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Key Points:

- The field correlations in turbulence are evaluated for the off-axis beam
- Increase in the receiver diagonal length causes the correlations to decrease
- Smaller displacement parameters and larger sizes exhibit larger correlations

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Field correlations for off-axis Gaussian laser beams in atmospheric turbulence

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Abstract The absolute field correlations in atmospheric turbulence are evaluated for the off-axis optical Gaussian beam incidence. Evaluations in the practical range of the source and the turbulent medium parameters show that an increase in the diagonal length at the receiver plane causes the absolute field correlations of the off-axis Gaussian beam to decrease. At a fixed receiver diagonal length, the off-axis Gaussian beams having smaller displacement parameters and larger source sizes exhibit larger absolute field correlations. Comparing the absolute field correlations of the off-axis Gaussian beams in atmospheric turbulence with their no turbulence counterparts, it is observed that the behavior of the absolute field correlation variations remains the same; however, the diminishing of the absolute field correlations in turbulence occurs at smaller diagonal lengths.

1. Introduction

In the recent years it became important to understand the various properties of different optical beam types after they propagate through atmosphere turbulence. The propagation in atmospheric turbulence of different incidences, such as the off-axis, flat-topped, annular, sinusoidal Gaussian, and the others, is investigated in detail. Research results in this respect cover the second order moments [*Fusco and Conan*, 2004; *Ji et al.*, 2008; *Dou et al.*, 2012; *Li et al.*, 2012; *Baykal and Eyyuboğlu*, 2007; *Ji et al.*, 2009; *Chen and Ji*, 2008; *Ghafary and Alavinejad*, 2011; *Baykal*, 2005] and the fourth order moments [*Baykal*, *et al.*, 2010, 2011; *Eyyuboğlu et al.*, 2008; *Baykal et al.*, 2009; *Arpali et al.*, 2008; *Eyyuboğlu and Baykal*, 2007]. The results obtained in these studies indicate that the incident field profiles can appreciably change the characteristics of the received beams when they propagate in the turbulent atmosphere. The second order results also play an important role in modeling of the atmosphere [*Moraes et al.*, 2014] and in remote sensing of layered random media [*Mudaliar*, 2013].

In this respect, spatial correlation of other atmospheric entities such as the rainfall [*Luini and Capsoni*, 2012] is studied. The second order field correlations [*Baykal*, 2012, 2011a, 2014; *Baykal et al.*, 2012] and the fourth order intensity correlations [*Baykal*, 2011b] of some types of optical excitations are scrutinized in atmospheric turbulence. The fourth order evaluations using the scintillations help to understand the system performance parameters such as the phase-locked loop error [*Forte*, 2012] and Global Positioning System signal [*Jiao et al.*, 2013]. In the current paper, we evaluate the absolute field correlations when an off-axis optical Gaussian beam propagates through a turbulent medium. In our earlier work we have examined the average intensity [*Baykal and Eyyuboğlu*, 2007] and the scintillation index behavior [*Baykal et al.*, 2010] of the off-axis Gaussian beam in atmospheric turbulence which reflect results at only one detector point, i.e., from these works it is not possible to obtain the field correlation values at two detector points. The field correlation formulation presented in the current work enables us to scrutinize the important field correlation information at two different detector points.

We face the practical significance of the field correlation formulations in the performance evaluations of the wireless optical telecommunication systems that have heterodyne detection and multiple-input multiple-output configurations. The performance characteristics of such systems involve the evaluations of the entities like the average intensity, scintillation index, and the bit error rate, which require knowledge about the field correlations. Our future work will involve the employment of these results in applications like heterodyne optical detection in order to improve the performance of optical wireless communication systems.

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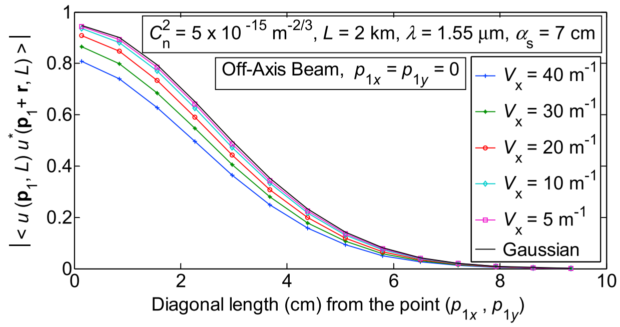

Figure 1. Field correlations for the off-axis Gaussian beam in atmospheric turbulence at various V_x in turbulence.

2. Formulation

The incident field for the off-axis Gaussian beam is defined as [*Baykal and Eyyuboğlu*, 2007]

$$u(s_x, s_y) = \exp\left[-\left(\frac{s_x^2}{2\alpha_s^2} + iV_x s_x + \frac{s_y^2}{2\alpha_s^2} + iV_y s_y\right)\right], \tag{1}$$

where (s_x, s_y) is the transverse source coordinate, α_s is the Gaussian source size, V_x and V_y are the x and y components of the displacement parameters, and $i = (-1)^{0.5}$.

