



Research article

Study of power law non-linearity in solitonic solutions using extended hyperbolic function method

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Abstract: This paper retrieves the optical solitons to the Biswas-Arshed equation (BAE), which is examined with the lack of self-phase modulation by applying the extended hyperbolic function (EHF) method. Novel constructed solutions have the shape of bright, singular, periodic singular, and dark solitons. The achieved solutions have key applications in engineering and physics. These solutions define the wave performance of the governing models. The outcomes show that our scheme is very active and reliable. The acquired results are illustrated by 3-D and 2-D graphs to understand the real phenomena for such sort of non-linear models.

Keywords: EHF method; power law non-linearity; optical solitons

Mathematics Subject Classification: 35Q51, 35Q53

1. Introduction

Solitons have much importance in various areas of science and nature, such as waves, geology, wave propagation, population ecology, fluid dynamics, computer science, biology, plasmas, heat, birefringent fibers, mechanics and optics. There are many mathematical models that efficiently demonstrate the properties of soliton transmission such as Radhakrishnan-Kundu-Lakshmanan equation [1], the resonant nonlinear Schrodinger's equation [2] and so on [3–10]. The key object for the presence of solitons in the optical fibers is to keep balance between group velocity dispersion (GVD) and non-linearity. In special conditions, there may be circumstances which produce small non-linearity and low GVD. Recently, Biswas and Arshed [11–16], estimated a precise creative

clue to grip the circumstances where non-linearity and GVD are small, which is presented in the model called BAE. This governing model can act as a feasible model to study the properties of solitons in crystals, Photonic crystal fiber, and optical fibers. Waves and solitons describe many real life phenomena. Several meaningful techniques have been recognized for solving wave structures of non-linear partial differential equations, such as the improved sub-equation scheme [17], expansion approach [18], extended Jacobi elliptic function expansion method [19], singular manifold method [20], modified fractional reduced differential transform method [21], modified (G'/G) -expansion approach [22], Sine-Gordon expansion method [23], extended modified mapping method [24], Hirota's bilinear method [25], the power series method [26], Lie symmetry method [27], Backlund transformation method [28], the generalized Darboux transformation [29], the Darboux transformation method [30], the extended Darboux transformation technique [31], the Hirota bilinear method [32], and the generalized Darboux transformation scheme [33]. This article studies the BAE with power law via the EHF method [34–36], which leads to the construction of singular, bright, dark-bright and periodic-singular optical soliton solutions. In the future, we will use fractional calculus to construct analytical solutions of nonlinear models under the Atangana-Baleanu derivative in Caputo sense (ABC) with the help of the Sumudu transform [37], group preserving scheme [38] and time-fractional Zakharov-Kuznetsov equation via the natural transform decomposition method with nonsingular kernel derivatives [39]. Studies on the physical behaviors of these non-linear models using different analytical methods have gained a lot of attention recently. In physics and engineering, a large number of non-linear dispersive models appear that have a large area of applications. These general solutions can also provide a useful help for researchers to study and understand the physical interpretations of systems.

The layouts of this paper are as follows. The governing model is described in Section 2. Analysis of the proposed EHF method is presented in Section 3. In Section 4 the EHF method is applied, and Section 5 consists of the findings and discussions. Conclusions of this paper are posted in Section 6.

2. Biswas-Arshed equation

The BAE with full non-linearity [11–16] is given by

$$iu_t + a_1 u_{xx} + a_2 u_{xt} + i(b_1 u_{xxx} + b_2 u_{xxt}) = i[\phi(|u|^{2n}u)_x + \sigma(|u|^{2n})_x u + \theta|u|^{2n}u_x], \quad (2.1)$$

where $u(x, t)$ and n are a complex-valued function and full non-linearity parameter, respectively. On the left of the (2.1), a_1 and a_2 indicate the temporal evolution, coefficient of GVD and coefficient of STD, respectively. Next, b_1 and b_2 are coefficients of 3OD and STD. θ and σ represent the effect of self-steepening and non-linear dispersion in the absence of SPM. This effect of dispersion and non-linearity provides the required balance for solitons, existence.

