

# Effect of source spatial partial coherence on the angle-of-arrival fluctuations for free-space optics links

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**Abstract.** The dependence of angle-of-arrival fluctuations on source coherence for free-space optics links in a turbulent atmosphere is examined. A monochromatic beam is taken, and the variation of the angle-of-arrival fluctuations for a spatially partially coherent source is investigated. Results are obtained for the currently used free-space optics links, which use infrared wavelengths of 0.85 and 1.55  $\mu\text{m}$  with link lengths of 3 and 5 km. The angle-of-arrival fluctuations are calculated and plotted against normalized source size and inner and outer scales of turbulence. It is observed that the angle-of-arrival fluctuations show behavior that is essentially independent of the degree of source partial coherence. In fact, as the source size increases, this dependence seems to almost disappear. It is further observed that mean square angle-of-arrival fluctuations become larger at greater propagation distances, at smaller inner scales of turbulence, and at larger outer scales of turbulence. However, the numerical values of the angle-of-arrival fluctuations found for all cases are not expected to degrade substantially the performance of a practical optical receiver having a field of view in the order of several milliradians. Our results presented here are compared with the existing theoretical and experimental work, and the range of applicability of our formulation is discussed. © 2006 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.2203371]

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## 1 Introduction

Free-space optics (FSO) communication has become increasingly popular in recent years. In broadband access applications, easy employment of FSO systems without the need for obtaining a license makes them attractive compared with fiber optics cable installations.

The effect of atmospheric turbulence on the variation in the angle of arrival of the mean wavefront has been studied in detail.<sup>1-5</sup> This effect may increase the field-of-view requirements in a direct-detection optical receiver. Angle-of-arrival fluctuations may also degrade the performance of a heterodyne system. The main studies made on the angle-of-arrival fluctuations, which calculate the phase-front variance around the average direction of the received beam, cover a coherent beam wave source that mimics the fundamental mode excitation of a laser incidence. The general conclusion in these studies is that the effect of angle-of-arrival fluctuations, even in strong turbulence, is minor for practical receivers with a finite field of view on the order of a few milliradians.

Lasers at infrared wavelengths are usually employed in the current applications of FSO broadband access communication systems, exhibiting basically coherent radiation if they operate in a single mode. However, multimode content in the laser excitation or nonlaser sources like LEDs will generate both temporally and spatially partially coherent

excitation. Recently we investigated<sup>6</sup> the angle-of-arrival fluctuations for a multimode laser source, concluding that the performance of a practical FSO receiver will not be affected substantially so long as the source excitation is confined to modes with indices below 20. In that study, it was pointed out that there was a need to also assess the dependence of the angle-of-arrival fluctuations on the degree of source coherence. The present work is undertaken to partially address this issue.

The question of interest here is to examine how the performance of a FSO communication system will be influenced, with respect to the angle-of-arrival fluctuations, when the source exhibits partial coherence. In this paper, we consider only the spatial partial coherence and do not take into account the temporal partial coherence. This is because temporal source coherence bandwidth of the source is practically much less than the carrier wavelength (narrowband), and the effect of atmospheric turbulence, especially in weakly turbulent regimes, is negligible in that case. Thus, our formulation represents quasimonochromatic incidence with central wavelength  $\lambda$  where the variation of the angle-of-arrival fluctuations for a spatially partially coherent source is investigated by introducing a source phase that changes spatially in a random manner.

Our results indicate that the angle-of-arrival fluctuations vary slightly, depending on the degree of the partial (spatial) coherence of the light source used. However, since the calculated magnitudes of the angle-of-arrival fluctuations stay on the order of tens of microradians, the performance

of a practical optical receiver, with a field of view of several milliradians, will not be affected. We note that our formulation cannot be extended to cover a completely incoherent source, since spatial partial coherence is introduced into the source by the random shifts and random tilts. Therefore an incoherent summation of the source modes is needed in order to mimic the perfectly incoherent source, which is not covered in this work.

