



Optical solitons to the Ginzburg–Landau equation including the parabolic nonlinearity

K. Hosseini¹ · M. Mirzazadeh² · L. Akinyemi³ · D. Baleanu^{4,5,6} · S. Salahshour⁷

Received: 11 September 2021 / Accepted: 7 June 2022 / Published online: 24 August 2022
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract

The major goal of the present paper is to construct optical solitons of the Ginzburg–Landau equation including the parabolic nonlinearity. Such an ultimate goal is formally achieved with the aid of symbolic computation, a complex transformation, and Kudryashov and exponential methods. Several numerical simulations are given to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. To the best of the authors' knowledge, the results reported in the current study, classified as bright and kink solitons, have a significant role in completing studies on the Ginzburg–Landau equation including the parabolic nonlinearity.

Keywords Ginzburg–Landau equation · Parabolic nonlinearity · Complex transformation · Kudryashov and exponential methods · Optical solitons

✉ K. Hosseini
kamyar_hosseini@yahoo.com

✉ M. Mirzazadeh
mirzazadehs2@gmail.com

D. Baleanu
dumitru@cankaya.edu.tr

¹ Department of Mathematics, Near East University TRNC, Mersin 10, Turkey

² Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, Rudsar, Vajargah, Iran

³ Department of Mathematics, Lafayette College, Easton, PA, USA

⁴ Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, 06530 Ankara, Turkey

⁵ Institute of Space Sciences, Magurele, Bucharest, Romania

⁶ Department of Medical Research, China Medical University, Taichung 40447, Taiwan

⁷ Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey

1 Introduction

Nonlinear partial differential (NLPD) equations have long been regarded as useful tools for describing a wide range of phenomena in a variety of scientific disciplines. As models for exploring real-world phenomena, NLPD equations play a fundamental role in the development of the contemporary world. Over the last several decades, one of the most significant challenges has been the development of new methods to construct exact solutions for NLPD equations. In recent years, several new, more powerful, and effective approaches have been established to retrieve exact solutions of NLPD equations, the sine-Gordon expansion method (Yan 1996; Yıldırım et al. 2021a, 2021b; Akbar et al. 2021), the (G'/G)-expansion method (Wang et al. 2008; Bekir 2008; Siddique et al. 2021; Bekir et al. 2021), the Sardar sub-equation method (Rezazadeh et al. 2020a, 2020b; Akinyemi 2021; Akinyemi et al. 2021a), the Kudryashov method (Kudryashov 2020a, 2020b, 2020c; Hosseini et al. 2021a), and the exponential method (He and Wu 2006; Ali and Hassan 2010; Hosseini et al. 2020a), are examples to mention.

As it is evident, the nonlinear Schrödinger equation is often used to simulate soliton dynamics in nonlinear optics. Many additional models, in contrast to the nonlinear Schrödinger equation, can be used as an alternative to such a classical model, for example, the Schrödinger–Hirota equation, the Chen–Lee–Liu equation, and many more. In the present study, the authors aim to conduct a study on the following Ginzburg–Landau equation including the parabolic nonlinearity (Biswas 2018; Arshed et al. 2019; Elboree 2020)

$$i \frac{\partial u(x, t)}{\partial t} + \alpha_1 \frac{\partial^2 u(x, t)}{\partial x^2} + (\alpha_2 |u(x, t)|^2 + \alpha_3 |u(x, t)|^4) u(x, t) - \frac{\alpha_4}{|u(x, t)|^2 u^*(x, t)} \left(2 |u(x, t)|^2 \frac{\partial^2 |u(x, t)|^2}{\partial x^2} - \left(\frac{\partial |u(x, t)|^2}{\partial x} \right)^2 \right) - \alpha_5 u(x, t) = 0, \quad (1)$$

and acquire its optical solitons using Kudryashov and exponential methods. In Eq. (1), $u(x, t)$ indicates the wave profile, and x and t denote spatial and temporal coordinates, respectively. Besides, α_1 is the GVD while α_4 is the coefficient of nonlinear terms, α_5 is the coefficient of detuning, and α_2 and α_3 relate to the parabolic nonlinearity. Optical solitons of the GL equation including the parabolic nonlinearity were derived by Biswas in (2018) with the help of the semi-inverse method. Arshed et al. (2019) employed the exponential method to obtain a series of optical soliton of the GL equation including the parabolic nonlinearity. Elboree (2020) used the $\exp(-\phi(\xi))$ method to report optical solitons of the GL equation including the parabolic nonlinearity. More works regarding the Ginzburg–Landau equation and its solitons can be found in Mirzazadeh et al. (2016); Rezazadeh 2018; Sulaiman et al. 2018; Osman et al. 2019; Hosseini et al. 2020b; Hosseini et al. 2021b; Ouahid et al. 2021).

