Active laser radar systems with stochastic electromagnetic beams in turbulent atmosphere

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Abstract: Propagation of stochastic electromagnetic beams through paraxial ABCD optical systems operating through turbulent atmosphere is investigated with the help of the ABCD matrices and the generalized Huygens-Fresnel integral. In particular, the analytic formula is derived for the cross-spectral density matrix of an electromagnetic Gaussian Schell-model (EGSM) beam. We applied our analysis for the ABCD system with a single lens located on the propagation path, representing, in a particular case, the unfolded double-pass propagation scenario of active laser radar. Through a number of numerical examples we investigated the effect of local turbulence strength and lens' parameters on spectral, coherence and polarization properties of the EGSM beam.

OCIS codes: (010.1300) Atmospheric propagation; (010.1330) Atmospheric turbulence; (030.1640) Coherence; (260.5430) Polarization

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1. Introduction

Over the past several decades, scalar partially coherent beams have found wide applications in optical projection, laser scanning, inertial confinement fusion, free space optical communications, imaging applications and nonlinear optics [1-9]. Gaussian Schell-model beam (GSM) is a conventional mathematical model for describing a typical scalar partially coherent beam whose spectral density and spectral degree of coherence are Gaussian functions [10-11]. Generation and propagation of a scalar GSM beam in various media and in imaging and non-imaging optical systems are now well understood [12-19].

In the past decades the two important properties of light waves: coherence and polarization were studied separately (cf. [1], [20]). After the unified theory of coherence and polarization was formulated [21] it became evident that these properties are interrelated. Scalar Gaussian Schell-model (GSM) beams were then extended to electromagnetic domain (called EGSM beams) and studied in details [21-38].

Propagation characteristics of different types of beams propagating in the turbulent atmosphere are of interest for optical communications, imaging and remote sensing applications [28, 33, 38-52]. In Refs. [28], [33] and [38] various statistical properties of EGSM beams propagating in the atmosphere have been studied. More importantly, it was found that under suitable conditions the EGSM beams may have reduced levels of intensity fluctuations (scintillations) compared to the scalar GSM beams (i.e. fully polarized GSM beam) [38], which makes them attractive for free-space optical communications.

In practice, atmospheric propagation is often combined with the passage of the beam through optical elements located within the transmitter or receiver system as well as anywhere in between. In such situations the ABCD matrix approach is used to characterize the effect of the optical elements on the beam [53]. To our knowledge no results have been reported up until now on propagation of EGSM beams through such systems operating in turbulence. In fact, little attention was paid even to interaction of laser beams with these systems [54-56] (see also [40]).

In this paper, we analyze various phenomena arising on propagation of an EGSM beam through a paraxial ABCD optical system in a turbulent atmosphere by deriving relating analytic formulas. To illustrate the usefulness of our analytic results we apply them to the case when the ABCD system consists of a single lens which may be located anywhere between the source and the receiving system [see Fig. 1(a)]. Such example may also be used for study of reflection of a beam from a mirror target in a bistatic mode [see Fig. 1(b)], the problem that was studied previously only in the framework of scalar theory [57]. We will pay special attention to spectral properties, and the states of coherence and polarization in such systems.

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2. Theory

Within the validity of the paraxial approximation, propagation of a laser beam through an astigmatic ABCD optical system situated in the turbulent atmosphere can be studied with the help of the following generalized Huygens-Fresnel integral [54-56]

$$E(\boldsymbol{\rho}_1, l) = -\frac{i}{\lambda[\det(\mathbf{B})]^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(\mathbf{r}_1, 0) \times \exp\left[-\frac{ik}{2}\left(\mathbf{r}_1^T \mathbf{B}^{-1}\mathbf{A}\mathbf{r}_1 - 2\mathbf{r}_1^T\mathbf{B}^{-1}\boldsymbol{\rho}_1 + \boldsymbol{\rho}_1^T \mathbf{D}\mathbf{B}^{-1}\boldsymbol{\rho}_1\right) + \Psi\left(\mathbf{r}_1, \boldsymbol{\rho}_1\right)\right] d\mathbf{r}_1, \quad (1)$$

where det stands for the determinant of a matrix, $E(\mathbf{r}_1, 0)$ and $E(\boldsymbol{\rho}_1, l)$ are the electric fields of the laser beam in the source plane (z=0) and the output plane (z=l), respectively. $\mathbf{r}_1^T = (x_1 \quad y_1)$ and $\boldsymbol{\rho}_1^T = (\rho_{1x} \quad \rho_{1y})$ with $\mathbf{r}_1$ and $\boldsymbol{\rho}_1$ being the position vectors in the source plane and output planes, $\Psi(\mathbf{r}_1, \boldsymbol{\rho}_1)$ is the Rytov perturbation being the random part of the complex phase of the beam induced by atmospheric fluctuations, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength of light. Here we note that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are the 2×2 sub-matrices of the astigmatic optical system [58, 59], satisfying the following Luneburg relations that describe the symplecticity of an astigmatic optical system [60]

$$(\mathbf{B}^{-1}\mathbf{A})^T = \mathbf{B}^{-1}\mathbf{A}, \quad (-\mathbf{B}^{-1})^T = (\mathbf{C} - \mathbf{D}\mathbf{B}^{-1}\mathbf{A}), \quad (\mathbf{D}\mathbf{B}^{-1})^T = \mathbf{D}\mathbf{B}^{-1}. \quad (2)$$