## 2 Formulation of the Angle-of-Arrival Fluctuations for a Partially Coherent Beam Wave

When light travels through the atmosphere, it experiences amplitude and phase fluctuations due to turbulence. In this paper, we need to find the phase fluctuations, which are governed by the phase structure function

$$\begin{aligned} D_S(\mathbf{p}_1, \mathbf{p}_2, L) &= \langle [S(\mathbf{p}_1, L) - S(\mathbf{p}_2, L)]^2 \rangle \\ &= B_S(\mathbf{p}_1, \mathbf{p}_1, L) + B_S(\mathbf{p}_2, \mathbf{p}_2, L) - 2B_S(\mathbf{p}_1, \mathbf{p}_2, L). \end{aligned} \quad (1)$$

Here  $\langle \rangle$  denotes the ensemble average over the statistics of the source and the medium;  $B_S$  and  $D_S$  are the phase correlation function and the phase structure function, respectively;  $S$  is the phase at the receiver transverse plane coordinate  $\mathbf{p}=(p_x, p_y)$ ; and  $L$  is the path length, i.e., the length of the atmospheric link.

In our earlier work,<sup>7</sup> the phase structure function for a spatially partially coherent beam-wave source,  $D_S(\mathbf{p}_1, \mathbf{p}_2, L)$ , was formulated. In this formulation, the laser source is taken as a Gaussian beam wave, and the spatial partial coherence of the source is introduced by a random phase model consisting of a random phase shift and random phase tilt. The source field can be written as

$$u(\mathbf{s}) = A \exp \left[ - \left( \frac{1}{2\alpha_s} + \frac{jk}{2F} \right) \mathbf{s}^2 \right] \exp(j\phi), \quad (2)$$

where  $A \exp[-(1/2\alpha_s + jk/2F)\mathbf{s}^2]$  represents the deterministic part, and  $\exp(j\phi)$  represents the random part, of the source field. Here  $\mathbf{s}$  is the transverse source coordinate,  $j=(-1)^{1/2}$ ,  $A$  is the amplitude,  $\alpha_s$  is the source beam size,  $k=2\pi/\lambda$  is the wave number,  $\lambda$  is the wavelength of operation, and  $F$  is the focal length (infinity for a collimated beam). Source spatial partial coherence is introduced through the random phase  $\phi$ , which is modeled as

$$\phi = a + i\mathbf{b} \cdot \mathbf{s}, \quad (3)$$

where  $a$  is the random phase shift and  $\mathbf{b}$  is a random vector introducing random tilt into the phase.

In Rytov method, the field at the point  $(\mathbf{p}, z)$  in the presence of turbulence is given by

$$u(\mathbf{p}, z) = u^{\text{FS}}(\mathbf{p}, z) \exp[\psi(\mathbf{p}, z)], \quad (4)$$

where

$$\begin{aligned} \psi(\mathbf{p}, z) &= \chi(\mathbf{p}, z) + jS(\mathbf{p}, z) \\ &= \frac{k^2}{2\pi u^{\text{FS}}(\mathbf{p}, z)} \int_0^L dz' \int_{-\infty}^{\infty} dp'_x \int_{-\infty}^{\infty} dp'_y n_1(\mathbf{p}', z') \\ &\quad \times u^{\text{FS}}(\mathbf{p}', z') \frac{\exp(jk|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} \end{aligned} \quad (5)$$

denotes the fluctuations of the complex amplitude in turbulence,  $u^{\text{FS}}(\mathbf{p}', z')$  is the field in free space (i.e., in the absence of turbulence) at  $\mathbf{r}'=(\mathbf{p}', z')=(p'_x, p'_y, z')$ ,  $n_1$  is the random part of the refractive index,  $\chi(\mathbf{p}, z)$  represents the log-amplitude fluctuations, and

$$S(\mathbf{p}, z) = \frac{1}{2j} [\psi(\mathbf{p}, z) - \psi^*(\mathbf{p}, z)]. \quad (6)$$