Using the extended Huygens Fresnel principle [*Feizulin and Kravtsov*, 1967], the received field of the off-axis Gaussian source beam given in equation (1), after it propagates in the turbulent atmosphere is found as

$$\begin{aligned} u\left(p_x, p_y\right) = \frac{k \exp(ikL)}{2\pi i L} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}s_x \mathrm{d}s_y\, u(s_x, s_y) \exp\left\{\frac{ik}{2L}\left[(s_x - p_x)^2 + \left(s_y - p_y\right)^2\right]\right\} \\ \times \exp\left[\psi\left(s_x, s_y, p_x, p_y\right)\right], \end{aligned} \tag{2}$$

where λ is the wavelength, $k = 2\pi/\lambda$ is the wave number, (p_x, p_y) is the transverse receiver coordinate, $\psi(s_x, s_y, p_x, p_y)$ is the Rytov solution of the random part of the complex phase of spherical wave, and L is the horizontal link length.

The field correlations at two different points in the receiver plane is found to be

$$< u\left(p_{1x}, p_{1y}\right) u^*\left(p_{2x}, p_{2y}\right) > . \tag{3}$$

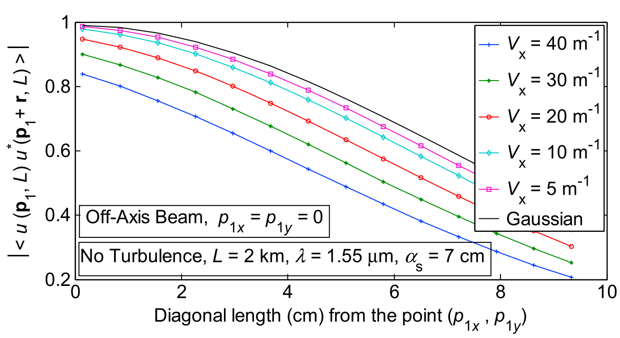

Figure 2. Field correlations for the off-axis Gaussian beam in the absence of atmospheric turbulence at various V_x.

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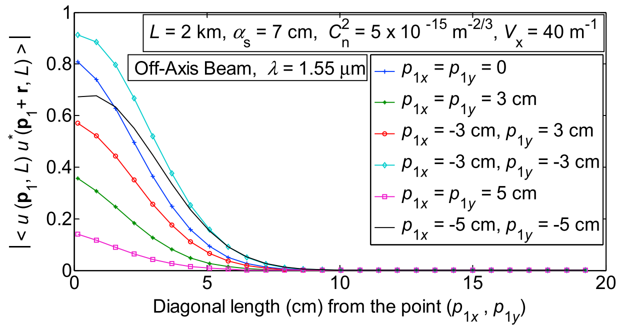

Figure 3. Field correlations for the off-axis Gaussian beam in atmospheric turbulence at various (p_x, p_y).

Here $< >$ is the ensemble average, (p_{1x}, p_{1y}) and (p_{2x}, p_{2y}) are the coordinates of the first and the second receiver points, and * denotes the complex conjugate. The turbulence term derived by using equation (2) is given by [*Wang et al.*, 1983]

$$<\exp(\psi + \psi^*)> = \exp\left\{-\rho_0^{-2}\left[\left(s_{1x} - s_{2x}\right)^2 + \left(s_{1y} - s_{2y}\right)^2 + \left(p_{1x} - p_{2x}\right)^2 + \left(p_{1y} - p_{2y}\right)^2 + \left(s_{1x} - s_{2x}\right)\right.\right.$$
$$\left.\left.\times \left(p_{1x} - p_{2x}\right) + \left(s_{1y} - s_{2y}\right)\left(p_{1y} - p_{2y}\right)\right]\right\}, \quad (4)$$

where $\rho_0 = \left(0.546 C_n^2 k^2 L\right)^{-3/5}$ is the coherence length and C_n^2 is the structure constant. $(p_{2x}, p_{2y}) = (p_{1x} + r_x, p_{1y} + r_y)$ is taken, and the diagonal length is defined as $\left(r_x^2 + r_y^2\right)^{0.5}$. Using equations (1), (2), and (4) in equation (3) and performing the integrations, we obtain

