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Research Article

n-Tupled Common Fixed Point Result in Fuzzy b-Metric Spaces

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1. Introduction

One of the several important settings in which fixed point theory has been explored is the fuzzy context [1, 2]. The introduction of the fuzzy set by Zadeh [3] was a turning point in the landscape of fuzzy mathematics. Fuzzy mathematics has improved enormously in the last two decades. Fuzzy set theory has many important applications in various fields of applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics and nervous system), image processing, control theory, and communication [4–9]. Kramosil [10] introduced the notion of fuzzy metric space. George and Veeramani [11] later on slightly modified Kramosil's definition of fuzzy metric space and also proved that every metric induces a fuzzy metric and every fuzzy metric induces Hausdorff topology. Sedghi and Shobe [12] introduced the concept of fuzzy b-metric space. Following this idea, many researchers analysed fixed point theory in fuzzy b-metric space via various contractive conditions [13–17].

Sessa [18] explored common fixed points of set-valued as well as single-valued mappings on a complete metric space under a contractive condition along with the commutativity concept. The concept of a common fixed point was later on generalized by many researchers [19–23]. Lakshmikantham and Bhaskar [24] initiated the concept of coupled fixed point. Ćirić and Lakshmikantham [25] established coupled coincidence and coupled common fixed point theorems in partially ordered metric spaces. Subsequently, many researchers explored coupled fixed point theory in various spaces [2, 12, 19, 25–28]. Hu [27] and Zhu and Xiao [2] gave a coupled fixed point theorem for contractions in fuzzy metric spaces. Vasuki [29] obtained common fixed point results in fuzzy metric spaces. Berinde and Borcut [30] presented the idea of a tripled fixed point and obtained some new tripled fixed point results using mixed g-monotone mapping. Their results generalize and extend the Bhaskar and Lakshmikantham's research for nonlinear mappings. Roldán et al. [31] investigated the multidimensional coincidence points between mappings. Roldán et al. [1] modified the concept of tripled fixed points and generalized the results of Berinde and Borcut [30] and Zhu and Xiao [2].

In this paper, we aim to generalize and extend the notion of coupled/tripled common fixed point by introducing the concept of n-tupled fixed point in fuzzy b-metric space. We established an existence and uniqueness theorem for contractive mapping in fuzzy b-metric space. Our main result generalizes and extends coupled and tripled fixed point theorems appearing in [1, 2, 13] to n-dimensional common fixed points in fuzzy b-metric space. Moreover, it can be particularized to complete metric spaces to obtain an n-tupled Brinde–Borcut type coincidence/fixed point result in a nonfuzzy domain.

The paper is organized as follows: Section 2 is devoted to recall the basic definitions and lemmas that will be crucial throughout the paper. In Section 3, we introduce the notions of an n-tupled common fixed point and an n-tupled coincidence point. Moreover, an existence and uniqueness theorem for mappings satisfying certain contractive conditions in fuzzy b-metric space has been proved. Section 4 is devoted to generalize the construction presented in Section 3 to nonfuzzy settings. Moreover, the idea is elaborated via a nontrivial example. In section 5, some applications of the main result of the paper are discussed, and a kind of Lipschitzian system and an integral system both for n variables have been solved.

2. Preliminaries

In this section, some terms and definitions are provided which will be used in the main work of this manuscript. Henceforth, $\mathbb{R}$ and $\mathbb{N}$ will denote the set of real numbers and positive integers, respectively, while S will stand for an arbitrary nonempty set. Arguments of a metric d and fuzzy metric $\mathfrak{T}$ will be represented by subscripts. For example, $d(u,v)$ and $\mathfrak{T}(x,y,\delta)$ will be represented by d_{uv} and $\mathfrak{T}_{xy}(\delta)$, respectively.

Definition 1 (see [32]). A map $*$: $[0,1]^2 \longrightarrow [0,1]$, such that $([0,1], \leq, *)$ is an ordered abelian topological monoid with unit 1, is called a continuous t-norm.

$x *_L y = \max\{x + y - 1, 0\}$, $x *_P y = xy$, and $x *_M y = \min\{x, y\}$ are examples of some frequently used continuous t-norm that satisfy $*_M \geq *_P \geq *_L$.

Definition 2 (see [33]). A continuous t-norm $*$ is said to be of H-type if the sequence $\{*^m x\}_{m=1}^{\infty}$ is equicontinuous at $x = 1$. That is, for all $\xi \in (0,1)$ there exists $\varsigma \in (0,1)$ such that $1 - \varsigma < x \leq 1$ implies that $*^m x > 1 - \xi$ for all $m \geq 1$, where the sequence $\{*^m x\}$ is defined as $*^1 x = x$ and $*^n x = (*^{n-1} x) * x$.

An important and most commonly used continuous t-norm of H-type is $*_M$ which satisfies $x *_M y \geq x *_P y$, $\forall x, y \in [0,1]$. The following lemma characterizes continuous t-norm to be of the H-type.

Lemma 1 (see [1]). *Let $*$ be a t-norm and $\epsilon \in (0,1]$. If*

$$a *_\varepsilon b = \begin{cases} a * b & ; \text{if } \max(a,b) \leq 1 - \varepsilon, \\ \min(a,b) & ; \text{if } \max(a,b) > 1 - \varepsilon. \end{cases} \tag{1}$$

Then, $*_\epsilon$ is a t-norm of theH-type.

Definition 3 (see [34]). The 3-tuple $(S, \mathfrak{T}, *)$ is called fuzzy metric space if S is a nonempty set, $*$ is a continuous t-norm, and $\mathfrak{F}$ is a fuzzy set on $S \times S \times (0,\infty)$ which satisfies the following conditions, for all $u, v, w \in S$ and $\delta, \rho > 0$.

(i) $[(FM1)]$ $\mathfrak{T}_{uv}(\delta) > 0$

(ii) $[(FM2)]$ $\mathfrak{T}_{uv}(\delta) = 1$, iff $u = v$

(iii) $[(FM3)]$ $\mathfrak{T}_{uv}(\delta) = \mathfrak{T}_{vu}(\delta)$

(iv) $[(FM4)]$ $\mathfrak{T}_{uw}(\delta + \rho) \geq \mathfrak{T}_{uv}(\delta) * \mathfrak{T}_{vw}(\rho)$

(v) $[(FM5)]$ $\mathfrak{T}_{uv}(.)$: $(0,\infty) \longrightarrow [0,1]$ is continuous

(vi) $[(FM6)]$ $\lim_{\delta \longrightarrow \infty} \mathfrak{T}_{uv}(\delta) = 1$

Definition 4 (see [12]). The 3-tuple $(S, \mathfrak{T}, *)$ is called fuzzy b-metric space (F b MS for short) if S is a nonempty set, $*$ is a continuous t-norm, and $\mathfrak{F}$ is a fuzzy set on $S \times S \times (0,\infty)$ which satisfies the following conditions, for all $u, v, w \in S$ and $\delta, \rho > 0$ and a given real number $b \geq 1$.

(i) $[(FM1)]$ $\mathfrak{T}_{uv}(\delta) > 0$

(ii) $[(FM2)]$ $\mathfrak{T}_{uv}(\delta) = 1$, iff $u = v$

(iii) $[(FM3)]$ $\mathfrak{T}_{uv}(\delta) = \mathfrak{T}_{vu}(\delta)$

(iv) $[(FM4)]$ $\mathfrak{T}_{uw}(\delta + \rho) \geq \mathfrak{T}_{uv}(\delta) * \mathfrak{T}_{vw}(\rho)$

(v) $[(FM5)]$ $\mathfrak{T}_{uv}(.)$: $(0,\infty) \longrightarrow [0,1]$ is continuous

(vi) $[(FM6)]$ $\lim_{\delta \longrightarrow \infty} \mathfrak{T}_{uv}(\delta) = 1$

Note that $\mathfrak{T}_{uv}(\delta)$ represents the degree of closeness between u and v with respect to $\delta > 0$. The fuzzy b-metric reduces to a fuzzy metric for $b = 1$. Therefore, the class of fuzzy b-metric spaces is larger than the class of fuzzy metric spaces. The following example shows that a fuzzy b-metric on a nonempty set S need not be a fuzzy metric.

Example 1 (see [13]). Let $S = \mathbb{R}$ and $\mathfrak{T}_{uv}(\delta) = e^{(-\|u-v\|^p/\delta)}$, for all $u, v \in S$ and $\delta > 0$ with $r * s = rs$. It can be easily verified that $(S, \mathfrak{T}, *)$ is a F b MS with $b = 2^{p-1}$. But for $p = 2$, $(S, \mathfrak{T}, *)$ is not a fuzzy metric space.

