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Evolutionary Mathematical Science, Fractional Modeling and Artificial Intelligence of Nonlinear Dynamics in Complex Systems

Yeliz Karaca[*,1] **and Dumitru Baleanu**[γ,α,2]
[*]University of Massachusetts Chan Medical School (UMASS), 55 Lake Avenue North, Worcester, MA 01655, USA, [γ]Çankaya University, Balgat Campus, Çankaya 06530, Ankara, Türkiye, [α]Institute of Space Science (ISS), 409 Atomistilor Street, Magurele, Ilfov 077125, Romania.

ABSTRACT Complex problems in nonlinear dynamics foreground the critical support of artificial phenomena so that each domain of complex systems can generate applicable answers and solutions to the pressing challenges. This sort of view is capable of serving the needs of different aspects of complexity by minimizing the problems of complexity whose solutions are based on advanced mathematical foundations and analogous algorithmic models consisting of numerous applied aspects of complexity. Evolutionary processes, nonlinearity and all the other dimensions of complexity lie at the pedestal of time, reveal time and occur within time. In the ever-evolving landscape and variations, with causality breaking down, the idea of complexity can be stated to be a part of unifying and revolutionary scientific framework to expound complex systems whose behavior is perplexing to predict and control with the ultimate goal of attaining a global understanding related to many branches of possible states as well as high-dimensional manifolds, while at the same time keeping abreast with actuality along the evolutionary and historical path, which itself, has also been through different critical points on the manifold. In view of these, we put forth the features of complexity of varying phenomena, properties of evolution and adaptation, memory effects, nonlinear dynamic system qualities, the importance of chaos theory and applications of related aspects in this study. In addition, processes of fractional dynamics, differentiation and systems in complex systems as well as the dynamical processes and dynamical systems of fractional order with respect to natural and artificial phenomena are discussed in terms of their mathematical modeling. Fractional calculus and fractional-order calculus approach to provide novel models with fractional-order calculus as employed in machine learning algorithms to be able to attain optimized solutions are also set forth besides the justification of the need to develop analytical and numerical methods. Subsequently, algorithmic complexity and its goal towards ensuring a more effective handling of efficient algorithms in computational sciences is stated with regard to the classification of computational problems. We further point out the neural networks, as descriptive models, for providing the means to gather, store and use experiential knowledge as well as Artificial Neural Networks (ANNs) in relation to their employment for handling experimental data in different complex domains. Furthermore, the importance of generating applicable solutions to problems for various engineering areas, medicine, biology, mathematical science, applied disciplines and data science, among many others, is discussed in detail along with an emphasis on power of predictability, relying on mathematical sciences, with Artificial Intelligence (AI) and machine learning being at the pedestal and intersection with different fields which are characterized by complex, chaotic, nonlinear, dynamic and transient components to validate the significance of optimized approaches both in real systems and in related realms.

KEYWORDS
Complex systems
Chaos theory
Chaos-order and complexity
Computational complexity
Fractal operators
Fractional calculus
Fractional dynamics
Evolutionary models
Fractional-order algorithms
Complex-valued neural networks
Nonlinear systems
Data science
Mathematical biology and medicine
Prediction of changes
Fractional integro-differentiation
Artificial Intelligence.

INTRODUCTION

Having existed as a notion since antiquity, complexity is a concept and scientific term that entails the nexus of the origin of complex components accompanied not only by meticulous and detailed computations but also causal processes. A complex system, in that regard, is one with multiple interactions emerging and occurring among interacting components with adapting, synchronizing, noisy, reacting, self-ordering, self-steering, self-similar, irregular, non-periodic and unpredictable elements, feedback loops as well as evolving features, amidst many others. Complexity starts when causality breaks down and it reveals many deep layers considering the structure with variational principles and far-reaching conditions including spontaneous order, nonlinearity, feedback, robustness, lack of central control, numerosity, hierarchical organization and emergence.

