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Common fixed points of generalized Meir-Keeler α-contractions

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Abstract

Motivated by Abdeljawad (Fixed Point Theory Appl. 2013:19, 2013), we establish some common fixed point theorems for three and four self-mappings satisfying generalized Meir-Keeler α-contraction in metric spaces. As a consequence, the results of Rao and Rao (Indian J. Pure Appl. Math. 16(1):1249-1262, 1985), Jungck (Int. J. Math. Math. Sci. 9(4):771-779, 1986), and Abdeljawad itself are generalized, extended and improved. Sufficient examples are given to support our main results.

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1 Introduction and preliminaries

The Meir-Keeler contractive condition [1] is one of the interesting aspects to study metrical fixed point theory, that is, for given $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\epsilon \leq d(x,y) < \epsilon + \delta \quad \Rightarrow \quad d(fx,fy) < \epsilon. \tag{1}$$

This contraction has further been generalized and studied by various authors (see [2–15]). Very recently, Abdeljawad [16] (see also [17]) established some fixed point results for α-contractive-type maps (due to Samet *et al.* [18]) to Meir-Keeler versions for single and a pair of maps. In this article, we prove some common fixed point theorems for three and four self-mappings satisfying generalized Meir-Keeler α-contractions. Thus, we provide an affirmative answer to the question of Abdeljawad (see [16], Remark 17).

Let us recall some definitions, which we will use in our main results.

Definition 1.1 (*cf.* [16, 18]) Let $f,g : X \to X$ be self-mappings of a set X, and let $\alpha : X \times X \to [0,\infty)$ be a mapping, then the mapping f is called α-admissible if

$$x,y \in X, \quad \alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha(fx,fy) \geq 1,$$

and the pair (f,g) is called α-admissible if

$$x,y \in X, \quad \alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha(fx,gy) \geq 1 \quad \text{and} \quad \alpha(gx,fy) \geq 1.$$

Definition 1.2 (*cf.* [19, 20]) Let f and g ($f \neq g$) be two self-mappings defined on a metric space (X,d), then f is called g-absorbing if there exists some real number $R > 0$ such that

$d(gx, gfx) \leq Rd(fx, gx)$ for all x in X. Analogously, g will be called f-absorbing if there exists some real number $R > 0$ such that $d(fx, fgx) \leq Rd(fx, gx)$ for all x in X. The pair of self-maps (f, g) will be called absorbing if it is both g-absorbing as well as f-absorbing. In particular, if we take g to be the identity map on X, then f is trivially I-absorbing. Similarly, I is also f-absorbing in respect to f.

Definition 1.3 (*cf.* [21]) Two self-mappings f and g of a metric space (X, d) are called reciprocally continuous if and only if $fgx_n \to ft$ and $gfx_n \to gt$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

2 Main results

We begin with the following definitions.

Definition 2.1 Let $f, g, T : X \to X$ be three self-mappings of a non-empty set X, and let $\alpha : T(X) \times T(X) \to [0, \infty)$ be a mapping, then the pair (f, g) is called α-admissible with respect to T (in short, (f, g) is α_T-admissible) if for all $x, y \in X$,

$$\alpha(Tx, Ty) \geq 1 \quad \text{implies that} \quad \alpha(fx, gy) \geq 1 \quad \text{and} \quad \alpha(gx, fy) \geq 1. \tag{2}$$

Definition 2.2 Let $f, g, S, T : X \to X$ be four self-mappings of a non-empty set X, and let $\alpha : S(X) \cup T(X) \times S(X) \cup T(X) \to [0, \infty)$ be a mapping, then the pair (f, g) is called α-admissible with respect to S and T (in short, (f, g) is $\alpha_{S,T}$-admissible) if for all $x, y \in X$,

$$\begin{aligned} &\alpha(Sx, Ty) \geq 1 \quad \text{or} \quad \alpha(Tx, Sy) \geq 1 \\ &\quad \text{implies that} \quad \alpha(fx, gy) \geq 1 \quad \text{and} \quad \alpha(gx, fy) \geq 1. \end{aligned} \tag{3}$$

Clearly, if $S = T = I$ (identity map), then the definitions above imply Definition 1.1.

In order to extend and improve the result contained in [16] for three self-mappings, we now introduce the concept of generalized Meir-Keeler α_T-contractive mappings as follows.

Definition 2.3 Let (X, d) be a metric space, and $f, g, T : X \to X$ are self-mappings. Then we say that the pair (f, g) is a generalized Meir-Keeler α_T-contractive pair of type m_3 (M_3, respectively) if given an $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\begin{aligned} &\epsilon \leq m_3(x, y) \ \big(M_3(x, y), \text{ respectively}\big) < \epsilon + \delta \\ &\quad \text{implies that} \quad \alpha(Tx, Ty) d(fx, gy) < \epsilon, \end{aligned} \tag{4}$$

where

$$m_3(x, y) = \max\left\{ d(Tx, Ty), \frac{1}{2}\big[d(Tx, fx) + d(Ty, gy)\big], \frac{1}{2}\big[d(Tx, gy) + d(Ty, fx)\big] \right\},$$

and

$$M_3(x, y) = \max\left\{ d(Tx, Ty), d(Tx, fx), d(Ty, gy), \frac{1}{2}\big[d(Tx, gy) + d(Ty, fx)\big] \right\}.$$

Definition 2.4 Let f, g, and T be three self-mappings on a metric space (X,d) such that $f(X) \cup g(X) \subseteq T(X)$. If for a point $x_0 \in X$, there exists a sequence $\{x_n\}$ such that $Tx_{2n+1} = fx_{2n}$, $Tx_{2n+2} = gx_{2n+1}$, $n = 0,1,2,\ldots$, then $\mathcal{O}(f,g,T,x_0) = \{Tx_n : n = 1,2,\ldots\}$ is called the orbit for (f,g,T) at x_0. The space (X,d) is called (f,g,T)-orbitally complete at x_0 iff every Cauchy sequence in $\mathcal{O}(f,g,T,x_0)$ converges to a point in X. X is called (f,g,T)-orbitally complete if it is so at every $x \in X$.