In obtaining the phase structure function by using the Rytov approximation, first using the source field given in Eq. (2),  $u^{\text{FS}}(\mathbf{p}', z')$  is found by utilizing the Huygens-Fresnel principle; then this  $u^{\text{FS}}(\mathbf{p}', z')$  is inserted in Eq. (5), integrations over  $p'_x, p'_y$  are performed, and  $\psi(\mathbf{p}, z)$  thus found is inserted into Eq. (6) to find the phase fluctuations  $S(\mathbf{p}, z)$ , from which the phase correlation function is evaluated as

$$B_S(\mathbf{p}_2, \mathbf{p}_2, L) = \langle S(\mathbf{p}_1, L) S(\mathbf{p}_2, L) \rangle. \quad (7)$$

Here  $\langle \rangle$  represents the ensemble average over both the medium and the source statistics. Employing Eq. (7) in Eq. (1), the phase structure function for a spatially partially coherent source in turbulence is obtained.

The remaining part of the derivation is quite lengthy, and it follows similar lines to those in Ref. 8, which covers coherent beams. Thus, here we point out the difference in the derivation introduced by the random source phase term given in Eq. (3), i.e., by the spatial partial coherence. Assuming that the medium and source statistics are independent, in taking the ensemble average in Eq. (7), for the source part we have the second-order source coherence function for the random portion of the field,

$$\langle \exp\{j[\phi(\mathbf{s}_1) - \phi(\mathbf{s}_2)]\} \rangle_s, \quad (8)$$

where  $\langle \rangle_s$  denotes the ensemble average over the source statistics. For Gaussian-distributed  $\phi$  with zero mean value, Eq. (8) becomes  $\exp[-\langle b^2 \rangle_s (\mathbf{s}_1 - \mathbf{s}_2)^2 / 4]$ . The degree of source partial coherence  $\rho_s$  is defined as  $\rho_s^2 = 1 / \langle b^2 \rangle_s$ , which is a measure indicating the distance over which the random phase at two source points stays correlated. Thus, within the model used, the second-order source coherence function for the random portion of the field given in Eq. (8) becomes

$$\exp \left[ - \frac{(\mathbf{s}_1 - \mathbf{s}_2)^2}{4\rho_s^2} \right], \quad (9)$$

where  $\rho_s$  has the dimension of distance [m], and  $\rho_s = \infty$  m and  $\rho_s = 0$  m will in turn correspond to the perfect spatially coherent source and completely incoherent source. When  $\rho_s$  is between these limits, then the source is said to be partially spatially coherent.

This model representing source spatial partial coherence has been used by several researchers,<sup>9,10</sup> and it is known to describe a multimode laser source.<sup>2</sup> A Gaussian Schell-model source model is also used to describe partial coherence and has been shown to be a good approximation for a multimode laser source.<sup>11</sup> The degree of coherence is also introduced as a function of two spatial points.<sup>12</sup> As seen from Eq. (9), the source partial coherence parameter  $\rho_s$  enters into the expression for the second-order source coherence function for the random portion of the field as a scale determining the exponential decay of the coherence function with respect to the square of the distance between two source points. In the source model we used for partial coherence, it is easier to obtain tractable numerical evaluations.

The function  $D_S(\mathbf{p}_1, \mathbf{p}_2, L)$  is rewritten below; the details of the source model and the derivation of  $D_S(\mathbf{p}_1, \mathbf{p}_2, L)$  can be found in Ref. 7:

$$D_S(\mathbf{p}_1, \mathbf{p}_2, L) = 4\pi^2 \operatorname{Re} \left[ \int_0^L d\eta \int_0^\infty \kappa d\kappa \left\{ [0.5J_0(2j\kappa P_1 \gamma_i) + 0.5J_0(2j\kappa P_2 \gamma_i) - J_0(\kappa P)] |H_1|^2 \right. \right. \\ \left. \left. - [1 - J_0(\kappa Q)] H^2 \right\} \Phi_n(\kappa) \right], \quad (10)$$