The substantial number of independent interacting components and multiple pathways through which the complex system can evolve further point out a number of the reasons causality breaks down as complexity starts. Along these lines, causality is relative, prone to fundamental variations that depend on perception, the environment, external factors, space, time and so (Karaca 2022c). The inherent complexity of the varying phenomena in complex systems needs to exceed a reductionist outlook of traditional science; thus, complexity requires an understanding extending across a class of complex problems with myriad of intricate and subtle attributes based on innovative and novel ways of thinking as well as applicable laws. In view of these, evolution, order and complexity can more conspicuously reveal the relationship between natural and social worlds, which actually reflects a modern way of thinking that challenges the dichotomy of natural and social. Complex problems in nonlinear dynamics necessitate the critical support of artificial phenomena so that each domain of complex systems addresses research aspects and theories towards the solutions to the pressing challenges which almost exceed the possible limits of human comprehension. This view is capable of serving the needs of different aspects of complexity by minimizing the problems of complexity whose solutions are based on advanced mathematical foundations and corresponding algorithmic models that are made up of numerous applied aspects of complexity.

Evolutionary processes, nonlinearity and all the other dimensions of complexity rest on time, reveal time and occur within time. In the ever-changing landscape and variations, with causality breaking down, the idea of complexity can be stated to be a part of unifying and revolutionary scientific framework for the understanding of complex systems whose behavior is challenging to predict and control with the ultimate goal of attaining a global understanding pertaining to many branches of possible states as well as high-dimensional manifolds, while at the same time keeping up with actuality along the evolutionary and historical path, which itself, has also been through different critical points on the manifold. In this sense, evolution is dependent upon exploration, innovation and causal learning. Yet, changing or removing the causes does not necessarily mean to be capable of removing or altering the outcomes, and hence, modern scientific way of thinking is catered towards the development of models benefiting from theoretical insights, local computations and task-related manifolds in spaces with high dimensions instead of just route learning the rules or representations of the world (Karaca 2022c). Within this scope, the exploration of the way patterns evolves in time spans across turbulent flow of fluids, geological formations, microstructures of materials, spatial organization of microbes, germs, even the behavior of genes, covering quantitative biology, physics, evolution, materials science, ecology, chemistry, neurology, applied mathematics, nonequilibrium statistical mechanics, among many other ones. These aspects point to the observation that nature can produce complex structures even in simple situations, and can obey simple laws even in complex situations (Goldenfeld and Kadanoff 1999).

The nonlinear character and being out of equilibrium are the important qualities characterizing complex systems which have a substantial number of interacting variables or elements which can be simple elements that interact in a nonlinear, either locally or globally (Mateos 2009). In complex systems, the local interactions between the components of the system lead to regularities in the overall, global behavior of the system that appear to be impossible to be derived in a rigorous and analytic way based on the knowledge of the local interactions, which point to both an empirical fact and mathematical intuition (Waldrop 1993). Nonlinear dynamical systems, reflected as a complex library of plethora of different behaviors, overflow with models of varying phenomena in complex systems. Given that, nonlinearity signifies a relationship that cannot be explained or modeled through a linear algebraic or differential combination of input or state variables. Hence is the reason why it is often possible to characterize nonlinear dynamical systems by highly unpredictable and dynamic behaviors, which proves to be challenging for analysis as they occur across different temporal and spatial planes (Karaca 2022a; Kia *et al.* 2017). Across this strand of thought, nonlinear science serves to reveal the nonlinear descriptions of broadly different systems, with a fundamental impact on complex dynamics.

Complex and nonlinear dynamical systems are regarded as thriving as models of natural phenomena, usually characterized by unpredictable behavior whose analysis is challenging to be performed due to occurring like the incidents in chaotic systems. The essence of the problem is rooted in exactly understanding which sort of information, particularly concerned with their long-term evolution and memory properties, can be expected to be derived from those systems. Correspondingly, complexity, chaos, order and evolution all unearth the relationship between natural and social worlds, representing a modern process of thinking (Karaca 2022b). Complexities require a horizon that takes into account the subtleties making their own means of solutions and applicability necessary and applicable. Evolvability in this sense is concerned with the species owing their existence to the capability of their ancestors with respect to evolving and adapting besides the correlation between the complexity of model, design, visualization and optimality. These perspectives are of utmost importance in the future science of complexity as well (Karaca 2022b).

