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Abstract. Intensity fluctuations of a crossbeam are evaluated in weak atmospheric turbulence. A crossbeam is defined as two asymmetrical Gaussian beams oriented perpendicular to each other, and one of these beams is wider along the *x*-axis whereas the other beam is wider along the *y*-axis. Our results indicate that in terms of the intensity fluctuations in weak turbulence, focused crossbeams offer favorable results when compared to the corresponding focused Gaussian beam intensity fluctuations. However, for collimated crossbeams, such a comparison is in favor of the collimated Gaussian beam.
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1 Introduction

It is well known that initial optical beam profiles result in different intensity fluctuations after propagating in a turbulent atmosphere. There exist numerous results in the literature describing the basic intensity fluctuations arising due to weak turbulence.[1–10] Effects of the beam profiles on the intensity fluctuations are studied for many types of symmetrical optical beams such as flat-topped,[11] annular,[12,13] cos-Gaussian,[13] higher-order sinusoidal Gaussian and annular,[14] dark hollow,[15] laser array,[16] and Bessel Gaussian.[17] The scintillations of some asymmetrical optical beams are also scrutinized.[18–20] We have reported the review of the intensity fluctuations in turbulence for different incidences.[21] It is interesting that some beam types show favorable intensity fluctuations under certain atmospheric conditions. Thus, it is important to understand the behavior of the intensity fluctuations of any possible incidence that may be employed in an atmospheric optics links. Previously, we have studied the propagation of crossbeams in a turbulent atmosphere.[22] A crossbeam is simply defined as a beam composed of two orthogonal asymmetrical Gaussian beams where each asymmetrical Gaussian beam has a wider source size along one of the orthogonal axes at the source plane. The asymmetry factor (AF) of the individual beam composing the crossbeam is defined as the ratio of the source size along the y-axis and x-axis. In this paper, by employing the Rytov method, the scintillation index of a crossbeam at the receiver plane is formulated after it propagates through a weakly turbulent atmosphere. Our evaluations are aimed at the comparison of the intensity fluctuations of the crossbeams, having different asymmetries, among themselves, and with the Gaussian beam intensity fluctuations. The results in this paper can be used in atmospheric optics link design when a crossbeam incidence is used.

2 Formulation

As also noted above, a crossbeam is obtained by superposing two asymmetrical Gaussian beams that are oriented perpendicular to each other. One Gaussian beam is wider along the x-axis and the other Gaussian beam is wider along the y-axis. The incident field at the laser exit plane ($z = 0$) for the crossbeam is thus given by[22]

$$u_\ell^{\mathrm{inc}}(s_x, s_y, z = 0) = \sum_{\ell=1}^{2} A_\ell \exp\left[-\frac{k}{2}(\beta_{x\ell} s_x^2 + \beta_{y\ell} s_y^2)\right], \quad (1)$$

where z is the axis of propagation, (s_x, s_y) presents the transverse x- and y-coordinates at the source plane ($z = 0$), A_ℓ is the amplitude of the ℓ'th Gaussian beam with $\ell = 1, 2$, $k = 2\pi/\lambda$, λ is the wavelength, $\beta_{x\ell} = (1/k\alpha_{sx\ell}^2) + (i/F_{x\ell})$, $\beta_{y\ell} = (1/k\alpha_{sy\ell}^2) + (i/F_{y\ell})$, $i = (-1)^{0.5}$, $\alpha_{sx\ell}$ and $F_{x\ell}$ are the source size and the focal length in the x-direction, and $\alpha_{sy\ell}$ and $F_{y\ell}$ are the source size and the focal length in the y-direction.

The limiting cases of our earlier formulations for the correlation functions of general type[23] and arbitrary beams[24] in weak turbulence yield the scintillation indices, which are the measure of the intensity fluctuations, of many types of beams such as the symmetrical flat-topped, annular, sinusoidal Gaussian, dark hollow, array, Bessel Gaussian and their higher-order and asymmetrical counterparts. Thus, using the log-amplitude correlation function, B_χ given by Eq. (13) of Ref. 23, multiplying it by 4 {which can be derived by starting from Eq. (13.13) of Ref. 1 and applying the approximation $\exp[4B_\chi(L)] \cong 1 + 4B_\chi(L)$ valid in weak turbulence} and employing Eq. (1), the scintillation index of crossbeams in weak turbulence is obtained as

$$m^2 = 4B_\chi(L) = 4\pi \mathrm{Re}\left[\int_0^L \mathrm{d}\eta \int_0^\infty \kappa \mathrm{d}\kappa \int_0^{2\pi} \mathrm{d}\theta [M_1(\eta, \kappa, \theta) + M_2(\eta, \kappa, \theta)] \Phi_n(\kappa)\right], \quad (2)$$

where $\Phi_n(\kappa)$ is the spectral density of the index of refraction fluctuations which is given by $0.033 C_n^2 \kappa^{-11/3}$ for Kolmogorov turbulence, C_n^2 being the structure constant