where  $\operatorname{Re}$  means the real part, and  $\mathbf{p}_1 = (p_{x_1}, p_{y_1})$  and  $\mathbf{p}_2 = (p_{x_2}, p_{y_2})$  are the transverse receiver coordinates of the first and second phase points, respectively. Integration over  $\eta$  takes into account the contribution of the fluctuations throughout the link length, whereas the integration over  $\kappa$  incorporates the contribution from all spatial scales of turbulence, i.e., over the turbulent blobs. In Eq. (10),  $J_0$  refers to the zeroth-order Bessel function of first kind,  $\Phi_n(\kappa)$  is the three-dimensional spectral density of the index of refraction fluctuations. The rest of the symbols are defined as

$$H^2 = -k^2 \exp - [j\gamma(L - \eta)\kappa^2/k],$$

$$|H_1|^2 = k^2 \exp(\beta_2 \kappa^2), \quad \beta_2 = -(L - \eta)^2 \xi_1,$$

$$P_m = (p_{x_m}^2 + p_{y_m}^2)^{1/2}, \quad m = 1, 2,$$

$$P = [(\gamma p_{x_1} - \gamma^* p_{x_2})^2 + (\gamma p_{y_1} - \gamma^* p_{y_2})^2]^{1/2},$$

$$Q = \gamma [(p_{x_1} - p_{x_2})^2 + (p_{y_1} - p_{y_2})^2]^{1/2},$$

$$\gamma = \frac{1 + j\alpha\eta}{1 + j\alpha L}, \quad \gamma_i = \operatorname{Im}(\gamma),$$

$$\alpha = \frac{1}{k\alpha_s^2} + \frac{j}{F},$$

$$\xi_1 = \frac{(\rho_s^2/L^2)[k^2\alpha_s^2(1 - L/F)^2 + (L/\alpha_s)^2] - 1}{(\rho_s^2/L^2)[k^2\alpha_s^2(1 - L/F)^2 + (L/\alpha_s)^2]}. \quad (11)$$

In Eq. (11), \* indicates the complex conjugate, and  $\operatorname{Im}$  the imaginary part.

The mean square angle-of-arrival fluctuation,  $\langle \alpha_a^2 \rangle$ , is related to the phase structure function through the expression below:

$$\langle \alpha_a^2 \rangle = \frac{D_S(\mathbf{p}_1 = 0, |\mathbf{p}_2| = R, L)}{(kR)^2}. \quad (12)$$

Light traveling through the atmosphere becomes incident on a receiver plane normal to the direction of propagation. Here the mean square angle-of-arrival fluctuations are evaluated at a radial distance  $R$  where the second phase point lies at  $\mathbf{p}_2 = (p_{x_2}, p_{y_2})$ ,  $|\mathbf{p}_2| = R$ , and the first phase point is at the origin  $\mathbf{p}_1 = (p_{x_1}, p_{y_1}) = (0, 0)$ .

The three-dimensional spectral density of the index of refraction fluctuations, described by the von Kármán spectrum, is given by

$$\Phi_n(\kappa) = \frac{0.033 C_n^2 \exp(-\kappa^2/\kappa_m^2)}{\kappa^2 + \kappa_0^2}. \quad (13)$$

