675861063	8.97927695
9.052777	11.59303	12.75558858	11.92728563	13.01321875	13.1906175	13.8454980
13.06909	17.06377	19.30560932	17.68472762	19.68319123	19.99387199	21.315293
18.86727	25.03428	29.1568508	26.13427148	29.70059605	30.22509227	32.7677855
27.23784	36.62969	43.94928083	38.51103481	44.72209143	45.58372588	50.3068297
39.32207	53.47791	66.128248	56.6101552	67.21537894	68.60163262	77.1394174
56.76753	77.93342	99.3363140	83.03881122	100.852767	103.046987	118.150615
81.95277	113.4006	148.9942263	3 121.5811084	151.0943093	154.5226722	180.776672

Figure 4.8: Computation results for each RK method for geotechnical engineering example

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V	W	х	Y	Z	AA	AB	AC
RUE Results	Error Euler	Error Heun	Error midpoint	Error Ralston's	Error third order	Error fourth order	Error fifth order
1.558393874	7.362589976	1.047421393	1.047421393	1.047421393	0.113501165		3.93667E-05
2.428591468	14.18310264	3.381611137	2.482685413	3.081969229	1.007695683	0.690569245	0.257654038
3.784702066	20.50144892	6.613271635	4.243003701	5.851617122	2.487636662	1.917774866	0.745082536
5.898056516	26.35460128	10.44538333	6.27273119	9.153118461	4.407525912	3.586943647	1.436625617
9.191495146	31.77681002	14.6526716	8.522654606	12.82332154	6.656863059	5.609904312	2.308853858
14.32396973	36.79980377	19.06555263	10.94934701	16.73198244	9.150752258	7.912277461	3.340356939
22.32238668	41.45297508	23.55758767	13.51458252	20.77582085	11.82308816	10.43129807	4.511584689
34.78707067	45.76355247	28.03568497	16.18480588	24.87360685	14.62173883	13.11400561	5.804700193
54.21195784	49.75675972	32.43245311	18.93065186	28.96210293	17.50511655	15.91573575	7.20344407
84.48358301	53.45596349	36.70023859	21.72651024	32.99271506	20.43971557	18.79885987	8.693009074
131.6586982	56.88281006	40.80648133	24.55013198	36.92873139	23.3983259	21.73172879	10.25992425
205.1761088	60.05735196	44.7301015	27.38227316	40.74304797	26.35872171	24.68778502	11.89194793

Figure 4.9: Computation results for exact results (true results) and true percent relative errors of each RK method for geotechnical engineering example

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U	V	w	x	Y	Z	AA	AB	AC
Fifth Order	TRUE Results	Error Euler	Error Heun	Error midpoint	Error Raiston's	Error third order	Error fourth order	Error fifth order
1 558394488	1 558393874	7.362589976		1.047421393	1.047421393	0.113501165	0.009916402	
2.422334104	2.428591468	14,18310264	3.381611137	2,482685413	3.081969229	1.007695683	0.690569245	0.257654
3 756502912		20 50144892		4 243003701	5.851617122	2 487636662	1 917774866	
5.813323526	5.898056516	26,35460128	10.44538333	6.27273119	9.153118461	4,407525912	3,586943647	
8.979276955		31,77681002	14.6526716		12 82332154	6 656863059	5.609904312	
13.84549801	14.32396973	36,79980377	19.06555263	10,94934701	16.73198244	9.150752258	7,912277461	3.340356
21.3152933		41,45297508	23.55758767	13.51458252	20.77582085	11.82308816	10.43129807	
32.76778551	34.78707067	45.76355247	28.03568497	16.18480588	24.87360685	14.62173883	13.11400561	5.804700
50.30682978	54.21195784	49.75675972	32,43245311	18.93065186	28.96210293	17.50511655	15,91573575	7.20344
77.13941747	84,48358301	53,45596349	36.70023859	21.72651024	32,99271506	20.43971557	18.79885987	8.6930090
118,1506155		56.88281006		24,55013198	36,92873139		21,73172879	
180.7766728	205.1761088	60.05735196	44,7301015	27.38227316	40.74304797	26.35872171	24,68778502	11.891947
Virmonical Deculte	Numer	son of True ical Solutic Method	ons of All	ryi+1Heun -yi+1Midpoint -yi+1Raiston's -yi+1Piorth Order -yi+1Pourth Order	100 55	Error versus		al Effort for
2	50 -0.4 0.1	0.6 X		yi+1Fifth Order - TRUE Results - yi+1Euler	20 15 10 5 0	·		Error_Euler

Figure 4.10: Graphical display of the computation results for geotechnical engineering example