Denoting the optical fields at the two arbitrary points $\mathbf{r}_1, \mathbf{r}_2$ in the source plane by $E(\mathbf{r}_1), E(\mathbf{r}_2)$ and the optical fields at the two arbitrary points $\boldsymbol{\rho}_1, \boldsymbol{\rho}_2$ in the output plane by $E(\boldsymbol{\rho}_1), E(\boldsymbol{\rho}_2)$, respectively, we may write the expressions for the cross-spectral density in the source and output planes as:

$$W(\mathbf{r}_1, \mathbf{r}_2, 0) = \left\langle E(\mathbf{r}_1, 0) E^*(\mathbf{r}_2, 0)\right\rangle, \quad \mathrm{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, l) = \left\langle E(\boldsymbol{\rho}_1, l) E^*(\boldsymbol{\rho}_2, l)\right\rangle. \quad (3)$$

Here "< >" denotes ensemble average. Using Eqs. (1) and (3) we find that the cross-spectral density of a scalar partially coherent beam propagating through a general astigmatic optical system is given by the expression

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, l) = \frac{1}{\lambda^2[\det(\mathbf{B})]^{1/2}[\det(\mathbf{B}^*)]^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(\mathbf{r}_1, \mathbf{r}_2, 0) \exp\left[-\frac{ik}{2}\left(\mathbf{r}_1^T\mathbf{B}^{-1}\mathbf{A}\mathbf{r}_1 - 2\mathbf{r}_1^T\mathbf{B}^{-1}\boldsymbol{\rho}_1 + \boldsymbol{\rho}_1^T\mathbf{D}\mathbf{B}^{-1}\boldsymbol{\rho}_1\right)\right] \times \exp\left[\frac{ik}{2}\left(\mathbf{r}_2^T\left(\mathbf{B}^*\right)^{-1}\mathbf{A}^*\mathbf{r}_2 - 2\mathbf{r}_2^T\left(\mathbf{B}^*\right)^{-1}\boldsymbol{\rho}_2 + \boldsymbol{\rho}_2^T\mathbf{D}^*\left(\mathbf{B}^*\right)^{-1}\boldsymbol{\rho}_2\right)\right]\left\langle \exp\left[\Psi\left(\mathbf{r}_1, \boldsymbol{\rho}_1\right) + \Psi^*\left(\mathbf{r}_2, \boldsymbol{\rho}_2\right)\right]\right\rangle d\mathbf{r}_1 d\mathbf{r}_2, \quad (4)$$

where "*" denotes the complex conjugate. The expression in the angular brackets in Eq. (4) can be expressed as [5, 42, 43, 54-55]

$$\left\langle \exp\left[\Psi\left(\mathbf{r}_1, \boldsymbol{\rho}_1\right) + \Psi^*\left(\mathbf{r}_2, \boldsymbol{\rho}_2\right)\right]\right\rangle = \exp\left[-\frac{\left(\mathbf{r}_1 - \mathbf{r}_2\right)^2}{\rho_0^2} - \frac{\left(\mathbf{r}_1 - \mathbf{r}_2\right)\left(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2\right)}{\rho_0^2} - \frac{\left(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2\right)^2}{\rho_0^2}\right], \quad (5)$$

ρ_0 being the coherence length of a spherical wave propagating in the turbulent medium given by the expression [54-56]

$$\rho_0 = \mathrm{Det}[\mathbf{B}]^{1/2}\left(1.46k^2C_n^2\int_0^l \mathrm{Det}[\mathbf{B}(z)]^{5/6} dz\right)^{-3/5}, \quad (6)$$

Here $\mathbf{B}(z)$ is the sub-matrix for back-propagation from output plane to propagation distance z [54-56], and C_n^2 is the structure constant of turbulent atmosphere. Here, following [42-56], we have applied the Kolmogorov turbulence spectrum and a quadratic approximation for wave structure function.

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After some arrangement Eq. (4) can be expressed in the tensor form as

$$W(\tilde{\boldsymbol{\rho}},l)=\frac{k^2}{4\pi^2\left[\det\left(\tilde{\mathbf{B}}\right)\right]^{1/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W\left(\tilde{\mathbf{r}},0\right)\exp\left[-\frac{ik}{2}\left(\tilde{\mathbf{r}}^T\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}\tilde{\mathbf{r}}-2\tilde{\mathbf{r}}^T\tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\rho}}+\tilde{\boldsymbol{\rho}}^T\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\rho}}\right)\right]$$
$$\times\exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T\tilde{\mathbf{P}}\ \tilde{\mathbf{r}}-\frac{ik}{2}\tilde{\mathbf{r}}^T\tilde{\mathbf{P}}\ \tilde{\boldsymbol{\rho}}-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\tilde{\mathbf{P}}\ \tilde{\boldsymbol{\rho}}\right]d\tilde{\mathbf{r}}, \tag{7}$$

where $d\tilde{\mathbf{r}}=d\mathbf{r}_1d\mathbf{r}_2$, $\tilde{\mathbf{r}}^T=\begin{pmatrix}\mathbf{r}_1^T & \mathbf{r}_2^T\end{pmatrix}$, $\tilde{\boldsymbol{\rho}}^T=\begin{pmatrix}\boldsymbol{\rho}_1^T & \boldsymbol{\rho}_2^T\end{pmatrix}$ and

$$\tilde{\mathbf{A}}=\begin{pmatrix}\mathbf{A} & 0\mathbf{I}\\ 0\mathbf{I} & \mathbf{A}^*\end{pmatrix},\ \tilde{\mathbf{B}}=\begin{pmatrix}\mathbf{B} & 0\mathbf{I}\\ 0\mathbf{I} & -\mathbf{B}^*\end{pmatrix},\ \tilde{\mathbf{C}}=\begin{pmatrix}\mathbf{C} & \mathbf{0I}\\ \mathbf{0I} & -\mathbf{C}^*\end{pmatrix},\ \tilde{\mathbf{D}}=\begin{pmatrix}\mathbf{D} & 0\mathbf{I}\\ 0\mathbf{I} & \mathbf{D}^*\end{pmatrix},\ \tilde{\mathbf{P}}=\frac{2}{ik\rho_0^2}\begin{pmatrix}\mathbf{I} & -\mathbf{I}\\ -\mathbf{I} & \mathbf{I}\end{pmatrix}, \tag{8}$$

I being a 2×2 unit matrix. In the absence of turbulence ($\rho_0\to\infty$, i.e., $C_n^2=0$), $\tilde{\boldsymbol{P}}=0$, Eq. (7) reduces to the generalized Collins formula for treating propagation of a partially coherent beam through a general astigmatic ABCD optical system in free space [16]. Due to its generality, Eq. (7) can be used to investigate the paraxial propagation of any partially coherent beam through a general astigmatic ABCD optical system in a turbulent atmosphere.