Assume that

$$u(x, t) = P(\eta)e^{i\Psi(x,t)}, \quad (2.2)$$

where $P(\eta)$ indicates the amplitude, and

$$\eta = x - ct, \quad \Psi(x, t) = -kx + wt + \epsilon, \quad (2.3)$$

where c and $\Psi(x, t)$ are the velocity and the phase component. k , w are frequency and wave number, accordingly, while ϵ is the phase constant of solitons. Inserting (2.2) and (2.3) in (2.1) and separating into real and imaginary parts, the real part gives

$$-(w + a_1k^2 + b_1k^3 - a_2wk - b_2wk^2)P + (a_1 + 3b_1k - a_2c - 2b_2ck - b_2w)P'' = k(\theta + \phi)P^{2n+1}, \quad (2.4)$$

while the imaginary part gives

$$\begin{aligned} (-3b_1k^2 + b_2ck^2 + 2b_2wk - 2a_1k - c + a_2ck + a_2w)P' + (-b_2c + b_1)P''' \\ = [\theta + 2n\sigma + (2n + 1)\phi]P^{2n}P'. \end{aligned} \quad (2.5)$$

Integrating (2.5) and taking the integration constant to be zero, we obtain

$$\begin{aligned} (-3b_1k^2 + b_2ck^2 + 2b_2wk - 2a_1k - c + a_2ck + a_2w)P + (-b_2c + b_1)P'' \\ = [\theta + 2n\sigma + (2n + 1)\phi]P^{2n}. \end{aligned} \quad (2.6)$$

To get the solutions, we use following substitution:

$$P = W^{1/2n}. \quad (2.7)$$

Equations (2.4) and (2.6), yield

$$\begin{aligned} (a_1 + 3b_1k - a_2c - b_2w - 2b_2ck)[(1 - 2n)(W')^2 + 2nWW''] \\ - 4n^2(w + a_1k^2 + b_1k^3 - a_2wk - b_2wk^2)W^2 - 4n^2k(\theta + \phi)W^3 = 0, \end{aligned} \quad (2.8)$$

$$\begin{aligned} (2n + 1)(b_1 - b_2c)[(1 - 2n)(W')^2 + 2nWW''] - 4n^2[2n\sigma + \theta + (2n + 1)\phi]W^3 \\ + (-2a_1k - 3b_1k^2 + b_2ck^2 + 2b_2wk + a_2ck - c + a_2w)4n^2(2n + 1)[(1 - 2n)(W')^2 \\ + 2nWW'']W^2 = 0. \end{aligned} \quad (2.9)$$

As $W(\xi)$ satisfies (2.8) and (2.9), the constraint conditions are as follows:

$$\begin{aligned} \frac{(a_1 - a_2c + 3b_1k - 2b_2ck - b_2w)}{(2n + 1)(b_1 - b_2c)} &= \frac{(\theta + \phi)k}{2n\sigma + \theta + (2n + 1)\phi} \\ &= \frac{-(w + a_1k^2 + b_1k^3 - a_2wk - b_2wk^2)}{(2n + 1)(b_2ck^2 - 3b_1k^2 + 2b_2wk - 2a_1k + a_2ck - c + a_2w)}. \end{aligned} \quad (2.10)$$

Now, (2.8) is analyzed using the extended hyperbolic function method.

Setting $A = b_2ck^2 - 3b_1k^2 + 2b_2wk - 2a_1k + a_2ck - c + a_2w$ and $B = w + a_1k^2 - a_2wk + b_1k^3 - b_2wk^2$, gives

$$A[(1 - 2n)(W')^2 + 2nWW''] - 4n^2BW^2 - 4n^2k(\theta + \phi)W^3 = 0. \quad (2.11)$$

3. The EHF method

Two phases of the EHF method are as follows:

Form 1: Let (2.11) have the solution

$$W(\eta) = \sum_{j=0}^N F_j \Phi^j(\eta), \quad (3.1)$$

where F_j are constants and $\Phi(\eta)$ satisfies the auxiliary ODE as

$$\frac{d\Phi}{d\eta} = \Phi \sqrt{\tau + \mu \Phi^2}, \quad \tau, \mu \in \mathbb{R}. \quad (3.2)$$

To find the number N , we use the balancing rule on (2.11). Substituting (3.1) in (2.11) along with (3.2) produces a system of algebraic equations for F_j ($0 \leq j \leq N$). A set of solutions is acquired by solving this system, that accepts (3.2) as