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in horizontal link, L is the link length, Re is the real part, η is the distance parameter, $\boldsymbol{\kappa} = \kappa e^{i\theta}$ is the spatial frequency vector with $\kappa = |\boldsymbol{\kappa}| = (\kappa_x^2 + \kappa_y^2)^{1/2}$ ($\kappa_x = \kappa \cos\theta$, $\kappa_y = \kappa \sin\theta$, $\mathrm{d}^2\boldsymbol{\kappa} = \mathrm{d}\kappa_x \mathrm{d}\kappa_y = \kappa \mathrm{d}\kappa \mathrm{d}\theta$), and θ being the amplitude and phase of the spatial frequency, respectively,

$$M_1(\eta,\kappa,\theta) = \frac{N(\eta,\kappa,\theta)N(\eta,-\kappa,\theta)}{D(L)D(L)}, \tag{3}$$

$$M_2(\eta,\kappa,\theta) = \frac{N(\eta,\kappa,\theta)N^*(\eta,\kappa,\theta)}{D(L)D^*(L)}, \tag{4}$$

$$N(\eta,\kappa,\theta) = \sum_{\ell=1}^{2} A_\ell \frac{ik}{\sqrt{(1+i\beta_{x\ell}L)}\sqrt{(1+i\beta_{y\ell}L)}} \times \exp\left(\frac{b_{3x\ell}}{2}\kappa^2\cos^2\theta\right)\exp\left(\frac{b_{3y\ell}}{2}\kappa^2\sin^2\theta\right), \tag{5}$$

$$D(L) = \sum_{\ell=1}^{2} A_\ell \frac{1}{(1+i\beta_{x\ell}L)^{1/2}}\frac{1}{(1+i\beta_{y\ell}L)^{1/2}}, \tag{6}$$

$$b_{3x\ell} = \frac{i\gamma_{x\ell}(\eta - L)}{k}. \tag{7}$$

$$\gamma_{x\ell} = \frac{(1+i\beta_{x\ell}\eta)}{(1+i\beta_{x\ell}L)}. \tag{8}$$

Here $b_{3y\ell}$ and $\gamma_{y\ell}$ are found by inserting y instead of x in Eqs. (7) and (8), and * denotes the complex conjugate. To check Eq. (2) for the Gaussian beam case, first Eq. (1) is reduced to a Gaussian beam incident field by removing the summation, taking $\beta_{x\ell} = \beta_{y\ell} = \beta = (1/k\alpha_s^2) + (i/F)$ in Eq. (1), where α_s and F are the source size and the focal length of the Gaussian beam, respectively. Inserting this Gaussian beam incident field expression into the $B_\chi(L)$ given by Eq. (13) of Ref. 23 which is the formula used in Eq. (2), and integrating over κ and θ, the variance of the log-amplitude fluctuations is obtained, which reduces to the well-known variance of the log-amplitude fluctuations for the Gaussian beam given by Eqs. (18)–(29) of Ref. 4. Since the scintillation index in weak turbulence is related to the variance of the log-amplitude fluctuations by a factor of 4, in the limiting case, Eq. (2) correctly reduces to the Gaussian beam scintillation index. In Sec. 3, we present the results obtained by using Eq. (2).

3 Results

In this section, all the figures show the variations of the scintillation index of crossbeams versus $\alpha_{sx1}/(\lambda L)^{1/2}$, i.e., the source size, normalized by the Fresnel zone, in the x-direction of the first Gaussian beam that forms the crossbeam. In each figure, the corresponding α_{sy1}, α_{sx2}, α_{sy2}, and the turbulence parameters are provided. $\mathrm{AF}_1 = \alpha_{sy1}/\alpha_{sx1}$ and $\mathrm{AF}_2 = \alpha_{sy2}/\alpha_{sx2}$ are the AFs of the first and the second asymmetrical Gaussian beam composing the crossbeam. Collimated and focused crossbeams are defined as the beams having $F_{x1} = F_{y1} = F_{x2} = F_{y2} = \infty$ and $F_{x1} = F_{y1} = F_{x2} = F_{y2} = L$, respectively. Except in Figs. 2 and 5 in which a single beam is taken, in all the plots, the amplitudes of both beams of the crossbeams are taken as $A_1 = A_2 = 0.5$.