Kudryashov and exponential methods have been designed as newly well-established methods to derive solitons of NLPD equations. In recent years, these methods have achieved much attention, especially from mathematicians and physicists. Akinyemi et al. (2021b) used the Kudryashov method to derive solitons of a Schrödinger equation involving spatio-temporal dispersions. Nisar et al. (2021) extracted solitons of a population equation with the beta-time derivative using the exponential method.

The structure of the present paper is as follows: In Sect. 2, a full description of Kudryashov and exponential methods are provided. In Sect. 3, the GL equation

including the parabolic nonlinearity is reduced in a 1D regime using a complex transformation. In Sect. 4, Kudryashov and exponential methods are used to retrieve optical solitons of the GL equation including the parabolic nonlinearity. Furthermore, Sect. 4 presents several numerical simulations to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. The article is concluded in Sect. 5.

2 Kudryashov and exponential methods

The current section gives a full description of Kudryashov and exponential methods. The Kudryashov method recommends a series as follows

$$U(\epsilon) = a_0 + a_1K(\epsilon) + a_2K^2(\epsilon) + \dots + a_NK^N(\epsilon), \quad a_N \neq 0, \quad (2)$$

as the solution of

$$O(U(\epsilon), U'(\epsilon), U''(\epsilon), \dots) = 0. \quad (3)$$

In series (2), $a_i, i = 0, 1, \dots, N$ are derived later, N is found through the balance principle, and $K(\epsilon)$ is

$$K(\epsilon) = \frac{1}{(A - B) \sin h(\epsilon) + (A + B) \cos h(\epsilon)},$$

satisfying

$$(K'(\epsilon))^2 = K^2(\epsilon)(1 - 4ABK^2(\epsilon)).$$

From Eqs. (2) and (3), we reach a consistent nonlinear system whose solution leads to solitons of Eq. (3).

Compared to the Kudryashov method, the exponential method seeks the following non-trivial solution

$$U(\epsilon) = \frac{a_0 + a_1a^\epsilon + a_2a^{2\epsilon} + \dots + a_Na^{N\epsilon}}{b_0 + b_1a^\epsilon + b_2a^{2\epsilon} + \dots + b_Na^{N\epsilon}}, \quad a_N \neq 0, b_N \neq 0, \quad (4)$$

as the solution of Eq. (3). In Eq. (4), the coefficients are acquired later and $N \in \mathbb{N}$.

As before, from Eqs. (4) and (3), we arrive at a consistent nonlinear system whose solution yields solitons of Eq. (3).

3 The model in its 1D regime

To reduce the governing model in a 1D regime, it is assumed that the model solution has the form

$$u(x, t) = U(\epsilon)e^{i\phi(x, t)},$$

where $U(\epsilon)$ indicates the shape of the pulse and

$$\epsilon = x - ct, \phi(x, t) = -kx + wt.$$

The above transformation causes

$$i(2k\alpha_1 + c) \frac{dU(\epsilon)}{d\epsilon} + (-\alpha_1 + 4\alpha_4) \frac{d^2U(\epsilon)}{d\epsilon^2} + (k^2\alpha_1 + w + \alpha_5)U(\epsilon) - \alpha_2U^3(\epsilon) - \alpha_3U^5(\epsilon) = 0. \quad (5)$$

From Eq. (5), one can acquire

$$(2k\alpha_1 + c) \frac{dU(\epsilon)}{d\epsilon} = 0, \quad (6)$$

$$(-\alpha_1 + 4\alpha_4) \frac{d^2U(\epsilon)}{d\epsilon^2} + (k^2\alpha_1 + w + \alpha_5)U(\epsilon) - \alpha_2U^3(\epsilon) - \alpha_3U^5(\epsilon) = 0. \quad (6)$$