Remark 1 (see [14]). For $u \neq v$ and $\delta > 0$, it is always true that $0 < \mathfrak{T}_{uv}(\delta) < 1$.

Lemma 2 (see [35]). *$\mathfrak{T}_{uv}(.)$ is nondecreasing for all $u, v \in S$.*

Definition 5 (see [36, 37]). In a fuzzy b-metric space $(S, \mathfrak{T}, *)$:

(1) A sequence $\{u_n\}$ converges to $u \in S$ if for every $\xi \in (0,1)$ and $\delta > 0$ there exists $n_\xi \in \mathbb{N}$ such that $\mathfrak{T}(u_n, u, \delta) > 1 - \xi$, $\forall n \geq n_\xi$

(2) $\{u_n\}_{n\in\mathbb{N}}$ is said to be Cauchy sequence if for every positive real number $\xi \in (0,1)$ and $\delta > 0$ there exists $n_\xi \in \mathbb{N}$ such that $\mathfrak{T}(u_n, u_m, \delta) > 1 - \xi$, $\forall m, n \geq n_\xi$

(3) A F b MS is said to be complete if every Cauchy sequence converges in it

Remark 2 (see [14]). In general, a fuzzy b-metric is not continuous.

In a fuzzy b-metric space, we have the following proposition.

Proposition 1 (see [37]). *Let $(S, \mathfrak{T}, *)$ be a F b MS and suppose a sequence $\{u_n\}$ converges to u, then*

$$\begin{aligned} \mathfrak{T}\left(u, v, \frac{\delta}{b}\right) &\leq \lim \sup_{n \longrightarrow \infty} \mathfrak{T}(u_n, v, \delta) \leq \mathfrak{T}(u, v, b\delta), \\ \mathfrak{T}\left(u, v, \frac{\delta}{b}\right) &\leq \lim \inf_{n \longrightarrow \infty} \mathfrak{T}(u_n, v, \delta) \leq \mathfrak{T}(u, v, b\delta). \end{aligned} \tag{2}$$

3. Main Results

Definition 6. Let $T\colon S^n \longrightarrow S$ and $\theta\colon S \longrightarrow S$, a point $({}_1u, {}_2u, {}_3u, \ldots, {}_nu) \in S^n$ is said to be the following:

(1) n-tupled fixed point of T if $T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = {}_1u, T({}_2u, {}_3u, \ldots, {}_nu, {}_1u) = {}_2u, \ldots, T({}_nu, {}_1u, {}_2u, \ldots, {}_{n-1}u) = {}_nu$

(2) n-tupled coincidence point of T and θ if $T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = \theta({}_1u), T({}_2u, {}_3u, , \ldots, {}_nu, {}_1u) = \theta({}_2u), \ldots, T({}_nu, {}_1u, {}_2u, \ldots, {}_{n-1}u) = \theta({}_nu)$

(3) n-tupled common fixed point of T and θ if $T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = \theta({}_1u), T({}_2u, {}_3u, , \ldots, {}_nu, {}_1u) = \theta({}_2u) = {}_2u, \ldots, T({}_nu, {}_1u, {}_2u, \ldots, {}_{n-1}u) = \theta({}_nu) = {}_nu$

Definition 7. Let $T\colon S^n \longrightarrow S$ and $\theta\colon S \longrightarrow S$ be two mappings. Then, T and θ are said to be commuting if $\theta T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = T(\theta({}_1u), \theta({}_2u), \theta({}_3u), \ldots, \theta({}_nu))$.

In our proof of main result, we will use the following lemmas.

Lemma 3. *Let $T\colon S^n \longrightarrow S$ and $\theta\colon S \longrightarrow S$ be mappings on a F b MS S, such that $T \equiv u$ is constant and θ is continuous and commuting with T. Then, $(u, u, u, \ldots, u)$ is a unique n-tupled common fixed point of T and θ.*

Proof. As T is constant on S^n, therefore, there exists $u \in S$ such that $T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = u$ for all ${}_1u, {}_2u, {}_3u, \ldots, {}_nu \in S$. Then, from T and θ being commuting, it can be deduced that

$$\begin{aligned} \theta(u) &= \theta T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) \\ &= T(\theta({}_1u), \theta({}_2u), \theta({}_3u), \ldots, \theta({}_nu)) = u. \end{aligned} \tag{3}$$

Therefore, $u = \theta(u) = T(u, u, u \ldots, u)$. That is, $(u, u, u \ldots, u)$ is an n-tupled common fixed point of T and θ. Let $({}_1v, {}_2v, {}_3v, \ldots, {}_nv)$ be another n-tupled common fixed point of T and θ such that $u \neq {}_kv$ for $k = 1, 2, \ldots, n$. Then,

$$\begin{aligned} 1 > \mathfrak{T}_{u, {}_kv}(\delta) &= \mathfrak{T}_{T(u,u,u,\ldots,u) T({}_kv, {}_{k+1}v, {}_{k+2}v, \ldots, {}_{n+1-k}v)}(\delta) \\ &= \mathfrak{T}_{u,u}(\delta) = 1. \end{aligned} \tag{4}$$

Which is a contradiction. Therefore, $(u, u, u \ldots, u)$ is a unique n-tupled common fixed point of T and θ.

Let $\mathfrak{B}$ denote the class of all increasing and continuous functions $\beta\colon [0,1] \longrightarrow [0,1]$ such that $\beta(\delta) > \delta$ for all $\delta \in (0,1)$ with $\beta(0) = 0$ and $\beta(1) = 1$. Then, the following lemma is used. □

Lemma 4. *Let $(S, \mathfrak{T}, *)$ be a complete F b MS with $b \geq 1$, where $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let $T\colon S^n \longrightarrow S$ and $\theta\colon S \longrightarrow S$ be such that*

$$\mathfrak{T}_{T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) T({}_1v, {}_2v, {}_3v, \ldots, {}_nv)}(\delta) \geq \beta\Big(\mathfrak{T}_{\theta({}_1u), \theta({}_1v)}(b^n\delta) * \mathfrak{T}_{\theta({}_2u), \theta({}_2v)}(\delta) * \mathfrak{T}_{\theta({}_3u), \theta({}_3v)}(b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_nu), \theta({}_nv)}(b^n\delta)\Big). \tag{5}$$

For some $\beta \in \mathfrak{B}$, for all ${}_1u, {}_2u, {}_3u, \ldots, {}_nu, {}_1v, {}_2v, {}_3v, \ldots, {}_nv \in S$ and $\delta > 0$. Suppose $({}_1u, {}_2u, {}_3u, \ldots, {}_nu)$ is an n-tupled coincidence point of T and θ. Then,

$$\begin{aligned} T({}_1u, {}_2u, {}_3u, \ldots, {}_nu)(\delta) &= \theta({}_1u) = \theta({}_2u) = T({}_2u, {}_3u, \ldots, {}_nu, {}_1u)(\delta) = \theta({}_3u) \\ &= T({}_3u, {}_4u, \ldots, {}_nu, {}_1u, {}_2u)(\delta) = \theta({}_4u) = \ldots = \theta({}_nu) = T({}_nu, {}_1u, {}_2u, \ldots, {}_{n-1}u)(\delta). \end{aligned} \tag{6}$$

Proof. Suppose on the contrary that there are at least two distinct integers p and q in $\{1, 2, \ldots, n\}$ such that $\theta({}_pu) \neq \theta({}_qu)$. Let $\mathfrak{T}_{\theta({}_pu), \theta({}_qu)}(b^n\delta) * \mathfrak{T}_{\theta}\ ({}_{p+1}u),\ \theta({}_{q+1}u)\ (b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_{n+1-p}u), \theta({}_{n+1-q}u)}\ (\delta) = \mathfrak{T}_{\theta({}_iu), \theta({}_4u)}\ (b^n\delta)$.. Then,

$$\begin{aligned} \mathfrak{T}_{\theta({}_iu), \theta({}_ju)}(\delta) &= \mathfrak{T}_{T({}_iu, {}_{i+1}u, {}_{i+2}u, \ldots, {}_{n+1-i}u), T({}_ju, {}_{j+1}u, {}_{j+2}u, \ldots, {}_{n+1-j}u)}(\delta) \\ &\geq \beta\Big(\mathfrak{T}_{\theta({}_iu), \theta({}_ju)}(b^n\delta) * \mathfrak{T}_{\theta({}_{i+1}u), \theta({}_{j+1}u)}(b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_{n+1-i}u), \theta({}_{n+1-j}u)}(b^n\delta)\Big) \\ &\geq \beta\Big(\mathfrak{T}_{\theta({}_ju), \theta({}_ju)}(b^n\delta)\Big) > \mathfrak{T}_{\theta({}_ju), \theta({}_ju)}(b^n\delta) \geq \mathfrak{T}_{\theta({}_ju), \theta({}_ju)}(b\delta), \end{aligned} \tag{7}$$

which is a contradiction. Hence,

$$T\left({}_1u,{}_2u,{}_3u,\ldots,{}_nu\right)(\delta)=\theta\left({}_1u\right)=\theta\left({}_2u\right)=T\left({}_2u,{}_3u,\ldots,{}_nu,{}_1u\right)(\delta)=\theta\left({}_3u\right)=$$
$$T\left({}_3u,{}_4u,\ldots,{}_nu,{}_1u,{}_2u\right)(\delta)=\theta\left({}_4u\right)=\ldots=\theta\left({}_nu\right)=T\left({}_nu,{}_1u,{}_2u,\ldots,{}_{n-1}u\right)(\delta). \tag{8}$$