Set 1: If $\tau > 0$ and $\mu > 0$,

$$\Phi(\eta) = -\sqrt{\frac{\tau}{\mu}} \operatorname{csch}(\sqrt{\tau}(\eta + \eta_0)). \quad (3.3)$$

Set 2: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\tau}{\mu}} \operatorname{sec}(\sqrt{-\tau}(\eta + \eta_0)). \quad (3.4)$$

Set 3: If $\tau > 0$ and $\mu < 0$,

$$\Phi(\eta) = \sqrt{\frac{\tau}{-\mu}} \operatorname{sech}(\sqrt{\tau}(\eta + \eta_0)). \quad (3.5)$$

Set 4: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\tau}{\mu}} \operatorname{csc}(\sqrt{-\tau}(\eta + \eta_0)). \quad (3.6)$$

Set 5: If $\tau > 0$ and $\mu = 0$,

$$\Phi(\eta) = \exp(\sqrt{\tau}(\eta + \eta_0)). \quad (3.7)$$

Set 6: If $\tau < 0$ and $\mu = 0$,

$$\Phi(\eta) = \cos(\sqrt{-\tau}(\eta + \eta_0)) + i \sin(\sqrt{-\tau}(\eta + \eta_0)). \quad (3.8)$$

Set 7: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = \pm \frac{1}{(\sqrt{\mu}(\eta + \eta_0))}. \quad (3.9)$$

Set 8: If $\tau = 0$ and $\mu < 0$,

$$\Phi(\eta) = \pm \frac{i}{(\sqrt{-\mu}(\eta + \eta_0))}. \quad (3.10)$$

Form 2: Adopting the same pattern as above, assume (3.1) satisfies the auxiliary ODE as follows:

$$\frac{d\Phi}{d\eta} = \tau + \mu\Phi^2, \quad \tau, \mu \in R. \quad (3.11)$$

Substituting (3.1) into (2.11) with (3.11) yields the value of N , and it provides a set equations. Assume (3.11) has the solutions as follows:

Set 1: If $\tau\mu > 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{\mu}} \tan(\sqrt{\tau\mu}(\eta + \eta_0)). \quad (3.12)$$

Set 2: If $\tau\mu > 0$,

$$\Phi(\eta) = -sn(\tau) \sqrt{\frac{\tau}{\mu}} \cot(\sqrt{\tau\mu}(\eta + \eta_0)). \quad (3.13)$$

Set 3: If $\tau\mu < 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{-\mu}} \tanh(\sqrt{-\tau\mu}(\eta + \eta_0)). \quad (3.14)$$

Set 4: If $\tau\mu < 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{-\mu}} \coth(\sqrt{-\tau\mu}(\eta + \eta_0)). \quad (3.15)$$

Set 5: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = -\frac{1}{\mu(\eta + \eta_0)}. \quad (3.16)$$

Set 6: If $\tau \in R$ and $\mu = 0$,

$$\Phi(\eta) = \tau(\eta + \eta_0). \quad (3.17)$$

Note: sn is the well-known sign function.

4. Application of the EHF method

Form 1: In this section, we utilize the above said method to solve the BAE with power law non-linearity. Using the balance principle in (2.11), yields $N = 2$, so (3.1) converts to

$$W(\eta) = F_0 + F_1\Phi(\eta) + F_2(\Phi(\eta))^2, \quad (4.1)$$

where F_0 , F_1 and F_2 are constants. Inserting (4.1) in (2.11), one attains a set of equations in F_0 , F_1 , F_2 , τ and μ is obtained. On working the set of equations, we achieve

$$F_0 = 0, \quad F_1 = 0,$$

$$F_2 = F_2, \quad \tau = -\frac{Bn^2}{2An - 2n - A},$$

$$\mu = -\frac{k n^2(\theta + \phi)F_2}{2An - 3n - A}. \quad (4.2)$$

Set 1: If $\tau > 0$ and $\mu > 0$,

$$u_1(x, t) = \left[F_2 \left(-\sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \operatorname{csch}\left(\sqrt{-\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x, t)}). \quad (4.3)$$

Set 2: If $\tau < 0$ and $\mu > 0$,

$$u_2(x, t) = \left[F_2 \left(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \sec\left(\sqrt{\frac{B n^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x, t)}). \quad (4.4)$$

Set 3: If $\tau > 0$ and $\mu < 0$,

$$u_3(x, t) = \left[F_2 \left(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \operatorname{sech}\left(\sqrt{-\frac{B n^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x, t)}). \quad (4.5)$$