Now we apply Eq. (7) to study propagation of an EGSM beam through a general astigmatic ABCD optical system in a turbulent atmosphere. The second-order statistical properties of the EGSM beam can be characterized by the 2×2 cross-spectral density matrix $\overleftrightarrow{\mathrm{W}}(\mathbf{r}_1,\mathbf{r}_2,0)$ specified at any two points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in the source plane with elements [21-25]

$$W_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2,0)=A_\alpha A_\beta B_{\alpha\beta}\exp\left[-\frac{\mathbf{r}_1^2}{4\sigma_a^2}-\frac{\mathbf{r}_2^2}{4\sigma_\beta^2}-\frac{\left(\mathbf{r}_1-\mathbf{r}_2\right)^2}{2\delta_{\alpha\beta}^2}\right],(\alpha=x,y;\beta=x,y) \tag{9}$$

Here A_α, $B_{\alpha\beta}=\left|B_{\alpha\beta}\right|\exp\left(i\phi_{\alpha\beta}\right)=B_{\beta\alpha}^*$, σ_α and $\delta_{\alpha\beta}$ are independent of position but, in general, depend on the frequency. In Eq. (9) and everywhere else in this paper we have omitted the dependence on the oscillation frequency for conciseness. The nine real parameters A_x, A_y, σ_x, σ_y, $\left|B_{xy}\right|$, ϕ_{xy}, δ_{xx}, δ_{yy} and δ_{xy} entering the general model are shown to satisfy several intrinsic constraints and obey some simplifying assumptions (e.g. the phase difference between the x- an y-components of the field is removable, i.e. $\phi_{\alpha\beta}=0$ [29,30]. The elements of the cross-spectral density matrix in Eq. (9) can alternatively be expressed in the following tensor form [16, 27]

$$W_{\alpha\beta}(\tilde{\mathbf{r}})=A_\alpha A_\beta B_{\alpha\beta}\exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T\mathbf{M}_{0\alpha\beta}^{-1}\tilde{\mathbf{r}}\right],\quad(\alpha=x,y;\beta=x,y), \tag{10}$$

where $k=2\pi/\lambda$ is the wave number, λ is the wavelength, $\tilde{\mathbf{r}}^T=\begin{pmatrix}\mathbf{r}_1^T & \mathbf{r}_2^T\end{pmatrix}$, and the 4×4 tensor has the form

$$\mathbf{M}_{0\alpha\beta}^{-1}=\begin{pmatrix}\frac{1}{ik}\left(\frac{1}{2\sigma_a^2}+\frac{1}{\delta_{\alpha\beta}^2}\right)\mathbf{I} & \frac{i}{k\delta_{\alpha\beta}^2}\mathbf{I}\\ \frac{i}{k\delta_{\alpha\beta}^2}\mathbf{I} & \frac{1}{ik}\left(\frac{1}{2\sigma_\beta^2}+\frac{1}{\delta_{\alpha\beta}^2}\right)\mathbf{I}\end{pmatrix}, \tag{11}$$

Substituting from Eq. (10) into Eq. (7), after some vector integration and tensor operations, we obtain (see Appendix A) the following expression for the elements of the cross-spectral density matrix of a EGSM beam after propagating through an astigmatic ABCD optical system in a turbulent atmosphere

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$$W_{\alpha\beta}(\tilde{\boldsymbol{\rho}},l)=\frac{A_\alpha A_\beta B_{\alpha\beta}}{\left[\det\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)\right]^{1/2}}\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\mathbf{M}_{1\alpha\beta}^{-1}\tilde{\boldsymbol{\rho}}\right]$$
$$\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\tilde{\mathbf{P}}\tilde{\boldsymbol{\rho}}-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{B}}^{-1T}-\frac{1}{4}\tilde{\mathbf{P}}^T\right)\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{-1}\tilde{\mathbf{P}}\tilde{\boldsymbol{\rho}}\right], \tag{12}$$

where

$$\mathbf{M}_{1\alpha\beta}^{-1}=\left(\tilde{\mathbf{C}}+\tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{D}}\tilde{\mathbf{P}}\right)\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1}, \tag{13}$$

In the absence of turbulence (when $C_n^2=0$, and hence, $\rho_0\to\infty$) $\tilde{\boldsymbol{P}}=0$. Equation (12) then reduces to the propagation formula for an EGSM beam passing through a general astigmatic ABCD optical system in free space [31], and Eq. (13) reduces to the known tensor ABCD law for a partially coherent beam [16, 31]. Equations (12) and (13) also can be applied to study propagation of an anisotropic EGSM beam whose $\mathbf{M}_{0\alpha\beta}^{-1}$ can be expressed as [35]

$$\mathbf{M}_{0\alpha\beta}^{-1}=\begin{pmatrix}\frac{1}{2ik}\left(\boldsymbol{\sigma}_a^2\right)^{-1}+\frac{1}{ik}\left(\boldsymbol{\delta}_{\alpha\beta}^2\right)^{-1} & \frac{i}{k}\left(\boldsymbol{\delta}_{\alpha\beta}^2\right)^{-1}\\ \frac{i}{k}\left(\boldsymbol{\delta}_{\alpha\beta}^2\right)^{-1} & \frac{1}{2ik}\left(\boldsymbol{\sigma}_\beta^2\right)^{-1}+\frac{1}{ik}\left(\boldsymbol{\delta}_{\alpha\beta}^2\right)^{-1}\end{pmatrix}, \tag{14}$$

where $\boldsymbol{\sigma}_a^2$, $\boldsymbol{\sigma}_\beta^2$ and $\boldsymbol{\delta}_{\alpha\beta}^2$ all are 2×2 matrices with transpose symmetry [15, 16].