The properties of evolution and adaptation can shed light on the understanding of past to interpret the present in a holistic way and to design future plans and schemes in an appropriate, adaptive, systematic and timely way. If there is a situation of getting stuck in between two extremes of order and chaos under uncertain and unpredictable conditions, complexity thinking and theory can render the systems be adaptive, respond to the world and act spontaneously. Furthermore, being cognizant of complex systems ensures the analysis of the essence of the problem by understanding the way systems self-organize their structures and self-regulate their dynamics and nonlinearity. On theoretical aspect, the theory

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[1] yeliz.karaca@ieee.org (**Corresponding author**)
[2] dumitru@cankaya.edu.tr

of complexity entails powerful evidence, providing elucidation to challenge mechanistic thinking to steer humanity toward the adaptation of an integrative way of thinking (Karaca 2022b).

Chaos theory, being an interdisciplinary theory, not only as a mathematical art but also as a means for practical engineering challenges, represents a set of techniques to analyze dynamical systems. As the founder of chaos theory, Edward Lorenz, put in summarized form: "*When the present determines the future, but the approximate present does not approximately determine the future*", which means that the systems are deterministic but a small change in initial conditions bring about evolution of a highly unpredictable behavior.

It should be noted that the deterministic behavior appears to be random and in that regard, chaos theory specifies the geometric patterns to be discerned in the seemingly random events of a complex system and introduces linear and nonlinear progressions. Initially originated from the simulation of dynamic systems in natural world, some practical applications of chaos include electronic circuit design, biological system analysis, information encryption, synchronization for communication and control, behavior prediction in complex systems, random number generators, modeling, parameter estimation of nonlinear systems, among others. The majority of the applications can benefit from the power of chaos theory in terms of modeling an irregular system with a deterministic equation which has high sensitivity to the initial condition.

Being able to utilize a chaotic system, which has sensitivity to initial condition, density of unstable periodic orbits in a chaotic attractor and topological transitivity, is treated as a library of different behaviors and patterns from which each behavior or pattern can be selected based on the related needs. These qualities pave the way to advances for understanding the physical world or design technology that interfaces with dynamical complex systems. As computation chaos was proposed, it was revealed that chaotic behavior exists not only in natural dynamic systems but also in the discretization process, including instances in which the original system manifests periodicity with its computational simulation being chaotic.

The investigation of the relationship between chaos theory and computational simulation shows that the precision of computer arithmetics has a remarkable impact on the final outcome of the simulation of a dynamic system where even a trivial change in arithmetic computation can modify the structure of orbits significantly. To put the defining idea of chaos differently, in chaotic systems, even minuscule uncertainties in measurements of initial position and momentum can lead to radical errors in the long-term predictions of the quantities. Yet, sensitive dependence upon initial conditions hints that even the tiniest errors in initial measurements in chaotic systems eventually produce critical errors in the prediction of the future motion of an object (Mitchell 2009).

In addition, a nonlinear dynamical system owns complex and flexible dynamics which combine different behaviors, and thus, it can be morphed for the implementation of different functions. Bifurcation parameters in a nonlinear dynamical system can be changed and the dynamics can be altered, namely qualitative behavior of the circuit. Hence, when a nonlinear system is in a chaotic regime, it is very sensitive to its initial conditions or state. Therefore, a change to its initial state can change the future state, and as a result, the type of function it builds. The number of these different functions that a nonlinear, complex system can implement exponentially increases by the evolution time (Kia *et al.* 2020).

In summary, this study puts forth the features of complexity of varying phenomena, properties of evolution and adaptation, memory effects, nonlinear dynamic system qualities, the importance of chaos theory and applications of related aspects. Processes of fractional dynamics, differentiation and systems in complex systems as well as the dynamical processes and dynamical systems of fractional order with respect to natural and artificial phenomena are discussed in terms of their modeling by ordinary or partial differential equations with integer order, ordinary and partial differential equations.

Fractional calculus and fractional-order calculus approach to provide novel models with fractional-order calculus as employed in machine learning algorithms to achieve optimized solutions is also discussed besides the justification of the need to develop analytical and numerical methods. Subsequently, algorithmic complexity and its goal to ensure a better handling of efficient algorithms in computational sciences is pointed out with regard to the classification of computational problems. Moreover, we set forth the importance of stochastic differential equations with respect to some of the real world problems for an effective and intensive addressing of real world problems which manifest randomness. Characteristics of and benefits derived from nonlinear science are elaborated on with a focus on their essential impact observed in complex dynamics, explores the implicit, latent and obscure dependence of schema, serving to reveal the nonlinear descriptions of widely different systems.

