Entropy **2014**, *16*, 6254-6262; doi:10.3390/e16126254

OPEN ACCESS
entropy
ISSN 1099-4300
www.mdpi.com/journal/entropy

Article

On a Local Fractional Wave Equation under Fixed Entropy Arising in Fractal Hydrodynamics

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External Editor: J. A. Tenreiro Machado

Received: 20 October 2014; in revised form: 20 November 2014 / Accepted: 24 November 2014 / Published: 28 November 2014

Abstract: In this paper, based on fixed entropy, the adiabatic equation of state in fractal flow is discussed. The local fractional wave equation for the velocity potential is also obtained by using the non-differential perturbations for the pressure and density of fractal hydrodynamics.

Keywords: wave equations; entropy; fractals; local fractional vector calculus; hydrodynamics

1. Introduction

The classical wave equation [1] is an important partial differential equation in the fields of acoustics, gravitation, chemistry, geophysics, electromagnetism and fluid dynamics. There are some physical wave types with examples of occurrence, such as acoustic, chemical, electromagnetic, gravitational, seismic, traffic flow and water waves. The description of the propagation of acoustic waves in the medium have been widely investigated; see e.g., [2,3]. The theory of the chemical waves was developed in [4,5]. The electromagnetic waves were reported by Rice [6] and Smith *et al.* [7]. The mathematical models of gravitational waves can be seen in [8,9], and the theory of seismic waves in geophysics was addressed in [10,11]. The kinematic waves of traffic flow were analyzed in [12–14], and the wave propagation analysis in water waves was reported in [15–17].

Recently, the theory of fractional calculus has successfully found many applications in the areas of physics and engineering [18,19]. The fractional wave equation was utilized to describe some crucial characteristics of the wave equation for a vibrating string with a differentiable term; see [20–24] and the references therein. The field equations for fractal media were considered to describe the hydrodynamic [25], electromagnetic [26] and magneto-hydrodynamic [27] behaviors of flows and the generalized Navier–Stokes equations for non-integer dimensional space [28]; also, see [29–31].

The framework of continuum mechanics of fractal media defined on Cantor sets was first proposed in references [32,33]. Differing from the former, the local fractional vector calculus was used to investigate the partial differential equations arising in mathematical physics [34–37]. For a review of this topic, we refer the reader to the results [38–41]. The local fractional wave equation was applied to present the fractal characteristics of the wave equation for a vibrating string with the non-differentiable terms [42–44] and the fractal waves on shallow water surfaces [45]. The aim of the present paper is to study linear and nonlinear wave equations under fixed entropy arising in hydrodynamics. This article is structured as follows. In Section 2, the basic theory of local fractional vector calculus is introduced. In Section 3, the local fractional linear wave equation in hydrodynamics is presented. In Section 4, the local fractional nonlinear wave equation is suggested. Finally, Section 5 is devoted to the main conclusions.

2. Local Fractional Vector Calculus

In this section, some definitions of local fractional vector derivative and integrals [26–28], which appear in the manuscript, are introduced.

Local fractional gradient of the scale function φ is [36]:

$$\nabla^{\alpha}\varphi=\lim_{dV^{(\gamma)}\to 0}\left(\frac{1}{dV^{(\gamma)}}\oiint_{S^{(\beta)}}\varphi d\mathbf{S}^{(\beta)}\right), \tag{1}$$

where $S^{(\beta)}$ is its bounding fractal surface and V is a small fractal volume enclosing P, and the local fractional surface integral is given by [34–37]:

$$\iint u\left(r_P\right)d\mathbf{S}^{(\beta)}=\lim_{N\to\infty}\sum_{P=1}^{N}u\left(r_P\right)\mathbf{n}_P\Delta S_P^{(\beta)}, \tag{2}$$

with N elements of area with a unit normal local fractional vector n_P, $\Delta S_P^{(\beta)}\to 0$ as $N\to\infty$ for $\gamma=\frac{3}{2}\beta=3\alpha$.