In the absence of an optical system but with presence of atmospheric turbulence, the transformation matrix between the source plane and the output plane is given by

$$\begin{pmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{pmatrix}=\begin{pmatrix}\mathbf{I} & l\mathbf{I}\\ 0 & \mathbf{I}\end{pmatrix}, \tag{15}$$

Eq. (12) reduces to expression

$$W_{\alpha\beta}(\tilde{\boldsymbol{\rho}},l)=\frac{A_\alpha A_\beta B_{\alpha\beta}}{\left[\det\left(\tilde{\mathbf{I}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)\right]^{1/2}}\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{P}}+\tilde{\mathbf{B}}\right)\tilde{\boldsymbol{\rho}}\right]$$
$$\times\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)^T\left(\mathbf{M}_1^{-1}+\tilde{\mathbf{B}}^{-1}+\tilde{\mathbf{P}}\right)^{-1}\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right], \tag{16}$$

with $\rho_0=\left(0.545C_n^2k^2l\right)^{-3/5}$. Equation (16) agrees well with existing propagation formula for a scalar partially coherent GSM beam for atmospheric propagation [48], and it can be applied to study all the second-order statistical properties of isotropic and anisotropic EGSM beams [61].

3. Focusing properties of an EGSM beam in a turbulent atmosphere

In this section we study the behavior of spectral density, spectral degree of coherence and the spectral degree of polarization (which we will call for the case under study the *focusing properties*) of an EGSM beam on propagation in a turbulent atmosphere by applying the formulae derived in section 2.

The propagation geometry is shown in Fig. 1(a). Here the transformation matrix of the total optical system between the source plane and the output plane has the form

$$\begin{pmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{pmatrix}=\begin{pmatrix}\mathbf{I} & f\mathbf{I}\\ 0 & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{I} & 0\mathbf{I}\\ -(1/f)\mathbf{I} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{I} & l_1\mathbf{I}\\ 0\mathbf{I} & \mathbf{I}\end{pmatrix}=\begin{pmatrix}0\mathbf{I} & f\mathbf{I}\\ -(1/f)\mathbf{I} & (1-l_1/f)\mathbf{I}\end{pmatrix}. \tag{17}$$

For $0<z\le l_1$, the transformation matrix for back-propagation from output plane to plane located at distance z from the source is given by

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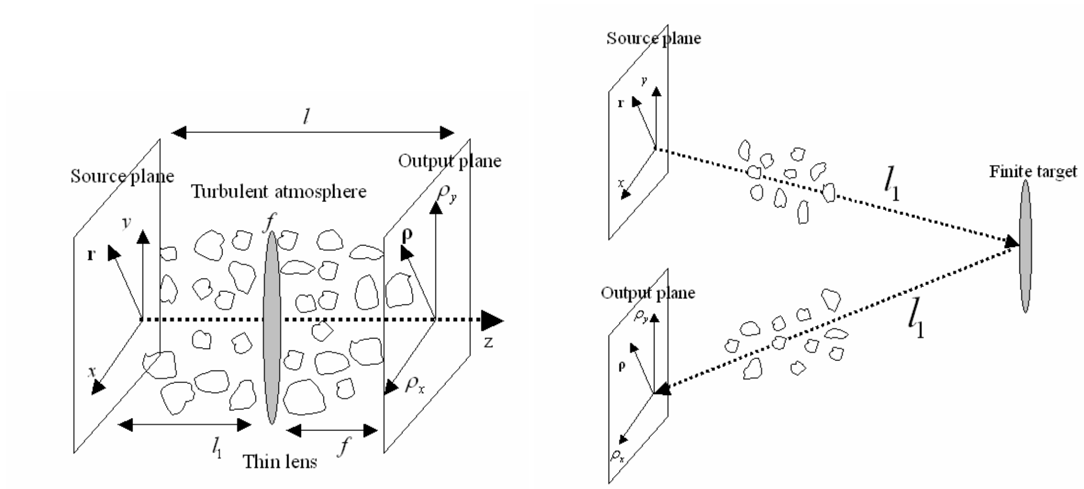
$$\begin{pmatrix} \mathbf{A}(z) & \mathbf{B}(z) \\ \mathbf{C}(z) & \mathbf{D}(z) \end{pmatrix} = \begin{pmatrix} \mathbf{I} & (l_1 - z)\mathbf{I} \\ 0 & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & 0\mathbf{I} \\ -(1/f)\mathbf{I} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & f\mathbf{I} \\ 0\mathbf{I} & \mathbf{I} \end{pmatrix} = \begin{pmatrix} (1+\dfrac{z-l_1}{f})\mathbf{I} & f\mathbf{I} \\ -(1/f)\mathbf{I} & 0\mathbf{I} \end{pmatrix}. \tag{18}$$

For $l_1 < z \le l_1 + f$, the transformation matrix for back-propagation from output plane to plane located at distance z from the source is given by

$$\begin{pmatrix} \mathbf{A}(z) & \mathbf{B}(z) \\ \mathbf{C}(z) & \mathbf{D}(z) \end{pmatrix} = \begin{pmatrix} \mathbf{I} & (f + l_1 - z)\mathbf{I} \\ 0\mathbf{I} & \mathbf{I} \end{pmatrix}. \tag{19}$$

