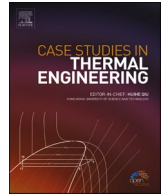




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Engine oil based MoS₂ Casson nanofluid flow with ramped boundary conditions and thermal radiation through a channel

Imran Siddique^a, Kashif Sadiq^a, Mohammed M.M. Jaradat^{b,*}, Rifaqat Ali^c, Fahd Jarad^{d,e,f,**}

^a Department of Mathematics, University of Management and Technology, Lahore, 54770, Pakistan

^b Mathematics Program, Department of Mathematics, Statistics and Physics, College of Arts and Sciences, Qatar University, 2713, Doha, Qatar

^c Department of Mathematics, College of Science and Arts, King Khalid University, Muhayil, 61413, Abha, Saudi Arabia

^d Department of Mathematics, Cankaya University, Etimesgut, Ankara, Turkey

^e Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia

^f Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan

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ABSTRACT

The modern era is a time to have cost-effective and energy-efficient technology. This demand has made nanotechnology the most effective field. The focus of this article is to increase the efficiency of engine oil (EO). The flow of EO-based Casson nanofluid containing Molybdenum disulfide (MoS₂) nanoparticles is investigated with ramped wall conditions and thermal radiation. Analytical results are calculated via the Laplace transform. The impact of physical parameters on isothermal and ramped conditions is illustrated graphically and discussed in detail. The researchers found that flow, mass, and energy can be controlled by using ramped conditions. The variation in concentration, temperature, and velocity is exponential for isothermal conditions and steady for ramped wall conditions. Finally, the results of Nusselt numbers, skin frictions, and Sherwood numbers on both walls of the channel for both isothermal and ramped conditions are graphically depicted and discussed. For higher values of time the results of ramped and isothermal wall conditions are identical. It is found that the nanoparticles of MoS₂ enhance the lubrication and heat transport rates of EO.

Nomenclature

$\tilde{C}(\tilde{y}, \tilde{t})$	Concentration (kg m ⁻³)
$\tilde{u}(\tilde{y}, \tilde{t})$	Velocity (m s ⁻¹)
$\tilde{T}(\tilde{y}, \tilde{t})$	Temperature (K)
k	Thermal Conductivity (W m ⁻¹ K ⁻¹)
g	Gravitational acceleration (m s ⁻²)
D	Mass diffusivity (m ² s ⁻¹)

* Corresponding author. Mathematics Program, Department of Mathematics, Statistics and physics, College of Arts and sciences, Qatar University, Doha, Qatar.

** Corresponding author. Department of Mathematics, Cankaya University, Etimesgut, Ankara, Turkey.

E-mail addresses: imransmsrazi@gmail.com (I. Siddique), kashifsaq2525@gmail.com (K. Sadiq), mmjst4@qu.edu.qa (M.M.M. Jaradat), rrafat@kku.edu.sa (R. Ali), fahd@cankaya.edu.tr (F. Jarad).

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<i>Gr</i>	Thermal Grashof number
<i>Pr</i>	Prandtl number
<i>c_p</i>	Specific heat (J kg ⁻¹ K ⁻¹)
<i>Gm</i>	Mass Grashof number
<i>R</i>	Chemical reaction (s ⁻¹)
<i>Sc</i>	Schmidt number
<i>Nu</i>	Nusselt number
<i>Sk</i>	Skin friction
<i>Sh</i>	Sherwood number

Greek Symbols

θ	Dimensionless temperature
μ	Dynamic Viscosity (kg m ⁻¹ s ⁻¹)
ρ	Density (kg m ⁻³)
βT	Thermal expansion (K ⁻¹)
φ	Nanoparticles volume fraction
ν	Kinematic viscosity (m ² s ⁻¹)
βC	Mass volumetric (K ⁻¹)
γ	Casson Parameter

Subscript

<i>f</i>	Fluid
<i>nf</i>	Nanofluid
<i>s</i>	Solid particles

1. Introduction

Non-Newtonian fluids are attracting researchers due to their applications in the industrial and technological sciences. Sugar solutions, blood, clay coating, paints, drilling mud, certain oils, and synthetic lubricants are different types of non-Newtonian fluids. These fluids cannot be defined by the Navier-Stokes equations due to their complex mathematical formulation. Jeffrey, Burger, Casson, Carreau, Maxwell, Oldroyd-B, Seely, Bulky and Eyring-Powell fluids are modeled to classify various non-Newtonian fluids according to their characteristics. The model of Casson [1] fluid helped to understand the features of drying oil deferments of ink used for printing. Casson fluids are shear-thinning fluids [2] and have infinite viscosity at zero shear stress and zero viscosity at infinite shear stress. Human blood, soup, honey, tomato sauce, and jelly are different Casson fluids.

For the last two decades, researchers have been investigating nanofluids due to their extraordinary characteristics. Conventional fluids like engine oil, water, ethylene glycol, sodium alginate, and kerosene oil have significant use in the application of heat transfer, but these reduce the rate of heat transport due to low thermal conductivities. The nanofluids are useful to save energy and reduce the temperature of the equipment [3–6]. For instance, the most common uses of nanofluids are as lubricants in the transportation of heavy machines, biomedical tools, heat exchangers, food processing, fuel cells, biomedicine, computer microchips, and coolants in automobiles [7,8]. Loganathan et al. [9] established the first analytical solution of convective nanofluid flow with radiation. The characteristics of carbon nanotubes in a fluid affected by partial slip were investigated by Reddy et al. [10]. Archana et al. [11] studied the incompressible and compressed flow of Casson nanofluid, which is used as a lubricant, between two parallel plates. The constitutive equations are included in the mathematical formulation. Reddy et al. [12] explored magnetohydrodynamic flow, thermal conduction, and diffusion nanofluid flow over a stretched sheet.