□

Theorem 1. *Let $(S,\mathfrak{T},*)$ be a complete F b MS, where $a*b=\min\{a,b\}$ for all $a,b\in[0,1]$. Let T: $S^n\longrightarrow S$ and θ: $S\longrightarrow S$ be such that $T(S^n)\subseteq\theta(S)$ and θ is continuous and commuting with T. Suppose for all ${}_1u,{}_2u,{}_3u,\ldots,{}_nu,{}_1v,{}_2v,{}_3v,\ldots,{}_nv\in S$ and $\delta>0$,*

$$\mathfrak{T}_{T\left({}_1u,{}_2u,{}_3u,\ldots,{}_nu\right)T\left({}_1v,{}_2v,{}_3v,\ldots,{}_nv\right)}(\delta)\geq\beta\Big(\mathfrak{T}_{\theta\left({}_1u\right),\theta\left({}_1v\right)}\left(b^n\delta\right)*\mathfrak{T}_{\theta\left({}_2u\right),\theta\left({}_2v\right)}(\delta)*\mathfrak{T}_{\theta\left({}_3u\right),\theta\left({}_3v\right)}\left(b^n\delta\right)*\ldots*\mathfrak{T}_{\theta\left({}_nu\right),\theta\left({}_nv\right)}\left(b^n\delta\right)\Big), \tag{9}$$

where $\beta\in\mathfrak{B}$, then T and θ have a unique n-tupled common fixed point.

Proof. If T is constant, then the proof of the theorem follows from Lemma 2. Suppose T is not constant on S^n. In this case, the proof is divided into three steps. □

Step 1. Definition of the sequences $\{\theta({}_1u_m)\}$, $\{\theta({}_2u_m)\}$, $\{\theta({}_3u_m)\},\ldots,\{\theta({}_nu_m)\}$.

Let ${}_1u_o,{}_2u_o,{}_3u_o,\ldots,{}_nu_o$ be arbitrary points of S. As $T(S^n)\subseteq\theta(S)$, therefore, there exist ${}_1u_1,{}_2u_1,{}_3u_1,\ldots,{}_nu_1\in S$ such that $\theta({}_1u_1)=T({}_1u_o,{}_2u_o,{}_3u_o,\ldots,{}_nu_o)$, $\theta({}_2u_1)=T({}_2u_o,{}_3u_o,\ldots,{}_nu_o,{}_1u_o)$, $\theta({}_3u_1)=T({}_3u_o,{}_4u_o,\ldots,{}_nu_o,{}_1u_o,{}_2u_o)$ and $\theta({}_nu_1),=T({}_nu_o,{}_1u_o,{}_2u_o,\ldots,{}_{n-1}u_o)$.

Again, as $T(S^n)\subseteq\theta(S)$, therefore, there exists ${}_1u_2,{}_2u_2,{}_3u_2,\ldots,{}_nu_2\in S$ such that

$$\theta\left({}_1u_2\right)=T\left({}_1u_1,{}_2u_1,{}_3u_1,\ldots,{}_nu_1\right),\theta\left({}_2u_2\right)=T\left({}_2u_1,{}_3u_1,\ldots,{}_nu_1,{}_1u_1\right),$$
$$\theta\left({}_3u_2\right)=T\left({}_3u_1,{}_4u_1,\ldots,{}_nu_1,{}_1u_o,{}_2u_1\right),\ldots,\theta\left({}_nu_2\right)=T\left({}_nu_1,{}_1u_1,{}_2u_1,\ldots,{}_{n-1}u_1\right). \tag{10}$$

Continuing in the same way, sequences $\{\theta({}_1u_m)\}$, $\{\theta({}_2u_m)\}$, $\{\theta({}_3u_m)\},\ldots,\{\theta({}_nu_m)\}$ can be constructed such that

$$\begin{aligned}
&\theta\left({}_1u_{m+1}\right)=T\left({}_1u_m,{}_2u_m,{}_3u_m,\ldots,{}_nu_m\right),\\
&\theta\left({}_2u_{m+1}\right)=T\left({}_2u_m,{}_3u_m,\ldots,{}_nu_m,{}_1u_m\right),\\
&\theta\left({}_3u_{m+1}\right)=T\left({}_3u_m,{}_4u_m,\ldots,{}_nu_m,{}_1u_m,{}_2u_m\right),\\
&\qquad\vdots\\
&\theta\left({}_nu_{m+1}\right)=T\left({}_nu_m,{}_1u_m,{}_2u_m,\ldots,{}_{n-1}u_m\right),\quad\text{where } m\in N\cup\{0\}.
\end{aligned} \tag{11}$$

Step 2. $\{\theta({}_1u_m)\}$, $\{\theta({}_2u_m)\}$, $\{\theta({}_3u_m)\},\ldots,\{\theta({}_nu_m)\}$ are Cauchy sequences.

$$\text{Let } \lambda_m(\delta)=\mathfrak{T}_{\theta\left({}_1u_m\right)\theta\left({}_1u_{m+1}\right)}(\delta)*\mathfrak{T}_{\theta\left({}_2u_m\right)\theta\left({}_2u_{m+1}\right)}(\delta)*\ldots*\mathfrak{T}_{\theta\left({}_nu_m\right)\theta\left({}_nu_{m+1}\right)}(\delta);\ \forall\delta>0. \tag{12}$$

Using (9), for all $\delta > 0$, we have

$$\begin{aligned}
\mathfrak{T}_{\theta({}_1u_m)\theta({}_1u_{m+1})}(\delta) &= \mathfrak{T}_{T({}_1u_{m-1},{}_2u_{m-1},\ldots,{}_nu_{m-1})T({}_1u_m,{}_2u_m,\ldots,{}_nu_m)}(\delta),\\
&\geq \beta\left(\mathfrak{T}_{\theta({}_1u_{m-1})\theta({}_1u_m)}(b^n\delta) * \mathfrak{T}_{\theta({}_2u_{m-1})\theta({}_2u_m)}(b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_{m-1})\theta({}_nu_m)}(b^n\delta)\right),\\
&= \beta(\lambda_{m-1}(b^n\delta)) > \lambda_{m-1}(b^n\delta),\\
\mathfrak{T}_{\theta({}_2u_m)\theta({}_2u_{m+1})}(\delta) &= \mathfrak{T}_{T({}_2u_{m-1},{}_3u_{m-1},\ldots,{}_nu_{m-1},{}_1u_{m-1})T({}_2u_m,{}_3u_m,\ldots,{}_nu_m,{}_1u_m)}(\delta),\\
&\geq \beta\left(\mathfrak{T}_{\theta({}_2u_{m-1})\theta({}_2u_m)}(b^n\delta) * \mathfrak{T}_{\theta({}_3u_{m-1})\theta({}_3u_m)}(b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_{m-1})\theta({}_nu_m)}(b^n\delta) * \mathfrak{T}_{\theta({}_1u_{m-1})\theta({}_1u_m)}(b^n\delta)\right),\\
&= \beta(\lambda_{m-1}(b^n\delta)) > \lambda_{m-1}(b^n\delta),\\
&\vdots\\
\mathfrak{T}_{\theta({}_2u_m)\theta({}_2u_{m+1})}(\delta) &= \mathfrak{T}_{T({}_nu_{m-1},{}_1u_{m-1},{}_2u_{m-1},\ldots,{}_{n-1}u_{m-1})T({}_nu_m,{}_1u_m,{}_2u_m,\ldots,{}_{n-1}u_m)}(\delta),\\
&\geq \beta\left(\mathfrak{T}_{\theta({}_nu_{m-1})\theta({}_nu_m)}(b^n\delta) * \mathfrak{T}_{\theta({}_1u_{m-1})\theta({}_1u_m)}(b^n\delta) * \ldots * \mathfrak{T}_{\theta({}_{n-1}u_{m-1})\theta({}_{n-1}u_m)}(b^n\delta)\right),\\
&= \beta(\lambda_{m-1}(b^n\delta)) > \lambda_{m-1}(b^n\delta),
\end{aligned} \tag{13}$$