$$< u\left(p_{1x}, p_{1y}\right) u^*\left(p_{1x} + r_x, p_{1y} + r_y\right) > = \frac{1}{16 M_1 M_2}\left(\frac{k}{L}\right)^2 \exp\left[\left(\frac{1}{4M_1} + \frac{1}{8 M_2 M_1^2 \rho_0^2}\right)\left(N_{1x}^2 + N_{1y}^2\right)\right]$$
$$\times \exp\left[\frac{N_{1x} N_{2x} + N_{1y} N_{2y}}{2 M_2 M_1 \rho_0^2} + \frac{N_{2x}^2 + N_{2y}^2}{4 M_2}\right], \quad (5)$$

where

$$M_1 = -\frac{ik}{2L} + \frac{1}{2\alpha_s^2} + \frac{1}{\rho_0^2}, \quad M_2 = \frac{ik}{2L} + \frac{1}{2\alpha_s^2} + \frac{1}{\rho_0^2} - \frac{1}{M_1 \rho_0^4}, \quad N_{1x} = -iV_x - \frac{ikp_{1x}}{L} + \frac{r_x}{\rho_0^2},$$
$$N_{1y} = -iV_y - \frac{ikp_{1y}}{L} + \frac{r_y}{\rho_0^2}, \quad N_{2x} = iV_x + \frac{ik}{L}(p_{1x} + r_x) - \frac{r_x}{\rho_0^2}, \quad N_{2y} = iV_y + \frac{ik}{L}\left(p_{1y} + r_y\right) - \frac{r_y}{\rho_0^2}$$

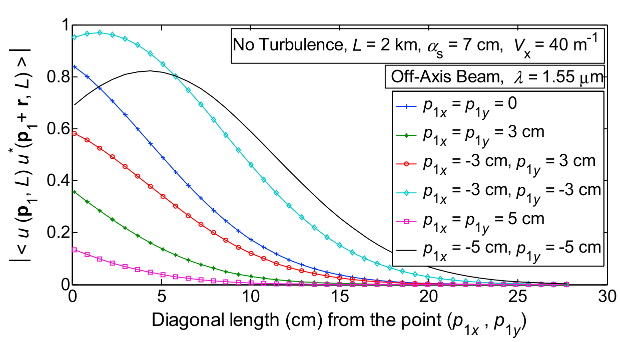

Figure 4. Field correlations for the off-axis Gaussian beam in the absence of atmospheric turbulence at various (p_x, p_y).

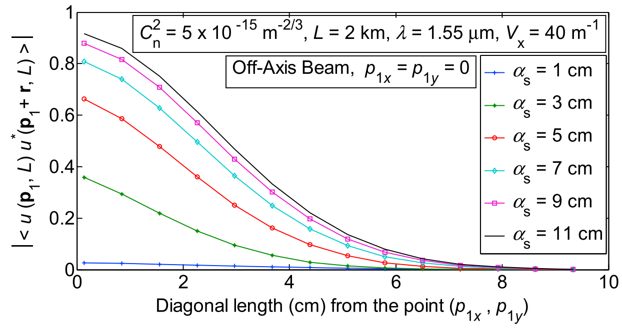

Figure 5. Field correlations for the off-axis Gaussian beam in weak atmospheric turbulence at various α_s.

3. Results

In this section, all the figures are plotted to show the variations of the absolute field correlations versus the diagonal length from the receiver point (p_{1x}, p_{1y}). Thus, | . | appearing in the vertical axes of the figures represents the absolute value. It is seen from Figure 1 that at a fixed diagonal length, smaller beam displacement parameter yields larger field correlations in the presence of turbulence. We note that the Gaussian beam in Figure 1 has $V_x = 0$. Figure 2 is plotted in the absence of atmospheric turbulence but keeping the other parameters the same as in Figure 1. The variation of the field correlation in the absence of turbulence is the same as in turbulence except that the vanishing of the field correlation versus the diagonal length occurs at larger diagonal lengths when there is no turbulence.

Figure 3 is plotted at various (p_x, p_y). It is seen that the field correlation of an off-axis beam in turbulence can show varying behavior depending not only on the diagonal length but also on the location of the two fields at the receiver plane. Figure 4 presents the no turbulence counterpart of Figure 3. When Figures 3 and 4 are compared, it is concluded that in the absence of atmospheric turbulence, the behavior of the field correlations is the same as in the presence of turbulence. However, in the absence of turbulence, much longer diagonal distance is required at the receiver plane in order to reach the same field correlation value of the equivalent turbulence case.

Figure 5 reveals that in a turbulent atmosphere, when the diagonal length at the receiver plane is kept constant, as the size of the optical off-axis Gaussian beam increases, the field correlations increase. Figure 6 reflects the no turbulence counterpart of Figure 5. As observed in Figures 2 and 4, in Figure 6, it is also observed that the variation of the field correlation in the absence of turbulence remains similar as in the case where there is turbulence. The difference is that the field correlation approaches zero at much larger diagonal lengths when there is no turbulence. As a common conclusion, we comment that in all the figures provided in this section, as the diagonal length increases, the field correlation of the off-axis beam decreases and approaches zero at a sufficiently large diagonal length.

