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Research Paper

Multi-complexiton and positive multi-complexiton structures to a generalized B-type Kadomtsev–Petviashvili equation

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ABSTRACT

Recently, Zhang et al. (International Journal of Modern Physics B 30 (2016) 1640029) constructed N-wave solutions of a generalized B-type Kadomtsev–Petviashvili (gbKP) equation using the linear superposition method. The authors' aim of the present paper is to derive multi-complexiton and positive multi-complexiton structures of the gbKP equation through considering N-wave solutions and applying specific systematic methods. To investigate the dynamical characteristics of positive multi-complexiton structures, particularly single and double positive complexitons, several two and three-dimensional simulations are formally considered. The results of the current research enrich the studies regarding the gbKP equation.

1. Introduction

Nowadays, the search for exact solutions to nonlinear partial differential equations (NLPEDs) is a very hot research topic, especially in nonlinear sciences. As mentioned many times, NLPEDs are effective tools in modeling a lot of phenomena from fluid mechanics to nonlinear optics. Today, capable methods, benefitting from particular packages for handling symbolic computations, have been used to deal with NLPEDs. Some of these methods are simplified Hirota's method [1–5], multiple exp-function method [6–10], and ansatz methods [11–15]. Here are some uses for these methods. Hosseini et al. [5] employed the simplified Hirota's method to derive dispersive waves of a generalized Hirota Bilinear Equation. Li et al. [10] obtained multiple waves of a generalized Kadomtsev–Petviashvili equation using the multiple exp-function method. Paul et al. [15] applied ansatz methods to acquire lump, rogue, and breather waves of a generalized Kadomtsev–Petviashvili–Boussinesq equation.

Recently, Zhou and Manukure [16] employed effective methods to construct multi-complexiton and positive multi-complexiton structures of the Hirota–Satsuma–Ito equation. Such well-established methods were utilized several times by other scholars to derive multi-complexiton and positive multi-complexiton structures of the B-type Kadomtsev–Petviashvili equation [17], the generalized breaking soliton equation [18], the asymmetric Nizhnik–Novikov–Veselov equation [19], and the generalized Boiti–Leon–Manna–Pempinelli equation [20]. Such achievements encouraged the authors for applying these methods to obtain multi-complexiton and positive multi-complexiton structures of the gbKP

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equation [21–25]

$$u_{zt} - u_{xxxy} - 3(u_x u_y)_x + 3u_{xx} = 0, \tag{1}$$

describing the evolution of shallow water waves with insignificant effects of surface tension and viscosity [26]. Some of the previous studies on Eq. (1) are as follows. Ma and Fan [21] used the linear superposition principle to construct N-soliton solutions of the gbKP equation. In another study conducted by Zhang et al. [22], the linear superposition method was utilized to extract N-wave solutions of the gbKP equation. Using the logarithmic transformation $u = 2(\ln f)_x$, the gbKP equation is written as

$$\left(D_z D_t - D_x^3 D_y + 3D_x^2\right) f.f = 0,$$

or in the equivalent form

$$(f_{tz} - f_{xxxy} + 3f_{xx})f - f_t f_z + f_{xxx} f_y + 3f_{xxy} f_x - 3\ f_{xx}\ f_{xy} - 3f_x^2 = 0. \tag{2}$$

It should be pointed out that for $z = y$, the gbKP equation reduces the bKP equation in (2+1) dimensions as follows [27,28]

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x + 3u_{xx} = 0.$$

The organization of the current paper is as follows: In Section 2, based on the methods adopted by Zhou and Manukure in [16], multi-complexiton and positive multi-complexiton structures of the gbKP equation are obtained. In Section 3, to investigate the dynamical characteristics of positive multi-complexiton structures, particularly single and double positive complexitons, several two and three-dimensional simulations are formally considered. A review of the achievements is given in the last section.

2. The gbKP equation: its multi-complexiton and positive multi-complexiton structures

In the present section, based on the methods used by Zhou and Manukure, multi-complexiton and positive multi-complexiton structures of the gbKP equation are constructed. First, we would like to mention that the following N-wave solution to the gbKP equation was derived in [22] using the linear superposition method

$$u = 2(\ln f)_x,\quad f = \sum_{i=1}^{N} \epsilon_i e^{k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t},$$

where $\epsilon_i,\quad i = 1, 2, \ldots, N$ and $k_i,\quad i = 1, 2, \ldots, N$ are constants.

