Influence of turbulence on the effective radius of curvature of radial Gaussian array beams

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Abstract: The analytical formula for the effective radius of curvature of radial Gaussian array beams propagating through atmospheric turbulence is derived, where coherent and incoherent beam combinations are considered. The influence of turbulence on the effective radius of curvature of radial Gaussian array beams is studied by using numerical calculation examples.

OCIS codes: (010.1300) Atmospheric propagation; (010.1330) Atmospheric turbulence; (140.3295) Laser beam characterization; (140.3298) Laser beam combining .

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20. To derive Eq. (4), we first introduce the new variables of integration by setting $\mathbf{u} = \left(\mathbf{r}_2' + \mathbf{r}_1'\right)/2$ and $\mathbf{v} = \mathbf{r}_2' - \mathbf{r}_1'$. Then we use the formulae $\int \exp(-\mathrm{i}2\pi xs)\mathrm{d}x = \delta(s)$, $\int x^2 \exp(-\mathrm{i}2\pi xs)\mathrm{d}x = -\delta''(s)/(2\pi)^2$, $\int f(x)\delta''(x)\mathrm{d}x = f''(0)$ and $\int f(x)\delta(x)\mathrm{d}x = f(0)$, where δ denotes the Dirac delta function and δ'' is its second derivative, and f is an arbitrary function and f'' is its second derivative.
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1. Introduction

The propagation of laser beams through atmospheric turbulence is a topic that has been of considerable theoretical and practical interest for a long time [1,2]. Much work has been carried out concerning the turbulence-induced changes in the spreading, the degree of polarization, the degree of coherence, and the spectrum of laser beams [3–8]. On the other hand, the propagation property of the radius of curvature of laser beams is an important subject. However, until now only few papers have dealt with the radius of curvature of laser beams both in free space and in turbulence. The radius of curvature of non-Gaussian and nonspherical beams has been defined in Ref. [9] by using the spherical wave front that best fits the actual wave front. But, it is difficult to obtain easy analytical expression for the radius of curvature of non-Gaussian and nonspherical beams because derivatives of the field are involved in the definition (see Eq. (16) in Ref. [9]). Besides, it is not easy to apply this definition to laser beams in turbulence, since the definition is given in terms of the field expression rather than the intensity expression. In 2002, the radius of curvature of Gaussian Schell-model beams in turbulence has been evaluated from mutual coherence expression [10]. The method used in Ref. [10] is only applicable to laser beams expressed by a single Gaussian term. However, in practice laser beams expressed by more than one term are often encountered, such as cosh-Gaussian, annular-Gaussian and array beams. Laser array beams have found wide applications in high-power system, inertial confinement fusion and high-energy weapons. Up to now a variety of linear, rectangular and radial laser arrays have been developed to achieve high system powers [11,12]. The influence of turbulence on the propagation properties of laser array beams has also been investigated [13–17]. In this paper, the radial Gaussian array beam is taken as a typical example of laser array beams. By using the definition of the effective curvature radius of an arbitrary field [18] and the propagation law of the beam matrix in terms of second moments of partially coherent beams in turbulence [19], the simple and analytical formula for the effective radius of curvature of radial Gaussian array beams propagating through atmospheric turbulence is derived. In addition, the influence of turbulence on the effective radius of curvature of radial Gaussian array beams is studied by using numerical calculation examples.

2. Analytical Formulae

2.1 Mean squared beam width in free space

Assume that a radial array beam consists of N equal elements, which are Gaussian beams and located symmetrically on a ring with radius r_0, and separation angle $\alpha_0 = 2\pi/N$. For the coherent combination, the cross-spectral density function of the radial Gaussian array beam in the plane $z = 0$ is expressed as

$$W(x_1', x_2', y_1', y_2', z=0) = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} \exp\left[-\frac{(x_1'\cos\alpha_p + y_1'\sin\alpha_p - r_0)^2 + (y_1'\cos\alpha_p - x_1'\sin\alpha_p)^2}{w_0^2}\right]$$

$$\times\exp\left[-\frac{(x_2'\cos\alpha_q + y_2'\sin\alpha_q - r_0)^2 + (y_2'\cos\alpha_q - x_2'\sin\alpha_q)^2}{w_0^2}\right], \tag{1}$$

where $\alpha_j = j\alpha_0$ ($j = p$, q = 0, 1, 2, …N-1), $N \geq 2$ and w_0 is the waist width of Gaussian beams.