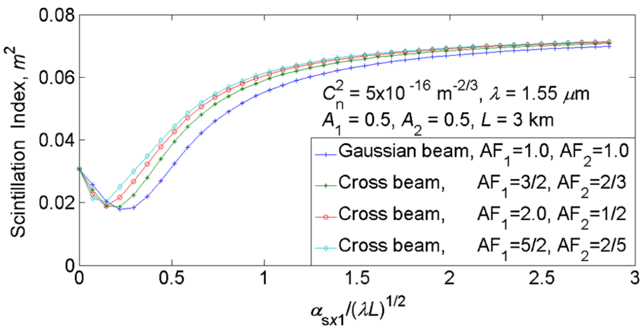

Fig. 1 Scintillation index of collimated crossbeams whose individual beams possess asymmetry factors (AFs) that are inverse of each other.

In Fig. 1, collimated crossbeams whose individual beams possess AFs that are inverse of each other, i.e., $\mathrm{AF}_1 = 1/\mathrm{AF}_2$, are chosen. Note that when $\mathrm{AF}_1 = \mathrm{AF}_2 = 1$, the collimated crossbeam reduces to the Gaussian beam and the corresponding curve in Fig. 1 correctly represents the collimated Gaussian beam intensity fluctuations in weak turbulence. It is observed in Fig. 1 that the variations of intensity fluctuations of the collimated crossbeams versus $\alpha_{sx1}/(\lambda L)^{1/2}$ are similar when compared to the variations of intensity fluctuations of the collimated Gaussian beam. This means, as implied by the Rytov solution, an increase in the source size first reduces the intensity fluctuations of

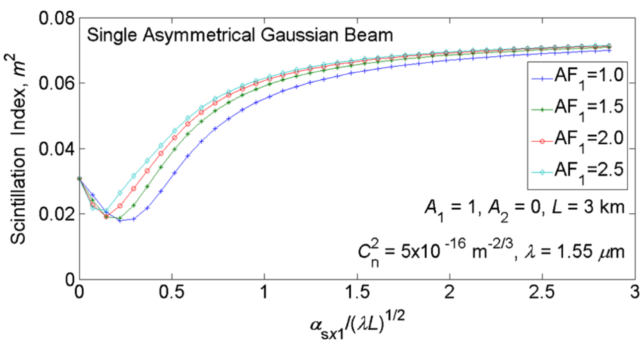

Fig. 2 Scintillation index of collimated asymmetrical Gaussian beams.

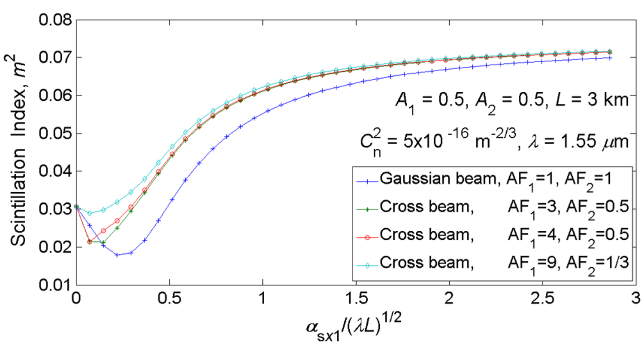

Fig. 3 Scintillation index of collimated crossbeams whose individual beams possess independent AFs.

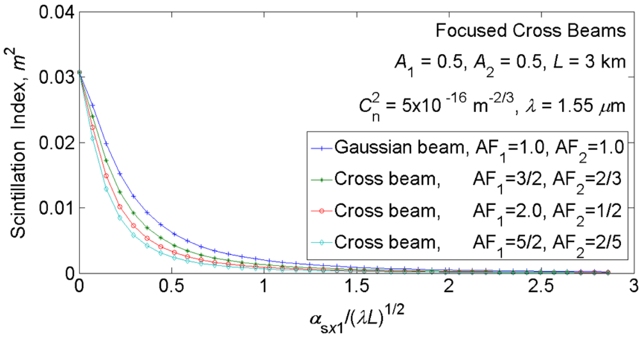

Fig. 4 Scintillation index of focused crossbeams whose individual beams possess AFs that are inverse of each other.