To increase heat transport properties, a term called “hybrid nanofluid”, which is a novel form of nanofluid, has recently been used. Solar cells, power generation engines, cooling devices, naval structures, medical, defense, transportation, and micro-fluidics are a few examples of nanofluids. Reddy et al. [13] examined the hybrid nanofluid flow across a revolving disc. Kumar et al. [14] used hybrid nanofluids to investigate the sustainability of heat transport increases caused by major characteristics variations of nanofluids under the influence of thermal radiation. Reddy et al. [15] used the Koo and Kleinstreuer model to examine the Blasius and Rayleigh-Stokes flow of aluminium alloys across a semi-infinite heated plate in Darcy-Forchheimer porous space under the impact of nonlinear radiation. Souayah [16] established a mathematical model for dusty hybrid nanofluid flow and heat transmission across a stretched sheet. An incompressible two-dimensional flow of hybrid dusty nanofluid in a Darcy-Forchheimer medium on a stretched sheet was studied by Reddy et al. [17]. Kumar et al. [18] investigated the nature of a moving frame hydrodynamic hybrid nanofluid by applying solar radiation.

The EO is used as a lubricant and has significant applications in the mechanical, chemical, and production industries. Meng et al. [19] investigated EO-silver nanofluid. Eswaraiah et al. [20] analyzed the results of the EO-graphene nanofluid. Aman et al. [21] calculated the analytical results of Maxwell nanofluid (MoS₂-EO) flow. Arif et al. [22] discussed the applications of EO-based nanofluids. The use of nanoparticles of various shapes helps to improve the efficiency of lubricants. The nanoparticles of MoS₂ are added to EO due to its robustness and low friction, which boost the lubricity and thermal conductivity of EO [23,24]. MoS₂ is a black, silvery solid that appears as the mineral molybdenite and contains atoms of sulphur and molybdenum and is classified in the class of inorganic

compounds. MoS₂ is comparatively unaffected and nonreactive by dilute oxygen and acids. Zhang et al. [25] experimentally prepared stable MoS₂ nanofluids. The major attraction of MoS₂ for researchers is its applications, mainly in two-dimensional (2D) electronic machines like amplifiers, logic circuits, and field-effect transistors [26,27]. Researchers also investigated factors like volumetric expansion, thermal conductivity, and heat capacity [28–33].

The use of ramped wall conditions is very significant in different sections of modern science and technology. For example, ramped velocity helps to evaluate the functioning of blood vessels and the heart, to diagnose blood-vascular diseases, and to determine treatment and find prognosis involving treadmill ergometry [34]. Despite the practical propositions, not enough study is done with ramped conditions. Ramped conditions play a vital role in managing the flow, mass, and energy of the fluid. Ahmed and Dutta [35] introduce ramped velocity and temperature for Newtonian fluid flow on a vertical moving plate. Ali et al. [36], Seth et al. [37] and Kao [38], they studied the impact of ramped wall temperature on diverse physical phenomena of modern science and found the results. Seth et al. [39–41] discussed mass and heat transport under the effect of ramped heating. Chandran et al. [42] analyzed mixed convection flow with ramped heating. Arif et al. [43] examined the EO-based fractionalized Casson fluid containing nanoparticles of graphene oxide and MoS₂ with ramped heating and ramped concentration.

The movement of machine parts produces additional heat owing to frictional forces in various operating systems, causing machine components to be unable to continue their functions and stop working before time. We have worked to develop the most significant qualities of lubricants for machines, such as EO, so that they can have a long service life and work for longer periods of time. Additives are critical components in oils and greases, as well as in the long-term performance of bearings and other machine components. Keeping these theories in mind, MoS₂ nanoparticles are used in the current investigation. The rate of heat transfer will be enhanced without reducing the thickness of EO. As a consequence, the life of machineries such as bearings, automotive engines, and turbines will be extended, and they will operate more efficiently. In the above literature, most of the investigations are on ramped temperature, and in very few, ramped concentration is included. There is no evidence of the results of the Casson nanofluid flow with ramped velocity. Also, most of the studies are for the flow on a single plate. Here the motivation is to discuss the flow of EO-MoS₂ Casson nanofluid in the channel with ramped wall conditions and thermal radiation. The particles of MoS₂ are dispersed in EO to prepare a nanofluid. Analytical results are calculated via the Laplace transform. The influence of physical parameters on isothermal and ramped conditions is depicted graphically and explained.

2. Mathematical model

Consider an incompressible, unsteady flow of Casson nanofluid inside two infinite vertical parallel plates (walls) with ramped boundary conditions and thermal radiation. Nanoparticles of MoS₂ are suspended in EO to have a nanofluid.

At $t^* = 0$ ($y^* = 0$ and $y^* = l$), the temperature, velocity, and concentration are uniform. At $0 < t^* < t_0^*$, the temperature, concentration, and velocity of left plate change temporarily to $T_l^* + (T_0^* - T_l^*)t^*/t_0^*$, $C_l^* + (C_0^* - C_l^*)t^*/t_0^*$, and $u_0 t^*/t_0^*$, respectively. After $y^* = l$, the system attains its initial position (see Fig. 1). We have made the following assumptions:

- Nanofluid is considered optically thick. Therefore the Rosseland approximations can be considered for radiation effects.
- The flow is being confined to $y^* > 0$.
- For $t^* > 1$, the ramped wall conditions changes to isothermal conditions.
- Velocity, concentration, and temperature are functions of y^* and t^* only.

Thermo-physical features of EO and MoS₂ are assumed constant and shown in Table 1.