Based on the Huygens-Fresnel principle, the intensity of the radial Gaussian array beam represented by Eq. (1) propagating in free space reads as

$$I(\mathbf{r}, z) = \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2 r_1' \iint \mathrm{d}^2 r_2' W(\mathbf{r}_1', \mathbf{r}_2', z=0) \exp\left\{\frac{\mathrm{i}k}{2z}[(\mathbf{r}_1'^2 - \mathbf{r}_2'^2) - 2\mathbf{r}\cdot(\mathbf{r}_1' - \mathbf{r}_2')]\right\}, \tag{2}$$

where $\mathbf{r}' \equiv (x', y')$, $\mathbf{r} \equiv (x, y)$, k is the wave number related to the wavelength λ by $k = 2\pi / \lambda$.

The mean squared beam width $< r^2 >$ is defined as [18]

$$< r^2 >= \iint r^2 I(\mathbf{r}, z)\mathrm{d}^2 r \Big/ \iint I(\mathbf{r}, z)\mathrm{d}^2 r. \tag{3}$$

Upon substituting from Eq. (2) into Eq. (3), and making use of the integral transform technique [20], after very tedious integral calculations we obtain

$$< r^2 >= G + (Q / k^2) z^2, \tag{4}$$

where

$$G = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} w_0^2 \left\{r_0'^2[1+\cos(\alpha_p - \alpha_q)] + 1\right\} S \Big/ 2\sum_{p=0}^{N-1}\sum_{q=0}^{N-1} S, \tag{5}$$

$$Q = \frac{2}{w_0^2}\sum_{p=0}^{N-1}\sum_{q=0}^{N-1} \left\{1 - r_0'^2[1-\cos(\alpha_p - \alpha_q)]\right\} S \Big/ \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} S, \tag{6}$$

$$S = \exp\left\{r_0'^2[1+\cos(\alpha_p - \alpha_q)]\right\}, \tag{7}$$

with $r_0' = r_0 / w_0$ being the inverse radial fill-factor [21].

2.2 Effective radius of curvature in turbulence

The transformation of the mean squared beam width of an arbitrary field by the optical ABCD system can be expressed as [18]

$$< \mathbf{r}_2^2 >=< (A\mathbf{r}_1 + B\boldsymbol{\theta}_1)^2 >, \tag{8}$$

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where A, B and D are elements of the transfer matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, $\boldsymbol{\theta} \equiv (\theta_x, \theta_y)$, $k\theta_x$ and $k\theta_y$ are the wave vector components along the x-axis and y-axis respectively. The parameters with the subscripts "1" and "2" denote those before and after the optical ABCD system respectively.

Comparing the propagation equation of the mean squared beam width of an arbitrary field (i.e., Eq. (8)) with that of an ideal Gaussian beam, the effective radius of curvature is defined by [18]

$$R = \langle r^2 \rangle \big/ \langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle. \tag{9}$$

When the beam waist is located in the plane $z = 0$, from Eq. (9) we obtain the free-space propagation equation of the effective radius of curvature of an arbitrary field, i.e.,

$$R = z + \frac{z_R^2}{z}, \tag{10}$$

where $z_R^2 = \langle r^2 \rangle_0 \big/ \langle \theta^2 \rangle_0$ is the Rayleigh range of an arbitrary field in free space [18], the angle brackets with the subscript 0 denote the second moments in the plane $z = 0$. The Rayleigh range is used in the theory of lasers to characterize the distance over which a beam may be considered effectively non-spreading. It is clear that the effective radius of curvature of an arbitrary field defined by Eq. (9) obeys the same free-space propagation equation as does the wavefront curvature of an ideal Gaussian beam.