Our first result is the following.

Theorem 2.1 *Let (X,d) be an (f,g,T)-orbitally complete metric space. Suppose that (f,g) is generalized Meir-Keeler α_T-contractive pair of type m_3 and satisfies the following conditions*:

(i) *(f,g) is α_T-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(Tx_0,fx_0) \geq 1$*;
(iii) *on the (f,g,T)-orbit of x_0, we have $\alpha(Tx_n,Tx_j) \geq 1$ for all n even and $j > n$ odd.*

Then $\{Tx_n\}$ is a Cauchy sequence. Moreover, if

(iv) *$\alpha(Tx_n,Tx_{n+1}) \geq 1$ for all n, and $Tx_n \to x$ implies that $\alpha(Tx_n,Tx) \geq 1$ for all n*;
(v) *one of the pairs (f,T) and (g,T) is absorbing as well as reciprocal continuous.*

Then f, g, and T have a common fixed point.

Proof Let $x_0 \in X$ such that $\alpha(Tx_0,fx_0) \geq 1$. Define the sequences $\{x_n\}$ and $\{Tx_n\}$ in X given by the rule

$$Tx_{2n+1} = fx_{2n}, \qquad Tx_{2n+2} = gx_{2n+1}, \quad n = 0,1,2,\ldots.$$

Since (f,g) is α_T-admissible, we have

$$\alpha(Tx_0,fx_0) = \alpha(Tx_0,Tx_1) \geq 1 \quad \Longrightarrow \quad \alpha(fx_0,gx_1) \geq 1 \quad \text{and} \quad \alpha(gx_0,fx_1) \geq 1,$$

which gives

$$\alpha(Tx_1,Tx_2) \geq 1.$$

Again by (i), we have

$$\alpha(Tx_1,Tx_2) \geq 1 \quad \Longrightarrow \quad \alpha(fx_1,gx_2) \geq 1 \quad \text{and} \quad \alpha(gx_1,fx_2) \geq 1,$$

which gives

$$\alpha(Tx_2,Tx_3) \geq 1.$$

Inductively, we have

$$\alpha(Tx_n,Tx_{n+1}) \geq 1, \quad n = 0,1,2,\ldots. \tag{5}$$

The fact that (f,g) is generalized Meir-Keeler α_T-contractive implies that

$$\alpha(Tx,Ty)d(fx,fy) < m_3(x,y) \quad \text{for each } x,y \in X, x \neq y. \tag{6}$$

Now, to obtain a common fixed point of f, g, and T, we take the following steps.

Step 1: We show that there exists a point $z \in X$ such that $Tx_n \to z$ as $n \to \infty$. For this, first, we claim that $\{Tx_n\}$ is a Cauchy sequence. Two cases arise: either $Tx_n = Tx_{n+1}$ for some n or $Tx_n \neq Tx_{n+1}$ for each n.

Case I: Suppose that $Tx_n = Tx_{n+1}$ for some n. We first assume that n is even, *i.e.*, $Tx_{2m} = Tx_{2m+1}$ but $Tx_{2m+1} \neq Tx_{2m+2}$, then by (6),

$$\begin{aligned} d(Tx_{2m+1}, Tx_{2m+2}) &= d(fx_{2m}, gx_{2m+1}) \\ &\leq \alpha(Tx_{2m}, Tx_{2m+1}) d(fx_{2m}, gx_{2m+1}) \\ &< \max\biggl\{ d(Tx_{2m}, Tx_{2m+1}), \frac{1}{2}\bigl[d(Tx_{2m}, fx_{2m}) + d(Tx_{2m+1}, gx_{2m+1})\bigr], \\ &\quad \frac{1}{2}\bigl[d(Tx_{2m}, gx_{2m+1}) + d(Tx_{2m+1}, fx_{2m})\bigr]\biggr\} \\ &= \max\biggl\{0, \frac{1}{2} d(Tx_{2m+1}, Tx_{2m+2}), \frac{1}{2} d(Tx_{2m}, Tx_{2m+2})\biggr\} \\ &= \frac{1}{2} d(Tx_{2m+1}, Tx_{2m+2}), \end{aligned}$$

which is a contradiction. Hence $Tx_{2m+1} = Tx_{2m+2}$. By proceeding in this way, we obtain $Tx_{2m+k} = Tx_{2m}$ for all $k \in \mathcal{N}$. Similar is the case when n is odd. Thus, we conclude that $\{Tx_n\}$ is a Cauchy sequence.