Substituting the expression for $\mathbf{B}(z)$ into Eq. (6), we obtain (after integration)

$$\rho_0 = \left[0.1825 C_n^2 k^2 (3f + 8l_1)\right]^{-3/5}. \tag{20}$$



(a) (b)

Fig. 1 (a). Focusing geometry, (b). Schematic of laser radar configuration

The spectral density and the degree of polarization of an EGSM beam at point are defined by the expressions

$$I(\boldsymbol{\rho}_1, l) = \mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, l) \tag{21}$$

and

$$P(\boldsymbol{\rho}_1, l) = \sqrt{1 - \frac{4\mathrm{Det}\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, l)}{\left[\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, l)\right]^2}}. \tag{22}$$

The spectral degree of coherence of the EGSM beam at a pair of transverse points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ is defined by the formula

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, l) = \frac{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, l)}{\sqrt{\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, l)\,\mathrm{Tr}\overleftrightarrow{W}(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, l)}}. \tag{23}$$

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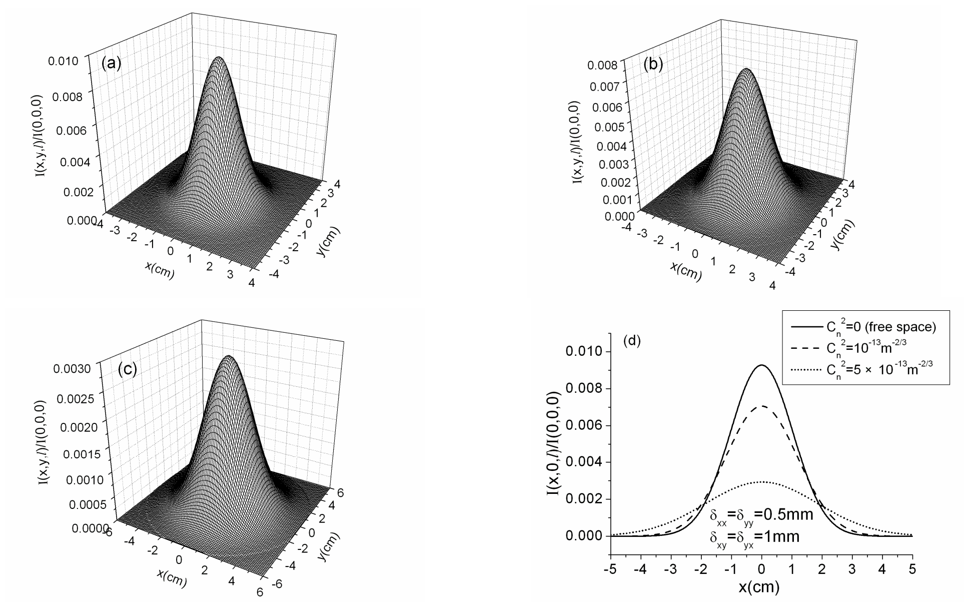

Fig. 2. Normalized intensity distribution and corresponding cross line (y=0) of an EGSM beam at the geometrical focal plane for three different values of the structure constant of turbulent atmosphere.

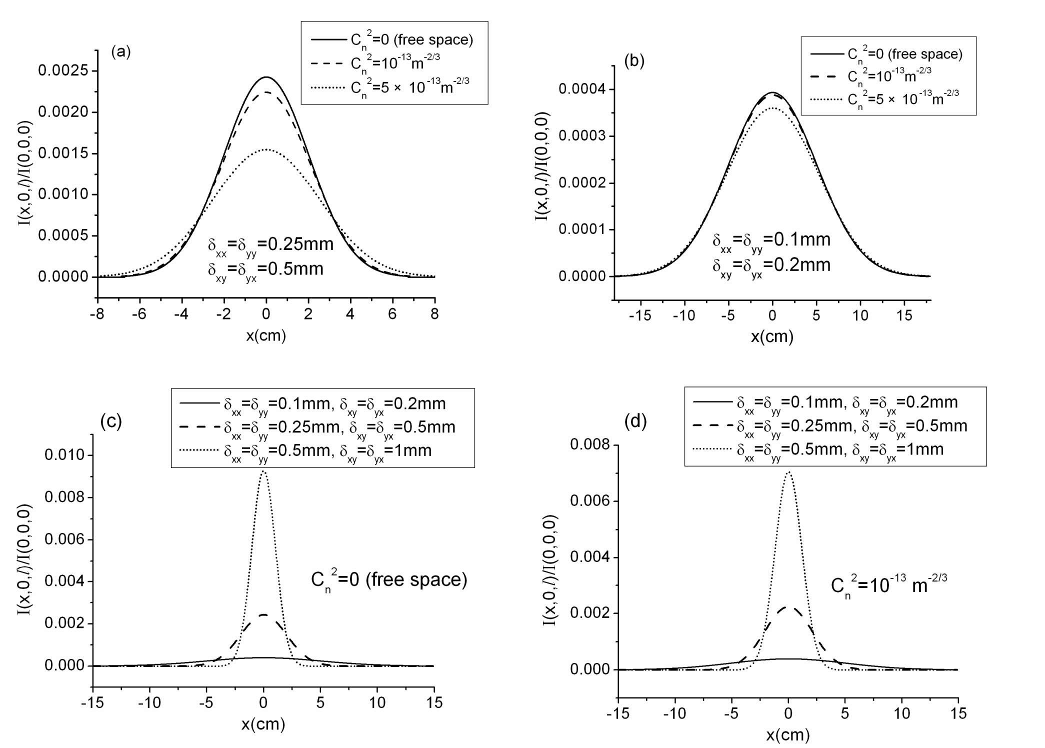

Fig. 3. Normalized intensity distribution (cross line, y=0) of an EGSM beam at the geometrical focal plane for different values of the structure constant of turbulent atmosphere and the source correlation coefficients