Set 4: If $\tau < 0$ and $\mu > 0$,

$$u_4(x, t) = \left[F_2 \left(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \operatorname{csc}\left(\sqrt{\frac{B n^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x, t)}). \quad (4.6)$$

Set 5: If $\tau > 0$ and $\mu = 0$,

$$u_5(x, t) = \left[F_2 \left(\exp\left(\sqrt{-\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x, t)}). \quad (4.7)$$

Set 6: If $\tau < 0$ and $\mu = 0$,

$$u_6(x, t) = \left[F_2 \left(\cos\left(\sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0)\right) + i \sin\left(\sqrt{\frac{B n^2}{2An - 2n - A}}(\eta + \eta_0)\right) \right)^2 \right]^{1/2n}$$

$$\times (e^{t\Psi(x,t)}). \quad (4.8)$$

Where $\Psi(x, t) = -kx + wt + \epsilon$, $\eta = x - ct$.

Form 2: Using the balance principle on (2.11), yields $N = 2$, so (3.1) gives

$$W(\eta) = F_0 + F_1\Phi(\eta) + F_2(\Phi(\eta))^2, \quad (4.9)$$

where F_0 , F_1 and F_2 are constants. Inserting (4.9) in (2.11) and comparing the coefficients of each polynomial of $\Phi(\eta)$ to zero, we retrieve a set of equations in F_0 , F_1 , F_2 , τ and μ .

Working on the set of equations, we acquire

$$F_0 = \frac{(A(1-2n) + 3n)B}{k(\theta A(1-2n) + 2n\theta + \phi A(1-2n) + 2n\phi)},$$

$$F_1 = 0, \quad F_2 = \frac{\mu^2(A(1-2n) + 3n)}{kn^2(\phi + \theta)},$$

$$\tau = -\frac{Bn^2}{(2n + A(1-2n))\mu}, \quad \mu = \mu. \quad (4.10)$$

Set 1: If $\tau\mu > 0$,

$$u_9(x, t) = \left[\frac{(A(1-2n) + 3n)B}{k(\theta A(1-2n) + 2n\theta + \phi A(1-2n) + 2n\phi)} + F_2 \left(\chi \sqrt{-\frac{Bn^2}{(2n + A(1-2n))\mu^2}} \tan\left(\sqrt{-\frac{Bn^2}{A + 2n - 2An}}(\eta + \eta_0) \right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x,t)}). \quad (4.11)$$

Set 2: If $\tau\mu > 0$,

$$u_{10}(x, t) = \left[\frac{(A(1-2n) + 3n)B}{k(\theta A(1-2n) + 2n\theta + \phi A(1-2n) + 2n\phi)} + F_2 \left(\chi \sqrt{-\frac{Bn^2}{(2n + A(1-2n))\mu^2}} \cot\left(\sqrt{-\frac{Bn^2}{A + 2n - 2An}}(\eta + \eta_0) \right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x,t)}). \quad (4.12)$$

Set 3: If $\tau\mu < 0$,

$$u_{11}(x, t) = \left[\frac{(A(1-2n) + 3n)B}{k(\theta A(1-2n) + 2n\theta + \phi A(1-2n) + 2n\phi)} + F_2 \left(\chi \sqrt{\frac{Bn^2}{(2n + A(1-2n))\mu^2}} \tanh\left(\sqrt{\frac{Bn^2}{A + 2n - 2An}}(\eta + \eta_0) \right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x,t)}). \quad (4.13)$$

Set 4: If $\tau\mu < 0$,

$$u_{12}(x, t) = \left[\frac{(A(1-2n) + 3n)B}{k(\theta A(1-2n) + 2n\theta + \phi A(1-2n) + 2n\phi)} + F_2 \left(\chi \sqrt{\frac{Bn^2}{(2n + A(1-2n))\mu^2}} \coth\left(\sqrt{\frac{Bn^2}{A + 2n - 2An}}(\eta + \eta_0) \right) \right)^2 \right]^{1/2n} \times (e^{t\Psi(x,t)}), \quad (4.14)$$

where $\chi = \text{sgn}\left(-\frac{Bn^2}{(2n + A(1-2n))\mu}\right)$, $\Psi(x, t) = -kx + wt + \epsilon$, $\eta = x - ct$.