Case II: Suppose that $Tx_n \neq Tx_{n+1}$ for all integers n. Applying (6), we have

$$\begin{aligned} d(Tx_{2n}, Tx_{2n+1}) &= d(gx_{2n-1}, fx_{2n}) \\ &\leq \alpha(Tx_{2n}, Tx_{2n-1}) d(fx_{2n}, gx_{2n-1}) \\ &< \max\biggl\{ d(Tx_{2n}, Tx_{2n-1}), \frac{1}{2}\bigl[d(Tx_{2n}, fx_{2n}) + d(Tx_{2n-1}, gx_{2n-1})\bigr], \\ &\quad \frac{1}{2}\bigl[d(Tx_{2n}, gx_{2n-1}) + d(Tx_{2n-1}, fx_{2n})\bigr]\biggr\} \\ &= \max\biggl\{ d(Tx_{2n}, Tx_{2n-1}), \frac{1}{2}\bigl[d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n-1}, Tx_{2n})\bigr], \\ &\quad \frac{1}{2}\bigl[d(Tx_{2n}, Tx_{2n}) + d(Tx_{2n-1}, Tx_{2n+1})\bigr]\biggr\} \\ &= d(Tx_{2n-1}, Tx_{2n}). \end{aligned}$$

Similarly, it can be shown that

$$d(Tx_{2n+1}, Tx_{2n+2}) < d(Tx_{2n}, Tx_{2n+1}).$$

Thus, $\{d(Tx_n, Tx_{n+1})\}$ is strictly decreasing sequence of positive real numbers, and, therefore, converges to a limit $r \geq 0$. If possible, suppose that $r > 0$. Then given $\delta > 0$, there exists a positive integer $N = N(\delta)$ such that

$$r \leq d(Tx_{2n}, Tx_{2n+1}) = d(fx_{2n}, gx_{2n-1}) < r + \delta \quad (\text{for all } n \geq N), \tag{7}$$

where $d(Tx_{2n}, Tx_{2n+1}) \leq m(x_{2n}, x_{2n+1})$. So by Eqs. (5) and (6), we have

$$d(fx_{2n}, gx_{2n+1}) < \alpha(Tx_{2n}, Tx_{2n+1})d(fx_{2n}, gx_{2n+1}) < r,$$

that is, $d(Tx_{2n+1}, Tx_{2n+2}) < r$, which is a contradiction. Hence

$$\lim_{n\to\infty} d(Tx_n, Tx_{n+1}) = 0. \tag{8}$$

We now show that $\{Tx_n\}$ is a Cauchy sequence.

Suppose that it is not. Then there exists an $\epsilon > 0$ such that for each positive integer m, n with $m > n > N$, we have $d(Tx_m, Tx_n) \geq 2\epsilon$. Choose a number δ, $0 < \delta < \epsilon$ for which contractive condition (4) is satisfied. Since $d(Tx_n, Tx_{n+1}) \to 0$, there exists integer $N = N(\delta)$ such that $d(Tx_i, Tx_{i+1}) < \frac{\delta}{6}$ for all $i \geq N$. With this choice of N, pick m, n with $m > n > N$ such that

$$d(Tx_m, Tx_n) \geq 2\epsilon > \epsilon + \delta, \tag{9}$$

in which it is clear that $m - n > 6$. Otherwise, we have

$$d(Tx_m, Tx_n) \leq \sum_{i=0}^{5} d(Tx_{n+i}, Tx_{n+i+1}) < \delta < \epsilon + \delta,$$

which contradicts (9). Also from (9), it follows that

$$d(Tx_m, Tx_{n+1}) > \epsilon + \frac{\delta}{3}.$$

Without loss of generality, we may assume that n is even. Suppose that

$$d(Tx_n, Tx_{m-1}) < \epsilon + \frac{\delta}{3},$$

then

$$\begin{aligned} d(Tx_n, Tx_m) &\leq d(Tx_n, Tx_{m-1}) + d(Tx_{m-1}, Tx_m) \\ &< \epsilon + \left(\frac{\delta}{3}\right) + \left(\frac{\delta}{6}\right) \\ &< \epsilon + \delta, \end{aligned}$$

which is a contradiction to (9). So we have

$$d(Tx_n, Tx_{m-1}) \geq \epsilon + \left(\frac{\delta}{3}\right).$$

Similarly, suppose that

$$d(Tx_n, Tx_{m-2}) < \epsilon + \left(\frac{\delta}{3}\right),$$

then

$$\begin{aligned} d(Tx_n, Tx_m) &\le d(Tx_n, Tx_{m-2}) + d(Tx_{m-2}, Tx_{m-1}) + d(Tx_{m-1}, Tx_m) \\ &< \epsilon + \left(\frac{\delta}{3}\right) + \left(\frac{\delta}{6}\right) + \left(\frac{\delta}{3}\right) \\ &< \epsilon + \delta, \end{aligned}$$

which is a contradiction to (9). So we have

$$d(Tx_n, Tx_{m-2}) \ge \epsilon + \left(\frac{\delta}{3}\right).$$

Thus, there exists the smallest odd integer $j > n$ such that

$$d(Tx_n, Tx_j) \ge \epsilon + \left(\frac{\delta}{3}\right), \tag{10}$$

and hence,

$$d(Tx_n, Tx_{j-2}) < \epsilon + \left(\frac{\delta}{3}\right).$$

Now,

$$\begin{aligned} d(Tx_n, Tx_j) &\le d(Tx_n, Tx_{j-2}) + d(Tx_{j-2}, Tx_{j-1}) + d(Tx_{j-1}, Tx_j) \\ &< \epsilon + \left(\frac{\delta}{3}\right) + \left(\frac{\delta}{6}\right) + \left(\frac{\delta}{6}\right) \\ &= \epsilon + \left(\frac{2\delta}{3}\right). \end{aligned}$$