The local fractional divergence of the vector function $\mathbf{u}$ is defined through [36]:

$$\nabla^{\alpha}\bullet\mathbf{u}=\lim_{dV^{(\gamma)}\to 0}\left(\frac{1}{dV^{(\gamma)}}\oiint_{S^{(\beta)}}\mathbf{u}\bullet d\mathbf{S}^{(\beta)}\right), \tag{3}$$

where the local fractional surface integral is given by [34–36]:

$$\iint \mathbf{u}\left(r_P\right)\cdot d\mathbf{S}^{(\beta)}=\lim_{N\to\infty}\sum_{P=1}^{N}\mathbf{u}\left(r_P\right)\cdot\mathbf{n}_P\Delta S_P^{(\beta)}, \tag{4}$$

with N elements of area with a unit normal local fractional vector n_P, $\Delta S_P^{(\beta)}\to 0$ as $N\to\infty$ for $\gamma=\frac{3}{2}\beta=3\alpha$.

3. The Local Fractional Linear Wave Equation

In this section, we discuss the local fractional linear wave equation under the fixed entropy. Let us consider the hydrodynamic equations arising in an ideal fluid.

The equation for the conservation of mass in fractal flow reads [36,37]:

$$\frac{\partial^\alpha \rho}{\partial t^\alpha}+\nabla^\alpha\cdot\sigma=0, \tag{5}$$

where the fractal density denotes $\rho\left(x,t\right)$ and the fractal flux denotes $\sigma\left(x,t\right)=\rho\left(x,t\right)\upsilon\left(x,t\right)$.

Cauchy's equation of motion of flows on Cantor sets was suggested as: [37]

$$\rho\frac{D^\alpha\upsilon}{Dt^\alpha}=-\nabla^\alpha p\left(\rho\right)+\rho b, \tag{6}$$

where $\frac{D^\alpha\upsilon}{Dt^\alpha}=\frac{\partial^\alpha\upsilon}{\partial t^\alpha}+\upsilon\cdot\nabla^\alpha\upsilon$.

From (6), making $b=0$ and $\nabla^\alpha\upsilon=0$, the linear local fractional Euler equation arising in hydrodynamics is suggested as:

$$\rho\frac{\partial^\alpha\upsilon}{\partial t^\alpha}+\nabla^\alpha p\left(\rho\right)=0. \tag{7}$$

We present the small perturbations for the pressure and density of hydrodynamics, namely,

$$p=p_0+\tilde{p}, \tag{8}$$

$$\rho=\rho_0+\tilde{\rho}. \tag{9}$$

Using (5), (7), (8) and (9), we obtain:

$$\frac{\partial^\alpha\tilde{\rho}}{\partial t^\alpha}+\nabla^\alpha\cdot\sigma=0, \tag{10}$$

and

$$\rho_0\frac{\partial^\alpha\upsilon}{\partial t^\alpha}+\nabla^\alpha\tilde{p}\left(\rho\right)=0, \tag{11}$$

where the adiabatic equation of state in fractal flow is given as:

$$p\left(r,t\right)=p_0+\frac{\tilde{\rho}^\alpha\left(r,t\right)}{\Gamma\left(1+\alpha\right)}\left[\frac{\partial^\alpha p}{\partial\rho^\alpha}\right]_s+O\left(\rho^\alpha\right), \tag{12}$$

with the constant entropy s.