Due to the first equation, one can find the soliton speed as

$$c = -2k\alpha_1.$$

Now, considering Eq. (6) and the transformation $U(\epsilon) = \sqrt{V(\epsilon)}$ results in

$$\begin{aligned} & (\alpha_1 - 4\alpha_4) \left(\frac{dV(\epsilon)}{d\epsilon} \right)^2 + (-2\alpha_1 + 8\alpha_4) \left(\frac{d^2V(\epsilon)}{d\epsilon^2} \right) V(\epsilon) \\ & + (4k^2\alpha_1 + 4w + 4\alpha_5) V^2(\epsilon) - 4\alpha_2 V^3(\epsilon) - 4\alpha_3 V^4(\epsilon) = 0. \end{aligned} \quad (7)$$

Using the balance principle, from $\left(\frac{dV(\epsilon)}{d\epsilon} \right)^2$ and $V^4(\epsilon)$ in Eq. (7), it is found that

$$2N + 2 = 4N \Rightarrow N = 1. \quad (8)$$

4 The model and its solitons

In the current section, Kudryashov and exponential methods are applied to acquire optical solitons of the GL equation including the parabolic nonlinearity. Furthermore, the present section gives several numerical simulations to explore the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons.

4.1 Employing the Kudryashov method

Based on Eqs. (2) and (8), the solution of Eq. (7) takes the following form

$$V(\epsilon) = a_0 + a_1 K(\epsilon), \quad a_1 \neq 0, \quad (9)$$

where a_0 and a_1 are unknown, and $K(\epsilon)$ has been defined in Sect. 2. After inserting Eq. (9) into Eq. (7), we reach a consistent nonlinear system as follows

$$12ABa_1^2\alpha_1 - 48ABa_1^2\alpha_4 - 4a_1^4\alpha_3 = 0,$$

$$16ABa_0a_1\alpha_1 - 64ABa_0a_1\alpha_4 - 16a_0a_1^3\alpha_3 - 4a_1^3\alpha_2 = 0,$$

$$4k^2a_1^2\alpha_1 - 24a_0^2a_1^2\alpha_3 - 12a_0a_1^2\alpha_2 + 4wa_1^2 - a_1^2\alpha_1 + 4a_1^2\alpha_4 + 4a_1^2\alpha_5 = 0,$$

$$8k^2a_0a_1\alpha_1 - 16a_0^3a_1\alpha_3 - 12a_0^2a_1\alpha_2 + 8wa_0a_1 - 2a_0a_1\alpha_1 + 8a_0a_1\alpha_4 + 8a_0a_1\alpha_5 = 0,$$

$$4k^2a_0^2\alpha_1 - 4a_0^4\alpha_3 - 4a_0^3\alpha_2 + 4wa_0^2 + 4a_0^2\alpha_5 = 0,$$

whose solution gives:

Case 1:

$$a_0 = \frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1,$$

$$w = -\frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB},$$

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = -\frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Thus, the following optical soliton to the GL equation including the parabolic nonlinearity is derived

$$u_1(x, t) = \sqrt{\frac{\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1 + a_1}{(A - B)\sin h(x + 2k\alpha_1 t) + (A + B)\cos h(x + 2k\alpha_1 t)}} \times e^{i\left(-kx - \frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB}t\right)},$$

where

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = -\frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Case 2:

$$a_0 = -\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1,$$

$$w = -\frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB},$$

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

$$\alpha_2 = \frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Accordingly, the following optical soliton to the GL equation including the parabolic nonlinearity is acquired

$$u_2(x, t) = \sqrt{-\frac{1}{4}\sqrt{4}\sqrt{\frac{1}{AB}}a_1 + a_1 \frac{1}{(A - B)\sinh(x + 2k\alpha_1 t) + (A + B)\cosh(x + 2k\alpha_1 t)}} \times e^{i\left(-kx - \frac{48ABk^2\alpha_4 + 4k^2a_1^2\alpha_3 + 12AB\alpha_5 + 5a_1^2\alpha_3}{12AB}t\right)},$$