Thus, there exists an odd integer $j \in (n, m)$ such that

$$\epsilon + \left(\frac{\delta}{3}\right) \le d(Tx_n, Tx_j) < \epsilon + \left(\frac{2\delta}{3}\right). \tag{11}$$

Since we have

$$\begin{aligned} &\epsilon < d(Tx_n, Tx_j) \le m_3(x_n, x_j) \\ &\quad = \max\left\{ d(Tx_n, Tx_j), \frac{1}{2}\big[d(Tx_n, fx_n) + d(Tx_j, gx_j)\big], \right. \\ &\quad \left. \frac{1}{2}\big[d(Tx_n, gx_j) + d(Tx_j, fx_n)\big] \right\} \\ &\quad < d(Tx_n, Tx_j) + \left(\frac{\delta}{6}\right) \\ &\quad < \epsilon + \delta. \end{aligned}$$

So, using (4) and assumption (iii), we get

$$d(fx_n, gx_j) \leq \alpha(Tx_n, Tx_j)d(fx_n, gx_j) < \epsilon,$$

that is, $d(Tx_{n+1}, Tx_{j+1}) < \epsilon$. But then

$$\begin{aligned} d(Tx_n, Tx_j) &\leq d(Tx_n, Tx_{n+1}) + d(Tx_{n+1}, Tx_{j+1}) + d(Tx_{j+1}, Tx_j) \\ &< \left(\frac{\delta}{6}\right) + \epsilon + \left(\frac{\delta}{6}\right) = \epsilon + \left(\frac{\delta}{3}\right), \end{aligned}$$

which contradicts (11). Therefore, $\{Tx_n\}$ is a Cauchy sequence. Since X is (f,g,T)-orbitally complete, so there exists a point $z \in X$ such that $Tx_n \to z$ as $n \to \infty$. Consequently, $fx_{2n} \to z$ and $gx_{2n+1} \to z$.

Step 2: We show that z is common fixed point of (f,g,T). In view of assumption (v), without loss of generality, let the pair (f,T) be absorbing and reciprocal continuous. Then the reciprocal continuity of f and T implies that

$$\lim_{n\to\infty} fTx_{2n} = fz \quad \text{and} \quad \lim_{n\to\infty} Tfx_{2n} = Tz.$$

Since T is f-absorbing, so there exists an $R > 0$ such that

$$d(fx_{2n}, fTx_{2n}) \leq Rd(fx_{2n}, Tx_{2n}).$$

Letting $n \to \infty$, we get $fTx_{2n} \to z$. Similarly, since f is T-absorbing, so we have

$$d(Tx_{2n}, Tfx_{2n}) \leq Rd(fx_{2n}, Tx_{2n}),$$

letting $n \to \infty$, we get $Tfx_{2n} \to z$. By the uniqueness of the limit, we have $z = fz = Tz$.

Now, suppose that $z \neq gz$, then by assumption (iv) and Eq. (6), we have

$$\begin{aligned} d(fx_{2n}, gz) &\leq \alpha(Tx_{2n}, Tz)d(fx_{2n}, gz) \\ &< \max\Big\{d(Tx_{2n}, Tz), \frac{1}{2}\big[d(Tx_{2n}, fx_{2n}) + d(Tz, gz)\big], \\ &\quad \frac{1}{2}\big[d(Tx_{2n}, gz) + d(Tz, fx_{2n})\big]\Big\}. \end{aligned}$$

Letting $n \to \infty$, we get $d(z,gz) \leq \frac{1}{2}d(z,gz)$, which implies that $z = gz$. Thus, z is a common fixed point of f, g, and T. This completes the proof of the theorem. □

By putting $f = g$ and $T = I$ (identity map) in Theorem 2.1, we get the following result as a corollary.

Corollary 2.1 *Let (X,d) be an f-orbitally complete metric space, where f is a self-mapping on X. Also, let $\alpha : X \times X \to [0,\infty)$ be a mapping. Assume the following*:

(i) *f is α-admissible*;

(ii) *there exists an $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$*;

(iii) *for given* $\epsilon > 0$, *there exists a* $\delta > 0$ *such that*

$$\epsilon \leq m_1(x,y) < \epsilon + \delta \quad \Longrightarrow \quad \alpha(x,y)d(fx,fy) < \epsilon,$$

$$\text{where } m_1(x,y) = \max\left\{d(x,y), \frac{1}{2}\left[d(x,fx) + d(y,fy)\right], \frac{1}{2}\left[d(x,fy) + d(y,fx)\right]\right\};$$

(iv) *on the* f*-orbit of* x_0, *we have* $\alpha(x_n, x_j) \geq 1$ *for all* n *even and* $j > n$ *odd.*
Then, f *has a fixed point in the* f*-orbit* $\{x_n\}$ *of* x_0, *or* f *has a fixed point* z *and* $\lim_{n\to\infty} x_n = z$.