(13) implies that for all $\delta > 0$ and $m \geq 0$,

$$\lambda_m(\delta) \geq \beta(\lambda_{m-1}(b^n\delta)). \tag{14}$$

Obviously,

$$\lambda_m(\delta) \geq \beta(\lambda_{m-1}(b^n\delta)) > \lambda_{m-1}(b^n\delta) \geq \lambda_{m-1}(\delta). \tag{15}$$

It means $\{\lambda_n(\delta)\}$ is an increasing sequence in $[0,1]$, and therefore, $\limsup_{n\longrightarrow\infty}\lambda_n(\delta) = \ell(\delta) \leq 1$ for all $\delta > 0$. If $\ell(\delta) < 1$, then by letting $n \longrightarrow \infty$ in (6), we get a contradiction $\ell(\delta) > \ell(\delta)$. Therefore, $\ell(\delta) = 1$, that is, for all $m, \delta \geq 0$,

$$\lim\sup_{n\longrightarrow\infty}\left(\mathfrak{T}_{(1u_m)\theta(1u_{m+1})}(\delta) * \mathfrak{T}_{(2u_m)\theta(2u_{m+1})}(\delta) * \ldots * \mathfrak{T}_{(nu_m)\theta(nu_{m+1})}(\delta)\right) = 1. \tag{16}$$

To show that the sequences $\{\theta({}_1u_m)\}$, $\{\theta({}_2u_m)\}$, $\{\theta({}_3u_m)\},\ldots,\{\theta({}_nu_m)\}$, where $m = 1,2,3,\cdots$, are Cauchy, first we prove that for every $\epsilon \in (0,1)$ there exist $p, q \in N$ such that $\mathfrak{T}_{\theta({}_1u_p)\theta({}_1u_q)}(\delta) * \mathfrak{T}_{\theta({}_2u_p)\theta({}_2u_q)}(\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_p)\theta({}_nu_q)}(\delta) > 1-\epsilon$. Suppose it is not true. Then, there exists some $\epsilon \in (0,1)$ such that for each $r \in N$, there exist integers $p(r)$ and $q(r)$ with $p(r) > q(r) \geq r$ such that

$$\mathfrak{T}_{\theta({}_1u_{p(r)})\theta({}_1u_{q(r)})}(\delta) * \mathfrak{T}_{\theta({}_2u_{p(r)})\theta({}_2u_{q(r)})}(\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_{p(r)})\theta({}_nu_{q(r)})}(\delta) \leq 1-\epsilon. \tag{17}$$

Let $p(r)$ be the least such positive integer which exceeds $q(r)$ and satisfies (17). Then,

$$\mathfrak{T}_{\theta({}_1u_{p(r)})\theta({}_1u_{q(r)})}(\delta) * \mathfrak{T}_{\theta({}_2u_{p(r)})\theta({}_2u_{q(r)})}(\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_{p(r)})\theta({}_nu_{q(r)})}(\delta) > 1-\epsilon. \tag{18}$$

Let $m_r(\delta) = \mathfrak{T}_{\theta({}_1u_{p(r)})\theta({}_1u_{q(r)})}(\delta) * \mathfrak{T}_{\theta({}_2u_{p(r)})\theta({}_2u_{q(r)})}(\delta) * \ldots * \mathfrak{T}_{\theta({}_nu_{p(r)})\theta({}_nu_{q(r)})}(\delta)$. Using (18) and (19) and properties $(FbM3)$ and $(FbM4)$, we have

$$1-\epsilon \geq m_r(\delta) \geq \mathfrak{T}_{\theta({}_1u_{q(r)})\theta({}_1u_{p(r)-1})}\left(\frac{\delta}{2b}\right) * \mathfrak{T}_{\theta({}_1u_{p(r)-1})\theta({}_1u_{p(r)})}\left(\frac{\delta}{2b}\right) * \mathfrak{T}_{\theta({}_2u_{q(r)})\theta({}_2u_{p(r)-1})}\left(\frac{\delta}{2b}\right) * \mathfrak{T}_{\theta({}_2u_{p(r)-1})\theta({}_2u_{p(r)})}\left(\frac{\delta}{2b}\right) * \dots * \mathfrak{T}_{\theta({}_nu_{q(r)})\theta({}_nu_{p(r)-1})}\left(\frac{\delta}{2b}\right) * \mathfrak{T}_{\theta({}_nu_{p(r)-1})\theta({}_nu_{p(r)})}\left(\frac{\delta}{2b}\right) > (1-\epsilon) * \lambda_r\left(\frac{\delta}{2b}\right). \tag{19}$$

Letting $r \longrightarrow \infty$ we get the contradiction

$$1-\varepsilon \geq \lim \sup_{r\longrightarrow\infty} m_r(\delta) > 1-\varepsilon. \tag{20}$$

Hence, $\{\theta({}_1u_m)\}$, $\{\theta({}_2u_m)\}$, $\{\theta({}_3u_m)\}, \dots, \{\theta({}_nu_m)\}$ are Cauchy sequences.

Step 3. T and θ have an n-tupled common fixed point.

As S is complete, so there will be some ${}_1u, {}_2u, {}_3u, \dots, {}_nu \in S$ such that $\lim_{m\longrightarrow\infty}\theta({}_1u_m) = {}_1u, \lim_{m\longrightarrow\infty}\theta({}_2u_m) = {}_2u, \dots, \lim_{m\longrightarrow\infty}\theta({}_nu_m) = {}_nu$. Due to continuity of θ, $\lim_{m\longrightarrow\infty}\theta\theta_1 u_m = \theta({}_1u), \lim_{m\longrightarrow\infty}\theta\theta({}_2u_m) = \theta({}_2u), \dots, \lim_{n\longrightarrow\infty}\theta\theta({}_nu_m) = \theta({}_nu)$. Also, θ commutes with T, therefore, $\theta\theta({}_1u_{m+1}) = \theta T({}_1u_m, {}_2u_m, \dots, {}_nu_m) = T(\theta({}_1u_m), \theta({}_2u_m), \dots, \theta({}_nu_m))$.

Using (9), we have

$$\begin{aligned} \mathfrak{T}_{\theta(\theta_1 u_{m+1})T({}_1u, {}_2u, {}_3u, \dots, {}_nu)}(\delta) &= \mathfrak{T}_{T\theta({}_1u_m), \theta({}_2u_m), \dots, \theta({}_nu_m))T({}_1u, {}_2u, {}_3u, \dots, {}_nu)}(\delta) \\ &\geq \beta\Big(\mathfrak{T}_{\theta(\theta({}_1u_m)), \theta({}_1u)}(b^n\delta) * \mathfrak{T}_{\theta(\theta({}_2u_m)), \theta({}_2u)}(b^n\delta) \\ &\quad * \mathfrak{T}_{\theta(\theta({}_3u_m)), \theta({}_3u)}(b^n\delta) * \dots * \mathfrak{T}_{\theta(\theta({}_nu_m)), \theta({}_nu)}(b^n\delta)\Big). \end{aligned} \tag{21}$$

Using Proposition 1 and (16), we have

$$\begin{aligned} \mathfrak{T}_{\theta({}_1u)T({}_1u, {}_2u, {}_3u, \dots, {}_nu)}(\delta) &\geq \lim \sup_{m\longrightarrow\infty} \mathfrak{T}_{T\theta({}_1u_m), \theta({}_2u_m), \dots, \theta({}_nu_m)T({}_1u, {}_2u, {}_3u, \dots, {}_nu)}\left(\frac{\delta}{b}\right) \\ &\geq \lim \sup_{m\longrightarrow\infty} \beta\left(\mathfrak{T}_{\theta(\theta({}_1u_m)), \theta({}_1u)}(b^{n-1}\delta) * \mathfrak{T}_{\theta(\theta({}_2u_m)), \theta({}_2u)}(b^{n-1}\delta) * \dots * \mathfrak{T}_{\theta(\theta({}_nu_m)), \theta({}_nu)}(b^{n-1}\delta)\right) \\ &\geq \left(\mathfrak{T}_{\theta({}_1u), \theta({}_1u)}(b^{n-1}\delta) * \mathfrak{T}_{\theta({}_2u), \theta({}_2u)}(b^{n-1}\delta) * \mathfrak{T}_{\theta({}_3u), \theta({}_3u)}(b^{n-1}\delta) * \dots * \mathfrak{T}_{\theta({}_nu), \theta({}_nu)}(b^{n-1}\delta)\right) \\ &= 1. \end{aligned} \tag{22}$$