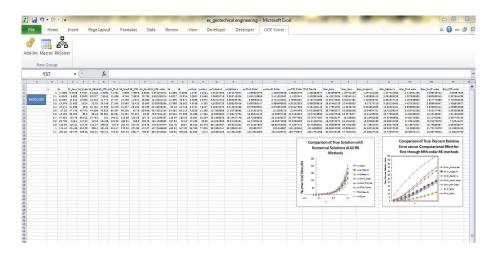


Figure 4.11: The spreadsheet image of full computation results for geotechnical engineering

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Figure 4.12: Userform in spreadsheet for mechanical engineering example

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Figure 4.13: Filled userform for mechanical engineering example

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		X		k1	k2_Heun	k2_Midpoint, Third and Fourth or	k2_Raiston's	k2_fifth order	k3_Third order	k3_Fourth order	k3_Fifth order	k4_Fourth order	k4_Fifth order	k5	k6
			0	9.8	8.90059	9.350293686	9.125440529	9.575146843	8.983132583	9.370929989	9.577726381	8.939966044	9.360375096	9.14814	8.94
			0.5	8.90059	8.12499	8.533426269	8.329209415	8.736408948	8.19112368	8.549041052	8.738292419	8.156022704	8.53955312	8.34661	8.1
	RKSOLVER		1	8.08372	7.41869	7.787740461	7.603528599	7.971397809	7.469476731	7.799767812	7.97268638	7.441297036	7.790959346	7.61556	5 7.43
			1.5	7.34182	6.77539	7.107051081	6.942077706	7.273610816	6.81193767	7.116673628	7.274393342	6.789659303	7.108216871	6.94877	6.78
			2	6.66801	6.18942	6.485708696	6.339106972	6.637120738	6.212807884	6.493893042	6.637475138	6.195523118	6.485513589	6.34056	6.18
			2.5	6.05604	5.65558	5.918553387	5.789385453	6.056524649	5.666894927	5.926081097	6.056519137	5.653797286	5.91755202	5.78577	5.64
			3	5.50024	5.16916	5.400872476	5.288153952	5.52689741	5.169467617	5,408367139	5.526591591	5,159841909			7 5.15
			3.5	4.99545	4.72588	4.928361896	4.831082233	5.043749286	4.716215096	4.936312726			4.926967059	4.81805	4.69
					4.32186		4.414230134	4.602987332	4.303209528						

Figure 4.14: Computation results for k's for mechanical engineering example

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	4.9	4.67515	4.67514684	4.67514684	4.6820256	4.681867783	4.6818706
9.35	029	8.93154	8.94185998	8.93498121	8.9508103	8.950329842	8.95175974
13.3	922	12.8071	12.8357302	12.8167774	12.842823	12.8419993	12.8460405
17.0	631	16.3364	16.3892557	16.3544403	16.39132	16.39024353	16.3978645
20.3	971	19.5508	19.6321101	19.5788115	19.626625	19.62547184	19.6374481
23.4	251	22.4787	22.5913868	22.5179473	22.576388	22.57539771	22.5923329
26.1	752	25.1461	25.291823	25.197372	25.265821	25.26527779	25.2876232
28.6	729	27.5764	27.756004	27.6403069	27.717913	27.71812971	27.7462025
30.9	414	29.7911	30.0045494	29.8678802	29.953626	29.95493061	29.9889308
33.0	017	31.8096	32.0562843	31.899316	31.992071	31.99479802	32.0348234

Figure 4.15: Computation results for each RK method for mechanical engineering example

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32.065765 2.918862 0.7987488 0.029567196 0.5190872 0.229821773 0.221317681 0.096	8 3.08	38 3.0	0829762	0.749385	7 0.03823	76016	0.4935966	0.20792992	0.203583685	0.090	310244
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Figure 4.16: Computation results for exact results (true results) and true percent relative errors of each RK method for mechanical engineering example

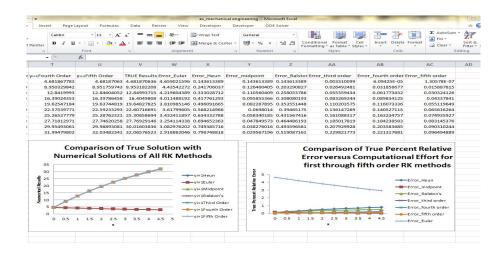


Figure 4.17: Graphical display of the computation results for mechanical engineering example

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Figure 4.18: The spreadsheet image of full computation results for mechanical engineering example

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Figure 4.19: Userform in spreadsheet for mixture problem

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Figure 4.20: Filled userform for mixture problem