Example 2.1 Let $X = [0,2]$ be endowed with the standard metric $d(x,y) = |x - y|$ for all $x, y \in X$. Define $f : X \to X$ by

$$fx = \begin{cases} 0 & \text{if } x \in \{0, \frac{1}{4}\}, \\ 1 & \text{if } x \in (0, \frac{1}{2}) - \{\frac{1}{4}\}, \\ \frac{3}{2} & \text{if } x \in [\frac{1}{2}, 2]. \end{cases}$$

Then f is not a Meir-Keeler contraction. To see this consider $\epsilon = \frac{1}{2}$, $x = \frac{1}{4}$, and $y = \frac{3}{4}$, then for any $\delta > 0$, we have $\epsilon \leq m_1(x,y) < \epsilon + \delta$, but $d(fx,fy) = d(0, \frac{3}{2}) = \frac{3}{2} > \epsilon$. However, f is a generalized Meir-Keeler α-contraction, where $\alpha : X \times X \to [0,\infty)$ is defined by

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x, y \in [\frac{1}{2}, 2], \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, f has two fixed points, namely $x = 0$ and $x = \frac{3}{2}$. Notice that $\alpha(\frac{3}{2}, 0) = 0 < 1$.

For the uniqueness of the fixed point of a generalized Meir-Keeler α-contractive mapping, we will consider the following hypothesis.

(H) For all fixed points x and y of (f, g, T), we have $\alpha(Tx, Ty) \geq 1$.

Theorem 2.2 *Adding condition* (H) *to the hypotheses of Theorem* 2.1 (*resp., Corollary* 2.1), *we obtain the uniqueness of the common fixed point of* f, g, *and* T.

Proof Let z be the common fixed point obtained as $Tx_n \to z$ and u is another common fixed point. Then, (6) and condition (H) yield to

$$\begin{aligned} d(z,u) &= d(fz,gu) \\ &\leq \alpha(Tz,Tu)d(fz,gu) \\ &< \max\left\{d(Tz,Tu), \frac{1}{2}\left[d(Tz,fz) + d(Tu,gu)\right], \frac{1}{2}\left[d(Tz,gu) + d(Tu,fz)\right]\right\} \\ &= d(z,u). \end{aligned}$$

Thus, we reach $d(z,u) < d(z,u)$, and hence $z = u$. □

The following example illustrates Theorem 2.2.

Example 2.2 Let $X = [2,20]$ and d be the usual metric on X. Define $f, g, T : X \to X$ as follows:

$$fx = \begin{cases} 3 & \text{if } x \in [2,4], \\ 2 & \text{if } x > 4, \end{cases} \qquad gx = \begin{cases} 2 & \text{if } x \in [2,3), \\ 3 & \text{if } x \geq 3, \end{cases} \qquad \text{and}$$

$$Tx = \begin{cases} 3 & \text{if } x = 3, \\ \frac{5}{2} & \text{if } x \in [2,20] - \{3,4\}, \\ 2 & \text{if } x = 4. \end{cases}$$

In this example the mappings f, g, and T do not satisfy the general Meir-Keeler contractive condition. To see this, consider $\epsilon = \frac{3}{4}$, $x = 3$ and $y \in [2,3)$, then for any $\delta > 0$, we have $\epsilon \leq m(x,y) < \epsilon + \delta$, but $d(fx, gy) = d(3,2) = 1 > \epsilon$. However, f, g, and T satisfy the generalized Meir-Keeler α-contractive condition (4) with the mapping $\alpha : T(X) \times T(X) \to [0,\infty)$ defined by

$$\alpha(u,v) = \begin{cases} 2 & \text{if } u, v \in \{2,3\}, \\ 0, & \text{otherwise.} \end{cases}$$

Also, all the hypotheses of Theorem 2.1 with condition (H) are satisfied, and clearly $x = 3$ is our unique common fixed point. Indeed, hypothesis (ii) is satisfied with $x_0 = 3$, and here $Tx_n = 3$ is a sequence, for which hypotheses (iii) and (iv) are satisfied. Also in view of the sequence $x_n = 3$, here both pairs (f, T) and (g, T) are reciprocal continuous as well as absorbing. Notice that $x = 3$ is the point of discontinuity of the mappings g and T.

Theorem 2.3 *The conclusion of Theorem* 2.1 *remains true if the assumption* (v) *of Theorem* 2.1 *is replaced by one of the following conditions*:

(a) $d(gx, Ty) \leq \max\{d(y, gx), d(y, Tx)\}$ *for all* $x, y \in X$ *with right-hand side positive.*

(b) $d(fx, Ty) \leq \max\{d(y, Tx), d(y, fx)\}$ *for all* $x, y \in X$ *with right-hand side positive.*

Proof In view of Theorem 2.1, we have that $\{Tx_n\}$ is a Cauchy sequence, and $Tx_n \to z \in X$ as $n \to \infty$, and, consequently, fx_{2n} and gx_{2n+1} also converge to z as $n \to \infty$.

Clearly, $Tx_n \neq z$ for infinitely many n. We can as well assume that $Tx_n \neq z$ for all n.

If (a) holds, then

$$d(gx_{2n+1}, Tz) \leq \max\{d(z, gx_{2n+1}), d(z, Tx_{2n+1})\}.$$

Letting $n \to \infty$, we get $d(z, Tz) \leq 0$, *i.e.*, $Tz = z$. If (b) holds, then also $Tz = z$.