5. Results and discussion

We illustrated the solutions of the BAE in the form of optical solitons using the EHF method. These solutions have applications in telecommunication to transfer information, as solitons have the proficiency to travel long spaces without distortion and without altering their shapes. In this work, we added selected graphical representations of some solutions of the BAE to dodge overcrowded the manuscript. Figures 1–6 represent 2-D and 3-D plots of some optical solitons of (2.1), with the help of parameters involved. Figures 1 and 6 demonstrate the solutions obtained given in (4.3) and (4.14), accordingly, which are singular soliton waves. Figures 2 and 4 demonstrate the solutions (4.4) and (4.11), which are periodic singular soliton waves. Solution (4.5) given by Figure 3, represents a bright soliton, and solution (4.13) provides a dark soliton and is shown by Figure 5. These results reveal that the bright solitons are favorable in watching the order of the solitons. It is helpful to the eliminate internet jam that is a big problem nowadays in industry. With the recent disease of COVID-19, everywhere commercial dealings are being conducted virtually, and it is necessary to preserve a continuous flow of Internet communications. Similarly, dark solitons are too encouraging for soliton communication in the presence of background wave. Though, singular solitons are just involved the figure of solitons and display a whole band of soliton solutions formed using the model.

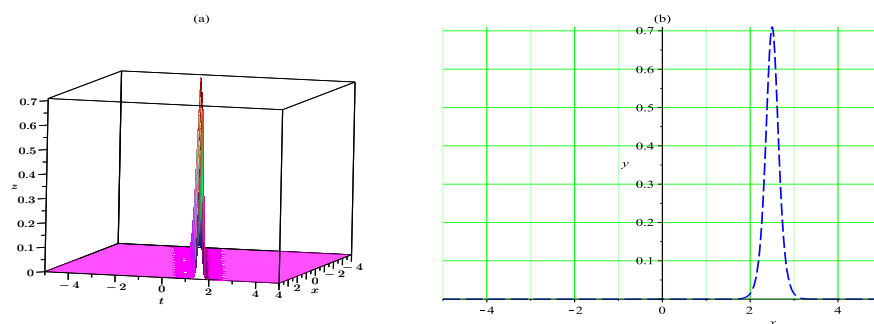


Figure 1. (a) 3D plot of singular soliton solution (4.3) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $c = 2.5$, $\epsilon = -4$. (b) 2D representation of (4.3) using $t = 1$.

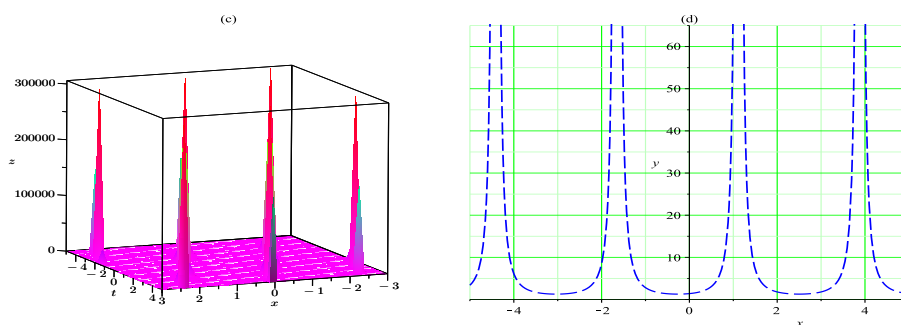


Figure 2. (c) 3D plot of periodic-singular soliton solution (4.4) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = 4$, $c = 2.5$. (d) 2D representation of (4.4) with $t = 1$.

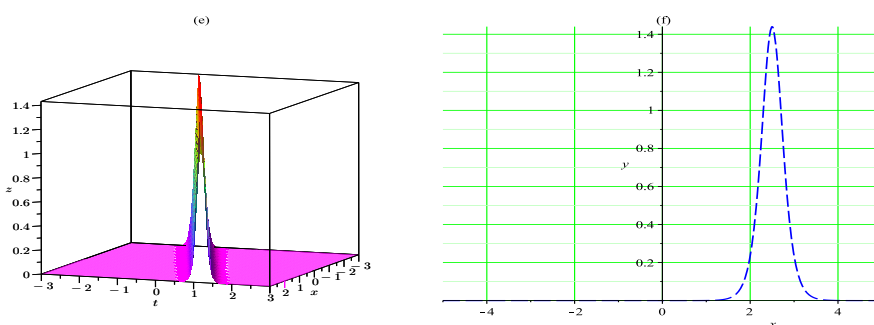


Figure 3. (e) 3D plot of bright soliton solution (4.5) for $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (f) 2D representation of (4.5) for $t = 1$.