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	0	-0.15	-0.15	0.149985	0.1499775	0.1499925		0.149985002	0.1499925	0.149970003	0.149985	0.15	
	0.02	-0.15	-0.1499	-0.149955006	-0.14994751	-0.149962504	-0.14994001	-0.149955007	-0.149962505	-0.149940012	-0.14995501	-0.1499	-0.
	0.04	-0.1499	-0.1499	-0.149925018	-0.14991752	-0.149932515	-0.14991003	-0.149925019	-0.149932515	-0.149910027	-0.14992502	-0.1499	-0.
	0.06	-0.1499	-0.1499	-0.149895036	-0.14988754	-0.149902531	-0.14988005	-0.149895037	-0.149902532	-0.149880048	-0.14989504	-0.1499	-0.
	0.08	-0.1499	-0.1499	-0.14986506	-0.14995757	-0.149972554	-0.14985008	-0.149865061	-0.149872554	-0.149950075	-0.14996506		-0.
	0.1	-0.1499	-0.1498	-0.14982509	-0.1498276	-0.149842582	-0.14982011	-0.149825091	-0.149842583	-0.149820108	-0.14983509	-0.1498	-0.1
	0.12	-0.1498	-0.1498	-0,149805126	-0.14979763	-0.149812617	-0.14979015	-0.149805127	-0.149812617	-0.149790147	-0.14980518		-0.
	0.14	-0.1498	-0.1498	-0.149775168	-0.14976768	-0.149782657	-0.14976019	-0.149775169	-0.149782658	-0.149760192	-0.14977517		-0.
	0.16	-0.1498	-0.1497	-0.149745216	-0.14973773	-0.149752704	-0.14973025	-0.149745217	-0.149752704	-0.149730243	-0.14974522		-0.
	0.18	-0.1497	-0.1497	-0.14971527	-0.14970778	-0.149722756	-0.1497003	-0.149715271	-0.149722757	-0.1497003	-0.14971527		-0.
	0.2	-0.1497	-0.1497	-0.14968523	-0.14967784	-0.149692815	-0.14967027	-0.149685221	-0.149692815		-0.14968522		-0.
	0.22	-0.1497	-0.1496	-0.149655296	-0.14964791	-0.149662879	-0.14964042	-0.149655397	-0.149662879	-0.149640482	-0.1496554		-0.
	0.24	-0.1496	-0.1496	-0.149625468	-0.14961799	-0.14963295	-0.14961051	-0.149625469	-0.14963295	-0.149610507	-0.14962547		-0.
	0.26	-0.1496	-0.1496	-0.149595546	-0.14958807	-0.149603026	-0.14958059	-0.149595547	-0.149603026	-0.149580587	-0.14959555		-0.
	0.28	-0.1496	-0.1496	-0.149565629	-0.14955815	-0.149573108	-0.14955068	-0.149565631	-0.149573109	-0.149550674			-0.
	0.3	-0.1496	-0.1495	-0.149535719	-0.14952824	-0.149543197	-0.14952077	-0.149535721	-0.149543197	-0.149520767	-0.14953572		-0.
	0.22	-0.1495	-0.1495	-0.149505815	-0.14949824	-0.149513291	-0.14949097	-0.149505817	-0.149512291	-0.149490866	-0.14950582		-0.
	0.24	-0.1495	-0.1495	-0.149475917	-0.14946844	-0.149483391 -0.149453498	-0.14946097	-0.149475918	-0.149482292	-0.149460971	-0.14947592		-0.
	0.36	-0.1495	-0.1494	-0.149446025	-0.14943855	-0.149453498	-0.14943108	-0.149446026	0.149453498	-0.149481082	-0.14944605		-0.
	0.50	-0.1494	-0.1494	-0.149386258	-0.14937879	-0.149393728	-0.14937132	-0.14935626	0.149393729	-0.149401198	-0.14938626		-0.
	0.42	-0.1494	-0.1494	-0.149386258	-0.14937879	-0.149393728	-0.1493/152	-0.149356386	-0.149363053	-0.1493/1321	-0.14935626		-0.
	0.44	-0.1492	-0.1493	-0.149226516	-0.14921905	-0.1492222982	-0.14921159	-0.149326517	-0.149333983	-0.149211585	-0.14932652		-0
	0.44	-0.1493	-0.1493	-0.149296653	-0.14928919	-0.149204119	-0.14928173	-0.149296655	-0.149304119	-0.149281725	-0.14929665	-0.1493	-0
	0.48	0.1493	0.1498	-0.149266797	0.14925988	-0.149274261	0.14925187	0.149266799	0.149274261	0.149251872	0.1492668		-0.