Now, suppose that $z \neq gz$. Since $Tx_{2n} \neq Tx_{2n+1}$, so by assumption (iv) and Eq. (6), we have

$$\begin{aligned} d(fx_{2n}, gz) &\leq \alpha(Tx_{2n}, Tz) d(fx_{2n}, gz) \\ &< \max\Big\{d(Tx_{2n}, Tz), \frac{1}{2}\big[d(Tx_{2n}, fx_{2n}) + d(Tz, gz)\big], \\ &\quad \frac{1}{2}\big[d(Tx_{2n}, gz) + d(Tz, fx_{2n})\big]\Big\}, \end{aligned}$$

letting $n \to \infty$, we get $d(z, gz) \leq \frac{1}{2} d(z, gz)$, which implies that $z = gz$.

Now, let $fz \neq z = Tz$, then again by the process above, we have

$$\begin{aligned} d(fz, gx_{2n+1}) &\leq \alpha(Tx_{2n+1}, Tz)d(fz, gx_{2n+1}) \\ &< \max\Big\{d(Tz, Tx_{2n+1}), \frac{1}{2}\big[d(Tz, fz) + d(Tx_{2n+1}, gx_{2n+1})\big], \\ &\quad \frac{1}{2}\big[d(Tz, gx_{2n+1}) + d(Tx_{2n+1}, fz)\big]\Big\}, \end{aligned}$$

letting $n \to \infty$, we get $d(fz, z) \leq \frac{1}{2}d(z, fz)$, which implies that $fz = z$. Thus, z is the common fixed point of f, g, and T. □

The following example demonstrates Theorem 2.3.

Example 2.3 Let $X = [0,1]$ and d be the usual metric on X. Define $f, g, T : X \to X$ as follows:

$$fx = \begin{cases} 0 & \text{if } x \in [0, \frac{1}{4}], \\ \frac{1}{20} & \text{if } x \in (\frac{1}{4}, \frac{1}{2}), \\ \frac{1}{4} & \text{if } x \in [\frac{1}{2}, 1], \end{cases} \qquad gx = \begin{cases} \frac{x}{3} & \text{if } x \in [0, \frac{1}{4}], \\ x & \text{if } x \in (\frac{1}{4}, \frac{1}{2}), \\ 0 & \text{if } x \in [\frac{1}{2}, 1], \end{cases} \quad \text{and}$$

$$Tx = \begin{cases} \frac{x}{3} & \text{if } x \in [0, \frac{1}{4}], \\ \frac{1}{4} & \text{if } x \in (\frac{1}{4}, \frac{1}{2}), \\ \frac{x}{2} & \text{if } x \in [\frac{1}{2}, 1]. \end{cases}$$

Here the mappings f, g, and T satisfy all the conditions of Theorem 2.3 with the mapping $\alpha : T(X) \times T(X) \to [0, \infty)$ defined by

$$\alpha(u, v) = \begin{cases} 1 & \text{if } (u, v) \in [0, \frac{1}{12}] \times [\frac{1}{4}, \frac{1}{2}], \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, none of the pairs (f, T) and (g, T) are reciprocal continuous. To see this consider the sequence $x_n = \frac{1}{2} + \frac{1}{n}$, then $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Tx_n = \frac{1}{4}$, but $\lim_{n\to\infty} fTx_n = \lim_{n\to\infty} f(\frac{1}{4} + \frac{1}{2n}) = \frac{1}{20} \neq 0 = f(\frac{1}{4})$. Therefore, (f, T) is not reciprocal continuous. To see that (g, T) is not reciprocal continuous, one can consider the sequence $y_n = \frac{1}{4} + \frac{1}{n}$. Here, the involved mappings satisfy condition (a) of Theorem 2.3, and they have the unique common fixed $x = 0$.

Remark 2.1 Theorem 2.3 generalizes and extends Theorem 1.2 of Rao and Rao [22].

Theorem 2.4 *Theorem* 2.1 *remains true if we replace* $m_3(x, y)$ *by* $M_3(x, y)$ *and condition* (iv) *by the following* (iv′):

(iv′) $\alpha(Tx_n, Tx_{n+1}) \geq 1$ *for all* n *and* $Tx_n \to x$ *implies that* $\alpha(Tx_n, Tx) \geq K$ *for all* n, *where* $K > 1$.

Proof The proof of $z = fz = Tz$ follows from Theorem 2.1. Now, suppose that $z \neq gz$, then by the help of condition (iv'), we have

$$\begin{aligned} d(fx_{2n}, gz) &\leq K^{-1}\alpha(Tx_{2n}, Tz)d(fx_{2n}, gz) < K^{-1}M_3(x_{2n}, z) \\ &= K^{-1}\max\Big\{d(Tx_{2n}, Tz), d(Tx_{2n}, fx_{2n}), d(Tz, gz), \\ &\quad \frac{1}{2}\big[d(Tx_{2n}, gz) + d(Tz, fx_{2n})\big]\Big\}. \end{aligned}$$

By letting $n \to \infty$, we conclude that $d(z, gz) \leq K^{-1}d(z, gz) < d(z, gz)$, and hence $z = gz$. Thus, z is a common fixed point of f, g, and T. □

Example 2.2 above also satisfies Theorem 2.4.

Remark 2.2 Theorem 2.4 generalizes and extends Theorem 1.3 of Rao and Rao [22].

By taking $T = I$ (identity map) in Theorem 2.4, we derive the following result as a corollary.