Hence, $T({}_1u, {}_2u, {}_3u, \dots, {}_nu)(\delta) = \theta({}_1u)$. Similarly, it can be shown that

$$\begin{aligned} T({}_2u, {}_3u, \dots, {}_nu, {}_1u)(\delta) &= \theta({}_2u), \\ T({}_3u, {}_4u, \dots, {}_nu, {}_1u, {}_2u)(\delta) &= \theta({}_3u), \\ &\vdots \\ T({}_nu, {}_1u, {}_2u, \dots, {}_{n-1}u) &= \theta({}_nu). \end{aligned} \tag{23}$$

Lemma 4 implies that

$$\begin{aligned} T({}_1u, {}_2u, {}_3u, \dots, {}_nu) &= \theta({}_1u) = \theta({}_2u) = \dots = \theta({}_nu) \\ &= T({}_nu, {}_1u, {}_2u, \dots, {}_{n-1}u). \end{aligned} \tag{24}$$

From (9) and (24) along with Proposition 1 and continuity of β, it comes out that

$$\begin{aligned} \mathfrak{T}_{{}_1u, \theta({}_1u)}(b\delta) &\geq \lim \sup_{m\longrightarrow\infty} \mathfrak{T}_{\theta({}_1u_{m+1}), \theta({}_1u)}(\delta), \\ &\geq \lim \sup_{m\longrightarrow\infty} \mathfrak{T}_{T({}_1u_m, {}_2u_m, \dots, {}_nu_m), T({}_1u, {}_2u, \dots, {}_nu)}(\delta), \\ &\geq \lim \sup_{m\longrightarrow\infty} \beta\Big(\mathfrak{T}_{\theta({}_1u_m), \theta({}_1u)}(b^n\delta) * \mathfrak{T}_{\theta({}_2u_m), \theta({}_2u)}(b^n\delta) * \dots * \mathfrak{T}_{\theta({}_nu_m), \theta({}_nu)}(b^n\delta)\Big), \\ &\geq \lim \sup_{m\longrightarrow\infty} \Big(\mathfrak{T}_{\theta({}_1u_m), \theta({}_1u)}(b^n\delta) * \mathfrak{T}_{\theta({}_2u_m), \theta({}_2u)}(b^n\delta) * \dots * \mathfrak{T}_{\theta({}_nu_m), \theta({}_nu)}(b^n\delta)\Big), \end{aligned} \tag{25}$$

applying Proposition 1 again, we get

$$\mathfrak{T}_{{}_1u,\theta({}_1u)}(b\delta) \geq \lim_{m\longrightarrow\infty} \sup\left(\mathfrak{T}_{\theta({}_1u),\theta({}_1u)}\left(b^{n-1}\delta\right) * \mathfrak{T}_{\theta({}_2u),\theta({}_2u)}\left(b^{n-1}\delta\right) * \mathfrak{T}_{\theta({}_3u),\theta({}_3u)}\left(b^{n-1}\delta\right) * \ldots * \mathfrak{T}_{\theta({}_nu),\theta({}_nu)}\left(b^{n-1}\delta\right)\right) = 1. \quad (26)$$

Therefore, $\theta({}_1u) = {}_1u$. Similarly, it can be shown that

$$\begin{aligned}\theta({}_2u) &= {}_2u,\\ \theta({}_3u) &= {}_3u, \ldots,\\ \theta({}_nu) &= {}_nu.\end{aligned} \quad (27)$$

From (24), it follows that

$$T({}_1u, {}_1u, {}_1u, \ldots, {}_1u) = \theta({}_1u) = {}_1u. \quad (28)$$

To show uniqueness, let $v \in S$ be another n-tupled common fixed point of T and θ such that $v \neq {}_1u$. Then,

$$\begin{aligned}\mathfrak{T}_{{}_1u,v}(\delta) &= \mathfrak{T}_{T({}_1u,{}_1u,{}_1u,\ldots,{}_1u),T(v,v,\ldots,v)}(\delta) \geq \beta\left(\mathfrak{T}_{\theta({}_1u),\theta v}(b^n\delta) * \mathfrak{T}_{\theta({}_1u),\theta v}(b^n\delta) * \cdots * \mathfrak{T}_{\theta({}_1u),\theta v}(b^n\delta)\right),\\ &= \beta\left(\mathfrak{T}_{\theta({}_1u),\theta v}(b^n\delta)\right) > \mathfrak{T}_{\theta({}_1u),\theta v}(b^n\delta) = \mathfrak{T}_{{}_1u,v}(b^n\delta) \geq \mathfrak{T}_{{}_1u,v}(\delta),\end{aligned} \quad (29)$$

which is a contradiction. Thus, T and θ have a unique n-tupled common fixed point. □

Taking $n = 2$ in the abovementioned theorem, we get the following corollary, which is the main result of [13].

Corollary 1. *Let $(S, \mathfrak{T}, *)$ be a complete F b MS, where $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let $T: S^2 \longrightarrow S$ and $\theta: S \longrightarrow S$ be such that $T(S^2) \subseteq \theta(S)$ and θ is continuous and commuting with T. Suppose for all ${}_1u, {}_2u, {}_3u, {}_4u \in S$ and $\delta > 0$,*

$$\mathfrak{T}_{T({}_1u,{}_2u)T({}_3u,{}_4u)}(\delta) \geq \beta\left(\mathfrak{T}_{\theta({}_1u),\theta({}_3u)}\left(b^2\delta\right) * \mathfrak{T}_{\theta({}_2u),\theta({}_4u)}\left(b^2\delta\right)\right), \quad (30)$$

where $\beta \in \mathfrak{B}$, then there exists a unique $u \in S$ such that $u = \theta(u) = T(u, u)$.

Corollary 2. *Let $(S, \mathfrak{T}, *)$ be a complete F b MS, where $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let $T: S^n \longrightarrow S$ and $\theta: S \longrightarrow S$ be such that $T(S^n) \subseteq \theta(S)$ and θ is continuous and commuting with T. Suppose for all ${}_1u, {}_2u, {}_3u, \ldots, {}_nu, {}_1v, {}_2v, {}_3v, \ldots, {}_nv \in S$ and $\delta > 0$,*

$$\mathfrak{T}_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}(\delta) \geq \left(\mathfrak{T}_{\theta({}_1u),\theta({}_1v)}\left(b^2\delta\right) * \mathfrak{T}_{\theta({}_2u),\theta({}_2v)}\left(b^2\delta\right) * \ldots * \mathfrak{T}_{\theta({}_nu),\theta({}_nv)}\left(b^2\delta\right)\right)^{\alpha}, \quad (31)$$

where $\beta \in \mathfrak{B}$, and $\alpha \in (0,1)$, then T and θ have a unique n-tupled common fixed point.

Proof. Proof follows from Theorem 1, by setting $\beta(t) = t^{\alpha}$. □

Corollary 3. *Let $(S, \mathfrak{T}, *)$ be a complete F b MS, where $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let $T: S^n \longrightarrow S$ and $\theta: S \longrightarrow S$ be such that $T(S^n) \subseteq \theta(S)$ and θ is continuous and commuting with T. Suppose for all ${}_1u, {}_2u, {}_3u, \ldots, {}_nu, {}_1v, {}_2v, {}_3v, \ldots, {}_nv \in S$ and $\delta > 0$,*

$$\begin{aligned}\mathfrak{T}_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}(\delta) \geq\ & 2\left(\mathfrak{T}_{\theta({}_1u),\theta({}_1v)}\left(b^2\delta\right) * \mathfrak{T}_{\theta({}_2u),\theta({}_2v)}\left(b^2\delta\right) * \ldots * \mathfrak{T}_{\theta({}_nu),\theta({}_nv)}\left(b^2\delta\right)\right)^{\alpha}\\ & - \left(\mathfrak{T}_{\theta({}_1u),\theta({}_1v)}\left(b^2\delta\right) * \mathfrak{T}_{\theta({}_2u),\theta({}_2v)}\left(b^2\delta\right) * \ldots * \mathfrak{T}_{\theta({}_nu),\theta({}_nv)}\left(b^2\delta\right)\right)^2,\end{aligned} \quad (32)$$

where $\beta \in \mathfrak{B}$, then T and θ have a unique n-tupled common fixed point.