The study further points out the neural networks, as descriptive models, for providing the means to gather, store and use experiential knowledge as well as Artificial Neural Networks (ANNs) in relation to their employment for handling experimental data in differing domains. Furthermore, evolutionary computational models and their role in retrieving applicable answers to optimization problems for various engineering areas, medicine, biology, mathematical science, applied disciplines and data science, among many others, is discussed along with an emphasis on mathematical sciences, with Artificial Intelligence (AI) and machine learning being at the pedestal and intersection with different fields which are characterized by complex, chaotic, nonlinear, dynamic and transient components in order to demonstrate the significance of novel approaches in real systems and related realms.

PROCESSES OF FRACTIONAL DYNAMICS, DIFFERENTIATION AND SYSTEMS IN COMPLEX SYSTEMS

Dynamical processes and dynamical systems of fractional order with respect to natural and artificial phenomena are modeled by ordinary or partial differential equations with integer order, which can be described aptly by employing ordinary and partial differential equations. Fractional calculus approach, remarkably, provides novel models through the introduction of fractional-order calculus to optimization methods, and thus, is employed in machine learning algorithms since this scheme is geared towards attaining optimized solutions by maximizing the model accuracy and minimizing functions like the computational burden. Hence, mathematical-informed frameworks can be employed for enabling reliable and robust understanding of various complex processes that involve a variety of temporal and spatial scales. This complexity requires a holistic understanding of different processes through multi-stage integrative models capable of capturing the significant attributes on the respective scales.

Fractional-order differential and integral equations, fractals, fractional integro-differentiation (non-integer order integro-differentiation), nonlinear time-delay systems, linear and nonlinear fractional ordinaries, nonlinear differential equations, integral fractional differential equations, partial differential equations and stochastic integral problems, in those regards, can provide the generalization of traditional integral and differential equations through the extension of the conceptions.

Analytical and numerical methods have been developed to solve ordinary and partial differential equations whose classes have been investigated in depth. Despite being useful to model some of the real world problems with efficiency, some classes of such differential equations have failed to replicate the observed facts due to complexities of several real world problems. For instance, since some real world problems manifest randomness that cannot be captured by these differential equations, stochastic differential equations can be suggested and are employed intensively in an effective way. Furthermore, algorithmic complexity, as a way of comparing the efficiency of an algorithm, can ensure a better grasping and designing of efficient algorithms in computational sciences while enabling the classification of computational problems based on their algorithmic complexity, as defined according to how the resources are required for the solution of the problem, including the execution time and scale with the problem size (Karaca 2022a). These sorts of approaches through the application of fractional-order calculus to optimization methods and the experimental results reveal the benefit maximizing the model's accuracy and minimizing the cost functions like the computational burden, as mentioned above, pointing toward the applicability of the methods in different domains which are characterized by complex, chaotic, nonlinear, irregular, asymmetric, dynamic and transient components.

Fractional (non-integer order) calculus, as an interdisciplinary field, is able to build mathematical models that are concise enough to describe the dynamic events occurring in complex elements, which is important to understand the underlying multiscale processes that arise when there are electrical stimulation or mechanical stress. Fractional calculus attends to the co-evolving entities, actual state properties, observations and patterns of complex systems in a spot-on manner with respect to nonlinear dynamic systems, modeling of complexity evolution, order of fractional chaotic as well as complex systems.

Fractional differential equations, associated with non-local phenomena, benefit from both qualitatively and quantitatively different properties as compared to the classical ones since the nonlocality of fractional calculus turns it into a sound means to unearth new properties of non-local phenomena. Furthermore, when compared with classical integer-order models, the preliminary advantage of fractional models is their potential use in chaotic dynamics in engineering and applied fields. Besides these points, the dynamics of fractional order systems have captured prominence through the development of fractional-order algorithm (Sun *et al.* 2022).

The dynamics of many systems, whether they be biological, economic, medical, physical, mechanical, electrical, thermal, and so forth, can be described in terms of differential equations. Fractional derivatives are extensively employed to model realistic systems since these derivatives are capable of modeling memory and hereditary effects observed in physical systems owing to their nonlocal nature. Systems that involve fractional derivatives can exhibit chaos, and when below a certain threshold value of fractional order derivative, the systems can show regular behavior. Being a nonlocal operator, contrary to ordinary derivative, the fractional derivative is particularly useful for modeling the system's both memory and hereditary properties (Baleanu *et al.* 2015).