To construct the positive multi-complexiton structure of the gbKP equation, the following exponential functions are considered

$$f_1 = \epsilon_1 e^{\sigma_1} + \epsilon_2 e^{\sigma_2} + \ldots + \epsilon_N e^{\sigma_N},$$
$$\sigma_i = k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t, \qquad 1 \le i \le N,$$

$$f_2 = \epsilon_1 e^{-\sigma_1} + \epsilon_2 e^{-\sigma_2} + \ldots + \epsilon_N e^{-\sigma_N},$$
$$-\sigma_i = (-k_i)x + (-k_i)^{-1} y + a_3(-k_i)^{-1} z + \frac{1}{a_3}(-k_i)^3 t, \qquad 1 \le i \le N,$$

where $k_i \neq 0,\quad i = 1, \ldots, N$. As the above exponential functions satisfy Eq. (2), therefore the following new function

$$\frac{1}{2}(f_1 + f_2) = \sum_{i=1}^{N} \epsilon_i \cosh\left(k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t\right),$$

satisfies Eq. (2). Furthermore, by considering $k_{N+1}, k_{N+2}, \ldots, k_{N+M}$ (as nonzero constants) and

$$f_1 = \epsilon_{N+1} e^{\sigma_{N+1}} + \epsilon_{N+2} e^{\sigma_{N+2}} + \ldots + \epsilon_{N+M} e^{\sigma_{N+M}},$$
$$\sigma_i = (Ik_i)x + (Ik_i)^{-1} y + a_3 (Ik_i)^{-1} z + \frac{1}{a_3}(Ik_i)^3 t = I\left(k_i x - k_i^{-1} y - a_3 k_i^{-1} z - \frac{1}{a_3} k_i^3 t\right),$$

$$f_2 = \epsilon_{N+1} e^{-\sigma_{N+1}} + \epsilon_{N+2} e^{-\sigma_{N+2}} + \ldots + \epsilon_{N+M} e^{-\sigma_{N+M}},$$
$$-\sigma_i = I\left((-k_i)x - (-k_i)^{-1} y - a_3(-k_i)^{-1} z - \frac{1}{a_3}(-k_i)^3 t\right),$$

it is found that

$$\frac{1}{2}(f_1 + f_2) = \sum_{i=N+1}^{N+M} \epsilon_i \cos\left(k_i x - k_i^{-1} y - a_3 k_i^{-1} z - \frac{1}{a_3} k_i^3 t\right),$$

satisfy Eq. (2). Now, the new function

$$f = \sum_{i=1}^{N} \epsilon_i \cosh\left(k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t\right) + \sum_{i=N+1}^{N+M} \epsilon_i \cos\left(k_i x - k_i^{-1} y - a_3 k_i^{-1} z - \frac{1}{a_3} k_i^3 t\right),$$

when $\epsilon_i > 0,\quad i = 1, 2, \ldots, N$ and $\sum_{i=1}^{N} \epsilon_i > \sum_{i=N+1}^{N+M} |\epsilon_i|$ is a positive function and accordingly, the positive compelexiton structure of the gbKP equation can be expressed as

$$u = 2(\ln f)_x,$$

where

$$f = \sum_{i=1}^{N} \epsilon_i \cosh\left(k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t\right) + \sum_{i=N+1}^{N+M} \epsilon_i \cos\left(k_i x - k_i^{-1} y - a_3 k_i^{-1} z - \frac{1}{a_3} k_i^3 t\right),$$
$$\epsilon_i > 0,\quad i = 1, 2, \ldots, N$$

and $\sum_{i=1}^{N} \epsilon_i > \sum_{i=N+1}^{N+M} |\epsilon_i|$.

In order to extract the multi-complexiton structure of the gbKP equation, it is supposed that k_i can be written as

$$k_i = k_{1i} + Ik_{2i}, \quad k_{1i}, k_{2i} \in \mathbb{R}, \quad i = 1, 2, \ldots, N.$$

By considering the above assumption, one can find

$$\sigma_i = k_i x + k_i^{-1} y + a_3 k_i^{-1} z + \frac{1}{a_3} k_i^3 t = \sigma_{i,1} + I\sigma_{i,2},$$
$$\bar{\sigma}_i = \bar{k}_i x + \bar{k}_i^{-1} y + a_3 \bar{k}_i^{-1} z + \frac{1}{a_3} \bar{k}_i^3 t = \sigma_{i,1} - I\sigma_{i,2}.$$