Corollary 2.2 *Let* (X, d) *be an* (f, g)*-orbitally complete metric space, where* f, g *are self-mappings of* X. *Also, let* $\alpha : X \times X \to [0, \infty)$ *be a mapping. Assume the following*:

(i) (f, g) *is* α*-admissible, and there exists an* $x_0 \in X$ *such that* $\alpha(x_0, fx_0) \geq 1$;

(ii) *for given* $\epsilon > 0$, *there exists a* $\delta > 0$ *such that*

$$\epsilon \leq M(x, y) < \epsilon + \delta \quad \textit{implies that} \quad \alpha(x, y)d(fx, gy) < \epsilon,$$

where

$$M(x, y) = \Big\{d(x, y), d(x, fx), d(y, gy), \frac{1}{2}\big[d(x, gy) + d(y, fx)\big]\Big\};$$

(iii) *on the* (f, g)*-orbit of* x_0, *we have* $\alpha(x_n, x_j) \geq 1$ *for all* n *even and* $j > n$ *odd*;

(iv) $\alpha(x_n, x_{n+1}) \geq 1$ *for* n, *and* $x_n \to x$ *implies that* $\alpha(x_n, x) \geq K$ *for all* n, *where* $K > 1$.

Then, the pair (f, g) *has a common fixed point provided it is absorbing as well as reciprocal continuous.*

Remark 2.3 Corollary 2.2 improves Theorem 8 contained in [16].

The next result is a common fixed point theorem for four self-mappings.

Theorem 2.5 *Let* f, g, S, *and* T *be four self-mappings on a complete metric space* (X, d) *such that* $f(X) \subseteq T(X)$ *and* $g(X) \subseteq S(X)$, *and they satisfy the following conditions*:

(i) *the pair* (f, g) *is* $\alpha_{S,T}$*-admissible*;

(ii) *there exists a point* $x_0 \in X$ *such that* $\alpha(Sx_0, fx_0) \geq 1$;

(iii) *for given* $\epsilon > 0$, *there exists a* $\delta > 0$ *such that*

$$\epsilon \leq m_4(x, y) < \epsilon + \delta \quad \Longrightarrow \quad \alpha(Sx, Ty)d(fx, gy) < \epsilon, \tag{12}$$

where

$$m_4(x,y) = \max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{1}{2}\big[d(fx,Ty) + d(gy,Sx)\big]\right\};$$

(iv) *there exists a sequence* $\{x_n\}$ *in* X *such that* $\alpha(Sx_n, Tx_j) \geq 1$ *for all* n *even and* $j > n$ *odd;*

Then f, g, S, *and* T *have a common fixed point provided both the pair* (f,S) *and* (g,T) *are absorbing as well as reciprocal continuous.*

Proof Let $x_0 \in X$ such that $\alpha(Sx_0, fx_0) \geq 1$. Define sequences $\{x_n\}$ and $\{y_n\}$ in X as

$$y_{2n} = fx_{2n} = Tx_{2n+1}; \qquad y_{2n+1} = gx_{2n+1} = Sx_{2n+2}.$$

This can be done since $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$.

Since (f,g) is $\alpha_{S,T}$-admissible, we have

$$\alpha(Sx_0, fx_0) = \alpha(Sx_0, Tx_1) \geq 1 \quad \Longrightarrow \quad \alpha(fx_0, gx_1) \geq 1 \quad \text{and} \quad \alpha(gx_0, fx_1) \geq 1,$$

which gives

$$\alpha(Tx_1, Sx_2) \geq 1 = \alpha(y_0, y_1) \geq 1.$$

Again by (i), we have

$$\alpha(Tx_1, Sx_2) \geq 1 \quad \Longrightarrow \quad \alpha(fx_1, gx_2) \geq 1 \quad \text{and} \quad \alpha(gx_1, fx_2) \geq 1,$$

which gives

$$\alpha(Sx_2, Tx_3) = \alpha(y_1, y_2) \geq 1.$$

Inductively, we obtain

$$\alpha(y_n, y_{n+1}) \geq 1, \quad n = 0, 1, 2, \ldots, \tag{13}$$

that is, $\alpha(Sx_{n+1}, Tx_{n+2}) \geq 1$, when n is odd and $\alpha(Tx_{n+1}, Sx_{n+2}) \geq 1$ when n is even.

By assumption (iii), we have

$$\alpha(Sx, Ty)d(fx, gy) < m_4(x,y). \tag{14}$$

Now, we claim that $\{y_n\}$ is a Cauchy sequence.

Case I: If $y_n = y_{n+1}$ for some n. We first assume that n is odd, *i.e.*, $y_{2m+1} = y_{2m+2}$ and suppose that $y_{2m+2} \neq y_{2m+3}$, then by applying (13) and (14), we get

$$\begin{aligned} d(y_{2m+2}, y_{2m+3}) &= d(fx_{2m+2}, gx_{2m+3}) \\ &\leq \alpha(Sx_{2m+2}, Tx_{2m+3})d(fx_{2m+2}, gx_{2m+3}) \end{aligned}$$

$$\begin{aligned}
&< \max\left\{d(Sx_{2m+2}, Tx_{2m+3}), d(fx_{2m+2}, Sx_{2m+2}), d(gx_{2m+3}, Tx_{2m+3}),\right.\\
&\quad \left.\frac{1}{2}\left[d(fx_{2m+2}, Tx_{2m+3}) + d(gx_{2m+3}, Sx_{2m+2})\right]\right\}\\
&= \max\left\{d(y_{2m+1}, y_{2m+2}), d(y_{2m+2}, y_{2m+1}), d(y_{2m+2}, y_{2m+3}),\right.\\
&\quad \left.\frac{1}{2}\left[d(y_{2m+2}, y_{2m+2}) + d(y_{2m+3}, y_{2m+1})\right]\right\}\\
&= \frac{1}{2}d(y_{2m+2}, y_{2m+3}),
\end{aligned}$$

a contradiction. Hence $y_{2m+2} = y_{2m+3}$. By proceeding in this manner, we obtain $y_{2m+k} = y_{2m+1}$ for all $k \geq 1$. Similarly, when we assume n as even, then we obtain $y_{2m+k} = y_{2m}$ for all $k \geq 1$, and so $\{y_n\}$ is a Cauchy sequence.