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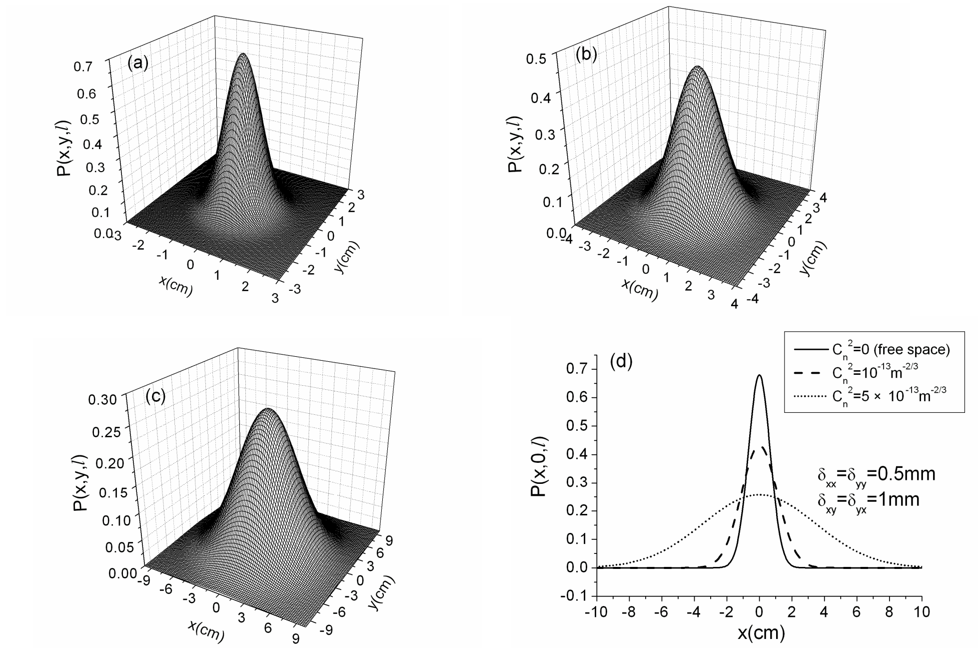

Fig. 4. Degree of polarization and corresponding cross line (y=0) of an EGSM beam at the geometrical focal plane for three different values of the structure constant of turbulent atmosphere.

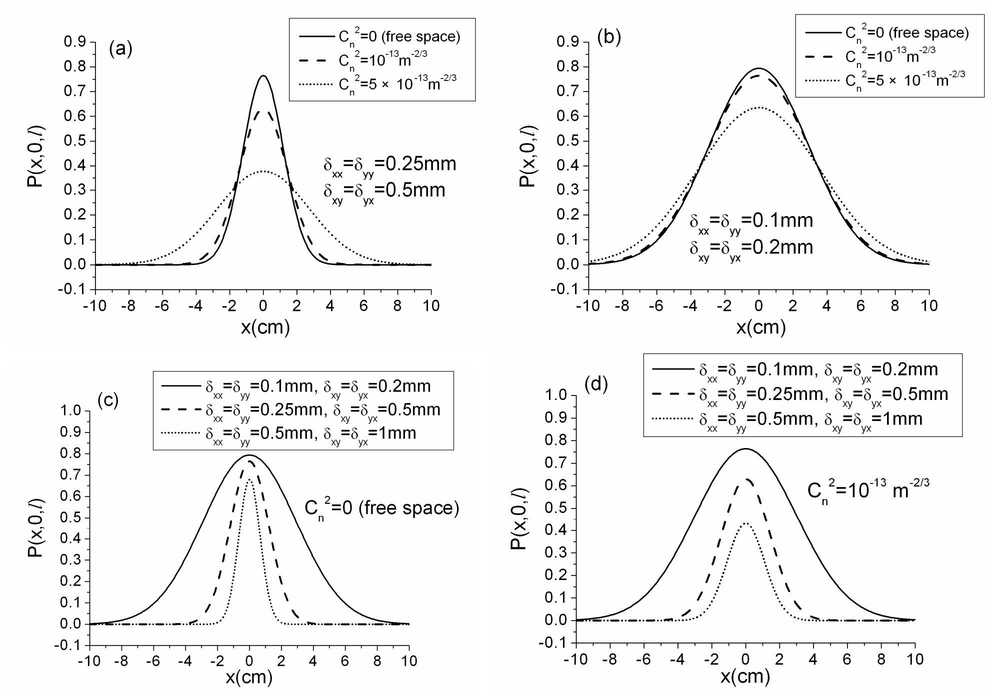

Fig. 5. Degree of polarization (cross line, y=0) of an EGSM beam at the geometrical focal plane for different values of the structure constant of turbulent atmosphere and the source correlation coefficients.