As e^{σ_i} and $e^{\bar{\sigma}_i}$ satisfy Eq. (2), consequently, the new function

$$f = \sum_{i=1}^{N} \left(\epsilon_i e^{\sigma_i} + \bar{\epsilon}_i e^{\bar{\sigma}_i}\right) = \sum_{i=1}^{N} e^{\sigma_{i,1}} \left(\epsilon_{i,1} \cos\ (\sigma_{i,2}) + \epsilon_{i,2} \sin\ (\sigma_{i,2})\right), \qquad \epsilon_{i,1}, \epsilon_{i,2} \in \mathbb{R},$$

satisfy Eq. (2). As a result, the multi-complexiton structure of the gbKP equation is as

$$u = 2(\ln f)_x,$$

where

$$f = \sum_{i=1}^{N} \left(\epsilon_i e^{\sigma_i} + \bar{\epsilon}_i e^{\bar{\sigma}_i}\right) = \sum_{i=1}^{N} e^{\sigma_{i,1}} \left(\epsilon_{i,1} \cos\ (\sigma_{i,2}) + \epsilon_{i,2} \sin\ (\sigma_{i,2})\right), \qquad \epsilon_{i,1}, \epsilon_{i,2} \in \mathbb{R}.$$

It is noteworthy that another N-wave solution of the gbKP equation was reported in [22] using the linear superposition method as follows

$$u = 2(\ln f)_x, \qquad f = \sum_{i=1}^{N} \epsilon_i e^{k_i x + k_i^{-1} y + a_3 k_i^3 z + \frac{1}{a_3} k_i^{-1} t},$$

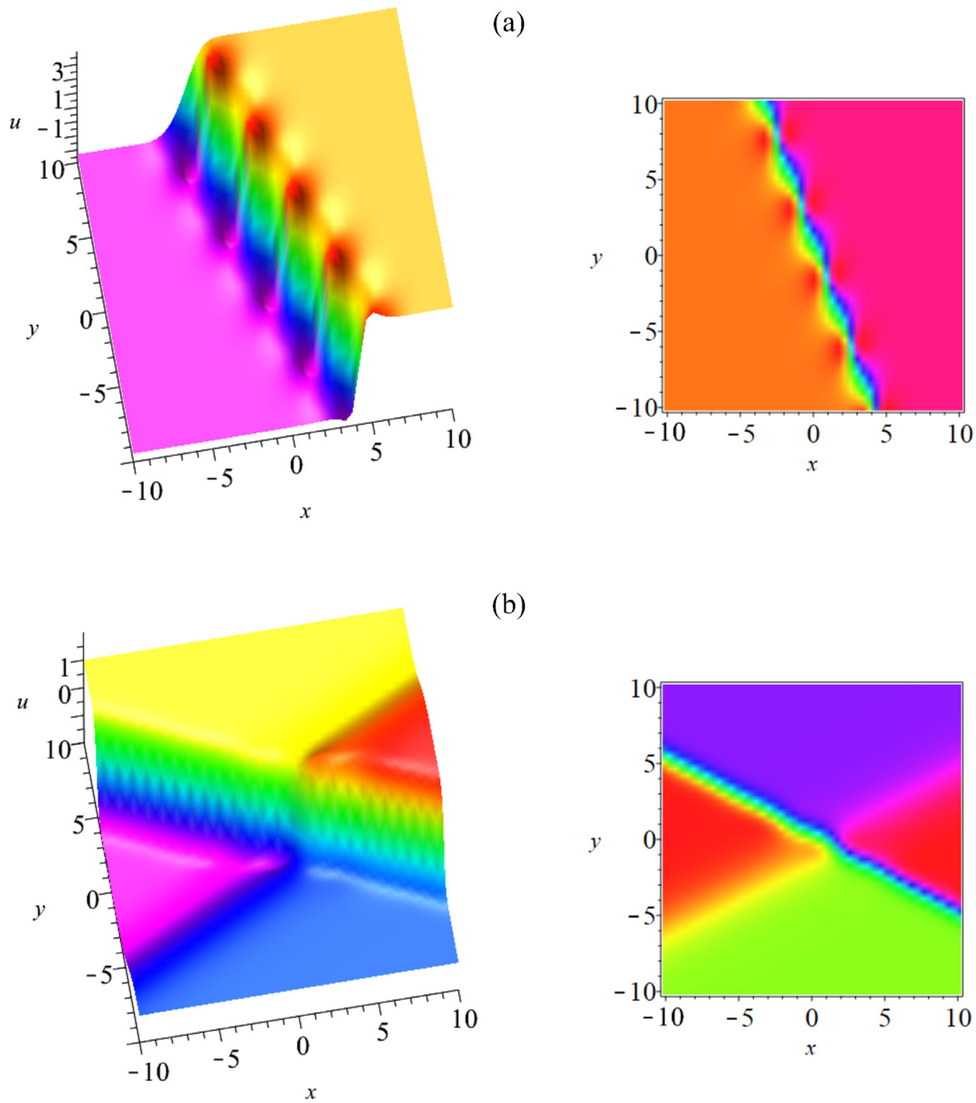

Fig. 1. . (a) Single positive complexiton (3); (b) Double positive complexiton (4).