The governing Eqs. are [43,44]:

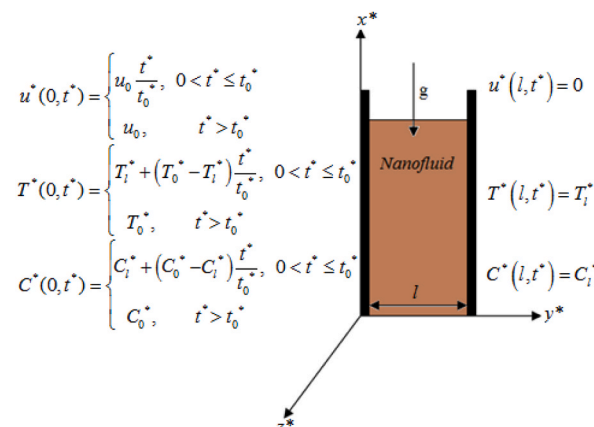


Fig. 1. Flow geometry.

Table 1
Thermo-physical properties of EO and MoS₂ [43].

Material	ρ	k	$\beta \times 10^{-5}$	c_p
Engine oil (EO)	884	0.144	70	1910
MoS ₂	5600	904.4	2.8424	397.21

$$\rho_{nf} \frac{\partial u^*}{\partial t^*} = \mu_{nf} \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u^*}{\partial y^{*2}} + g(\rho\beta_C)_{nf}(C^*(y^*, t^*) - C_l^*) + g(\rho\beta_T)_{nf}(T^*(y^*, t^*) - T_l^*), \tag{1}$$

$$(\rho c_p)_{nf} \frac{\partial T^*}{\partial t^*} = k_{nf} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*}, \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D_{nf} \frac{\partial^2 C^*}{\partial y^{*2}}, \tag{3}$$

with corresponding conditions are [45]:

$$u^*(y^*, 0) = 0, T^*(y^*, 0) = T_l^*, C^*(y^*, 0) = C_l^*, 0 \leq y^* \leq l, \tag{4}$$

$$u^*(0, t^*) = \begin{cases} u_0 \frac{t^*}{t_0^*}, & 0 < t^* \leq t_0^*; \\ u_0, & t^* > t_0^* \end{cases}, T^*(0, t^*) = \begin{cases} T_l^* + (T_0^* - T_l^*) \frac{t^*}{t_0^*}, & 0 < t^* \leq t_0^*; \\ T_0^*, & t^* > t_0^* \end{cases}, \tag{5}$$

$$C^*(0, t^*) = \begin{cases} C_l^* + (C_0^* - C_l^*) \frac{t^*}{t_0^*}, & 0 < t^* \leq t_0^*; \\ C_0^*, & t^* > t_0^* \end{cases},$$

$$u^*(l, t^*) = 0, T^*(l, t^*) = T_l^*, C^*(l, t^*) = C_l^*. \tag{6}$$

The characteristics of nanofluids are [46,47]:

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\varphi)^{2.5}}, \frac{\rho_{nf}}{\rho_f} = \varphi \frac{\rho_s}{\rho_f} + (1-\varphi), \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} = \varphi \frac{(\rho c_p)_s}{(\rho c_p)_f} + (1-\varphi), \tag{7}$$

$$\frac{(\rho\beta_T)_{nf}}{(\rho\beta_T)_f} = \varphi \frac{(\rho\beta_T)_s}{(\rho\beta_T)_f} + (1-\varphi), \frac{(\rho\beta_C)_{nf}}{(\rho\beta_C)_f} = \varphi \frac{(\rho\beta_C)_s}{(\rho\beta_C)_f} + (1-\varphi), \tag{8}$$

$$\frac{D_{nf}}{D_f} = (1-\varphi), k_{nf} = k_f \frac{[2k_f + k_s - 2\varphi(k_f - k_s)]}{[2k_f + k_s + \varphi(k_f - k_s)]}. \tag{9}$$

Non-dimensional variables, functions, and parameters are

$$u = \frac{u^*}{u_0}, t = \frac{t^*}{t_0^*}, t_0^* = \frac{l^2}{\nu_f}, y = \frac{y^*}{l}, \theta = \frac{T^* - T_l^*}{T_0^* - T_l^*}, C = \frac{C^* - C_l^*}{C_0^* - C_l^*}, \alpha_1 = \frac{\rho_f}{\rho_{nf}(1-\varphi)^{2.5}} \left(1 + \frac{1}{\gamma} \right),$$

$$\alpha_2 = Gr \frac{(\beta_T)_{nf}}{(\beta_T)_f}, \alpha_3 = Gm \frac{(\beta_C)_{nf}}{(\beta_C)_f}, \alpha_4 = \frac{1}{Pr} \left(\frac{k_{nf}}{k_f} + Nr \right) \frac{(\rho c_p)_f}{(\rho c_p)_{nf}}, \alpha_5 = \frac{1-\varphi}{Sc},$$

$$Gm = \frac{g(\beta_C)_f(C_0 - C_l)d^2}{U_0\nu_f}, Gr = \frac{g(\beta_T)_f(T_0 - T_l)d^2}{U_0\nu_f}, Pr = \frac{(\rho c_p)_f \nu_f}{k_f}, Sc = \frac{\nu_f}{D_f}. \tag{10}$$

By substituting Eq. (10) to Eqs. (1)–(6), we get

$$\frac{\partial u(y, t)}{\partial t} = \alpha_1 \frac{\partial^2 u(y, t)}{\partial y^2} + \alpha_2 \theta(y, t) + \alpha_3 C(y, t), \tag{11}$$

$$\frac{\partial \theta(y, t)}{\partial t} = \alpha_4 \frac{\partial^2 \theta(y, t)}{\partial y^2}, \tag{12}$$

$$\frac{\partial C(y, t)}{\partial t} = \alpha_5 \frac{\partial^2 C(y, t)}{\partial y^2}. \quad (13)$$

$$u(y, 0) = \theta(y, 0) = C(y, 0) = 0; \quad 0 \leq y \leq 1, \quad (14)$$

$$u(0, t) = \theta(0, t) = C(0, t) = \begin{cases} t, & 0 < t \leq 1; \\ 1, & t > 1 \end{cases} = H(t)t - H(t-1)(t-1), \quad (15)$$