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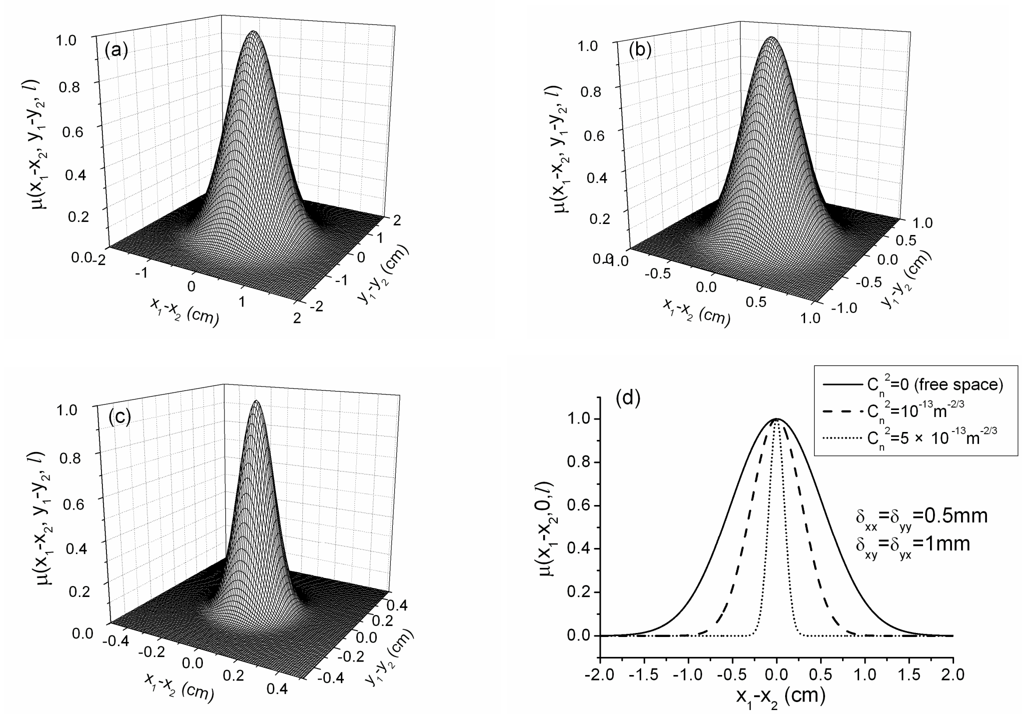

Fig. 6. Spectral degree of coherence and corresponding cross line (y_1-y_2=0) of an EGSM beam at the geometrical focal plane for three different values of the structure constant of turbulent atmosphere.

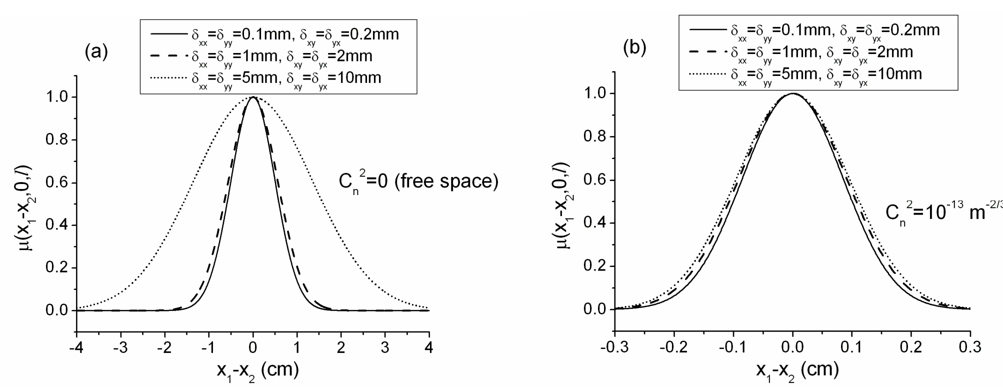

Fig. 7. Spectral degree of coherence (cross line, y_1-y_2=0) of an EGSM beam at the geometrical focal lane for different values of the structure constant of turbulent atmosphere and the source correlation coefficients.

On substituting from Eqs. (17) and (20) into Eqs. (12), (13) and (21)-(23), we can calculate the statistical properties of an EGSM beams at the geometrical focal plane in a turbulent atmosphere. For all the figures in this paper, the parameters of the source of the beam and of the optical system are chosen to be

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$A_x = A_y = 0.707,\quad B_{xy} = B_{yx} = 0.2,\ \sigma_x = \sigma_y = 1mm,\ \lambda = 590nm,\ f = 50m$ and $l_1 = 4.95km$. The polarization properties are uniform across the source plane with $\mathrm{P}(\mathbf{r}_1, 0) = 0.2$.

Figures 2 and 3 show the normalized intensity distribution and corresponding cross line (y=0) of an EGSM beam at the geometrical focal plane for different values of the structure constant of turbulent atmosphere and of the source correlation coefficients. One can see ~~from~~ that all the statistical properties of the EGSM beam in turbulent atmospheric are closely related to the structure constant C_n^2 and the source correlation coefficients. It is clear from Fig. 2 that the intensity distribution of the EGSM beam at the geometrical focal plane is of Gaussian distribution, and its width increases as the value of the structure constant C_n^2 increases (i.e., the local strength of atmospheric turbulence increases), which shows that an EGSM beam can be focused more tightly in free space than in turbulent atmosphere. From Fig. 2 (d) and Figs. 3(a) and 3(b) one finds that an EGSM beam with lower values of the source correlation coefficients is less affected by the atmospheric turbulence than that with higher values of the source correlation coefficients, which is similar to the fact that a scalar GSM beam with lower degree of coherence is less affected by the atmospheric turbulence [5, 42, 43, 46, 48]. One also finds from Figs. 3(c) and 3(d) that source correlation coefficients control the intensity distribution of the focused EGSM beam both in free space and in turbulent atmosphere, and an EGSM beam with higher values of the source correlation coefficients can be focused more tightly, which is also similar to the fact that a scalar GSM beam with higher coherence can be focused more tightly [1].

Figures 4 and 5 show the degree of polarization and corresponding cross line (y=0) of an EGSM beam at the geometrical focal plane for different values of the structure constant of turbulent atmosphere and the source correlation coefficients. One finds that the initial uniformly polarized EGSM beam becomes non-uniformly polarized after focusing, and the degree of polarization is of Gaussian profile. It is evident from Fig. 4 that as the strength of atmospheric turbulence increases, the width of the Gaussian profile increases, the value of the on-axis polarization decreases while the value of the off-axis polarization increases gradually. From Figs. 5(a) - 5(d), one finds that the shape of the Gaussian profile is affected differently by the refractive index structure parameter C_n^2 and by the source correlation coefficients: with increase in C_n^2 the distribution becomes shorter and flatter, with increase in source correlations it becomes higher and narrower. The later statement is valid in free space as well.