Define the quantity:

$$c^2=\left[\frac{\partial^\alpha p}{\partial\rho^\alpha}\right]_s, \tag{13}$$

then from (12), we have:

$$p\left(r,t\right)=p_{0}+\frac{\tilde{\rho}^{\alpha}\left(r,t\right)}{\Gamma\left(1+\alpha\right)}c^{2}+O\left(\rho^{\alpha}\right). \tag{14}$$

Hence, the linear equations in fractal flow reads as:

$$\frac{\partial^{\alpha}\tilde{\rho}}{\partial t^{\alpha}}+\rho_{0}\nabla^{\alpha}\cdot\upsilon=0, \tag{15}$$

$$\rho_{0}\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}}+\nabla^{\alpha}\tilde{p}\left(\rho\right)=0, \tag{16}$$

$$\tilde{p}=\tilde{\rho}c^{2}. \tag{17}$$

Following (15) and (16), we obtain:

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial^{\alpha}\tilde{\rho}}{\partial t^{\alpha}}+\rho_{0}\nabla^{\alpha}\cdot\upsilon\right)=\nabla^{\alpha}\cdot\left(\rho_{0}\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}}+\nabla^{\alpha}\tilde{p}\left(\rho\right)\right), \tag{18}$$

which leads to:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}}=\nabla^{2\alpha}\tilde{p}\left(\rho\right), \tag{19}$$

where $\nabla^{2\alpha}=\nabla^{\alpha}\cdot\nabla^{\alpha}$ [36,37].

Submitting (17) into (19) gives the local fractional wave equation:

$$\nabla^{2\alpha}\tilde{p}\left(\rho\right)-\frac{1}{c^{2}}\frac{\partial^{2\alpha}\tilde{p}}{\partial t^{2\alpha}}=0. \tag{20}$$

If the density ρ is not independent of t, then we obtain the local fractional wave equation:

$$\rho\nabla^{\alpha}\cdot\left(\frac{1}{\rho}\nabla^{\alpha}p\right)-\frac{1}{c^{2}}\frac{\partial^{2\alpha}p}{\partial t^{2\alpha}}=0. \tag{21}$$

The local fractional wave equation for the particle velocity also is suggested as:

$$\frac{1}{\rho}\nabla^{\alpha}\left(c^{2}\rho\nabla^{\alpha}\cdot\upsilon\right)-\frac{\partial^{2\alpha}\upsilon}{\partial t^{2\alpha}}=0, \tag{22}$$

or

$$\nabla^{\alpha}\left(c^{2}\nabla^{\alpha}\cdot\upsilon\right)-\frac{\partial^{2\alpha}\upsilon}{\partial t^{2\alpha}}=0. \tag{23}$$

Define the local fractional velocity potential:

$$\upsilon=\nabla^{\alpha}\varphi. \tag{24}$$

Submitting (24) into (23), we obtain:

$$\nabla^{\alpha}\left(c^{2}\nabla^{2\alpha}\varphi-\frac{\partial^{2\alpha}\varphi}{\partial t^{2\alpha}}\right)=0, \tag{25}$$

which reduces to the local fractional wave equation for the velocity potential:

$$c^{2}\nabla^{2\alpha}\varphi-\frac{\partial^{2\alpha}\varphi}{\partial t^{2\alpha}}=0. \tag{26}$$

For a semi-infinite fractal string with a fixed end, the local fractional linear wave equation in the one-dimensional case can be written as [43,44]:

$$\frac{\partial^{2\alpha}\varphi}{\partial x^{2\alpha}} - \frac{\partial^{2\alpha}\varphi}{\partial t^{2\alpha}} = 0, \tag{27}$$

with the initial value conditions:

$$\varphi(0,t) = \gamma(t), \quad \frac{\partial^{\alpha}\varphi}{\partial t^{\alpha}} = \kappa(t), \tag{28}$$

and the boundary value conditions:

$$u(x,0) = f(x), \quad \frac{\partial^{\alpha}u(x,0)}{\partial x^{\alpha}} = g(x). \tag{29}$$

4. The Local Fractional Nonlinear Wave Equation

In this section, we present the local fractional nonlinear wave equation under fixed entropy arising in fractal hydrodynamics.