$$u(1, t) = \theta(1, t) = C(1, t) = 0. \quad (16)$$

3. Solution of the problem

Applying Laplace transform to Eqs. 11–13, (15), (16) and using (14), we obtain

$$\alpha_1 \frac{\partial^2 \bar{u}(y, \tau)}{\partial y^2} - \tau \bar{u}(y, \tau) = -\alpha_2 \bar{\theta}(y, \tau) - \alpha_3 \bar{C}(y, \tau), \quad (17)$$

$$\alpha_4 \frac{\partial^2 \bar{\theta}(y, \tau)}{\partial y^2} - \tau \bar{\theta}(y, \tau) = 0, \quad (18)$$

$$\alpha_5 \frac{\partial^2 \bar{C}(y, \tau)}{\partial y^2} - q \bar{C}(y, \tau) = 0, \quad (19)$$

and

$$\bar{u}(0, \tau) = \bar{\theta}(0, \tau) = \bar{C}(0, \tau) = \tau^{-2}(1 - e^{-\tau}), \quad \bar{u}(1, \tau) = \bar{\theta}(1, \tau) = \bar{C}(1, \tau) = 0. \quad (20)$$

The solution of Eqs. 17–19 subject to Eq. (20), gives

$$\begin{aligned} \bar{u}(y, \tau) = (1 - e^{-\tau}) & \left[\left(\frac{1}{\tau} + \frac{\alpha_2 \alpha_4}{(\alpha_1 - \alpha_4) \tau^2} + \frac{\alpha_3 \alpha_5}{(\alpha_1 - \alpha_5) \tau^2} \right) \frac{\sinh \left[(1-y) \sqrt{\frac{\tau}{\alpha_1}} \right]}{\tau \sinh \left[\sqrt{\frac{\tau}{\alpha_1}} \right]} \right. \\ & \left. - \frac{\alpha_2 \alpha_4}{(\alpha_1 - \alpha_4) \tau^2} \frac{\sinh \left[(1-y) \sqrt{\frac{\tau}{\alpha_4}} \right]}{\tau \sinh \left[\sqrt{\frac{\tau}{\alpha_4}} \right]} - \frac{\alpha_3 \alpha_5}{(\alpha_1 - \alpha_5) \tau^2} \frac{\sinh \left[(1-y) \sqrt{\frac{\tau}{\alpha_5}} \right]}{\tau \sinh \left[\sqrt{\frac{\tau}{\alpha_5}} \right]} \right], \end{aligned} \quad (21)$$

$$\bar{\theta}(y, \tau) = (1 - e^{-\tau}) \frac{\sinh \left[(1-y) \sqrt{\frac{\tau}{\alpha_4}} \right]}{\tau^2 \sinh \left[\sqrt{\frac{\tau}{\alpha_4}} \right]}. \quad (22)$$

$$\bar{C}(y, \tau) = (1 - e^{-\tau}) \frac{\sinh \left[(1-y) \sqrt{\frac{\tau}{\alpha_5}} \right]}{\tau^2 \sinh \left[\sqrt{\frac{\tau}{\alpha_5}} \right]}. \quad (23)$$

The inverse Laplace transform of Eqs. 21–23, gives

$$u(y, t) = u_0(y, t) - H(t-1)u_0(y, t-1), \quad (24)$$

$$\theta(y, t) = \sum_{n=0}^{\infty} \int_0^t f(y, \alpha_4, \tau) d\tau - H(t-1) \sum_{n=0}^{\infty} \int_0^t f(y, \alpha_4, \tau-1) d\tau, \quad (25)$$

$$C(y, t) = \sum_{n=0}^{\infty} \int_0^t f(y, \alpha_5, \tau) d\tau - H(t-1) \sum_{n=0}^{\infty} \int_0^t f(y, \alpha_5, \tau-1) d\tau, \quad (26)$$

where

$$u_0(y, t) = \sum_{n=0}^{\infty} \int_0^t \left(1 + \frac{\alpha_2 \alpha_4 \tau}{(\alpha_1 - \alpha_4)} + \frac{\alpha_3 \alpha_5 \tau}{(\alpha_1 - \alpha_5)} \right) f(y, \alpha_1, t - \tau) d\tau$$

$$- \frac{\alpha_2 \alpha_4}{(\alpha_1 - \alpha_4)} \sum_{n=0}^{\infty} \int_0^t (t - \tau) f(y, \alpha_4, \tau) d\tau - \frac{\alpha_3 \alpha_5}{(\alpha_1 - \alpha_5)} \sum_{n=0}^{\infty} \int_0^t (t - \tau) f(y, \alpha_5, \tau) d\tau, \tag{27}$$

and

$$F(y, \alpha_k, q) = \frac{\sinh\left(\frac{(1-y)}{\sqrt{\alpha_k}} \sqrt{q}\right)}{q \sinh\left(\frac{1}{\sqrt{\alpha_k}} \sqrt{q}\right)} = \sum_{n=0}^{\infty} \left[\frac{e^{-\frac{(2n+y)}{\sqrt{\alpha_k}} \sqrt{q}}}{q} - \frac{e^{-\frac{(2n+2-y)}{\sqrt{\alpha_k}} \sqrt{q}}}{q} \right], \quad k = 1, 4, 5. \tag{28}$$

$$f(y, \alpha_k, t) = \operatorname{erfc}\left(\frac{2n+y}{2\sqrt{\alpha_k t}}\right) - \operatorname{erfc}\left(\frac{2n+2-y}{2\sqrt{\alpha_k t}}\right), \quad k = 1, 4, 5. \tag{29}$$

4. Nusselt numbers, skin friction, and Sherwood numbers

The Nusselt numbers, skin frictions, and Sherwood numbers on both walls of the channel can express as [43,48].