where

$$\alpha_1 = \frac{12AB\alpha_4 + a_1^2\alpha_3}{3AB},$$

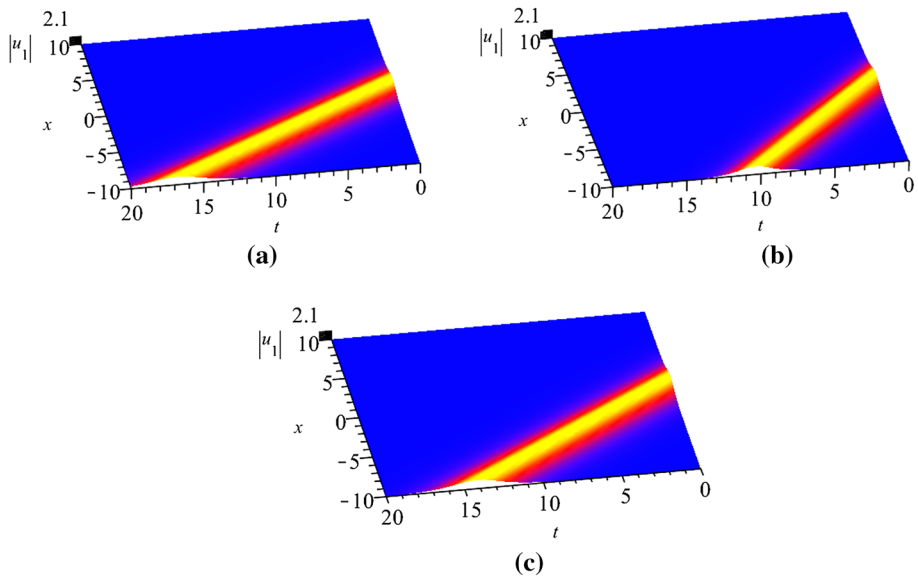


Fig. 1 **a** 3D representation of $|u_1(x, t)|$ for Set 1; **b** 3D representation of $|u_1(x, t)|$ for Set 2; **c** 3D representation of $|u_1(x, t)|$ for Set 3

$$\alpha_2 = \frac{2a_1\alpha_3\sqrt{4}}{3AB\sqrt{\frac{1}{AB}}}.$$

Several numerical simulations are presented in Fig. 1 to explore the influence of the coefficients of nonlinear terms on the dynamical features of $|u_1(x, t)|$. The following sets

Set 1 : $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 1, \alpha_4 = 1, \alpha_5 = 1, k = 0.03\}$,

Set 2 : $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 2, \alpha_4 = 1, \alpha_5 = 1, k = 0.03\}$,

Set 3 : $\{A = 0.1, B = 0.5, a_1 = 1, \alpha_3 = 1, \alpha_4 = 1.5, \alpha_5 = 1, k = 0.03\}$,

have been used to carry out this goal. A series of bright solitons (nontopological waves) are observed in Fig. 1.

4.2 Employing the exponential method

Since $N \in \mathbb{N}$, we choose $N = 1$. Such a selection leads to

$$V(\epsilon) = \frac{a_0 + a_1 a^\epsilon}{b_0 + b_1 a^\epsilon}, a_1 \neq 0, b_1 \neq 0, \tag{10}$$

where a_0, a_1, b_0 , and b_1 are unknown. After setting Eq. (10) in Eq. (7), the following consistent nonlinear system is acquired

$$4k^2 a_0^2 \alpha_1 b_0^2 + 4wa_0^2 b_0^2 - 4a_0^4 \alpha_3 - 4a_0^3 \alpha_2 b_0 + 4a_0^2 \alpha_5 b_0^2 = 0,$$

$$\begin{aligned} &2(\ln(a))^2 a_0^2 \alpha_1 b_0 b_1 - 8(\ln(a))^2 a_0^2 \alpha_4 b_0 b_1 - 2(\ln(a))^2 a_0 a_1 \alpha_1 b_0^2 \\ &+ 8(\ln(a))^2 a_0 a_1 \alpha_4 b_0^2 + 8k^2 a_0^2 \alpha_1 b_0 b_1 + 8k^2 a_0 a_1 \alpha_1 b_0^2 + 8wa_0^2 b_0 b_1 \\ &+ 8wa_0 a_1 b_0^2 - 16a_0^3 \alpha_1 \alpha_3 - 4a_0^3 \alpha_2 b_1 - 12a_0^2 a_1 \alpha_2 b_0 + 8a_0^2 \alpha_5 b_0 b_1 + 8a_0 a_1 \alpha_5 b_0^2 = 0, \end{aligned}$$