Here  $C_n^2$  is the structure constant;  $\kappa_m = 5.92/\ell_0$ , where  $\ell_0$  is the inner scale of turbulence; and  $\kappa_0 = 1/L_0$ , where  $L_0$  is the outer scale of turbulence. For  $R$  much smaller than the inner scale of turbulence, the integrand in Eq. (10) exists due to the presence of the second term in the series expansions of the Bessel functions. Thus by setting  $J_0(z) \approx 1 - 0.25z^2$  for small  $z$ , it is possible to reduce Eq. (10) to a single integral. To achieve this, Eq. (17) in Appendix II and the approximation in Appendix I of Ref. 13 are employed. Also, since  $L_0 \gg \ell_0$ , and further applying the change of variable  $\eta = Lt$ , the angle-of-arrival fluctuations for a spatially partially coherent beam-wave light source is obtained as

$$\langle \alpha_a^2 \rangle = 0.1628 C_n^2 L \kappa_0^{1/3} \int_0^1 dt \left( -7.2 + 5.5663 \right. \\ \times \left\{ \frac{\kappa_m^2}{\kappa_0^2 [1 + \kappa_m^2 L^2 (1-t)^2 \xi_1]} \right\}^{1/6} |\gamma|^2 \\ \left. + \operatorname{Re} \left( -7.2 + 5.5663 \left\{ \frac{k\kappa_m^2}{\kappa_0^2 [k + j\gamma L (1-t)\kappa_m^2]} \right\}^{1/6} \gamma^2 \right) \right). \quad (14)$$

Since the angle-of-arrival fluctuations are evaluated for  $R$  much smaller than the inner scale of turbulence, we present below the limiting form of Eq. (14) when  $\ell_0 \rightarrow 0$  and consequently  $\kappa_0 \rightarrow \infty$ :

$$\langle \alpha_a^2 \rangle = 0.906 C_n^2 L \int_0^1 dt [L^{-1/3} (1-t)^{-1/3} \xi_1^{-1/6} |\gamma|^2 \\ + L^{-1/6} (1-t)^{-1/6} k^{1/6} |\gamma|^{11/6} \cos \theta], \quad (15)$$

where

$$|\gamma| = (\gamma_r^2 + \gamma_i^2)^{1/2}, \quad \theta = \frac{\pi}{12} - \frac{11}{6} \tan^{-1} \left( \frac{\gamma_i}{\gamma_r} \right),$$

$$\gamma_r = \frac{1 + b_1^2 t}{1 + b_1^2}, \quad \gamma_i = \frac{b_1(t-1)}{1 + b_1^2}, \quad b_1 = \frac{L}{k\alpha_s^2}. \quad (16)$$

Note that due to the new parameter  $b_1$  appearing in Eq. (16), we have had to redefine  $\gamma_i$  that was originally introduced in Eq. (11).

From the definitions given in Eq. (11), it is obvious that  $\xi_1$  must be positive; otherwise the integrand in Eq. (10) would not converge. Consequently a limitation is imposed on  $\rho_s$ :

$$\frac{\alpha_s}{\rho_s} < \left[ 4\pi^2 \frac{\alpha_s^4}{(\lambda L)^2} + 1 \right]^{1/2}. \quad (17)$$

This restriction on  $\rho_s$  prevents us for obtaining the angle-of-arrival fluctuations for a completely incoherent source. The parameter  $\alpha_s/\rho_s$  in Eq. (17) is dimensionless, since both the source size ( $\alpha_s$ ) and the degree of source partial coherence ( $\rho_s$ ) have the dimensions of meters.

Here we note that, even though Eq. (15) represents the limiting form of the angle-of-arrival fluctuations for zero inner scale, in our numerical evaluations we used Eq. (14) to introduce also the effect of the finite inner scale on the angle-of-arrival fluctuations.