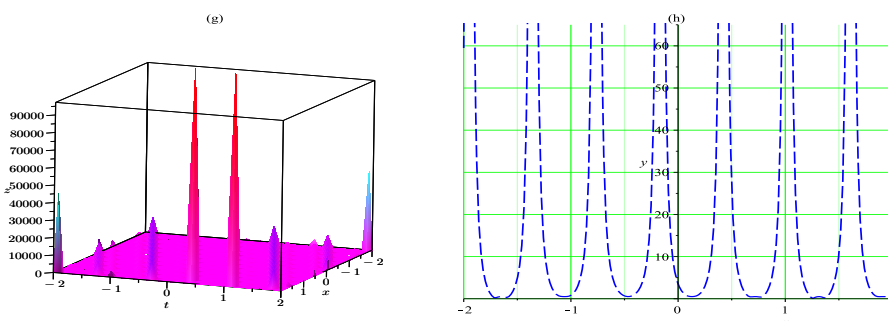


Figure 4. (g) 3D graph of periodic-singular soliton solution (4.11) for $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (h) 2D representation of (4.11) for $t = 1$.

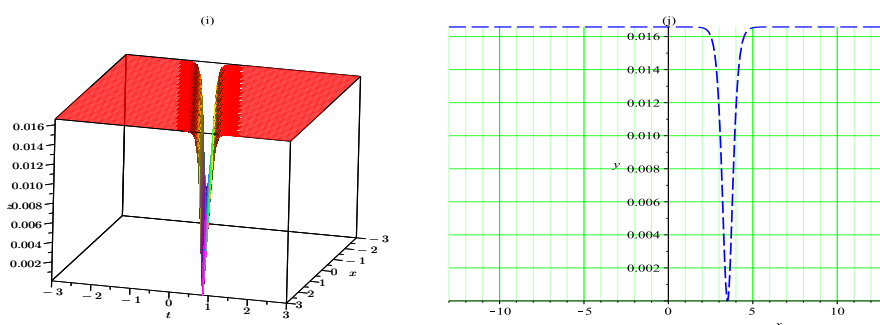


Figure 5. (i) 3D graph of singular soliton solution (4.13) for $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (j) 2D representation of (4.13) for $t = 1$.

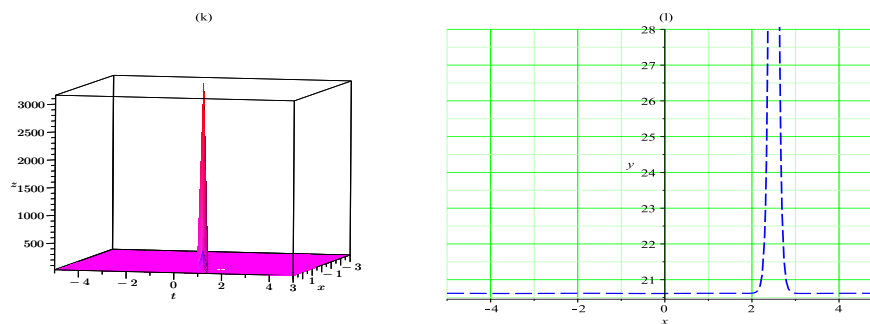


Figure 6. (k) 3D graph of periodic-singular soliton solution (4.14) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$, $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (l) 2D representation of (4.14) for $t = 1$.

6. Conclusions

In this paper, we have successfully employed the EHF method on the BAE and constructed the bright, periodic, dark, singular and combo solitons. These constructed results have much importance in many areas of non-linear sciences, such as physics, birefringent fibers, applied mathematics, optical fibers, engineering, pulse propagation and many more. The outcomes of this study are motivating and improve our understanding of the optical soliton solutions. By these outcomes, we can recognize that the present method is appropriate, useful and skilled for attaining the exact solutions of such kind of problems. The acquired results are fresh, correct and not reported earlier in literature.

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Conflict of interest

The authors declare that they have no conflict of interest.

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