$$\begin{aligned} &-(\ln(a))^2 a_0^2 \alpha_1 b_1^2 + 4(\ln(a))^2 a_0^2 \alpha_4 b_1^2 + 2(\ln(a))^2 a_0 a_1 \alpha_1 b_0 b_1 \\ &- 8(\ln(a))^2 a_0 a_1 \alpha_4 b_0 b_1 - (\ln(a))^2 a_1^2 \alpha_1 b_0^2 + 4(\ln(a))^2 a_1^2 \alpha_4 b_0^2 \\ &+ 4k^2 a_0^2 \alpha_1 b_1^2 + 16k^2 a_0 a_1 \alpha_1 b_0 b_1 + 4k^2 a_1^2 \alpha_1 b_0^2 + 4wa_0^2 b_1^2 + 16wa_0 a_1 b_0 b_1 \\ &+ 4wa_1^2 b_0^2 - 24a_0^2 a_1^2 \alpha_3 - 12a_0^2 a_1 \alpha_2 b_1 + 4a_0^2 \alpha_5 b_1^2 - 12a_0 a_1^2 \alpha_2 b_0 + 16a_0 a_1 \alpha_5 b_0 b_1 + 4a_1^2 \alpha_5 b_0^2 = 0, \end{aligned}$$

$$\begin{aligned} &-2(\ln(a))^2 a_0 a_1 \alpha_1 b_1^2 + 8(\ln(a))^2 a_0 a_1 \alpha_4 b_1^2 + 2(\ln(a))^2 a_1^2 \alpha_1 b_0 b_1 - 8(\ln(a))^2 a_1^2 \alpha_4 b_0 b_1 + 8k^2 a_0 a_1 \alpha_1 b_1^2 \\ &+ 8k^2 a_1^2 \alpha_1 b_0 b_1 + 8wa_0 a_1 b_1^2 + 8wa_1^2 b_0 b_1 - 16a_0 a_1^3 \alpha_3 - 12a_0 a_1^2 \alpha_2 b_1 + 8a_0 a_1 \alpha_5 b_1^2 - 4a_1^3 \alpha_2 b_0 + 8a_1^2 \alpha_5 b_0 b_1 = 0, \end{aligned}$$

$$4k^2 a_1^2 \alpha_1 b_1^2 + 4wa_1^2 b_1^2 - 4a_1^4 \alpha_3 - 4a_1^3 \alpha_2 b_1 + 4a_1^2 \alpha_5 b_1^2 = 0,$$

whose solution gives

$$a_0 = 0,$$

$$w = -\frac{12(\ln(a))^2 k^2 \alpha_4 b_1^2 + (\ln(a))^2 a_1^2 \alpha_3 + 3\alpha_5 (\ln(a))^2 b_1^2 - 4k^2 a_1^2 \alpha_3}{3(\ln(a))^2 b_1^2},$$

$$\alpha_1 = \frac{4(3b_1^2 (\ln(a))^2 \alpha_4 - a_1^2 \alpha_3)}{3(\ln(a))^2 b_1^2},$$

$$\alpha_2 = -\frac{4a_1 \alpha_3}{3b_1}.$$

Consequently, the following optical soliton to the GL equation including the parabolic nonlinearity is obtained

$$u_3(x, t) = \sqrt{\frac{a_1 a^{x+2k\alpha_1 t}}{b_0 + b_1 a^{x+2k\alpha_1 t}}} e^{i\left(-kx - \frac{12(\ln(a))^2 k^2 \alpha_4 b_1^2 + (\ln(a))^2 a_1^2 \alpha_3 + 3\alpha_5 (\ln(a))^2 b_1^2 - 4k^2 a_1^2 \alpha_3}{3(\ln(a))^2 b_1^2} t\right)},$$