On the other hand, the general formulae of the second moments $\langle r^2 \rangle$ and $\langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle$ of partially coherent beams propagating through atmospheric turbulence can be expressed as [19]

$$\langle r^2 \rangle = \langle r^2 \rangle_0 + 2 \langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle_0 \, z + \langle \theta^2 \rangle_0 \, z^2 + (4/3)Tz^3, \tag{11}$$

$$\langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle = \langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle_0 + \langle \theta^2 \rangle_0 \, z + 2Tz^2, \tag{12}$$

where

$$T = \pi^2 \int_0^\infty \kappa^3 \Phi_n(\kappa) \mathrm{d}\kappa. \tag{13}$$

with Φ_n being the spatial power spectrum of the refractive index fluctuations of the turbulent atmosphere.

Upon substituting from Eqs. (11)-(12) into Eq. (9), and considering $\langle r^2 \rangle_0 = A$, $\langle \mathbf{r} \cdot \boldsymbol{\theta} \rangle_0 = 0$ and $\langle \theta^2 \rangle_0 = B / k^2$ which can be derived from comparing Eq. (4) with Eq. (11) for $T = 0$, we obtain the effective radius of curvature of radial Gaussian array beams propagating through atmospheric turbulence for the coherent combination case, i.e.

$$R = \frac{A + (B/k^2)z^2 + (4/3)Tz^3}{(B/k^2)z + 2Tz^2}. \tag{14}$$

Similarly, for the case of incoherent combination, the effective radius of curvature of radial Gaussian array beams in turbulence can be obtained as

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$$R = \frac{w_0^2\left(1/2 + r_0'^2\right) + (2/k^2 w_0^2)z^2 + (4/3)Tz^3}{(2/k^2 w_0^2)z + 2Tz^2}. \tag{15}$$

From Eqs. (14) and (15), we have $R_\infty = \lim_{z\to\infty} R = z$ in free space. It implies that the wavefront can be regarded as the spherical surface when the free-space propagation distance is large enough. Equation (15) indicates that for the incoherent combination, R increases with increasing r_0', but R is independent of N.

3. Numerical calculation results and analysis

The numerical calculation results are given in Figs. 1-3 to show the influence of turbulence on the effective radius of curvature R, where $\lambda = 1.06\mu\text{m}$ and $w_0 = 0.01\text{m}$ are kept fixed. In the numerical calculations, Tatarskii spectrum is adopted, i.e., $\Phi_n(\kappa) = 0.033C_n^2\kappa^{-11/3}\exp\left(-\kappa^2/\kappa_m^2\right)$, where $\kappa_m = 5.92/l_0$, l_0 is the turbulence inner scale, C_n^2 is the refraction index structure constant. If the typical value of $l_0 = 0.01\text{m}$ is taken, from Eq. (13) yields $T = 7.6113C_n^2$.

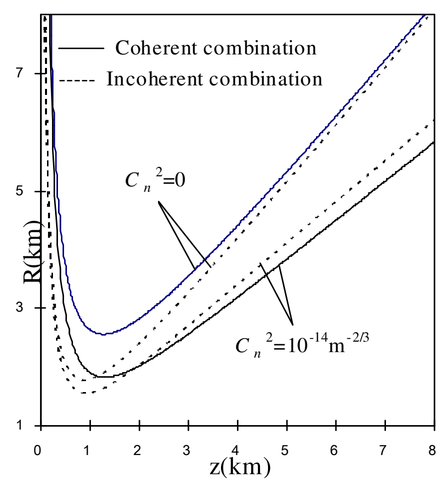

Fig. 1. Effective radius of curvature R versus the propagation distance z, $r_0' = 2$, $N = 15$