	0.5	-0.1475	-0.1492	-0 149236947	-0.14922948	0 149744409	0 14922205	0 1472 35745	0 147744407	-0.149222025	-0.14923695		-0
	0.52	-0.1492	-0.1492	-0.149207102	-0.14919964	-0.149214563	-0.14919219	-0.149207104	-0.149214564	-0.149192163	-0.1492071		-0
	0.54	-0.1492	-0.1492	-0.149177264	-0.1491698	-0.149104723	-0.14916235	-0.149177265	-0.149104724	-0.149162340	-0.14917726		-0.
	0.56	-0.1492	-0.1491	-0.149147421	-0.14913997	-0.14915489	-0.14913252	-0.149147433	-0.14915489	-0.149132518	-0.14914742		-0.
	0.58	-0.1491	-0.1491	-0.149117605	-0.14911015	-0.149125062	-0.1491027	-0.149117606	-0.149125062	-0.149102695	-0.14911761	-0.1491	-0.
	0.6	-0,1491	-0.1491	-0.149087784	-0.14908033	-0.149095239	-0.14907288	-0.149087786	-0.14909524	-0.149072877	-0.14908779	-0.1491	-0,
	0.62	-0.1491	-0.149	-0.14905797	-0.14905052	-0.149065423	-0.14904307	-0.149057971	-0.149065424	-0.149043066	-0.14905797	-0.1491	-5
	0.64	-0.149	-0.149	-0.149025161	-0.14902071	-0.149035613	-0.14901326	-0.149028163	-0.149035614	-0.14901326	-0.14902816		-0
	0.66	-0.149	-0.149	-0.140990359	-0.14099091	-0.149005809	-0.14090346	-0.14099036	-0.149005809	-0.14090346			-0
	0.68	-0.149	-0.149	-0.148968562	-0.14896111	-0.148976011	-0.14995267	-0.148968562	-0.148976011	-0.149953666	-0.14896856	-0.149	-
	0.7	-0.149	-0.1489	-0.148938771	-0.14893182	-0.148946219	-0.14892388	-0.148938773	-0.148946219	-0.148923879	-0.14893877		-0.
	0.72	-0.1489	-0.1489	-0.148908986	-0.14890154	-0.148916433	-0.1488941	-0.148908988	-0.148916433	-0.148894097	-0.14890899		-0.
	0.74	-0.1489	-0.1489	-0.148879207	-0.14887176	-0.148886652	-0.14886432	-0.148879209	-0.148886652	-0.148864321	-0.14887921	-0.1489	-0.
	0.76	-0.1489	-0.1488	-0.148849435	-0.14884199	-0.148856878	-0.14883455	-0.148849436	-0.148856878	-0.148834551	-0.14884944		-0.
	0.78	-0.1499	-0.1400	-0.140019660	-0.14991223	-0.148827109	-0.14000479	-0.140019669	-0.14992711	-0.148804787	-0.14991967	-0.1488	-0.
	0.8	-0.1488	-0.1498	-0.149799907	-0.14979247	-0.148797247	-0.14877502	-0.148789908	-0.149797247	-0.148775029	-0.14979991		-0.
	0.82	-0.1488	-0.1487	-0.148760152	-0.14875271	-0.148767591	-0.14874528	-0.148760153	-0.148767591	-0.148745277	-0.14876015		-0.
	0.84	-0.1487	-0.1487	-0.148730403	-0.14872297	-0.14873784	-0.14871553	-0.148730404	-0.14875784	-0.148715531	-0.1487304		-0.
	0.86	-0.1487	-0.1487	-0.14870066	-0.14869322	-0.148708095	-0.14868579	-0.148700661	-0.148708096	-0.148685791	-0.14870066		-0.
	0.68	-0.1487	-0.1487	-0.148670922	-0.14866349	-0.148678357	-0.14865606	-0.148670924	-0.148678357	-0.148656057	-0.14867092		-0.
	0.9	-0.1497	-0.1496	-0.148641191	-0.14963376	-0.148648624	-0.14962633	-0.149641193	-0.148648624	-0.148626329	-0.14864119		-0.
	0.92	-0.1486	-0.1486	-0.148611466	-0.14860402	-0.148618897	-0.14859661	-0.148611468	-0.148618897	-0.148596606	-0.14861147		-0.
	0.94	-0.1486	-0.1486	-0.148581747	-0.14857432	-0.148589177	-0.14856689	-0.148581748	-0.148589177	-0.14856689	-0.14858175		-0.
	0.96	-0.1486	-0.1485	-0.148552033	-0.14854461	-0.148559462		-0.148552035	-0.148559462	-0,14853718	-0.14855203	-0,1485	-0.1