where

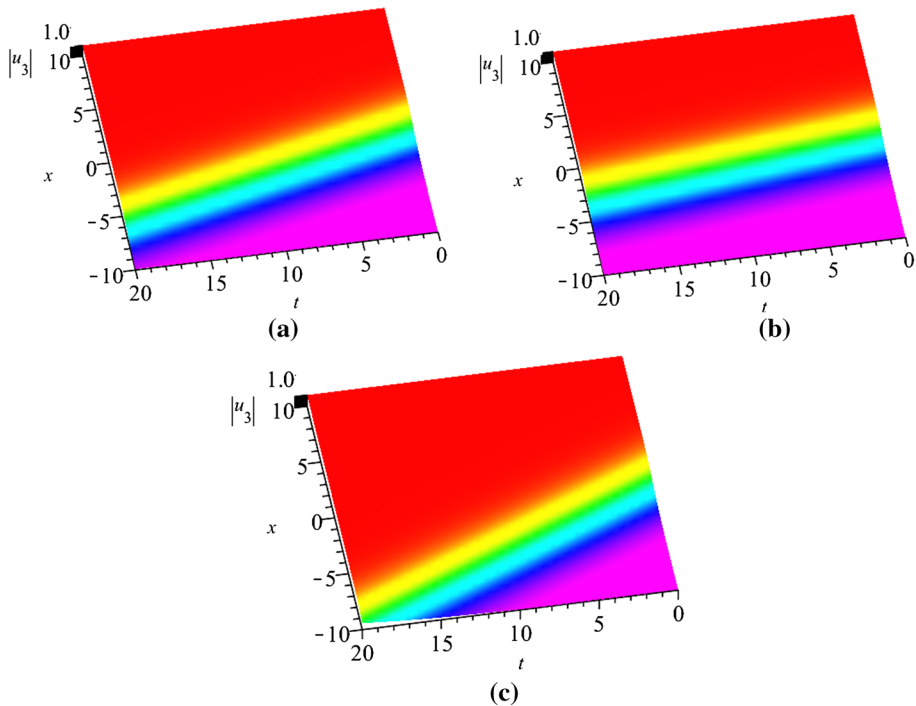


Fig. 2 **a** 3D representation of $|u_3(x, t)|$ for Set 1; **b** 3D representation of $|u_3(x, t)|$ for Set 2; **c** 3D representation of $|u_3(x, t)|$ for Set 3

$$\alpha_1 = \frac{4(3b_1^2(\ln(a))^2\alpha_4 - a_1^2\alpha_3)}{3(\ln(a))^2b_1^2},$$

$$\alpha_2 = -\frac{4a_1\alpha_3}{3b_1}.$$

Figure 2 presents some numerical simulations to show the influence of the coefficients of nonlinear terms on the dynamical features of $|u_3(x, t)|$. The following sets

$$\text{Set 1 : } \{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 1, \alpha_4 = 1, \alpha_5 = 1, k = 0.05, a = 2.7\},$$

$$\text{Set 2 : } \{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 2, \alpha_4 = 1, \alpha_5 = 1, k = 0.05, a = 2.7\},$$

$$\text{Set 3 : } \{a_1 = 1, b_0 = 1, b_1 = 1, \alpha_3 = 1, \alpha_4 = 1.5, \alpha_5 = 1, k = 0.05, a = 2.7\},$$

have been applied to achieve this aim. Several kink solitons (shock waves) are seen in Fig. 2.

5 Conclusion

In the present paper, the authors acquired optical solitons to the Ginzburg–Landau equation including the parabolic nonlinearity by employing the Kudryashov and exponential methods. As a result, a series of optical solitons, classified as bright and kink solitons, to the governing model was formally listed. Some numerical simulations were considered to examine the influence of the coefficients of nonlinear terms on the dynamical features of the obtained optical solitons. The current study's findings proved the superior performance of Kudryashov and exponential methods in dealing with the Ginzburg–Landau equation including the parabolic nonlinearity. It is worth mentioning that the authors' task for future works is adopting other well-designed methods (Kilic and Inc 2015, 2017; Inc et al. 2016; Tchier et al. 2016, 2017) to seek new optical solitons of the Ginzburg–Landau equation including the parabolic nonlinearity.

Funding The authors have not disclosed any funding.

Declarations

Conflict of interest The authors declare no conflict of interest.