Proof. It is adequate to set $\beta(t) = 2t - t^2$ in Theorem 1, □

Remark 3. Coincidence point of T and θ is not necessarily unique. For example, if $T \equiv c \equiv \theta$, where $c \in S$ is constant, then every point ${}_1u, {}_2u, {}_3u, \ldots, {}_nu \in S^n$ is coincidence point of T and θ.

Example 2. Let $S=\mathbb{R}$ and $\mathfrak{T}(x,y,\delta)=e^{(-|x-y|/\delta)}$ for all $x,y\in S$ and $\delta>0$ with $r*s=\min\{r,s\}$ where $r,s\in[0,1]$. It can be easily verified that $(S,\mathfrak{T},*)$ is a complete F b MS with $b=2$. Let $\alpha,\lambda,\gamma>0$ be such that $\max\{\alpha,\gamma\}\leq(\lambda/128)$. Define $T\colon S^4\longrightarrow S$, $\theta\colon S\longrightarrow S$ as $T(x,y,z,w)=\alpha(x-z)+\gamma(y-w)$, $\theta(u)=\lambda u$ and $\beta(\delta)=\sqrt{\delta}$ for all $u,x,y,z,w\in S$ and $\delta>0$. Obviously θ is continuous, T and θ are commuting and $T(S^4)=\mathbb{R}=\theta(S)$. Also,

$$\begin{aligned}
\mathfrak{T}_{TxyzwTmnpq}(\delta)&=e^{|\alpha(x-m)-\alpha(z-p)+\gamma(y-n)-\gamma(w-q)|(-1/\delta)},\\
&\geq e^{(\alpha|x-m|+\alpha|z-p|+\gamma|y-n|+\gamma|w-q|)(-1/\delta)}\\
&\geq e^{(|x-m|/\delta+|z-p|/\delta+|y-n|/\delta+|w-q|/\delta)(-4max\{\alpha,\gamma\})},\\
&\geq e^{(|x-m|/\delta+|z-p|/\delta+|y-n|/\delta+|w-q|/\delta)(-\lambda/128)},\\
&\geq e^{\max\{|x-m|/\delta+|z-p|/\delta+|y-n|/\delta+|w-q|/\delta\}(-4\lambda/128)}\\
&\geq\sqrt{\min\{e^{-|\lambda x-\lambda m|/16\delta},e^{-|\lambda z-\lambda p|/16\delta},e^{-|\lambda y-\lambda n|/16\delta},e^{-|\lambda w-\lambda q|/16\delta}\}}\\
&=\sqrt{\mathfrak{T}_{\theta x\theta m}(2^4\delta)*\mathfrak{T}_{\theta y\theta n}(2^4\delta)*\mathfrak{T}_{\theta z\theta p}(2^4\delta)*\mathfrak{T}_{\theta w\theta q}(2^4\delta)}.
\end{aligned}\tag{33}$$

That is, all the conditions of Theorem 1 are fulfilled. Therefore, $(0,0,0,0)$ is a unique quadrupled common fixed point of T and θ.

4. Consequences

It is known that if $(S,\mathfrak{T},*)$ is a F b MS and $\diamondsuit$ is a continuous t-norm such that $u*v\geq u\diamondsuit v,\forall u,v\in[0,1]$, then $(S,\mathfrak{T},\diamondsuit)$ is F b MS. Since for any t-norm $*$, it is always true that $*_M\geq *$, therefore, $(S,\mathfrak{T},*_M)$ is F b MS implies $(S,\mathfrak{T},*)$ is F b MS. As Fb MS is a generalization of b-metric space, therefore, from a given b-metric space, a F b MS can be considered in different ways.

Example 3 (see [1]). Let (S,d) be a b-metric space. For $\delta>0$ and $u\neq v$, the following is defined:

$$\mathfrak{T}^d_{uv}(\delta)=\delta/\delta+d_{uv}$$

$$\mathfrak{T}^e_{uv}(\delta)=e^{-d_{uv}/\delta}$$

$$\mathfrak{T}^c_{uv}(\delta)=\begin{cases}0, & \text{if }\delta\leq d_{uv};\\ 1, & \text{if }\delta>d_{uv}.\end{cases}$$

It is known that each of $(S,\mathfrak{T}^d,*_P)$, $(S,\mathfrak{T}^d,*_M)$, $(S,\mathfrak{T}^e,*_M)$, and $(S,\mathfrak{T}^c,*_M)$ is a F b MS.

Moreover, the completeness of (S,d) implies the completeness of any one of these F b MSs and vice versa. With this approach, many important results for b-metric space can be deduced from the corresponding results in a fuzzy setting. In the following theorem the b-metric space (S,d) is viewed as the crisp F b MS $(S,\mathfrak{T}^c,*_M)$.

Theorem 2. *Let (S,d) be a complete b-metric space, $T\colon S^n\longrightarrow S$ and $\theta\colon S\longrightarrow S$ be given mappings such that $T(S^n)\subseteq\theta(S)$ and θ is continuous and commuting with T. Suppose T and θ satisfy some of the following conditions for all ${}_1u,{}_2u,{}_3u,\ldots,{}_nu,{}_1v,{}_2v,{}_3v,\ldots,{}_nv\in S$:*

$$\begin{aligned}
&d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}\leq\kappa\max\left(d_{\theta({}_1u)\theta({}_1v)},d_{\theta({}_2u)\theta({}_{21}v)},\ldots,d_{\theta({}_nu)\theta({}_nv)}\theta({}_nv)\right)\text{for some }\kappa\in(0,1),\\
&d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}\leq\kappa\left[\alpha_1d_{\theta({}_1u)\theta({}_1v)}+\alpha_2d_{\theta({}_2u)\theta({}_{21}v)}+\cdots+\alpha_nd_{\theta}({}_nu)\theta({}_nv)\right]\\
&\qquad\text{for some }\kappa\in(0,1)\text{ and some }\alpha_1,\alpha_2\cdots\alpha_n\in\left[0,\frac{1}{n}\right]\text{ where }n\in\mathbb{N}.\\
&d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}\leq\alpha_1d_{\theta({}_1u)\theta({}_1v)}+\alpha_2d_{\theta({}_2u)\theta({}_{21}v)}+\cdots+\alpha_nd_{\theta({}_nu)\theta({}_nv)}\\
&\qquad\text{for }\alpha_1,\alpha_2\cdots\alpha_n\in[0,1)\text{ such that }\sum_{i=1}^n\alpha_i<1.
\end{aligned}\tag{34}$$

Then, there exists a unique $u \in S$ such that $u = \theta u = T(u, u, u, \ldots, u)$.

Proof. Consider $\mathfrak{T}^c$ as defined in example 3. As (S, d) is complete, $(S, \mathfrak{T}^c_{uv}, *_M)$ is complete. We prove (2), for $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_n = (1/n)$, $\delta > 0$ and $* = *_M$. In case any one of $\mathfrak{T}^c_{\theta({}_1u)\theta({}_1v)}, \mathfrak{T}^c_{\theta({}_2u)\theta({}_{21}v)}, \ldots, \mathfrak{T}^c_{\theta({}_nu)\theta({}_nv)}$ is 0, then (2) obvious. Suppose

$$\begin{aligned}
&\mathfrak{T}^c_{\theta({}_1u)\theta({}_1v)}(\delta) = \mathfrak{T}^c_{\theta({}_2u)\theta({}_2v)}(\delta) = \cdots = \mathfrak{T}^c_{\theta({}_nu)\theta({}_nv)}(\delta) = 1,\\
&\Rightarrow d_{\theta({}_1u)\theta({}_1v)} < \delta, d_{\theta({}_2u)\theta({}_2v)} < \delta, \ldots, d_{\theta({}_nu)\theta({}_nv)} < \delta.\\
&\Rightarrow \delta > \max\left(d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right).
\end{aligned} \tag{35}$$

Now (a)

$$\begin{aligned}
\kappa\delta &> \kappa \max\left(d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right)\\
&\geq d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}.\\
&\Rightarrow \mathfrak{T}^c_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}(\kappa\delta) = 1.
\end{aligned} \tag{36}$$

Hence, (2) is satisfied.