where $\epsilon_i,\quad i=1,2,\ldots,N$ and $k_i,\quad i=1,2,\ldots,N$ are constants. In a similar manner, the following multi-complexiton and positive multi-complexiton structures to the gbKP equation are constructed

$$u=2(\ln f)_x,$$

where

$$f=\sum_{i=1}^{N}\left(\epsilon_i e^{\sigma_i}+\epsilon_i e^{\bar{\sigma}_i}\right)=\sum_{i=1}^{N}e^{\sigma_{i,1}}(\epsilon_{i,1}\cos\ (\sigma_{i,2})+\epsilon_{i,2}\sin\ (\sigma_{i,2})),$$

$$\sigma_i=k_ix+k_i^{-1}y+a_3k_i^3z+\frac{1}{a_3}k_i^{-1}t=\sigma_{i,1}+I\sigma_{i,2},$$

$$k_i=k_{1i}+Ik_{2i},\qquad k_{1i},k_{2i},\epsilon_{i,1},\epsilon_{i,2}\in\mathbb{R},$$

and

$$u=2(\ln f)_x,$$

where

$$f=\sum_{i=1}^{N}\epsilon_i\cosh\left(k_ix+k_i^{-1}y+a_3k_i^3z+\frac{1}{a_3}k_i^{-1}t\right)+\sum_{i=N+1}^{N+M}\epsilon_i\cos\left(k_ix-k_i^{-1}y-a_3k_i^3z-\frac{1}{a_3}k_i^{-1}t\right),$$

$$\epsilon_i>0,\quad i=1,2,\ldots,N$$

and $\sum_{i=1}^{N}\epsilon_i>\sum_{i=N+1}^{N+M}|\epsilon_i|$.

3. Simulations and discussion

In the current section, to investigate the dynamical characteristics of positive multi-complexiton structures, particularly single and double positive complexitons, several two and three-dimensional simulations are formally considered. Particularly, the first positive multi-complexiton structure for

$$\{N=1,\ M=1,\ \epsilon_1=2,\ \epsilon_2=1,\ k_1=1.65,\ k_2=-1,\ a_3=-1,\ z=1,\ t=0\},$$

$$\{N=2,\ M=2,\ \epsilon_1=5,\ \epsilon_2=5,\ \epsilon_3=1,\ \epsilon_4=1,\ k_1=0.5,\ k_2=-1,\ k_3=-1,\ k_4=1,\ a_3=-1,\ z=1,\ t=0\},$$

can be written as

$$u=\frac{2(3.3\sinh(1.65x+0.6060606061y-0.6060606061)-\sin(x-y+1))}{2\cosh(1.65x+0.6060606061y-0.6060606061)+\cos(x-y+1)},\qquad(3)$$