$$Nu_{0,1} = - \frac{k_{nf}}{k_f} \frac{\partial \theta(y, t)}{\partial y} \Big|_{y=0,1} \tag{30}$$

$$Sk_{0,1} = - \frac{\mu_{nf}}{\mu_f} \left(1 + \frac{1}{\gamma} \right) \frac{\partial u(y, t)}{\partial y} \Big|_{y=0,1} \tag{31}$$

$$Sh_{0,1} = - \frac{D_{nf}}{D_f} \frac{\partial C(y, t)}{\partial y} \Big|_{y=0,1} \tag{32}$$

5. Graphical results and discussions

The flow of EO-MoS₂ nanofluid is analyzed graphically in the section. Figs. 2–4 show the impacts of variation of physical parameters volume fraction (φ), Casson parameter (γ), Schmidt number (Sc), Prandtl number (Pr) and Grashof numbers (Gr and Gm) on non-dimensional velocity, concentration, and temperature fields. Furthermore, the figures also show the comparison of results for both ramped and isothermal conditions. The results of ramped boundary conditions are applicable for time $0 < t < 1$, isothermal conditions are suitable for time $t > 1$. The results show that the variations with ramped conditions are steady compare to isothermal conditions, which are exponential. For higher values of time ($t > 1$) the results of ramped and isothermal wall conditions are identical.

Figs. 2(a), 3(a) and 4(a) show how φ affects concentration, energy, and momentum profiles. The concentration and velocity profiles are decreasing due to the thickness caused by the higher density of MoS₂ particles, the temperature of the nanofluid rises due to the superior thermal conductivity of MoS₂. Fig. 2(b) illustrates that the concentration decreases by increasing Sc .

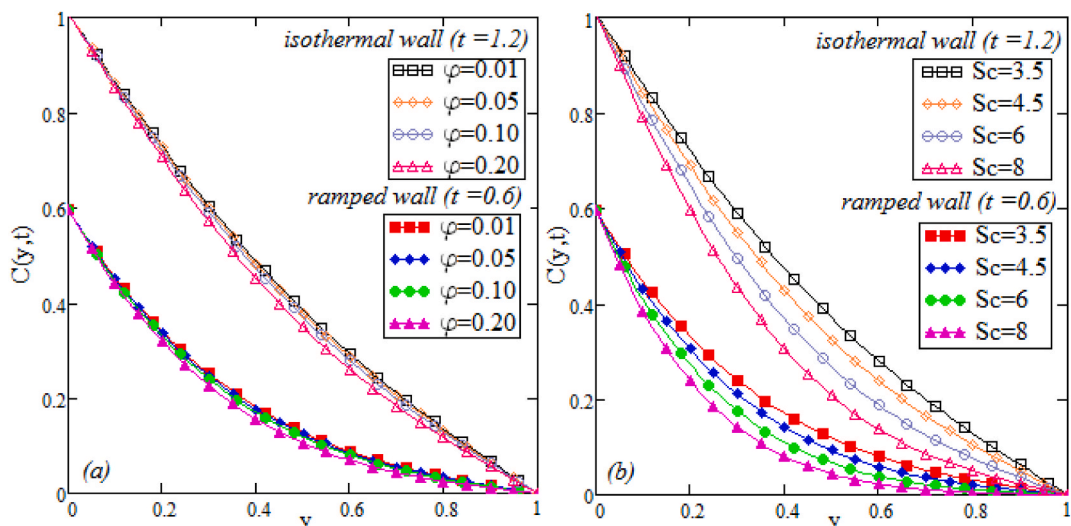


Fig. 2. Variation of concentration when $Sc = 3.3, \varphi = 0.04$.