References

- Akbar, M.A., Akinyemi, L., Yao, S.W., Jhangeer, A., Rezazadeh, H., Khater, M.M.A., Ahmad, H., Inc, M.: Soliton solutions to the Boussinesq equation through sine-Gordon method and Kudryashov method. *Results Phys.* **25**, 104228 (2021)
- Akinyemi, L.: Two improved techniques for the perturbed nonlinear Biswas–Milovic equation and its optical solitons. *Optik* **243**, 167477 (2021)
- Akinyemi, L., Rezazadeh, H., Shi, Q.H., Inc, M., Khater, M.M.A., Ahmad, H., Jhangeer, A., Ali Akbar, M.: New optical solitons of perturbed nonlinear Schrödinger–Hirota equation with spatio-temporal dispersion. *Results Phys.* **29**, 104656 (2021a)
- Akinyemi, L., Hosseini, K., Salahshour, S.: The bright and singular solitons of (2+1)-dimensional nonlinear Schrödinger equation with spatio-temporal dispersions. *Optik* **242**, 167120 (2021b)

- Ali, A.T., Hassan, E.R.: General \exp_q function method for nonlinear evolution equations. *Appl. Math. Comput.* **217**, 451–459 (2010)
- Arshed, S., Biswas, A., Mallawi, F., Belic, M.R.: Optical solitons with complex Ginzburg–Landau equation having three nonlinear forms. *Phys. Lett. A* **383**, 126026 (2019)
- Bekir, A.: Application of the (G/G) -expansion method for nonlinear evolution equations. *Phys. Lett. A* **372**, 3400–3406 (2008)
- Bekir, A., Shehata, M.S.M., Zahran, E.H.M.: New optical soliton solutions for the thin-film ferroelectric materials equation instead of the numerical solution. *Comput. Methods Differ. Equ.* **10**, 158–167 (2021). <https://doi.org/10.22034/cmde.2020.38121.1677>
- Biswas, A.: Chirp-free bright optical solitons and conservation laws for complex Ginzburg–Landau equation with three nonlinear forms. *Optik* **174**, 207–215 (2018)
- Elboree, M.K.: Optical solitons for complex Ginzburg–Landau model with Kerr, quadratic-cubic and parabolic law nonlinearities in nonlinear optics by the $\exp(-\phi(\xi))$ expansion method. *Pramana J. Phys.* **94**, 139 (2020)
- He, J.H., Wu, X.H.: Exp-function method for nonlinear wave equations. *Chaos Solitons Fractals* **30**, 700–708 (2006)
- Hosseini, K., Mirzazadeh, M., Rabiei, F., Baskonus, H.M., Yel, G.: Dark optical solitons to the Biswas–Arshed equation with high order dispersions and absence of self-phase modulation. *Optik* **209**, 164576 (2020a)
- Hosseini, K., Mirzazadeh, M., Osman, M.S., Al Qurashi, M., Baleanu, D.: Solitons and Jacobi elliptic function solutions to the complex Ginzburg–Landau equation. *Front. Phys.* **8**, 225 (2020b)
- Hosseini, K., Salahshour, S., Mirzazadeh, M.: Bright and dark solitons of a weakly nonlocal Schrödinger equation involving the parabolic law nonlinearity. *Optik* **227**, 166042 (2021a)
- Hosseini, K., Mirzazadeh, M., Baleanu, D., Raza, N., Park, C., Ahmadian, A., Salahshour, S.: The generalized complex Ginzburg–Landau model and its dark and bright soliton solutions. *Eur. Phys. J. Plus* **136**, 709 (2021b)
- Inc, M., Ates, E., Tchier, F.: Optical solitons of the coupled nonlinear Schrödinger’s equation with spatiotemporal dispersion. *Nonlinear Dyn.* **85**, 1319–1329 (2016)
- Kilic, B., Inc, M.: On optical solitons of the resonant Schrödinger’s equation in optical fibers with dual-power law nonlinearity and time-dependent coefficients. *Waves Random Complex Media* **25**, 334–341 (2015)
- Kilic, B., Inc, M.: Optical solitons for the Schrödinger–Hirota equation with power law nonlinearity by the Bäcklund transformation. *Optik* **138**, 64–67 (2017)
- Kudryashov, N.A.: Method for finding highly dispersive optical solitons of nonlinear differential equation. *Optik* **206**, 163550 (2020a)
- Kudryashov, N.A.: Highly dispersive solitary wave solutions of perturbed nonlinear Schrödinger equations. *Appl. Math. Comput.* **371**, 124972 (2020b)
- Kudryashov, N.A.: Highly dispersive optical solitons of the generalized nonlinear eighth-order Schrödinger equation. *Optik* **206**, 164335 (2020c)
- Mirzazadeh, M., Ekici, M., Sonmezoglu, A., Eslami, M., Zhou, Q., Kara, A.H., Milovic, D., Majid, F.B., Biswas, A., Belic, M.: Optical solitons with complex Ginzburg–Landau equation. *Nonlinear Dyn.* **85**, 1979–2016 (2016)
- Nisar, K.S., Ciancio, A., Ali, K.K., Osman, M.S., Cattani, C., Baleanu, D., Zafar, A., Raheel, M., Azeem, M.: On beta-time fractional biological population model with abundant solitary wave structures. *Alex. Eng. J.* **61**, 1996–2008 (2021). <https://doi.org/10.1016/j.aej.2021.06.106>
- Osman, M.S., Ghanbari, B., Machado, J.A.T.: New complex waves in nonlinear optics based on the complex Ginzburg–Landau equation with Kerr law nonlinearity. *Eur. Phys. J. Plus* **134**, 20 (2019)
- Ouahid, L., Abdou, M.A., Owyed, S., Inc, M., Abdel-Baset, A.M., Yusuf, A.: New optical solitons for complex Ginzburg–Landau equation with beta derivatives via two integration algorithms. *Indian J. Phys.* **96**, 2093–2105 (2021). <https://doi.org/10.1007/s12648-021-02168-0>
- Rezazadeh, H.: New solitons solutions of the complex Ginzburg–Landau equation with Kerr law nonlinearity. *Optik* **167**, 218–227 (2018)
- Rezazadeh, H., Inc, M., Baleanu, D.: New solitary wave solutions for variants of (3+1)-dimensional Wazwaz–Benjamin–Bona–Mahony equations. *Front. Phys.* **8**, 332 (2020a)
- Rezazadeh, H., Abazari, R., Khater, M.M.A., Inc, M., Baleanu, D.: New optical solitons of conformable resonant nonlinear Schrödinger’s equation. *Open Phys.* **18**, 761–769 (2020b)
- Siddique, I., Jaradat, M.M.M., Zafar, A., Mehdi, K.B., Osman, M.S.: Exact traveling wave solutions for two prolific conformable M -fractional differential equations via three diverse approaches. *Results Phys.* **28**, 104557 (2021)