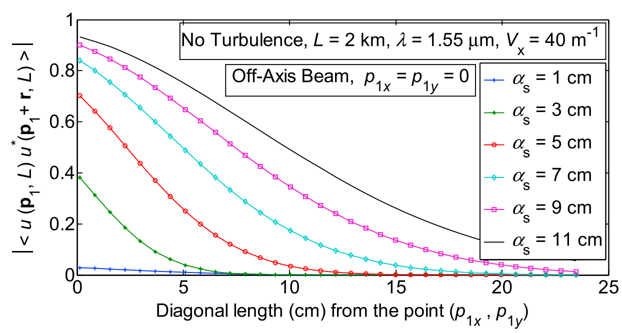

Figure 6. Field correlations for the off-axis Gaussian beam in the absence of atmospheric turbulence at various α_s.

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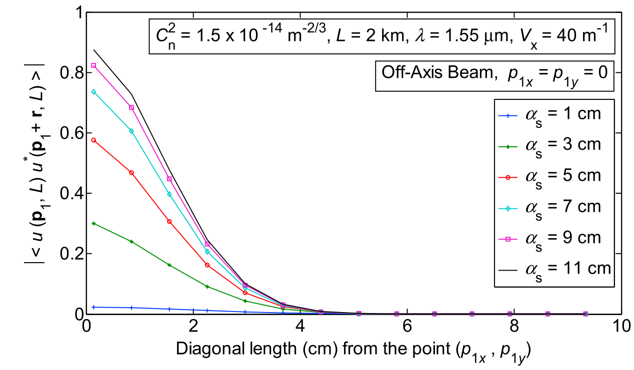

Figure 7. Field correlations for the off-axis Gaussian beam in moderate atmospheric turbulence at various α_s.

Figures 1, 3, and 5 are for weak turbulence, i.e., the chosen medium parameters fulfill the condition that the plane wave scintillation index, $1.23C_n^2k^{7/6}L^{11/6}$ is appreciably smaller than unity. Since the second order solutions are also applicable in moderate and strong turbulence, using the same formula given by equation (5), Figures 7 and 8 are introduced to cover the ranges of moderate ($1.23C_n^2k^{7/6}L^{11/6}$ is around unity) and strong turbulence ($1.23C_n^2k^{7/6}L^{11/6}$ is appreciably larger than unity) regimes, respectively. In Figures 7 and 8, in order to make a fair comparison, the same parameters as in Figure 5 are chosen, except that the structure constants are changed to be 1.5×10^{-14} m$^{-2/3}$ for the moderate and 1.5×10^{-13} m$^{-2/3}$ for the strong turbulence. Comparison of Figures 5, 7, and 8 shows that the behavior of the field correlations versus the diagonal length at the receiver plane do not change; however, at the same diagonal length value, as expected, the field correlations decrease as the turbulence strength increases. In other words, to reach the same field correlation value, the required diagonal length at the receiver plane becomes smaller as the turbulence becomes stronger.

4. Conclusion

The field correlation for the off-axis Gaussian beams is formulated and evaluated at the receiver plane after these beams propagate through the turbulent atmosphere. At the fixed diagonal length at the receiver plane, off-axis Gaussian beam field correlations become larger at smaller source displacement parameters and at larger off-axis source sizes. In turbulence, the variation of the off-axis Gaussian field correlations is found to vary depending on the location of the two field points at the receiver plane. Comparison of the field correlations in the presence and the absence of turbulence reveals that the trend of the field correlations in both cases is almost the same, except that in no turbulence, the field correlations along the diagonal length at the receiver plane diminish at a smaller rate. For all the cases of interest, an increase in the diagonal length at the receiver plane is found to reduce the field correlations.

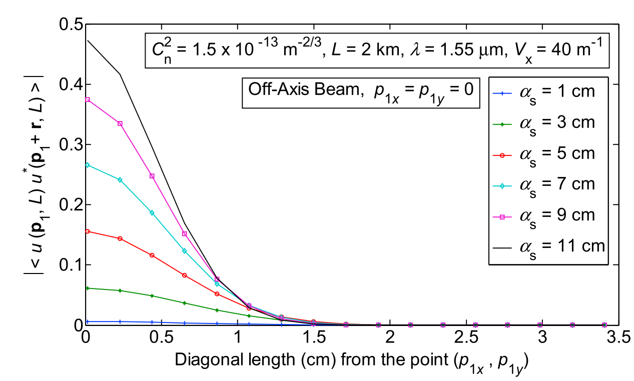

Figure 8. Field correlations for the off-axis Gaussian beam in strong atmospheric turbulence at various α_s.

Acknowledgments
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