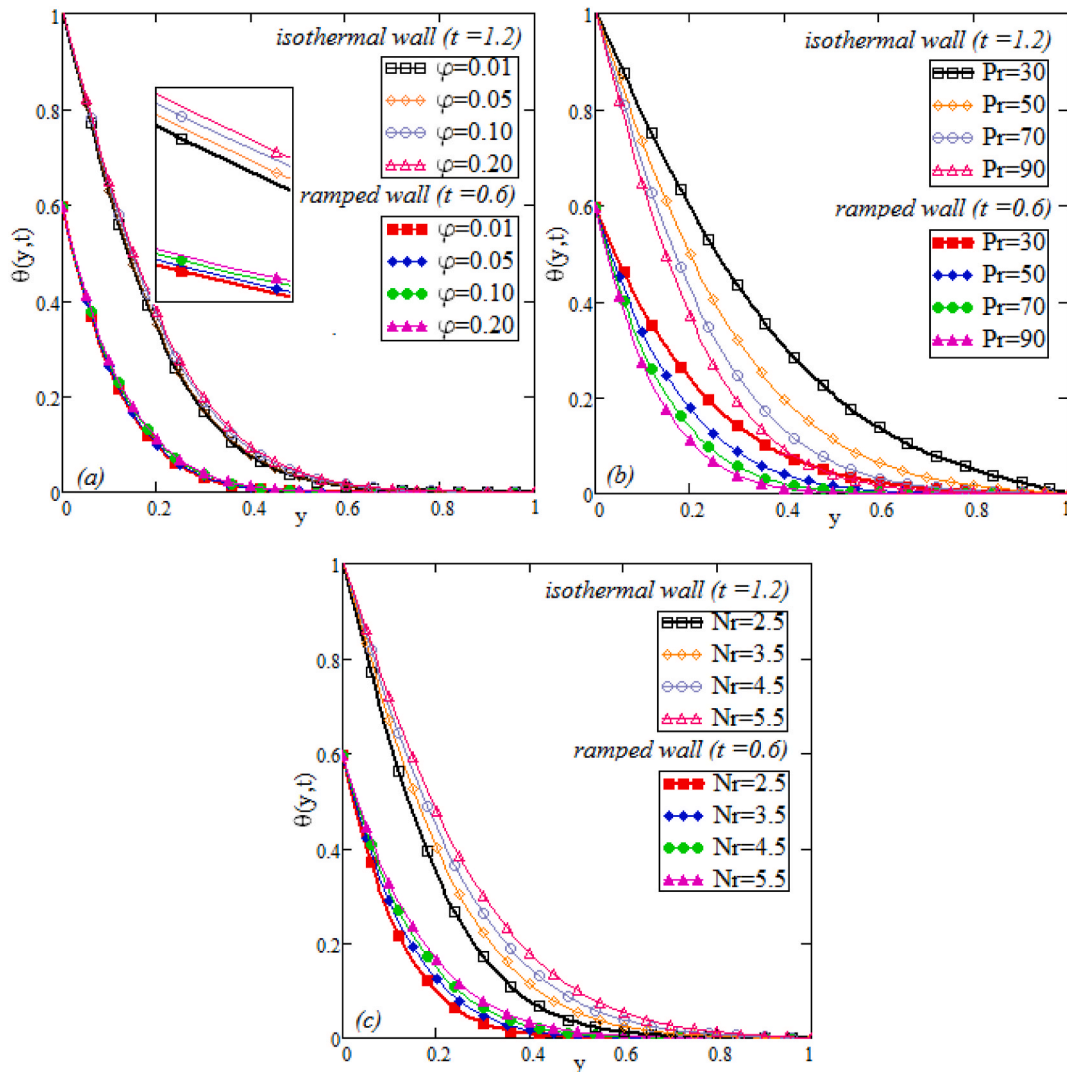


Fig. 3. Variation of temperature when $\phi = 0.04, Pr = 100, Nr = 2.5$.

Fig. 3(b) illustrates that the viscosity of the nanofluid rises for greater values of Pr which reduces the temperature. Fig. 3(c) demonstrates that the increase in temperature at higher thermal radiation. Fig. 4(b) and (c) show the influence of buoyancy forces (Gr and Gm) on velocity. The rise in these forces increases the flow of the nanofluid. The influence of γ on the velocity profile is seen in Fig. 4(d). For increasing values of γ , the velocity increases exponentially for the isothermal conditions and steadily for the ramped conditions. With ramped and isothermal conditions, flow, energy, and concentration are compared. It has been proven that ramped velocity, concentration, and temperature are less than those obtained under isothermal circumstances. As a result, the ramped wall conditions are more suitable.

Fig. 5 depicts the rate of heat transport on both walls of the channel with isothermal and ramped conditions. An exponential rise is found in heat transfer by increasing ϕ .

Fig. 6 demonstrates the fluctuation of skin frictions on both walls of the channel with isothermal and ramped conditions. An increase in skin friction is noticed by increasing ϕ .

Fig. 7 illustrates the rate of mass transport on both walls of the channel with isothermal and ramped conditions. The rate of mass transfer increases by increasing ϕ .

Fig. 8 illustrates the comparison of present results with existing results of Kashif et al. [45]. It is concluded that in the absence of chemical reactions (K, R), heat generation (Q), thermal radiation (Nr), and Casson Parameter ($\gamma \rightarrow \infty$) the results are identical.

6. Conclusions

Analytical results are calculated via the Laplace transform. The influence of physical parameters on isothermal and ramped conditions is illustrated graphically and discussed in detail. The researchers found that flow, mass, and energy can be controlled by using