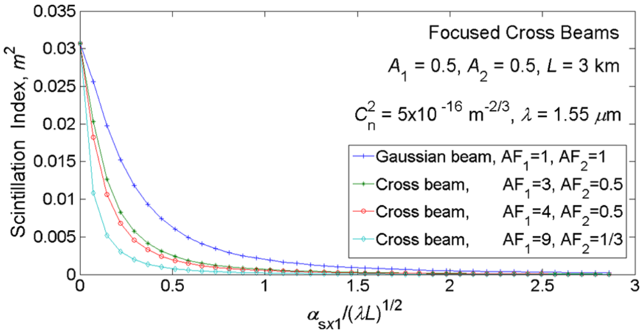

Fig. 6 Scintillation index of focused crossbeams whose individual beams possess independent AFs.

the collimated crossbeams, but after a dip is reached, the intensity fluctuations of the collimated crossbeams start to increase until the plane wave intensity fluctuations are reached, after which the scintillations stay at the same value. For all $\alpha_{sx1}/(\lambda L)^{1/2}$ larger than 0.25, the collimated crossbeam intensity fluctuations are larger than the collimated Gaussian beam intensity fluctuations. To explain this, we consider the beam wander, which is one factor influencing the scintillations. The collimated crossbeam composed of two beams is expected to exhibit larger beam wander as compared to a single collimated Gaussian beam, which results in the larger scintillations for the collimated crossbeams. The collimated crossbeams having larger asymmetry, i.e., having larger AF_1, and thus smaller AF_2, exhibit larger intensity fluctuations. Another observation from Fig. 1 is that at the very small and very large source sizes, similar to the collimated Gaussian beam, all the collimated crossbeams attain the same spherical and plane wave intensity fluctuations. In Fig. 2, we report the intensity fluctuations of single collimated asymmetrical Gaussian beam, i.e., collimated crossbeams having only one collimated Gaussian beam component. It is seen from Fig. 2 that the beams having larger asymmetry show larger intensity fluctuations when the source size is large [$\alpha_{sx1} > 0.25(\lambda L)^{1/2}$], but when the source size is small to satisfy $\alpha_{sx1} < 0.25(\lambda L)^{1/2}$, the situation flips and the beams with larger AF have lower scintillation indices. In Fig. 3, we repeat Fig. 1, but this time without taking the AFs of the individual beams as inverse of each other, i.e., the asymmetry of each individual beam is independent of each other. The results obtained from Fig. 3 are the same as the results obtained from Fig. 1.

Figures 4–6 are drawn by employing the same parameters used in Figs. 1–3, respectively, except the crossbeams are taken to be focused. In Fig. 4, AFs of the individual beams of the focused crossbeams are chosen to be inverse of each other, i.e., $\mathrm{AF}_1 = 1/\mathrm{AF}_2$. In Fig. 4, it is seen that the variations of intensity fluctuations of the focused crossbeams versus $\alpha_{sx1}/(\lambda L)^{1/2}$ are similar when compared to the variations of intensity fluctuations of the focused Gaussian beam. For the focused Gaussian and focused crossbeams, increase in the source size monotonically reduces the intensity fluctuations until the focused plane wave intensity fluctuation values are reached; eventually the scintillations merge to zero. For all $\alpha_{sx1}/(\lambda L)^{1/2}$, the focused crossbeam intensity fluctuations are smaller than the focused Gaussian beam intensity fluctuations. Beam wander effect in shaping the scintillations is more important in the focused case. When a single focused Gaussian beam is considered, the beam wander is more critical in shaping the scintillations as compared to a focused crossbeam composed of two focused beams. For the focused crossbeam, beam wandering effect on the scintillation index for one of the beams could be compensated for by the beam wandering effect on the other beam. This results in a smaller overall beam wandering effect, and thus a smaller scintillation index is obtained for the focused crossbeam when compared to the focused Gaussian beam scintillations. Intensity fluctuations of the focused crossbeams that have larger asymmetry, i.e., larger AF_1, thus smaller AF_2, are smaller. The intensity fluctuations of a single focused asymmetrical Gaussian beam are shown in Fig. 5; it is seen that the larger asymmetry beams exhibit smaller intensity fluctuations. Figure 6 has the same parameters as in Fig. 4, except that the asymmetry of each

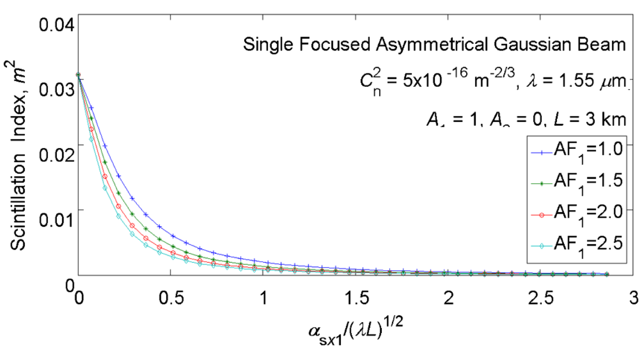

Fig. 5 Scintillation index of focused asymmetrical Gaussian beams.