Fractional calculus is frequently opted for mathematical modeling to analyze the evolutionary systems which are known to have memory effect on dynamics. To extend ordinary calculus to fractional calculus, there are different ways, with the related common definitions, Riemann-Liouville fractional integral and derivative, the Grünwald-Letnikov fractional integral and derivative as well as the Caputo fractional derivative as being some of them (Karaca and Baleanu 2022b). Within this framework, Caputo definition is preferred to be used to solve differential equations, which is denoted as per Equation 1:

$$D_{\alpha}^{m} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{0}^{t} \frac{f^{(m)}(\tau)}{(t-\o)^{\alpha+1-m}} d\tau \tag{1}$$

The fundamental results on fractional integral and derivatives of the power function $(t-t0)\beta\ for\ \beta > -1$ are the case and for the Caputo's derivative, Equation 2 is employed in the following manner (see (Karaca and Baleanu 2022b) for further details):

$$D_{t_0}^{\alpha}(t-t_0)^{\beta} = \begin{cases} 0 & \beta \epsilon \{0,1,...,m-1\} \\ \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(t-t_0)^{\beta-\alpha} & \beta > m-1 \\ nonexisting & otherwise \end{cases} \tag{2}$$

Taken together, fractional calculus, equations with fractional derivatives, integrals and differences prove to be powerful tools for the description of local processes in time and space with different nonlocality types (Diethelm *et al.* 2022).

Nonlinear science, which has had a central impact on complex dynamics, explores the implicit, latent and obscure dependence of schema, serving to reveal the nonlinear descriptions of widely different systems (Karaca 2022a). As a substitution for fractional derivatives, the memory-dependent derivative reflects the memory effect distinctively, and as an application, representative processes are remodeled with it, considering the temporal-spatial evolution mechanisms of the related complex processes. Accordingly, the theory of fractional differentiation and applications constitutes a significant component of nonlinear analysis to be able to address highly diverse real-life problems. Analysis and control of fractional order nonlinear systems can appear to be another pressing challenge with the observation of unknown inputs and concepts used and derived analytically. Being a noteworthy attribute of complex systems, nonlinearity represents the various interactions between the variables in a nonlinear fashion.

Emergence, feedback, adaptiveness, irreducibility, chaos, operating between order and chaos, having multilayers of structure as well as self-organization seem to be among some of the other related attributes. Multiple nonlinear complex systems manifest phenomena in which oscillations enhance periodic behaviors and the system's synchronization. Accordingly, fractional-order control as a field of control theory employs the fractional-order integrator as part of the control system design compilation, through which well-established control methods and strategies can be generalized and improved (Monje *et al.* 2010). More complex physical problems, on the other hand, require advanced mathematical operators of differentiation.

The concept of non-local operators of differentiation some of which are power law, exponential decay law and generalized Mittag-Leffler law, has been employed by different engineering and medicine fields, applied sciences and technology owing to the fact that they have the capability of integrating more complex natural aspects into mathematical equations (Kober 1940; Baleanu and Karaca 2022). Thus, the extension of classical and modern control theories to integrative and novel perspectives ensures the development of algorithms applicable both in integer and non-integer order systems. Therefore, fractional and integral equations, fractional discrete calculus, fuzzy fractional calculus as well as fractional dynamics concern not only theoretical aspects but also the applications related to fractional differentiation extending broadly across other mathematical models within mathematical sciences and engineering mathematics (i.e.: entropy, fractals, wavelet, quantum, etc.) ranging along mathematical analysis, numerical analysis, chaos, image and/or signal analysis, bifurcation, data analysis, time series analysis, medicine, neurology, bioengineering, economics, finance, control systems, artificial intelligence, mathematical biology, biotechnology, genetics, nanotechnology, and so forth.