### 3 Results and Discussion

Taking a collimated beam, Eq. (14) is numerically evaluated and used in plotting the root mean square of the angle-of-arrival fluctuations,  $\langle \alpha_a^2 \rangle^{1/2}$ , versus the source size normalized with respect to the Fresnel zone,  $(\alpha_s/\lambda L)^{1/2}$ . In our graphs, two infrared wavelengths are employed, representing the most commonly utilized wavelengths in the current FSO links. As defined earlier,  $\rho_s = \infty$  and  $\rho_s = 0$  correspond to perfect spatial coherence and complete incoherence, respectively. To be more accurate, we note that, because  $\rho_s$  is physically a measure indicating the distance over which the random phase at two source points stays correlated, for a fixed size source, as  $\rho_s$  increases, the source becomes more coherent, and as  $\rho_s$  decreases, the source becomes less coherent. On the other hand, when  $\rho_s$  is fixed, as the source size increases, the source becomes less coherent, and as the source size decreases, the source becomes more coherent. This means the scale  $\rho_s$  by itself does not accurately describe the spatial partial coherence of the source; the ratio of  $\rho_s$  to the source size more accurately determines the partial coherent regime. For example, when the value of  $\rho_s = 0.03$  taken in the plots is considered, it gives the distance at which the random source phase stays correlated. Relative value of  $\rho_s = 0.03$  compared to the corresponding source size determines more accurately the partial coherence property.

In Fig. 1, the variations of  $\langle \alpha_a^2 \rangle^{1/2}$  are displayed for an FSO link operating at  $\lambda = 0.85 \mu\text{m}$ , and for  $L = 3$  km as well as for  $L = 5$  km. Figure 2 shows the same variations of  $\langle \alpha_a^2 \rangle^{1/2}$  as Fig. 1, but this time the wavelength of operation,  $\lambda$ , is raised to  $1.55 \mu\text{m}$ .

As observed from Figs. 1 and 2, within the permitted range of values for  $\rho_s$  according to Eq. (17), the behavior of the angle-of-arrival fluctuations is virtually independent of the degree of source partial coherence. Indeed, in both cases, as the source size increases (eventually approaching

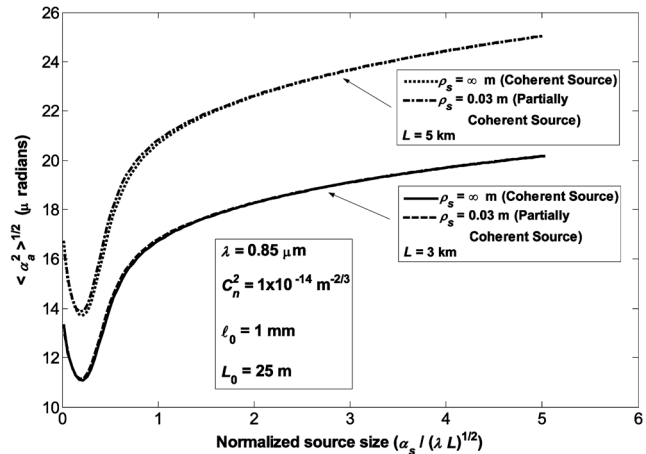


Fig. 1 Mean square angle-of-arrival fluctuations versus normalized source size at a wavelength of  $0.85 \mu\text{m}$  and link lengths of 3 and 5 km.

the limit of plane wave), the dependence on the source coherence seems to almost disappear. By comparing Fig. 1 against Fig. 2, it is seen that, for the given source and propagation parameters, the mean square angle-of-arrival fluctuations become less at longer wavelengths. In any case, however, longer distances have the overall effect of increasing the angle-of-arrival fluctuations. From the numerical values placed along the vertical axes of these graphs, we are able to deduce that angle-of-arrival fluctuations will not heavily degrade the performance of a practical optical receiver having a field of view on the order of several milliradians.

We note that in our solution, very small value of  $\rho_s$  that make  $\xi_1$  in Eq. (11) very small yield unreasonable angle-of-arrival fluctuation values, as also can be seen from Eq. (15). This is a limitation of our solution, that we cannot approach the incoherent limit (i.e.,  $\rho_s \rightarrow 0$ ). The results seen in Figs. 1 and 2 that the angle-of-arrival fluctuations are essentially independent of the degree of source partial coherence might be because that we cannot reach the incoher-