In view of (6), making $b = 0$ the local fractional Euler equation is presented as:

$$\rho\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}} + \nabla^{\alpha}p(\rho) + \rho\upsilon\cdot\nabla^{\alpha}\upsilon = 0. \tag{30}$$

The adiabatic equation of state reads as:

$$\begin{aligned} &p(r,t) \\ &= p_0 + \tfrac{\tilde{\rho}^{\alpha}(r,t)}{\Gamma(1+\alpha)}\left[\tfrac{\partial^{\alpha}p}{\partial\rho^{\alpha}}\right]_s + \tfrac{\tilde{\rho}^{2\alpha}(r,t)}{\Gamma(1+2\alpha)}\left[\tfrac{\partial^{2\alpha}p}{\partial\rho^{2\alpha}}\right]_s + O\left(\rho^{2\alpha}\right) \\ &= p_0 + \tfrac{\tilde{\rho}^{\alpha}(r,t)}{\Gamma(1+\alpha)}c^2 + \tfrac{2c\tilde{\rho}^{2\alpha}(r,t)}{\Gamma(1+2\alpha)}\left[\tfrac{\partial^{\alpha}c}{\partial\rho^{\alpha}}\right]_s + O\left(\rho^{2\alpha}\right) \\ &\approx p_0 + \tfrac{\tilde{\rho}^{\alpha}}{\Gamma(1+\alpha)}c^2, \end{aligned} \tag{31}$$

where s is the fixed entropy, and the speed of fractal sound in an ideal fluid is:

$$c = \sqrt{\left[\frac{\partial^{\alpha}p}{\partial\rho^{\alpha}}\right]_s}. \tag{32}$$

From (6) and (30), we get:

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left[\frac{\partial^{\alpha}\tilde{\rho}}{\partial t^{\alpha}} + \nabla^{\alpha}\cdot(\rho\upsilon)\right] = \nabla^{\alpha}\cdot\left(\rho\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}} + \nabla^{\alpha}p(\rho) + \rho\upsilon\cdot\nabla^{\alpha}\upsilon\right), \tag{33}$$

which refers to:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} + \nabla^{\alpha}\cdot\left[\frac{\partial^{\alpha}(\rho\upsilon)}{\partial t^{\alpha}}\right] = \nabla^{\alpha}\cdot\left(\rho\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}} + \nabla^{\alpha}p(\rho) + \rho\upsilon\cdot\nabla^{\alpha}\upsilon\right), \tag{34}$$

Making use of (34), we give:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} + \nabla^{\alpha}\cdot\left(\frac{\rho\partial^{\alpha}\upsilon}{\partial t^{\alpha}} + \frac{\upsilon\partial^{\alpha}\rho}{\partial t^{\alpha}}\right) = \nabla^{\alpha}\cdot\left(\rho\frac{\partial^{\alpha}\upsilon}{\partial t^{\alpha}} + \nabla^{\alpha}p(\rho) + \rho\upsilon\cdot\nabla^{\alpha}\upsilon\right), \tag{35}$$

which leads to:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} + \nabla^{\alpha} \cdot \left[\frac{\upsilon\partial^{\alpha}\rho}{\partial t^{\alpha}}\right] = \nabla^{\alpha} \cdot \left(\nabla^{\alpha} p\left(\rho\right) + \rho\upsilon \cdot \nabla^{\alpha}\upsilon\right). \tag{36}$$

Setting:

$$\nabla^{\alpha} \cdot \left[\frac{\upsilon\partial^{\alpha}\rho}{\partial t^{\alpha}}\right] = 0, \tag{37}$$

from (36), we obtain:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} = \nabla^{\alpha} \cdot \left(\nabla^{\alpha} p\left(\rho\right) + \rho\upsilon \cdot \nabla^{\alpha}\upsilon\right). \tag{38}$$

In view of (5), we write (38) as:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} = \nabla^{2\alpha} p + \nabla^{2\alpha}\left(\rho\upsilon \cdot \upsilon\right). \tag{39}$$