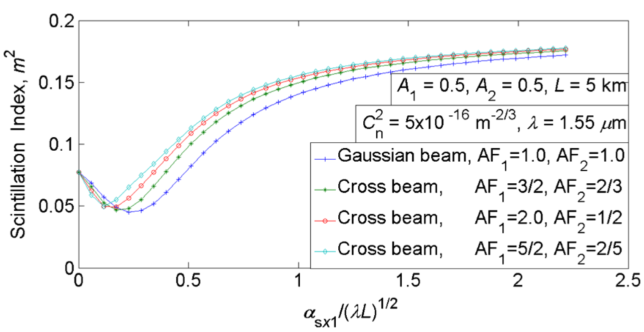

Fig. 7 Scintillation index, evaluated at a larger link length than in Fig. 1, of collimated crossbeams whose individual beams possess AFs that are inverse of each other.

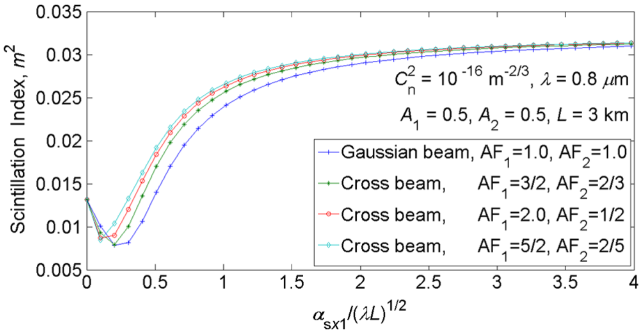

Fig. 8 Scintillation index, evaluated at smaller structure constant and wavelength than in Fig. 1, of collimated crossbeams whose individual beams possess AFs that are inverse of each other.

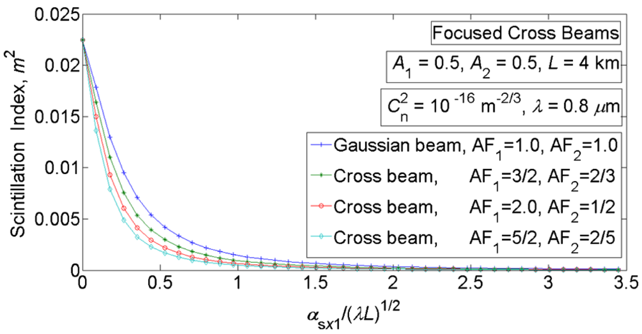

Fig. 9 Scintillation index, evaluated at smaller structure constant, wavelength, and larger link length than in Fig. 4, of focused crossbeams whose individual beams possess AFs that are inverse of each other.

individual beam is taken to be independent of each other. The results obtained from Fig. 6 follow the same trends of the results found in Fig. 4.

In Figs. 7 and 8, Fig. 1 is repeated for the same collimated crossbeams but at larger link length and at smaller structure constant, wavelength, respectively. Results obtained from Figs. 7 and 8 indicate that the change in the turbulence parameters naturally changes the scintillation index, however, it does not vary the trend in the intensity fluctuations of the crossbeams. Finally, Fig. 9 is presented to observe the effects of the turbulence parameters on the intensity fluctuations of the focused crossbeams. It is observed that the variations in the turbulence parameters do not change the trend in the intensity fluctuations of the focused crossbeams.

4 Conclusion

The intensity fluctuations of crossbeam incidences are investigated in weakly turbulent atmosphere by using the Rytov solution. In weak turbulence, the crossbeam is found to exhibit a similar intensity fluctuations trend as the Gaussian beam intensity fluctuations. Especially when the source size is larger than the Fresnel zone, the intensity fluctuations of the collimated crossbeams attain larger values as the asymmetry of the collimated crossbeam increases. For the focused crossbeams, however, for all the source sizes, the intensity fluctuates less as the focused crossbeam possesses higher asymmetries. Our results presented in this paper can be utilized in atmospheric optics, atmospheric imaging, and adaptive optics applications when crossbeam incidences are employed. In such applications, compared to the case when focused Gaussian beams are employed, the use of the focused asymmetrical crossbeams will serve to minimize the effects of weak turbulence. Higher asymmetry in the focused crossbeam will further improve the scintillation performance in these applications.

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