Figure 4.21: Computation results for k's for mixture problem

View		eloper	Developer	ODE Solver	-	
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Las Euler	14.997	14.9970005	Y Raiston's	14.9970003	14.9970003	yas Fifth Order
14.994	14.004	14.9940012	14.9940012	14.9940012	14,9940012	14.994001
14.991	14 221	14 9910027	14.9910027	14,9910027	14 9910027	14,991002
14.955	14,955	14,9550045	14,9550045	14,9550045	14,9550045	14,955004
14.905	14,903	14,9850075	14,9850075	14,9850075	14,9850075	14,9850071
14.952	14,982	14,9520105	14.9520105	14,9520105	14,9820108	14,982010
14.979	14,979	14,9790147	14,97901469	14,97901469	14,97901469	14,9790145
14.975	14,970	14,9760192	14,97501919	14,97001919	14,97601919	14,9700191
14.973	14.973	14.9730243	14.97302429	14,97302429	14.97302429	14.97302421
14.97	24.97	14.97003	14.97002998	14.97002998	14.97002998	14.97002991
14.997	34.997	14.9970393	14.99703927	14.99703927	14.99703927	14.99703921
3.4	14.0004	14.0040402	14.004817	14.004317	14.00404017	14.00040401
14.901	14.9011	14.9010507	14.98105088	14.90105000	14.98105088	14.90105004
14.0581	14.0581	14.0580587	14.95805875	14.05805875	14.95805875	14.05805871
14.9551	14.9551	14.9550674	14.95506745	14.95506743	14,95506743	14.95506741
14.9521	34.9523	14,9520767	14.95207672	14,95207672	14,95207672	14.95207672
14.9491	14.9491	14.9490866	14.9490866	14.9490866	14.9490866	14.9490864
14.9461	14.9461	14.9460971	14.94609709	14.94609708	14.94609708	14.94609700
14.9431	14.9431	14.9431082	14.94310817	14.94310816	14.94310816	14.94310814
14.9401	14.9401	14.9401198	14.94011984	14.94011984	14.94011984	14.94011984
14.9371	14.9371	14.9371321	14.93713212	14.93713212	14.93713212	14.93713211
14.9341	14.9341	14.934145	14.93414499	14.93414499	14.93414499	14.93414491
14.9312	14.9312	14.9311585	14.93115840	14.93115849	14.93115840	14.93115844
3.4. 0.2.0.2	3.43. 00.21.00.21	14.0281725	14.02817258	14.02817258	14.02817258	1.4.02817251
14.9292	14.0292	14.0251872	14.02518710	14.02518710	14.02518710	14.02518714
14.9222	14.9222	14.9222024	14.92220246	14.92220245	14.92220245	14.9222024
14.9192	14.9192	14.0102185	14.01021851	14.01021851	14.01021851	14.0102185
14.9162	14.9162	14.9162346	14,91623477	14,91623477	14.91623477	14.91623474
14.9132	14.9133	14.9132518	14.91325182	14.91325182	14.91325182	14.9132518
14.9103	14.9103	14.9102695	14.91026947	14.91026947	14.91026947	14.91026944
14.9073	14.9073	14,9072877	14.90728771	14,90728771	14.90728771	14.9072877
14.9043	14.9043	14.9043065	14,90430656	14,90430655	14.90430655	14,90132591
14.9993	14.09013	14.090340	14.00132500	14.090332500	14.00132500	14.0013250
14.8954	14.8924	14.8953000	14.89536666	14,89536665	14.89536665	14.8953666
14.8924	2.0.892.0	14.8923879	14.89238788	14.89238787	14.89238787	14.8923878
14.8894	Ad. BERDA	14.8894097	10.0000007	10.0000007	10 8880087	14 8884088
2.4.888.84	3.4.888.04	14.8884321	3.4.8884423232	14.88843211	3.4.888.43233	3.4.888843233
14.8834	1.0.00000	1d BERGERI	14.88345513	14.88345512	14.88345512	14.8834551
14. BBOR	14.8803	14.8804787	14.88047874	14.88047873	14.88047875	14.88047871
14.8775	14.8775	14,8775029	14,87750294	14,87750293	14,87750295	14,87750293
14.8745	14.8745	14,8745277	14.87452774	14,67452773	14.87452775	14.87452773
14.0715	14.0710	14.0713331	14.07133313	14.07155312	14.07155312	14.07155313
14.0000	14.0000	14.0005791	14.00057912	14,05057911	14.05057911	14.000579:
14.0000	14.0000	14.0050057	14.0050057	14.00500509	14.00500509	14.00300301
14.0020	14.0020	14.0020329	14.00203200	14,06263207	14.00203207	14.00203204
14.0390	14.0397	14.8596606	14.05966065	14.85966064	14.85966064	14.85966063
14.8297	14.8597	14.850089	14.85008902	14.850089	14.850089	14.85008
			14.85271798	14.85371799	14.85371790	14.8937179