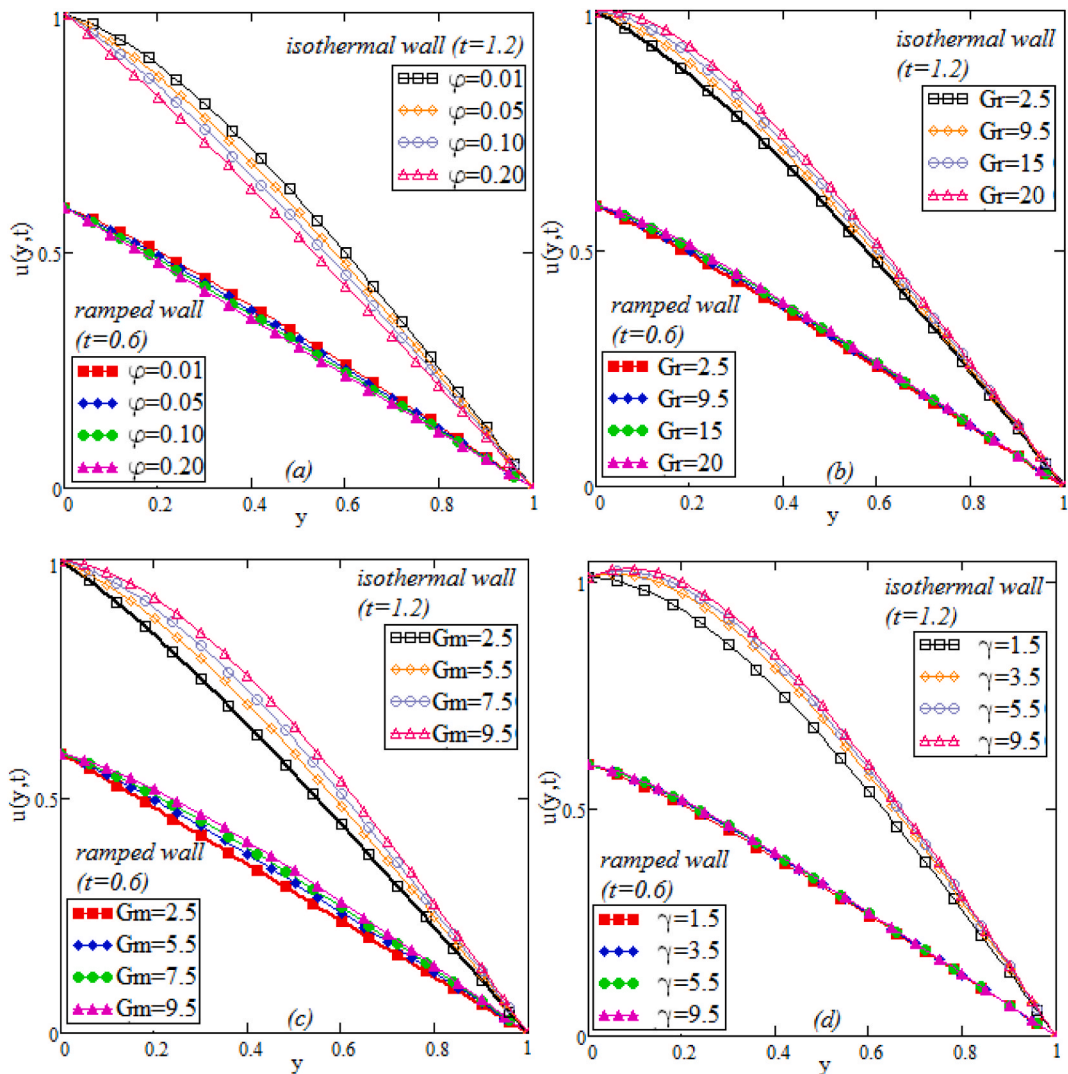


Fig. 4. Variation of velocity when $\phi = 0.04, Gr = 4.5, Pr = 100, Sc = 1.3, \gamma = 0.5, Gm = 5.2, Nr = 2.5$.

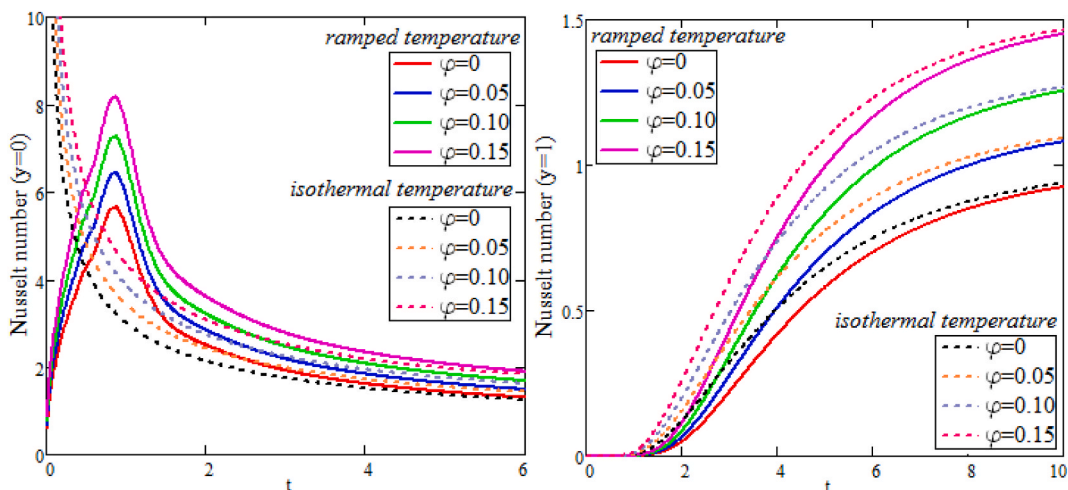


Fig. 5. Effects of ϕ on Nusselt numbers when $Pr = 100, Nr = 2.5$.

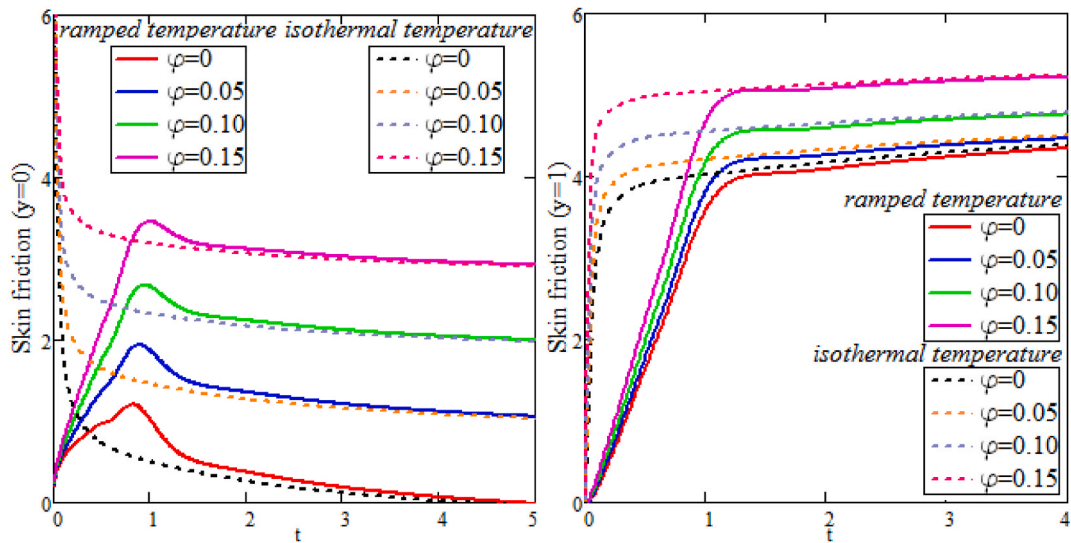


Fig. 6. Effects of φ on skin frictions when $Gr = 4.5, Pr = 100, Sc = 1.3, \gamma = 0.5, Gm = 5.2, Nr = 2.5$.