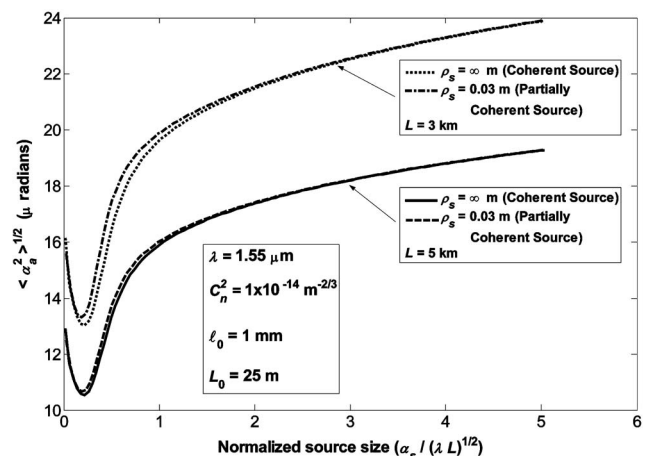
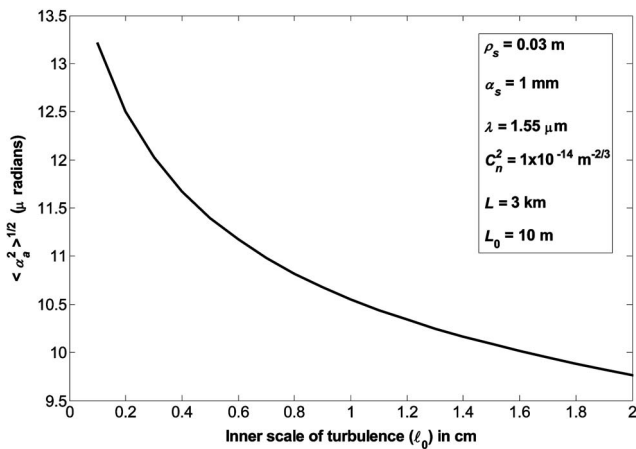


Fig. 2 Mean square angle-of-arrival fluctuations versus normalized source size at a wavelength of  $1.55 \mu\text{m}$  and link lengths of 3 and 5 km.



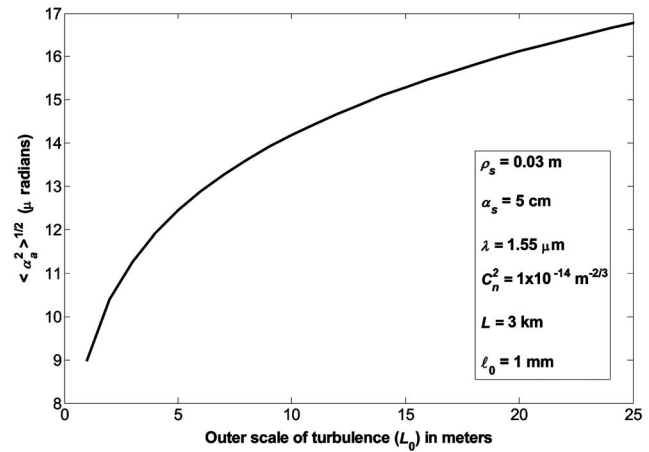
**Fig. 3** Mean square angle-of-arrival fluctuations versus inner scale of turbulence at constant source partial coherence, source size, wavelength, and link length.

ent regime, in which the angle-of-arrival fluctuations can be differentiated more than in the relatively coherent regime.

The evaluations for the two sets in Figs. 1 and 2 are made in such a manner that in each case, the horizontal axis is normalized with respect to the related link length and wavelength. It is observed in Figs. 1 and 2 that for intermediate-size beams the mean square angle-of-arrival fluctuations decrease sharply. This recalls the recent work by Baker and Benson,<sup>14</sup> where they point out that there is a deficiency in the analytical theory (Rytov approximation) when beam wander has to be considered, and due to this deficiency, for intermediate-size beams, the predicted on-axis scintillation index shows discrepancies when compared with the numerical wave-optics simulation. However, it is not easy to make one-to-one comparison of the intermediate-size portions of our Figs. 1 and 2 with the work in Ref. 14, since we deal basically with the phase structure function, whereas the on-axis scintillation in weak turbulence is based on the log-amplitude variance.