Figure 4.22: Computation results for each RK method for mixture problem

Develope	r ODE So	lver.					~ 🕝 =
~	w	×	×	7		AB	AC
~	~	×	Y	Z	AA	AB	AC
RUEResults	Error Euler	Error Heun	Errer mideeint	France Option and	France shill and south an	Error fourth order	Error fifth order
14,9970003	2.000278-06	1.33358-10	1.333488-10	1.333488-10	1.184478-14	o	C C C C C C C C C C C C C C C C C C C
14,9940012	4 000535-05	4 66755-10	2.666675-10	4.000535-10	6.665195-11	6.66756E-11	3.112245-11
14,9910027	6.00085-06	1.00026-09	2.99932E-10	8.001016-10	1.999725-10	2.000075-10	9,225026-11
14,9880048	9.001075-06	1.73375-09	5.22168E-10	1.222526-09	2.999655-10	4.000246-10	1.86696-10
14.9850075	1.000135-05	2.66748-09	6.66362E-10	2.000325-09	6.66658E-10	6.66729E-10	8.11155-10
14.9820108	1.200165-05	3.80116-02	7.995146-10	2.800495-09	1.00006E-09	1.000135-09	4.667355-10
14.97901469	1.400195-05	5.1355-09	2.326375-10	3.73407E-09	1.40016E-09	1.40024E-09	6.53441E-10
14.97601919	1.600216-05	6.66916-09	1.065716-09	4.801026-09	1.866996-09	1.867026-09	8.712515-10
14.97802429	1.800246-05	8.40BBE-09	1.198785-09	6.00146-09	2.400555-09	2.400555-09	1.120185-09
14.97002998	2.000278-05	1.03388-08	1.331738-09	7.33528-09	3.000868-09	3.000795-09	1.400236-09
14.96703627	2 200226-05	1.24725-08	1.464695-09	8.802428-09	3.66794E-09	3.66776E-09	1.711416-09
14,96404217	2.400225-05	1.48075-08	1.59765-09	1.040216-08	4.401785-09	4.401466-09	2.05275-09
14.96105066	2.600355-05	1.78426-08	1.780475-09	1.218726-08	5.202395-09	5.201895-09	2.927115-09
14 75805875	2.800375-05	2.00785-08	1.86331E-09	1.000075-00	6.06385-03	6.063085-03	2,831655-09
14.95506743	3.00045-05	2 30145-08	1 99615-09	1.600575-08	7.004016-09	7.00301E-09	3.267316-09
14,95207672	3.200436-05	2.6156-09	2.128875-09	1.814025-08	8.005046-09	8.003725-09	2.72415-09
14,9490866	3.400455-05	2.94865-08	2.261585-09	2.040826-08	9.072885-09	9.071186-09	4.231995-09
14,94609708	3.600488-05	3.30238-08	2.394268-09	2,280978-08	1.020768-08	1.020548-08	4.761048-09
14,94310816	3.800518-05	3.50256-08	2.526886-09	2 5 3 6 6 5 - 0 5	1.140916-08	1.140655-08	5.321176-09
14.94011984	4.000536-05	4.06975-08	2.659486-09	2.801215-08	1.267755-08	1.267426-08	5.912455-09
14.93713211	4.200566-05	449255-09	2 792025-09	2.08156-08	1.401285-08	1.400895-08	6.524855-09
10.93010099	4.400595-05	9.91745-08	2.924545-09	3.375055-08	1.541495-08	1.541035-08	7.188375-09
14.23115846	4.600615-05	5.37135-08	3.057016-09	3.661265-08	1.666395-08	1.687855-08	7.873028-09
14.92017252	4.800646-05	5.04525-00	2.102446-02	4.002215-08	1.041995-00	1.041255-00	0.500795-09
14,92518719	5.000675-05	6.23926-08	2.221855-09	4.225826-08	2.002275-08	2.001526-08	9.22575-09
14.92220245	5.200695-05	6.85325-08	3.45418E-09	9.682795-08	2.169255-08	2.16845-08	1.011875-08
14.91921031	5.400728-05	7.38735-08	3.586516-09	5.043128-08	2.342926-08	2.341955-08	1.092296-08
14.91623476	5.60075E-05	7.94155-08	3.718795-09	5.416816-08	2.523295-08	2.522105-00	1.176325-08
14,91225181	5.800775-05	8.51575-08	2.851025-09	5.802855-08	2.710255-08	2.70916-08	1.262466-08
14,91026946	6.00085-05	9.10996-08	8.988215-09	6.20425E-08	2.90411E-08	2.902716-08	1.858715-08
14.90728771	6 200825-05	772425-08	4.115376-09	6.618012-08	3 104575-05	3,1035-08	1.447085-08
14.90430655	6.400858-05	1 03595-07	4.247486-09	7.045146-08	3.311726-08	3.309985-08	1.54356E-08
14.90132598	6.600885-05	1.10126-07	4.279566-09	7.485626-08	2.525585-08	2.523646-08	1.642156-08
14.89834602	6.80098-05	1.16886-07	4.5116E-09	7.939476-08	8.746145-08	8.743995-08	1.745865-08
14,89536664	7.000938-05	1.23828-07	4.643598-09	8,406688-08	3,97348-08	3.971038-08	1.851672-08
14.89536664	7 200955-05	1.30975-07	4 775535-09	8.887265-08	4.207365-08	4,20476E-08	1.96061E-08
14.88940969	7.400985-05	1.28225-07	4.907456-09	9.291215-09	4.448025-08	4,445185-08	2.072655-08
14,8864321	7.601016-05	1.45866-07	5.039326-09	9.888526-08	4.69546-08	4.69236-08	2.187815-08
10.88305512	7.80104E-05	1.53515-07	5.171155-09	1.040925-07	4.949485-08	9.94515-08	2.305085-08
14.88047872	8.001065-05	1.61565-07	5.302956-09	1.094325-07	5.210275-08	5,206595-08	2,427475-08
14 87750292	8 201095-05	1 69715-07	5 43475-09	1 149075-07	5 477775-00	5 473785-08	2 551975-08
14.87452772	8.401126-05	1.78075-07	5.566428-09	1.205146-07	5.751995-08	5.747665-08	2.679585-08
14.87155311	8.601145-05	1.86628-07	5.698095-09	1.262565-07	6.032915-08	6.02828E-08	2.8103E-08
14,8685791	8.80117E-05	1.25375-07	5.629736-09	1.321315-07	6.320546-08	6.315516-08	2,244145-08
14.06560560	9.00128-05	2.04325-07	5.961316-09	1.30146-07	6.614895-08	6.609475-08	2.08115-08
14.86263286	9.201225-05	2.12486-07	6.09286E-09	1.442836-07	6.915955-08	6.910126-08	2.221165-08
14.85966063	9.401255-05	2.22846-07	6.22438E-09	1.50568-07	7.228786-08	7.217495-08	8.864846-08
14 656666777	9.601285-05	2 32395-07	6.35586E-09	1 56575-07	7.538228-08	7,531546-08	3 510545-08
14.05371795	2.80135-05	2.42156-07	6.487295-09	1.635146-07	7.052446-08	7,85235-08	3.660046-08