(b) As

$$\begin{aligned}
d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)} &\leq \kappa\left[\alpha_1 d_{\theta({}_1u)\theta({}_1v)} + \alpha_2 d_{\theta({}_2u)\theta({}_2v)} + \cdots + \alpha_n d_{\theta({}_nu)\theta({}_nv)}\right]\\
&\leq \kappa\left[\frac{1}{n} d_{\theta({}_1u)\theta({}_1v)} + \frac{1}{n} d_{\theta({}_2u)\theta({}_2v)} + \cdots + \frac{1}{n} d_{\theta({}_nu)\theta({}_nv)}\right],\\
&= \frac{\kappa}{n}\left[d_{\theta({}_1u)\theta({}_1v)} + d_{\theta({}_2u)\theta({}_2v)} + \cdots + d_{\theta({}_nu)\theta({}_nv)}\right]\\
&\leq \frac{\kappa}{n} n \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right],\\
&= \kappa \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right] \leq \kappa\delta,\\
&\Rightarrow \mathfrak{T}^c_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}(\kappa\delta) = 1.
\end{aligned} \tag{37}$$

Hence, (2) is true.

(c) Let $\kappa = \sum_{i=1}^{n} \alpha_i < 1$. Then,

$$\begin{aligned}
d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)} &\leq \alpha_1 d_{\theta({}_1u)\theta({}_1v)} + \alpha_2 d_{\theta({}_2u)\theta({}_2v)} + \cdots + \alpha_n d_{\theta({}_nu)\theta({}_nv)}\\
&\leq \alpha_1 \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right]\\
&\quad + \alpha_2 \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right]\\
&\quad + \alpha_3 \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right]\\
&\quad \vdots\\
&\quad + \alpha_n \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right]\\
&= \sum_{i=1}^{n} \alpha_i \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right]\\
&= \kappa \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right] \leq \kappa\delta\\
&= \kappa \max\left[d_{\theta({}_1u)\theta({}_1v)}, d_{\theta({}_2u)\theta({}_2v)}, \ldots, d_{\theta({}_nu)\theta({}_nv)}\right] \leq \kappa\delta,\\
&\Rightarrow \mathfrak{T}^c_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)}(\kappa\delta) = 1.
\end{aligned} \tag{38}$$

Hence, (2) is true. □

Corollary 4 (see [1], Theorem 15). *Let (S, d) be a complete metric space and $T\colon S^3 \longrightarrow S$ and $\theta\colon S \longrightarrow S$ be such that $T(S^3) \subseteq \theta(S)$ and θ is continuous and commuting with T. Suppose T and θ satisfy some of the following conditions for all ${}_1u, {}_2u, {}_3u, {}_1v, {}_2v, {}_3v \in S$:*

$$d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v)} \leq \kappa \max\left(d_{\theta({}_1u)\theta({}_1v)} + d_{\theta({}_2u)\theta({}_2v)} + \cdots + d_{\theta({}_3u)\theta({}_3v)}\right) \text{for some } \kappa \in (0,1),$$

$$d_{T({}_1u,{}_2u,{}_3u)T({}_1v,{}_2v,{}_3v)} \leq \kappa\left[\alpha_1 d_{\theta({}_1u)\theta({}_1v)} + \alpha_2 d_{\theta({}_2u)\theta({}_2v)} + \alpha_3 d_{\theta({}_3u)\theta({}_3v)}\right] \text{for some } \alpha_1, \alpha_2, \alpha_3 \in \left[0, \frac{1}{3}\right] \text{and } \kappa \in (0,1). \tag{39}$$

$$d_{T({}_1u,{}_2u,{}_3u)T({}_1v,{}_2v,{}_3v)} \leq \alpha_1 d_{\theta({}_1u)\theta({}_1v)} + \alpha_2 d_{\theta({}_2u)\theta({}_2v)} + \alpha_3 d_{\theta({}_3u)\theta({}_3v)} \text{ for } \alpha_1, \alpha_2, \alpha_3 \in [0,1) \text{ such that } \sum_{i=1}^{3} \alpha_i < 1.$$

Then, there exists a unique $u \in S$ such that $u = \theta u = T(u, u, u)$.

Proof. Proof is similar to that of Theorem 2. □

Example 4. Let $S = \mathbb{R}$, and $d_{uv} = |u - v|$ for all $u, v \in \mathbb{R}$. Let $T\colon S^n \longrightarrow S$ and $\theta\colon S \longrightarrow S$ be defined as $T({}_1u, {}_2u, {}_3u, \ldots, {}_nu) = \sum_{i=1}^{n}(\eta_i)({}_iu) + \epsilon/\mathscr{K}$ and $\theta(u) = u$ for all ${}_1u, {}_2u, {}_3u, \ldots, {}_nu, \eta_1, \eta_2, \ldots, \eta_n, u, \epsilon, K \in \mathbb{R}$. Then,

$$\begin{aligned} d_{T({}_1u,{}_2u,{}_3u,\ldots,{}_nu)T({}_1v,{}_2v,{}_3v,\ldots,{}_nv)} &= \frac{1}{K}\left|\sum_{i=1}^{n} \eta_i({}_iu - {}_iv)\right|, \\ &\leq \sum_{i=1}^{n} \frac{\eta_i}{K}\left|{}_iu - {}_iv\right| = \sum_{i=1}^{n} \alpha_i \left|{}_iu - {}_iv\right|, \\ &= \sum_{i=1}^{n} \alpha_i \left|\theta({}_iu) - \theta({}_iv)\right| = \sum_{i=1}^{n} \alpha_i d_{\theta({}_iu)-\theta({}_iv)}. \end{aligned} \tag{40}$$

That is, the condition of Theorem 2 (c) is fulfilled.

Moreover, for n-tupple $({}_0u, {}_0u, \ldots, {}_0u)$, where ${}_0u = (\epsilon/K - \sum_{i=1}^{n} \eta_i)$, we have

$$\begin{aligned} T({}_0u, {}_0u, \ldots, {}_0u) &= \frac{\sum_{i=1}^{n}(\eta_i){}_0u + \epsilon}{K} = \frac{\sum_{i=1}^{n}(\eta_i)(\epsilon/K - \sum_{i=1}^{n}(\eta_i)) + \epsilon}{K}, \\ &= \frac{\epsilon\sum_{i=1}^{n}(\eta_i) + \epsilon(K - \sum_{i=1}^{n}(\eta_i))}{K(K - \sum_{i=1}^{n}(\eta_i))}, \\ &= \frac{\epsilon}{K - \sum_{i=1}^{n}\eta_i} = {}_0u = \theta({}_0u). \end{aligned} \tag{41}$$

Hence, $({}_0u, {}_0u, \ldots, {}_0u)$ is a unique n-tupled common fixed point of T and θ.

5. Applications

5.1. Lipschitzian Systems. Let $g_1, g_2, \ldots, g_n\colon \mathbb{R} \longrightarrow \mathbb{R}$ be Lipschitzian mappings on $\mathbb{R}$ (equipped with the Euclidean metric) with Lipschitz constants $c_1, c_2, c_3, \ldots, c_n$, respectively, that is, for each g_i where $1 \leq i \leq n$, there exists a corresponding real number c_i where $1 \leq i \leq n$ such that $|g_i(u) - g_i(v)| \leq |c_i u - v|$ for all $u, v \in \mathbb{R}$. Define $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ as $f(u) = \mu_1 g_1(u) + \mu_2 g_2(u) + \cdots + \mu_n g_n(u)$ for all $u \in \mathbb{R}$, where $\mu_i \in \mathbb{R}$ for $1 \leq i \leq n$. Then,

$$\begin{aligned} |f(u) - f(v)| &= \sum_{i=1}^{n} \left|\mu_i g_i({}_iu) - g_i({}_iv)\right|, \\ &\leq \sum_{i=1}^{n} \left|\mu_i c_i u - v\right|. \end{aligned} \tag{42}$$

That is, f is itself a Lipschitzian mapping with $c_f = \sum_{i=1}^{n} |\mu_i c_i|$. If $c_f = \sum_{i=1}^{n} |\mu_i c_i| < 1$, then f is contraction and by Banach contraction principle there is a unique $u_o \in \mathbb{R}$ such that $f(u_o) = u_o$. Define $T\colon S^n \longrightarrow S$ by

$$T({}_1u,{}_2u,{}_3u,\ldots,{}_nu) = \mu_1 g_1({}_1u) + \mu_2 g_2({}_2u) + \cdots + \mu_n g_n({}_nu), \quad \text{for all } {}_1u,{}_2u,{}_3u,\ldots,{}_nu \in \mathbb{R}. \tag{43}$$