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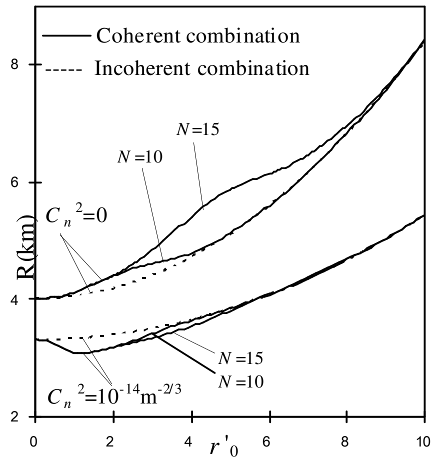

Fig. 2. Effective radius of curvature R versus the inverse radial fill-factor r_0', z = 4km.

It is noted that in Figs. 1-3, two combinations of the sources, i.e., coherent and incoherent beam arrays are considered. From Figs. 1-3 it can be seen that for the two types of beam combination R decreases due to turbulence. But the decrease of R is generally larger for the coherent combination than that for the incoherent combination, i.e., for the incoherent combination R is less sensitive to turbulence than that for the coherent combination. Figure 1 shows that in free space for the coherent combination R is always larger than that for the incoherent combination, while for the coherent combination R may be smaller than that for the incoherent combination as the propagation distance z increases in turbulence. Figure 1 also indicates that, there exists the minimum $R_{\min}$ as z increases, and position $z_{\max}$ of $R_{\min}$ is further away from the source plane for the coherent combination than that for the incoherent combination. From Fig. 2 it can be seen that there may exist a minimum of R as r_0' varies for the coherent combination in turbulence. It has been shown that this result is valid only when the strength of turbulence is strong enough, and the propagation distance is long enough. In addition, for the two types of beam combination R approaches the same value when r_0' is large enough. Figure 3 shows that for the coherent combination, R increases with increasing N in free space, but R decreases with increasing N in turbulence. Furthermore, R tends to its asymptotical value when N is large enough, and for the two types of beam combination R approaches the same value when N is small enough.

We note that a radial array beam is quite different from an annulus beam even when N is large enough since it is composed of multi-beams that are combined coherently or incoherently. Thus, propagation of radial array and annulus beams are different.

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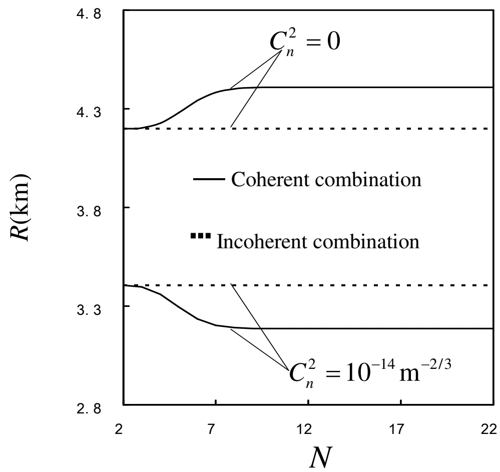

Fig. 3. Effective radius of curvature R versus the beam number N, z = 4km, $r_0' = 2$.

4. Conclusions

In summary, the simple and analytical formula for the effective radius of curvature R of radial Gaussian array beams in turbulence has been derived in this paper. It has been shown that for the two types of beam combination R decreases due to turbulence. However, for the incoherent combination R is less sensitive to turbulence than that for the coherent combination. In free space R for the coherent combination is always larger than that for the incoherent combination, but this situation may reverse as the propagation distance increases in turbulence. For the coherent combination, R increases with increasing the beam number N in free space, but R decreases with increasing N in turbulence. In addition, there may exist a minimum of R as the inverse radial fill-factor r_0' varies when the strength of turbulence is strong enough and the propagation distance is long enough. The method used in this paper would be useful for studying the effective radius of curvature of an arbitrary single laser beam or an arbitrary array laser beam in free space or in turbulence.

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