Figure 4.23: Computation results for exact results (true results) and true percent relative errors of each RK method for mixture problem

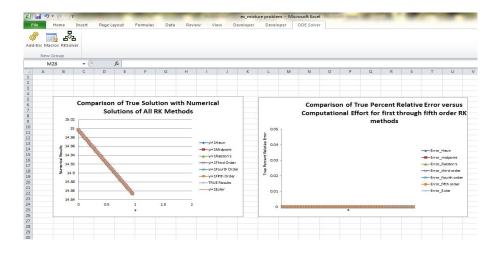


Figure 4.24: Graphical display of the computation results for mixture problem

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-	A		-0 Longer				-0.1499900022		-0.1-0070000 -0				14 MMT 14		LA SUPPORT	LA SUPPORT	LA SETTIONS	14 SUPTOOLS	14 9970009	3 000278-08		2.444476.22	1.000425.10	5.584478-04	A MONTH I	amoni		
VER	3.04 -0.14004 -0	14941	-0.140008018	-0.140017811	-0.1400000038	-0.14000002	-0.140000000		-0.048830007 -0				14.001 14.		14.0013027	14 8830007	14.0053037	14 3830007	14 8810007	4.00040-04	1.00030-08	3 000000-53	8.000.000-0.0	1.000730-00	3 00007E-30	9.338000-11		
_	0.08 -0.14988 -0	1-945	-0.14948300	-0.149907500	-0.149871034	-0.149900179	-0.149900001	-0.149871554	-0.1-0400079 -0		0 14980 -0 1498	14 385	14 947 14	*******	14 3850075	14 9490079	14 3850079	14 9830075	14 3430075	1 000148-03	1.007978-08	8.054509-10	1.000409-09	8.555548-10	6 667196-51	8 11156-10		
	8 1 -0 14000 -0		-0.14000000			-0.14070018	-D LANGERED		A DESCRIPTION OF				LA BER LA		LA GEODINE	TA STOCKED	LA GEDOLINE	LA BESCADE	14 STOCKER	1 200142-04	1 100105-00	7 0041-02.53	3 724270-08	1.000042.00	1.000126-00	A 887962.10		
	114 -014979 -0		-0.149770108		-0.149790877		-0.149772089	-0.249790828	-0.049780080 -0				14.978 14	#1905.80 S	A STRUCTURE	14 PROCESS	14 PROCESCO	14 PROCESS	14 PROCESS	1 000110-02	6.00000-09	1.000710-00	4.805228-08	1.00000-00	1.807080-08	8.713518-30		
													14.67 1	4.47083	LA STOCOGRE					1 006275-05								
	11 -1140° -0		-0.14042030		-0.140400418	-0.149670366	-0.149426331	-0.149490928	-0.040670040 -0				14.847 14	NTONS 1	A 84709417	14 84708407	14.86703607	14.04703427	14 84703407	1 100306-08	1.147240-08	1.46460-00	1000000	1007040-00	3.467768-00	1.711412-00		
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	1 14 -1 14041 -1	Labor	-0.140600520	-D LODGEDOM	-0.140670106	-0.140500577	-0.140540421	-0.140670008	- Constants -	- LANGERSON -	A LINES of LINES	14 6425	A 9581 14	CARDINAL I	A 66306742	14 peacears 14 peacers	14.05206276	LA DEBORATE	14.852062752	3 00040-05	1.007764.04	1.00010.00	1.40071-04	A Design in	7.00002-00	2 221452-00		
	0.82 -0.14900 -0	1-0-0	-0.149009415		-0.1495(2091	-0.149490908	-0.149505817	-0.1495(4091	-0.149490988 -0	149503410	-0.1490 -0.1494	14 9495	19491 11		A SANDOWN	14 BARDONNE	In process	14 9490800	14 BARDINE	1-00-00-00	1.0-00-00	1.101540-09	1.00000-00	1.000705-00	\$ CT1140-CH	A 101000-CB		
	0.30 -0.34940 -		-0.049-08139	-0.140408027	-0.149-0361	-0.549401303	-0.34945834	-0.34940365	-0.049405388 -0	1.149-02039 -	0.14942 -0.1494	14,3402 0	4,0402, 24,	HOLDS D	A BACLINGA	14 PHOLENEH	14.84003884	14.94001984	14 PHOLINE	4 000038-02	4.089730-08	1.00440-09	1.00330-08	1.107700-08	1.007+00-00	3.313400-09		