PROCESSES OF MATHEMATICAL SCIENCE, ARTIFICIAL INTELLIGENCE AND APPLICATIONS IN COMPLEX SYSTEMS

Complexity, as a highly correlated nonlinear phenomena evolving along an extensive array of timescales and length scales, poses challenges for technical analyses, theoretical modeling and numerical simulations in many domains. It becomes critical to control the underlying systems and processes across their spatio-temporal evolution. Thus, data, concerning biological, financial, physical or technological complex systems, can be rendered manageable through computer simulations that employ the effective nonlinear dynamic methods.

The attempt to understand a complex system with multiple interacting components, human body, for example, or weather patterns, financial market, living systems concern two factors, one of which is chaos and the other one being complexity. An important finding of modern chaos theory is that even though complex systems may be predictable in the short term, that would not be the case for the long term since there exists an element of uncertainty and unpredictability in all complex systems (Schueler 1996). In other words, complex interactions may make the prediction of long-term outcomes almost impossible; and complexity constitutes complex interacting systems with new emergent properties that make them more than the sum of their parts (Clegg 2020). A slight disturbance in the chaotic system could make one be unable to specify the future state with precision, meaning its evolution could not be predictable, which points to an intrinsic uncertainty situation (Sanjuán 2021). These significant findings of chaos theory have accordingly been transposed to other disciplines.

Chaos, which is a long-term aperiodic and random-like behavior, is exhibited by many nonlinear complex dynamic systems, which requires the revealing of accessible and applicable paths into abundance of complexity and the superfluity of experimental processes to generate novel, diverse and robust means. The use of predictive tools, Artificial Intelligence (AI) and machine learning techniques has made the number of applications possible, including the prediction of mechanisms ranging extensively from living organisms to other interactions across incredible spectra. These techniques combined with fractal analysis highlight the fractal dimension and measurement of lacunarity both on local and global scale as well as the entire volume of the samples being handled. Machine learning, as a subset of AI, signifies the methods which are capable of learning from experience, which enables the performance of designated tasks such as detection, recognition, iteration, diagnosis, optimization and prediction.

Machine learning is employed in different domains of complex systems within nonlinear dynamic processes, which involve the identification of the basic system structure, for instance network nodes and links, as well as the exploration of dynamic behavior of nonlinear systems like determining exponents, prediction of future evolution and inferring causality of interactions. Reservoir computing and long short-term memory which are machine learning processes are usually dynamical in nature, whose understanding of when, how and why to function well based on data can potentially be addressed by employing tools from dynamical systems theory (Tang *et al.* 2020).

Neural networks and physical systems have emergent collective computational abilities and their architecture is capable of producing an emergent associative memory. By providing an explicit physical interpretation, efficiency for practical applications and more manageable computational complexity, fractional mathematics and Artificial Intelligence (AI) can capture the history of dynamical effects existent in different natural and artificial phenomena, proving to be essential modes with their conceptions supporting a productive interplay in the exploration of the structure and functions with respect to complex system dynamics (Karaca 2022c). To be able to capture and observe the dynamic variations in complex systems, distributions can also be employed. For instance, heavy-tailed distributions, as found throughout many naturally occurring phenomena, are particularly preferable concerning their use in stochastic dynamical models for extreme events which display the presence of outliers and possibility of extreme values.

Machine learning and control theory, with high technological impacts, comprise gateways that are in proximity with each other in the complex landscape of the universe of mathematics. Involving the prompting of a dynamical system from an initial configuration to the final one over extended ranges of time and frequency through aptly designed and applicable controls, the notion of controllability allows the disclosing of the pathways between the disciplines. Hence, control theory can be stated to lie on the pillars of machine learning.

Machine learning and data science, as integrative domains, entail an optimized method to calculate mathematics and meta-mathematics derivatives in minimum timeframe with maximum efficiency; hence, evolutionary computation, in computer science, constitutes a cluster of algorithms for global optimization, which draws its inspiration from nature and biological evolutionary processes. Evolutionary computational models own a sophisticated searching technological foundation besides a mathematical problem optimization tool taking up less time and reducing complexity so that the precise and applicable answer to optimization problems for various engineering, mathematical and applied disciplines can be sought and found. Accordingly, AI and machine learning, situated at the core, can extend broadly across the related mathematical models within the framework of mathematical sciences and engineering mathematics as well as across the intersection of different fields.