Obviously, $T(\overbrace{u,u,\ldots,u}^{n}) = f(u)$ for all $u \in \mathbb{R}$. Moreover,

$$\begin{aligned}\left|T({}_1u,{}_2u,{}_3u,\ldots,{}_nu) - ({}_1v,{}_2v,{}_3v,\ldots,{}_nv)\right| &= \sum_{i=1}^{n}\left|\mu_i g_i({}_iu) - g_i({}_iv)\right|,\\ &\le \sum_{i=1}^{n}\left|\mu_i c_i u - v\right|,\\ &= \sum_{i=1}^{n} c_f\left|{}_iu - {}_iv\right|,\\ &\le c_f \max_{1\le i\le n}\left|{}_iu - {}_iv\right|.\end{aligned} \tag{44}$$

If $c_f < 1$, then by Theorem 2 (a), there exists a unique $u \in \mathbb{R}$ such that

$$u = f(u) = T(\overbrace{u,u,\ldots,u}^{n}). \tag{45}$$

Corollary 5. *Let $g_1, g_2, \ldots, g_n\colon \mathbb{R} \longrightarrow \mathbb{R}$ be Lipschitzian mappings furnished with the Euclidean metric, and $\mu_1, \mu_2, \ldots, \mu_n \in \mathbb{R}$ be such that $C = \sum_{i=1}^{n} |\mu_i c_i| < 1$. Then, the system*

$$(H) = \begin{cases}\mu_1 g_1({}_1u) + \mu_2 g_2({}_2u) + \cdots + \mu_{n-1} g_{n-1}({}_{n-1}u) + \mu_n g_n({}_nu) = {}_1u,\\ \mu_1 g_1({}_2u) + \mu_2 g_2({}_3u) + \cdots + \mu_{n-1} g_{n-1}({}_nu) + \mu_n g_n({}_1u) = {}_2u,\\ \mu_1 g_1({}_3u) + \mu_2 g_2({}_4u) + \cdots + \mu_{n-1} g_{n-1}({}_1u) + \mu_n g_n({}_2u) = {}_3u,\\ \vdots\\ \mu_1 g_1({}_nu) + \mu_2 g_2({}_1u) + \cdots + \mu_{n-1} g_{n-1}({}_{n-2}u) + \mu_n g_n({}_{n-1}u) = {}_nu,\end{cases} \tag{46}$$

has a unique solution $(\overbrace{{}_ou,{}_ou,\ldots,{}_ou}^{n})$, where ${}_ou$ is a unique real solution of $\sum_{i=1}^{n} \mu_i g_i(u) = u$.

Example 5. Consider the following:

$$(J) = \begin{cases}30sinx - \dfrac{28}{1+y^2} + \sqrt{z} + cosw^2 + 150 = 72x - 15tan^{-1}t,\\[2ex] 30siny - \dfrac{28}{1+z^2} + \sqrt{w} + cost^2 + 150 = 72y - 15tan^{-1}x,\\[2ex] 30sinz - \dfrac{28}{1+w^2} + \sqrt{t} + cosx^2 + 150 = 72z - 15tan^{-1}y,\\[2ex] 30sinw - \dfrac{28}{1+t^2} + \sqrt{x} + cosy^2 + 150 = 72w - 15tan^{-1}z,\\[2ex] 30sint - \dfrac{28}{1+x^2} + \sqrt{y} + cosz^2 + 150 = 72t - 15tan^{-1}w.\end{cases} \tag{47}$$

Let $g_1(u) = 5 + sinu, g_2(u) = 1/1 + u^2, g_3(u) = \sqrt{u}, g_4(u) = \cos(u^2)$ and $g_5(u) = tan^{-1}u$. g_1, g_2, g_3, g_4 and g_5 are Lipschitzian mappings with $c_{g_1} = c_{g_3} = c_{g_4} = c_{g_5} = 1$ and $c_{g_2} = (3\sqrt{3}/8)$. Let $\mu_1 = (5/12), \mu_2 = (-7/18), \mu_3 = \mu_4 = (1/72)$ and $\mu_5 = (5/24)$. Then,

$$\sum_{i=1}^{5} |\mu_i| c_i = \frac{5}{12} - \left(\frac{3\sqrt{3}}{8}\right)\left(\frac{7}{18}\right) + \frac{1}{72} + \frac{1}{72} + \frac{5}{24}, \\ = 0.40018703501 < 1. \tag{48}$$

System (J) has a unique solution (u, u, u, u, u), where $u \approx 2.549220382$ is a unique solution of $30sinu - 28/1 + u^2 + \sqrt{u} + cosu^2 + 150 = 72u - 15tan^{-1}u$.

5.2. Integral Systems. Let $I = [a, b]$, where $a, b \in \mathbb{R}$ with $a < b$ and $S = \mathscr{L}^1(I)$ with $d_1(g, h) = \int_I |g(t) - h(t)| dt$ where $\int$ is Lebesgue integral. Then, $(\mathscr{L}^1(I), d_1)$ is a complete metric space. Let $\kappa, \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n \in \mathbb{R}$ and let $H: \mathbb{R}^n \longrightarrow \mathbb{R}$ be such that $H(0, 0, 0, \ldots, 0) = 0$ and satisfying

$$\left|H({}_1u, {}_2u, {}_3u, \ldots, {}_nu) - H({}_1v, {}_2v, {}_3v, \ldots, {}_nv)\right| \le \kappa \sum_{i=1}^{n} \lambda_i \left|{}_iu - {}_iv\right| \text{ for all } ({}_1u, {}_2u, {}_3u, \ldots, {}_nu), ({}_1v, {}_2v, {}_3v, \ldots, {}_nv) \in \mathbb{R}^n. \tag{49}$$

If $C \in \mathbb{R}$, we will look for mappings $g_1, g_2, g_3, \ldots, g_n \in \mathscr{L}^1(I)$ such that

$$g_i(x) = C + \int_{[a,x]} H_{g_1 g_2 g_3 \cdots g_n}(t) dt, \tag{50}$$

is satisfied for all $x \in I, i = 1, 2, 3, \ldots, n$ (arguments of H are represented by subscripts).

Let for all $g_1, g_2, g_3, \ldots, g_n \in \mathscr{L}^1(I)$ and all $x \in I$, $J: \mathscr{L}^1(I)^n \longrightarrow \mathscr{L}^1(I)$ be defined as follows:

$$J_{g_1 g_2 g_3 \cdots g_n}(x) = C + \int_{[a,x]} H_{g_1 g_2 g_3 \cdots g_n}(t) dt. \tag{51}$$

Then,

$$\begin{aligned} d_1\left(J_{g_1 g_2 g_3 \cdots g_n}(x), J_{h_1 h_2 h_3 \cdots h_n}(x)\right) &= \int_I \left|J_{g_1 g_2 g_3 \cdots g_n}(x), J_{h_1 h_2 h_3 \cdots h_n}(x)\right| dx, \\ &\le \int_I \left(\int_{[a,x]} H_{g_1 g_2 g_3 \cdots g_n}(t) dt - \int_{[a,x]} H_{h_1 h_2 h_3 \cdots h_n}(t) dt\right) dx \\ &\le \int_I \left(\int_{[a,x]} \kappa \sum_{i=1}^{n} \lambda_i \left|g_i(t) - h_i(t)\right| d(t)\right) dx \\ &\le \kappa \sum_{i=1}^{n} \lambda_i \int_I \left(\int_I \left|g_i(t) - h_i(t)\right| d(t)\right) dx \\ &= \kappa \sum_{i=1}^{n} \lambda_i \int_I d_1(g_i, h_i) dx \\ &= \kappa (b - a) \sum_{i=1}^{n} \lambda_i d_1(g_i, h_i). \end{aligned} \tag{52}$$

If, then by 4.1 (c), (50) has a unique solution of the form $(g_0, g_0, g_0, \ldots, g_0) \in \mathscr{L}^1(I)^n$.

6. Conclusion

We generalized the concept of tripled fixed point by introducing n-tupled fixed points and established an n-tupled unique fixed point result in fuzzy b-metric space. This generalization may be helpful for further investigation and applications.

We conclude this paper by indicating, in the form of open questions, some directions for further investigation and work.

(1) Can the condition of continuity of θ in Theorem 1 be relaxed?

(2) If the answer to 1 is yes, then what hypotheses on S and θ are needed to guarantee the existence of the n-tupled common fixed points T and θ?

(3) Can the concept of n-tupled coincidence point be extended to more than two mappings?

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this article.

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