Using (31), we have:

$$\nabla^{2\alpha} p = \nabla^{2\alpha}\left(p_0 + \frac{\tilde{\rho}^{\alpha}}{\Gamma\left(1+\alpha\right)}c^2\right), \tag{40}$$

which leads to:

$$\nabla^{2\alpha} p = \nabla^{2\alpha}\left(\frac{c^2}{\Gamma\left(1+\alpha\right)}\tilde{\rho}^{\alpha}\right). \tag{41}$$

In view of (39), we write:

$$\nabla^{2\alpha}\left(\rho\upsilon \cdot \upsilon\right) \approx \frac{1}{\rho_0}\nabla^{2\alpha} c^2\left(\tilde{\rho}^{2\alpha}\right). \tag{42}$$

From (39), we obtain:

$$\frac{\partial^{2\alpha}\tilde{\rho}}{\partial t^{2\alpha}} = \nabla^{2\alpha}\left(\frac{c^2}{\Gamma\left(1+\alpha\right)}\tilde{\rho}^{\alpha}\right) + \frac{1}{\rho_0}\nabla^{2\alpha} c^2\left(\tilde{\rho}^{2\alpha}\right). \tag{43}$$

Making $\phi = \frac{\tilde{\rho}^{\alpha}}{\rho_0}$, from (39), we have:

$$\rho_0\frac{\partial^{2\alpha}\phi}{\partial t^{2\alpha}} = \rho_0\nabla^{2\alpha} c^2\left(\frac{\phi}{\Gamma\left(1+\alpha\right)} + \phi^2\right), \tag{44}$$

which reduces to the local fractional nonlinear wave equation:

$$\frac{\partial^{2\alpha}\phi}{\partial t^{2\alpha}} = \nabla^{2\alpha} c^2\left(\frac{\phi}{\Gamma\left(1+\alpha\right)} + \phi^2\right). \tag{45}$$

If the fractal dimension α is one, then (45) can be written as: [46]

$$\frac{\partial^{2}\phi}{\partial t^{2}} = \nabla^{2} c^2\left(\phi + \phi^2\right). \tag{46}$$

The local fractional nonlinear wave equation (45) can be written as:

$$\frac{\partial^{2\alpha}\phi}{\partial t^{2\alpha}} = \eta\nabla^{2\alpha}\phi + \mu\phi\nabla^{2\alpha}\phi, \tag{47}$$

where $\eta = \frac{c^2}{\Gamma(1+\alpha)}$ and $\mu = 2c^2$.

Following (47), we have the local fractional nonlinear wave equation in the one-dimensional case:

$$\frac{\partial^{2\alpha}\phi}{\partial t^{2\alpha}} = \eta\frac{\partial^{2\alpha}\phi}{\partial x^{2\alpha}} + \mu\phi\frac{\partial^{2\alpha}\phi}{\partial x^{2\alpha}} \tag{48}$$

subject to the initial-boundary value conditions:

$$\phi(0,t) = \gamma(t), \quad \frac{\partial^{\alpha}\phi}{\partial t^{\alpha}} = \kappa(t), \tag{49}$$

$$\phi(x,0) = f(x), \quad \frac{\partial^{\alpha}\phi(x,0)}{\partial x^{\alpha}} = g(x). \tag{50}$$

5. Conclusions

In this work, the linear and nonlinear wave equations in hydrodynamics via the local fractional vector calculus are derived using the non-differential perturbations for the pressure and density of the fractal hydrodynamics; both the local fractional linear and nonlinear wave equations for the velocity potential under fixed entropy are obtained, and the local fractional linear and nonlinear wave equations in the one-dimensional case are also discussed.

Acknowledgments

The authors would like to thank the referees for their useful comments and remarks.

Author Contributions

All authors contributed equally to the computation and intepretation of this work. All authors have read and approved the final manuscript.

Conflict of Interest

The authors declare no conflict of interest.

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