Fractional-order calculus is concerned with the differentiation and integration of non-integer orders, and fractional calculus (FC), based on fractional-order thinking, is the quantitative analysis of functions using non-integer order integration and differentiation. Therein, the order can be a complex number, a real number of the function of a variable. Owing to these features, FC, entailed by complexity, can enhance the processing of complex signals, improve the control of complex systems extend the enabling of the potential for creativity, considering the fact that an observable phenomenon that is represented by a fractal function has integer-order derivatives which diverge. Hence, complex phenomena, no matter if they are natural or engineered, need to be described by fractional dynamics as such point of view is to be consulted for the characterization and regulation of complexity (West 2022). In view of these attributes, innovative approaches to machine learning with the introduction of fractional-order calculus have started to be extensively used as optimization methods used in machine learning algorithms. By performing the training of the models, making inferences and solving optimization problems, machine learning techniques are geared towards maximizing model's accuracy and minimizing cost functions. In short, these techniques provide flexible options for the analysis and prediction of changes that could occur in the dynamics of complex and chaotic systems.

Life, being the most complex physical system in the universe, at all scales, requires the understanding of the massive complexity encompassing its origin, structure, dynamics, adaptation and organization. The number of substructures and interacting pathways of each of the substructure along with the other ones as well as neurons determine the degree of complexity. Neural networks, as descriptive models provide the means to gather, store and use experiential knowledge; and are designed in such a way that they can emulate different operations of the human brain. A neuron is an imitation of the observations occurring in the human brain which is composed of interconnected neurons that transmit electrochemical elements, which forms the basis of all neural networks (Karaca and Baleanu 2022a).

A synapse is the connection between nodes, or neurons, in an artificial neural network (ANN); and in this configuration, the strength or amplitude controls this connection between the nodes, which is called the synaptic weight. There exists a complex structure with multiple connections as multiple synapses can connect with the same neurons, in which each synapse has a different level of trigger or impact on whether the neuron is fired and activates the next neuron. With relation to machine learning, a synapse is often referred to as a node in machine learning; yet, the artificial neurons, which output a value from a continuous function, do not fire, unlike biological neurons. To put differently, the main different point between a biological process and artificial process is concerned with the level of control imposed on the input values. Thus, the resulting output of the nodes is utilized as the input for the next layer of nodes, in an ongoing process, throughout the neural network brain until the final output layer is reached (Karaca and Baleanu 2022a) (see Figure 1).

One major ongoing challenge of integrating fractional calculus in cases of complexity is the effective use of empirical, numerical, experimental and analytical methods to be able to tackle complexity. In that regard, ANNs, including a family of nonlinear computational methods, can be employed to handle experimental data in various domains as a result of their capability of managing complex computations to direct their progressive application towards serving the applicable and timely solutions of practical problems.

The specific neural architectures are generated by AI, as instantiated in the brain, can provide applicable answers to the problems of cognition through the understanding of the way architectures implement cognitive processes. Thereby, if programmed properly and adjusted to the data at hand appropriately, AI enables availability at all times, providing more prompt decision-making processes, digital assistance, new inventions and rapid pattern analysis of large datasets while also reducing human error. To attain these goals, Bidirectional Encoder Representations from Transformers (BERT), as one of the applicable methods of Natural Language Processing (NLP) can be employed, with the related models which have the main stages like pre-training and fine-tuning. In the former one, the model is trained on unlabeled data over different pre-training tasks and in the latter stage, the BERT model is initialized with the pre-trained parameters (Karaca *et al.* 2022) (see Figure 2).

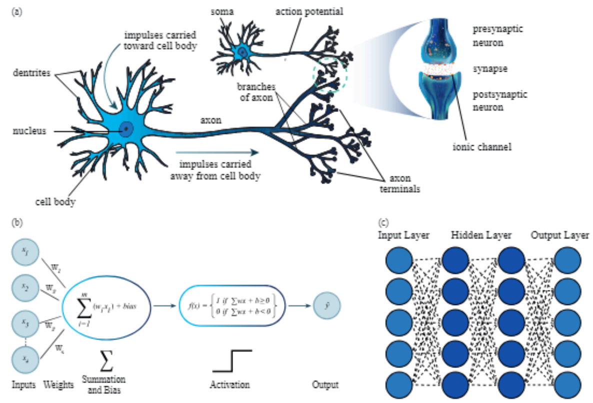

Figure 1 Biological versus artificial neuron interconnection from inputs at dendritic compartments to outputs at axon terminals: (a) the basic building blocks of biological neural processing with neurons and synapses. (b) artificial implementation of a neuron, as electrically excitable cell producing action potential. (c) synaptic interrelation among input layer, hidden layers and output layer (Karaca and Baleanu 2022a).