Figures 6 and 7 show the spectral degree of coherence and corresponding cross lines (y_1-y_2=0) of an EGSM beam versus the spatial difference vectors x_1-x_2 and y_1-y_2 at the geometrical focal plane for different values of the structure constant of turbulent atmosphere and the source correlation coefficients. One can see from these figures that the spectral degree of coherence is of Gaussian profile. The width of the Gaussian profile decreases as the value of the structure constant increases, which means the atmospheric turbulence degrades the coherence of the EGSM beam. Similar phenomenon is known for laser (coherent) Gaussian beams [62]. One also finds from Fig. 7 that the initial source correlation coefficients have obvious influence on the spectral degree of coherence of the focused EGSM beam in free space, while in turbulent atmosphere their influence is small although the width of the Gaussian distribution increases gradually as the value of the structure constant increases. This is caused by the fact the influence of atmospheric turbulence on spectral degree of coherence largely surpasses the influence of the initial source correlation coefficients at sufficiently large propagation distances from the source.

4. Summary

We have derived laws for the cross-spectral density matrix of an EGSM beam propagating through a paraxial ABCD optical system in the turbulent atmosphere based on the generalized Huygens-Fresnel integral with the help of a tensor method. In particular, we have obtained the closed-form propagation formula for the cross-spectral density matrix of an EGSM beam. The

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statistical properties of an EGSM beam focused by a thin lens in the turbulent atmosphere have been studied as a numerical example. We have found that the beam spot of an EGSM beam with higher values of the initial source correlation coefficients can be focused more tightly, the initial uniformly polarized EGSM beam will become non-uniformly polarized after focusing, the atmospheric turbulence will degrade the coherence of the EGSM beam, and the EGSM beam with lower values of the source correlation coefficients is less affected by the atmospheric turbulence. The focusing properties of an EGSM beam can be-closely controlled by the structure constant of the turbulent atmosphere and the statistical properties of the EGSM beam.

Our results might find uses in optimization of bistatic LIDAR systems.

Appendix A. Derivation of propagation Eq. (12)

Substituting Eq. (10) into Eq. (7) and after some arrangement, we obtain

$$\begin{aligned}
W(\tilde{\boldsymbol{\rho}},l) &= \frac{k^2 A_\alpha A_\beta B_{\alpha\beta}}{4\pi^2\left[\det\left(\tilde{\mathbf{B}}\right)\right]^{1/2}}\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}+\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right]\\
&\quad\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)\tilde{\mathbf{r}}\right]\exp\left[ik\tilde{\mathbf{r}}^T\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right]d\tilde{\mathbf{r}}\\
&= \frac{k^2 A_\alpha A_\beta B_{\alpha\beta}}{4\pi^2\left[\det\left(\tilde{\mathbf{B}}\right)\right]^{1/2}}\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}+\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right]\\
&\quad\times\exp\left[\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)^T\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{-1}\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right]\\
&\quad\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{ik}{2}\left|\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{1/2}\tilde{\mathbf{r}}-\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{-1/2}\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right|\right]d\tilde{\mathbf{r}},
\end{aligned}\tag{A1}$$

Then after applying the integral formula

$$\int_{-\infty}^{\infty}\exp(-ax^2)\mathrm{d}x=\sqrt{\pi/a},\tag{A2}$$

Eq. (A1) reduces (after vector integration) to the expression

$$\begin{aligned}
W(\tilde{\boldsymbol{\rho}},l) &= \frac{A_\alpha A_\beta B_{\alpha\beta}}{\left[\det\left(\tilde{\mathbf{B}}\right)\right]^{1/2}\left[\det\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)\right]^{1/2}}\exp\left[-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}+\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right]\\
&\quad\exp\left[\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)^T\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{-1}\left(\tilde{\mathbf{B}}^{-1}-\frac{1}{2}\tilde{\mathbf{P}}\right)\tilde{\boldsymbol{\rho}}\right],
\end{aligned}\tag{A3}$$

By applying the following operations

$$\left[\det\left(\tilde{\mathbf{B}}\right)\right]^{-1/2}\left[\det\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)\right]^{-1/2}=\left[\det\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)\right]^{-1/2},\tag{A4}$$

$$\begin{aligned}
\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}-\tilde{\mathbf{B}}^{-1T}\left(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{P}}\right)^{-1}\tilde{\mathbf{B}}^{-1} &= \tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}-\tilde{\mathbf{B}}^{-1}\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1}\\
&= \left[\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)-\tilde{\mathbf{B}}^{-1}\right]\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1}\\
&= \left[\tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}+\tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{D}}\tilde{\mathbf{P}}-\tilde{\mathbf{B}}^{-1}\right]\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1}\\
&= \left(\tilde{\mathbf{C}}+\tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{D}}\tilde{\mathbf{P}}\right)\left(\tilde{\mathbf{A}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1},
\end{aligned}\tag{A5}$$

and by setting

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$$\mathbf{M}_{1\alpha\beta}^{-1} = \left(\tilde{\mathbf{C}} + \tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1} + \tilde{\mathbf{D}}\tilde{\mathbf{P}}\right)\left(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1} + \tilde{\mathbf{B}}\tilde{\mathbf{P}}\right)^{-1}, \tag{A6}$$

then Eq. (A3) reduces to Eq. (12) in the text. In (A5) and (A6) we have used the Luneburg relations (Eq. (2)) and the relations $\tilde{\mathbf{B}}^{-1T} = \tilde{\mathbf{B}}^{-1}$ and $\tilde{\mathbf{P}}^{T} = \tilde{\mathbf{P}}$.

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