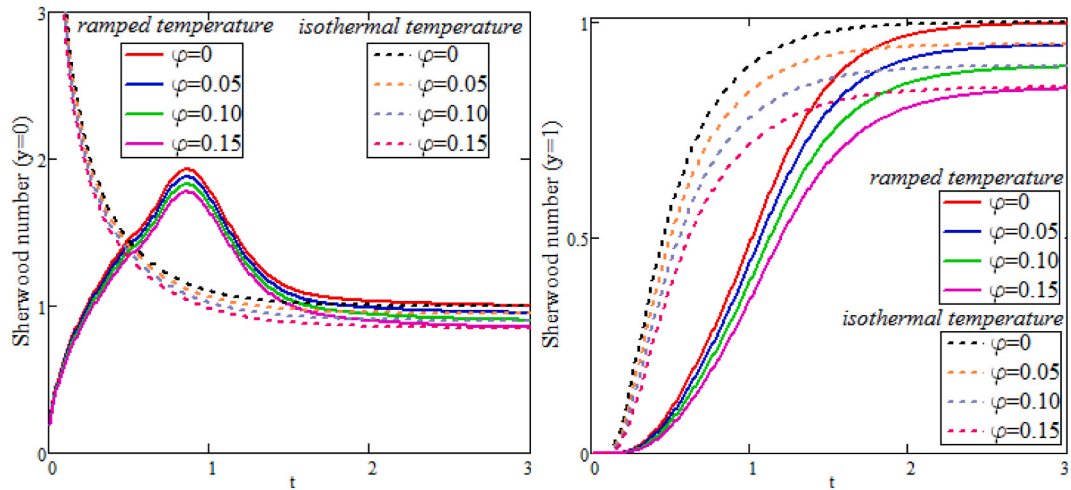


Fig. 7. Effects of φ on Sherwood numbers when $Sc = 3.3$.

ramped conditions. The variation in concentration, temperature, and velocity is exponential for isothermal boundary conditions and steady for ramped conditions. Finally, the results of Nusselt numbers on both walls ($y = 0$ and $y = 1$) of the channel for both ramped conditions and isothermal conditions are graphically depicted and discussed. MoS_2 nanoparticles enhance the lubrication and heat transport rates of EO. An integral transform (Laplace) is used to calculate the results. A nanofluid is prepared by adding nanoparticles of MoS_2 to EO. The noteworthy outcomes for velocity, concentration, temperature, and Nusselt numbers for both ramped and isothermal conditions are graphically highlighted and explained in depth.

The following are the most important outcomes of this research:

- Isothermal boundary conditions have higher concentrations, speeds, and temperatures than ramped boundary conditions.
- Ramped conditions reduce the Casson parameter's effect.
- With ramped wall conditions, variations in concentration, velocity, and temperature fields can be controlled.
- For increasing values of γ, Gr, Gm , the velocity field rises, while for increasing values of φ , the velocity field decreases.
- As Pr grows, the temperature drops, and as φ rises, the temperature increases.
- As Sc increases, the concentration of the ramped wall decreases.
- The Nusselt numbers skin frictions rise with the rise of φ .
- The Sherwood numbers decrease with the increase of φ .
- For higher values of t the results of ramped and isothermal wall conditions are identical.

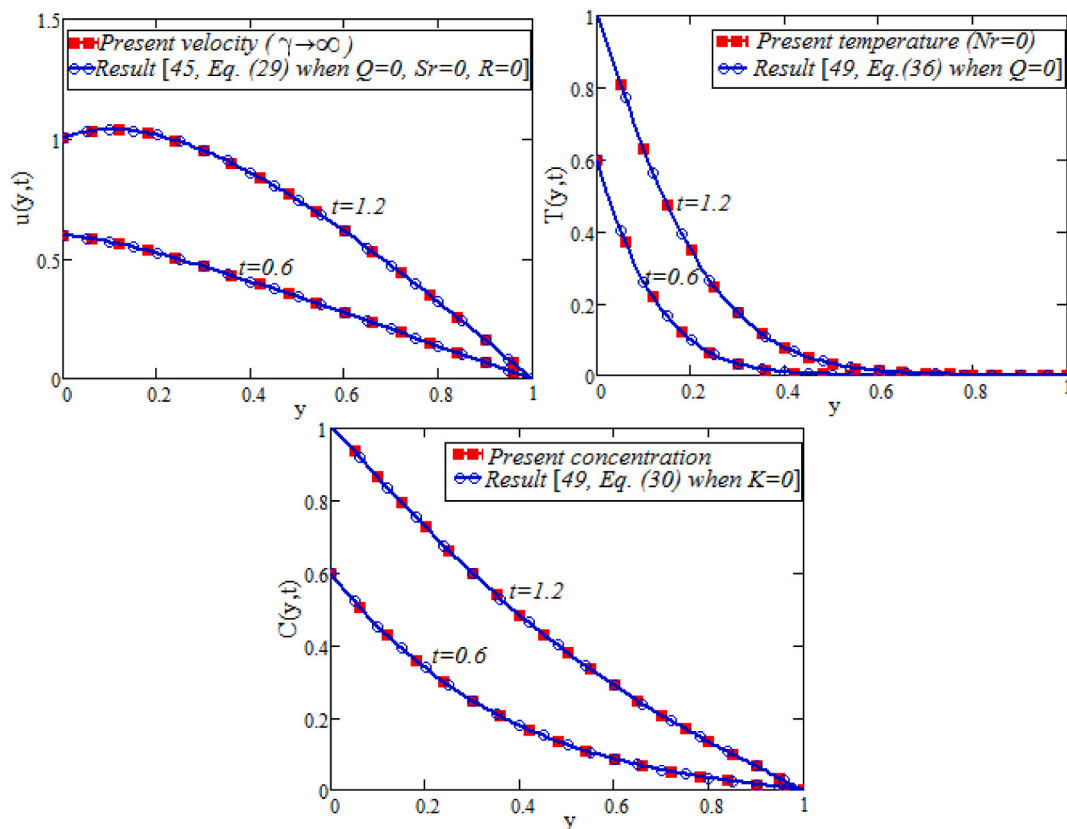


Fig. 8. Comparison of results with [49].

CRediT author statement

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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