- Sulaiman, T.A., Baskonus, H.M., Bulut, H.: Optical solitons and other solutions to the conformable space-time fractional complex Ginzburg–Landau equation under Kerr law nonlinearity. *Pramana J. Phys.* **91**, 58 (2018)
- Tchier, F., Aslan, E.C., Inc, M.: Optical solitons in parabolic law medium: Jacobi elliptic function solution. *Nonlinear Dyn.* **85**, 2577–2582 (2016)
- Tchier, F., Inc, M., Kilic, B., Akgül, A.: On soliton structures of generalized resonance equation with time dependent coefficients. *Optik* **128**, 218–223 (2017)
- Wang, M., Li, X., Zhang, J.: The (G/G) -expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. *Phys. Lett. A* **372**, 417–423 (2008)
- Yan, C.: A simple transformation for nonlinear waves. *Phys. Lett. A* **224**, 77–84 (1996)
- Yıldırım, Y., Biswas, A., Khan, S., Guggilla, P., Alzahrani, A.K., Belic, M.R.: Optical solitons in fiber Bragg gratings with dispersive reflectivity by sine-Gordon equation approach. *Optik* **237**, 166684 (2021a)
- Yıldırım, Y., Biswas, A., Dakova, A., Khan, S., Moshokoa, S.P., Alzahrani, A.K., Belic, M.R.: Cubic-quartic optical soliton perturbation with Fokas–Lenells equation by sine-Gordon equation approach. *Results Phys.* **26**, 104409 (2021b)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.