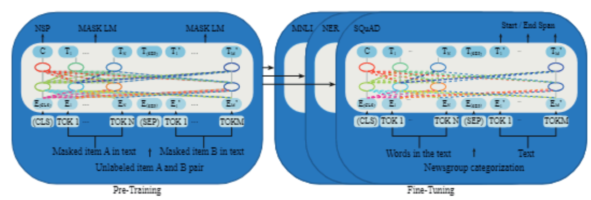

Figure 2 BERT model's pre-training and fine-tuning stages (Karaca *et al.* 2022).

All in all, the advanced theory and applications of computability enables one to distinguish complexity classes of problems, considering the order of corresponding functions that describe computational time pertaining to their algorithms or computational problems. This refers to the essential requirement of being concerned not only with the complexity of universal problem solving but also with the complexity of knowledge-based programs (Mainzer and Mainzer 1997). Through that perspective in conjunction with advanced mathematical modeling with AI support, it would be possible to generate applicable cumulative answers and customized solutions to real world problems in a novel, constructive, adaptive and flexible way on the ever-evolving global landscape.

CONCLUDING REMARKS AND FUTURE DIRECTIONS

The use of different predictive mathematical modeling, Artificial Intelligence (AI) and machine learning techniques has made the number of applications possible, including the prediction of mechanisms ranging extensively from life at simple level to other interactions getting gradually complex across incredible spectra. It is possible to observe that nature can produce complex structures even in simple situations, and can obey simple laws even in complex situations. Consequently, complex problems in nonlinear dynamics require the precarious support of artificial phenomena along with interpretability and predictability in order that each domain of complex systems can yield applicable answers and solutions to the pressing challenges of our current era. This view can help to cater the needs of different aspects of complexity by significantly diminishing the problems of complexity whose solutions are based on advanced mathematical foundations and corresponding algorithmic models that include numerous applied aspects of complexity.

Constituting the prompting of a dynamical system from an initial configuration to the final one over extended ranges of time and frequency through aptly designed and applicable controls, the notion of controllability allows the disclosing of the pathways across the related disciplines. Furthermore, processes of fractional dynamics, differentiation and systems in complex systems as well as the dynamical processes and dynamical systems of fractional order with relation to natural and artificial phenomena are important in terms of their modeling by ordinary or partial differential equations with integer order, ordinary and partial differential equations. Fractional calculus and fractional-order calculus approach to provide novel models with fractional-order calculus as employed in machine learning algorithms to achieve optimized solutions is also noteworthy considering the need to develop analytical and numerical methods.

Machine learning, a subset of AI, referring to the methods capable of learning from experience, enables the performance of designated tasks such as detection, recognition, iteration, diagnosis, optimization and prediction. It is employed in different domains of complex systems within nonlinear dynamic processes that involve the identification of the basic system structure, such as network nodes and links, as well as the exploration of dynamic behavior of nonlinear systems like determining exponents, prediction of future evolution and inferring causality of interactions. Besides conventional methods like approximation, estimation, convergence, stability analysis, and so forth, it is important to capture the latent aspects of complex nonlinear dynamic structures so that prediction can be made possible.

Accurate and prompt predictive processes can ensure foreseeing to be put in practice in order to cater the needs of our era with the landscape being transient and ever-evolving. These pressing challenges and needs point to the importance of yielding applicable solutions to problems for various engineering areas, medicine, biology, mathematical science, applied disciplines and data science, among many others. These points have been discussed in detail in this study along with an emphasis on power of predictability, relying on mathematical sciences and engineering mathematics with Artificial Intelligence (AI) and machine learning being at the pedestal and intersection with different fields characterized by complex, chaotic, nonlinear, dynamic and transient components to reveal the significance of optimized approaches in real systems and related realms. Significant developments based on these critical points and perspectives can pave the way for the future research towards optimized applicable solutions of unsolved problems arising as formidable challenges of our ever-evolving and fast-changing global landscape.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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