	0.40 -0.1400* -0	Logian	-0.140006004	-1.1499-9915	-0.140000042	-0.149201452	-0.14020607	-0 140203000	distants of		- Lesis - 6 Loss	ba sanh in	4 9343 L	-	A SECLORY	14 BRILLING	14 STATES		14 BRILLING	4 400402-05	1.017000-00	3.057016-00	2.00000.00	1.600000.00	1.007000-00	7.472006-00		
	2.48 -0 14908 -0	14935	-0.140000000	-0.149023888	-0.149274081	-0.149130873	-0.149308799	-0.349274083	-0.1-01218*** -0		0.14908 -0.1492				A STOLAT	14 STREETS	14 STREETS	14 PERSONAL PROPERTY.	14 STREETS	5.00079-02	5.000000-00	8 811408-09	1.00000-08	1.000179-04	1.001508-08	5 100 TO -CR		
								-0.140214044																				
	0.04 -0.04008 -0	14916	-0.040077004	-0.1490@88004	-0.14858-713	-0.149040301	-0.549577585	-0.348584704	-	1.140077368 -	0.1405* -0.1403	14,8080 1	4,0000 14	10022-0	A 20000477	14 SOLDATE	14 81610477	14 20822-10	14 ROED-TH	3 800730-02	1040400-08	3 718790-09	140803-08	1 100000-00	1.00000-00	1.170308-08		
	14 4146 4	14807	-0.140007104 -0.14002787	-0.1400000000	-0.140006120	-0.14001046	-0.140007784	-0.14909934	-0.048048088 -0	1 140007108	0.14000 -0.1400	14,0070 1	4 8043 14	Second 1	A BOADORDE	14 80128775	14.80138773	14 BOX DOTE	14.80128715	4.000402-04	A 104086-08	4.114270-00	1000408	3 104170-04	3 3000-00	1.040000-00		
	1.04 -0.14904 -0		-0.1-9009001		-0.149090818		-0.149009088	-0.149080814			0 14902 -0 1492				A 90121299	14 PELETER	14 90121209		14 PELETON	0.000000-02	1.101818-0*	* 379000-09	1 40000-08	1 202249-04	1 1110-0-0	1.040130-08		
	0.7 -0.1400 -0	1480	-0.140008771	-0.140001314	-0.048048008		-0.148808773	-0.048948008	-0.04883879 -0	1.148008773 -	0.14800 -0.1488	14,8804 1	4.8824 14.	4812979 2	A BECORTER	14 BRIDETET	14.88138797	14 80036797	14 BELLETER	7.00048-08	1.309690-07	4.775220-09	1.007080-08	4.307360-08	4.004768-08	1.000020-00		
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Figure 4.25: The spreadsheet image of full computation results for mixture problem

This spreadsheet solver is so user-friendly that users (students, educators and also beginner users of Excel and VBA) only require to click RKSOLVER button and enter relevant information in userform to perform all computations for the complete solution of IVPs efficiently without typing any commands in the spreadsheet.

It is hoped that this spreadsheet solver can be used as a marking scheme for users who need the complete solutions of IVPs numerically and analytically with comparison of them in terms of error at the same time. Lastly, it is hoped that this spreadsheet solver could serve as not only a numerical IVP tool but also an analytical IVP tool with a comparison of them that is convenient for the community of engineering educators and students.

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