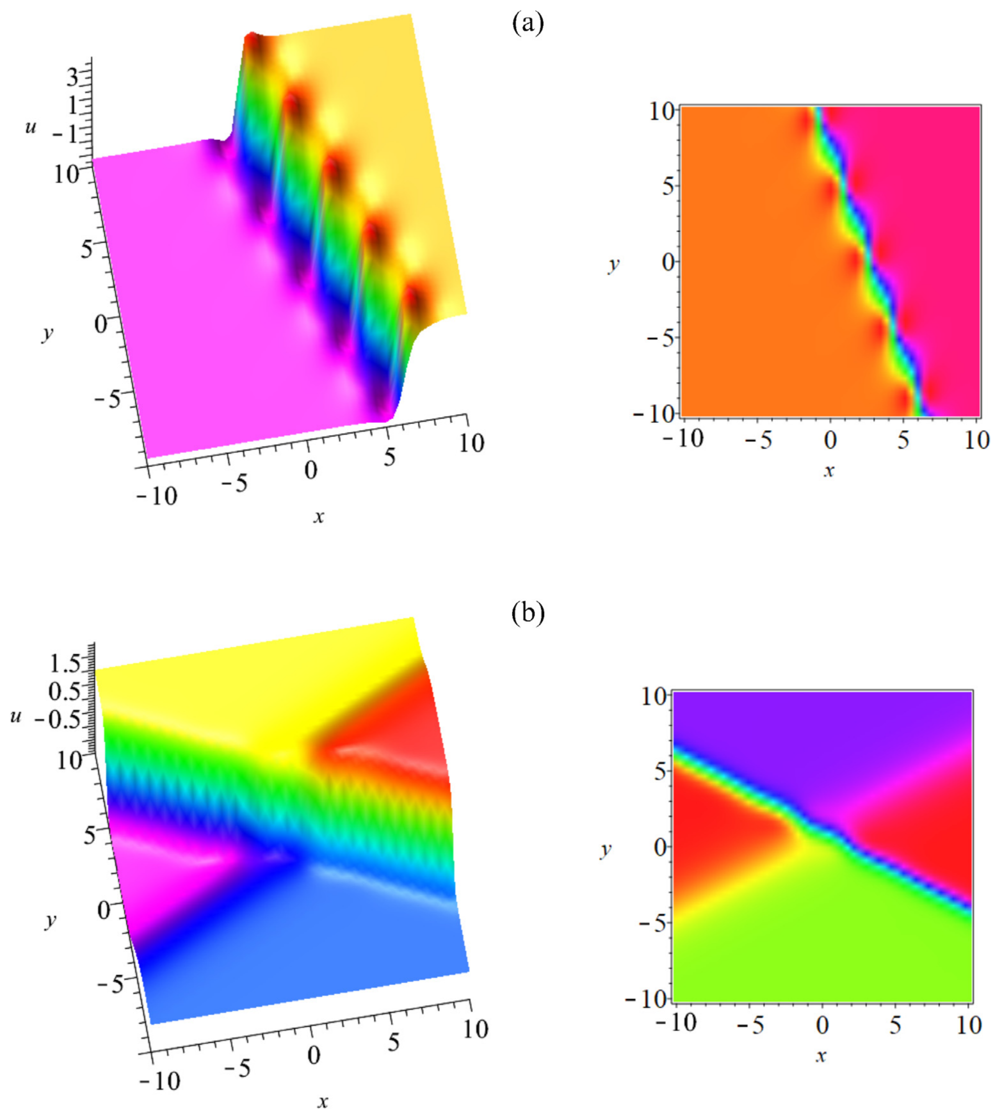

Fig. 2. . (a) Single positive complexiton (5); (b) Double positive complexiton (6).

$$u = \frac{2(2.5\sinh(0.5x+2y-0.125)+5\sinh(x+y-1)-2\sin(x-y+1))}{5\cosh(0.5x+2y-0.125)+5\cosh(x+y-1)+2\cos(x-y+1)}. \tag{4}$$

The dynamical behavior of the above single and double positive complexitons has been shown on the x-y plane in Fig. 1.

As another investigation, the second positive multi-complexiton structure for

$$\{N=1,\ M=1,\ \epsilon_1=2,\ \epsilon_2=1,\ k_1=1.65,\ k_2=-1,\ a_3=-1,\ z=1,\ t=0\},$$

$$\{N=2,\ M=2,\ \epsilon_1=5,\ \epsilon_2=5,\ \epsilon_3=1,\ \epsilon_4=1,\ k_1=0.5,\ k_2=-1,\ k_3=-1,\ k_4=1,\ \ a_3=-1,\ z=1,\ t=0\},$$

can be written as

$$u = \frac{2(3.3\sinh(1.65x+0.6060606061y-4.492125)-\sin(x-y+1))}{2\cosh(1.65x+0.6060606061y-4.492125)+\cos(x-y+1)}, \tag{5}$$

$$u = \frac{2(2.5\sinh(0.5x+2y-2)+5\sinh(x+y-1)-2\sin(x-y+1))}{5\cosh(0.5x+2y-2)+5\cosh(x+y-1)+2\cos(x-y+1)}. \tag{6}$$

Fig. 2 shows the dynamical behavior of the above single and double positive complexitons on the x-y plane.

4. Conclusion

In the present paper, the authors conducted a detailed investigation on the evolution of shallow water waves in a generalized B-type Kadomtsev–Petviashvili equation. More specifically, based on the N-wave solutions of the governing model (derived by Zhang et al.) and the methods used by Zhou and Manukure, multi-complexiton and positive multi-complexiton structures of the gbKP equation were successfully constructed. Additionally, the dynamical features of positive multi-complexiton structures, especially single and double positive complexitons, were examined through considering some two and three-dimensional plots. The achievements of the authors in the current paper play a significant role in completing the research on the gbKP equation. As a new task, the authors are interested in applying other methods [29–41] to acquire other wave structures of the gbKP equation.

Declaration of Competing Interest

The authors declare no conflict of interest.

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