Figures 1 and 2 are compared with the existing theoretical and experimental results for the root-mean-square angular displacement of a coherent beam.<sup>5</sup> Even though the comparison could only be made under different source and medium parameters, we can comment that the angle of arrival in Figs. 1 and 2 shows a similar trend to the one obtained theoretically in Ref. 5 for large-size coherent beam. This trend changes both theoretically and experimentally, especially in the range of small and intermediate beam sizes, which might be attributed to the possible limitations of the Rytov analytical theory as predicted in Ref. 14.

Next we investigate the dependence of angle-of-arrival fluctuations on the inner scale of turbulence. For this purpose, Fig. 3 shows the variations of  $\langle \alpha_a^2 \rangle^{1/2}$  against  $\ell_0$ . It is seen that  $\langle \alpha_a^2 \rangle^{1/2}$  becomes smaller for larger  $\ell_0$ . However, the absolute values of  $\langle \alpha_a^2 \rangle^{1/2}$  still remain in the microradian range. The decrease in the angle-of-arrival fluctuations with the increase in the inner scale of turbulence is attributed to the fact that the contribution to the phase fluctuations coming from the larger spatial frequencies of the refractive in-



**Fig. 4** Mean square angle-of-arrival fluctuations versus outer scale of turbulence at constant source partial coherence, source size, wavelength, and link length.

dex spectral density is smaller when inner scale is large. Figure 4 illustrates the variation of  $\langle \alpha_a^2 \rangle^{1/2}$  against the outer scale  $L_0$ . It is observed from Fig. 4 that the changes in the outer scale have a larger impact on the angle-of-arrival fluctuations.

We reiterate that the preceding results are constrained to the permitted range of values for  $\rho_s$  as defined by Eq. (16). To overcome the limitation on the degree of source spatial coherence  $\rho_s$ , we need to consider modal analyses with random phase introduced for each mode. Through such an analysis, one can obtain universally applicable results that will also cover the completely incoherent regime.

In Ref. 2, phase fluctuations of a multimode laser field are examined in a turbulent atmosphere where the phase fluctuations of the wave are split into two parts: the random phase distortions in a homogeneous medium due to source partial coherence, and the random phase distortions due to the turbulent atmosphere. Then the theoretical predictions are compared with the experimental data obtained under incoherent thermal source excitation. Unfortunately, a direct comparison of our work with the results in Ref. 2 is not possible.

Comparing the results of this work to our earlier findings,<sup>6</sup> we see that the magnitude of angle-of-arrival fluctuations remains on the order of several microradians both for a source with multimode content and for a spatially partially coherent source. Thus, from the point of view of angle-of-arrival fluctuations, neither the multimode content nor the spatial partial coherence of the source can manifest itself as a serious design consideration for an FSO link.

## 4 Conclusion

In this study, we have investigated the effect of fluctuations in the angle of arrival in FSO and its relation to source coherence and inner and outer scales of turbulence. It is found that perfectly coherent and partially coherent sources are not expected to have a serious affect on the receiver performance. However, as is known, depending on the turbulence strength, partial coherence of the source affects the second- and fourth-order moments of the field to a certain extent. This fact may not seem in line with our findings.

The discrepancy may arise because we cannot extend our solution to more incoherent sources where a noticeable difference in the angle-of-arrival fluctuations occurs from those obtained with a relatively coherent source. The dependence of the angle-of-arrival fluctuations of a partially coherent source on the inner-scale and outer-scale values of turbulence is still not major. To extend the analysis so that it includes the limit of completely incoherent sources (e.g., LEDs), a mathematical formulation involving incoherent multimode analysis is required, which is not covered in this paper. An extension of our formulation for angle-of-arrival fluctuations in strong turbulence by using a different source partial coherence model covering an arbitrary degree of coherence is foreseen.

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