Case II: If $y_n \neq y_{n+1}$ for each n. Applying (13) and (14), we get

$$\begin{aligned}
d(y_{2n}, y_{2n+1}) &= d(fx_{2n}, gx_{2n+1})\\
&\leq \alpha(Sx_{2n}, Tx_{2n+1})d(fx_{2n}, gx_{2n+1})\\
&< \max\left\{d(Sx_{2n}, Tx_{2n+1}), d(fx_{2n}, Sx_{2n}), d(gx_{2n+1}, Tx_{2n+1}),\right.\\
&\quad \left.\frac{1}{2}\left[d(fx_{2n}, Tx_{2n+1}) + d(gx_{2n+1}, Sx_{2n})\right]\right\}\\
&= \max\left\{d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n}),\right.\\
&\quad \left.\frac{1}{2}\left[d(y_{2n}, y_{2n}) + d(y_{2n+1}, y_{2n-1})\right]\right\}\\
&= d(y_{2n-1}, y_{2n}).
\end{aligned}$$

Similarly, we obtain $d(y_{2n-1}, y_{2n}) < d(y_{2n-2}, y_{2n-1})$. Thus, $\{d(y_n, y_{n+1})\}$ is a strictly decreasing sequence of positive numbers, and, therefore, tends to a limit $r \geq 0$. If possible, suppose that $r > 0$. Then given $\delta > 0$, there exists a positive integer N such that for each $n \geq N$, we have

$$r \leq d(y_{2n}, y_{2n+1}) = d(Tx_{2n+1}, Sx_{2n+2}) < r + \delta, \tag{15}$$

where $d(Sx_{2n+2}, Tx_{2n+1}) \leq m_4(x_{2n+2}, x_{2n+1})$. Then by applying (14), we have

$$d(fx_{2n+2}, gx_{2n+1}) \leq \alpha(Sx_{2n+2}, Tx_{2n+1})d(fx_{2n+2}, gx_{2n+1}) < r,$$

that is, $d(y_{2n+2}, y_{2n+1}) < r$, which is a contradiction, and hence,

$$\lim_{n\to\infty} d(y_n, y_{n+1}) = 0. \tag{16}$$

Now, we show that $\{y_n\}$ is a Cauchy sequence. Suppose that it is not, then there exists an $\epsilon > 0$ such that for each integer N, there exist integers $m > n > N$ such that $d(y_m, y_n) \geq 2\epsilon$.

Choose a number δ, $0 < \delta < \epsilon$, for which contractive condition (12) is satisfied. By virtue of (16), there exists an integer N such that $d(y_i, y_{i+1}) < \frac{\delta}{6}$ for all $i \geq N$. With this choice of N, pick integers $m > n > N$ such that

$$d(y_m, y_n) \geq 2\epsilon > \delta + \epsilon, \tag{17}$$

in which it is clear that $m - n > 6$. Also from (17), it follows that $d(y_m, y_{n+1}) > \epsilon + \frac{\delta}{3}$.

If not, then

$$\begin{aligned} d(y_m, y_n) &\leq d(y_m, y_{n+1}) + d(y_{n+1}, y_n) \\ &< \epsilon + \left(\frac{\delta}{3}\right) + \left(\frac{\delta}{6}\right) < 2\epsilon, \end{aligned}$$

which is a contradiction. Without loss of generality, we can assume that n is even. From (17), there exists the smallest odd integer $j > n$ such that

$$d(y_n, y_j) \geq \epsilon + \left(\frac{\delta}{3}\right), \tag{18}$$

and hence $d(y_n, y_{j-2}) < \epsilon + \frac{\delta}{3}$. So we have

$$\begin{aligned} d(y_n, y_j) &\leq d(y_n, y_{j-2}) + d(y_{j-2}, y_{j-1}) + d(y_{j-1}, y_j) \\ &< \epsilon + \left(\frac{\delta}{3}\right) + \left(\frac{\delta}{6}\right) + \left(\frac{\delta}{6}\right) \\ &= \epsilon + \left(\frac{2\delta}{3}\right). \end{aligned}$$

Thus, there exists an odd integer $j \in (n, m)$ such that

$$\epsilon + \left(\frac{\delta}{3}\right) \leq d(y_n, y_j) < \epsilon + \left(\frac{2\delta}{3}\right). \tag{19}$$

Therefore, we have

$$\begin{aligned} \epsilon < d(y_n, y_j) &= d(Tx_{n+1}, Sx_{j+1}) \leq m_4(x_{j+1}, x_{n+1}) \\ &= \max\Big\{ d(Sx_{j+1}, Tx_{n+1}), d(fx_{j+1}, Sx_{j+1}), d(gx_{n+1}, Tx_{n+1}), \\ &\quad \frac{1}{2}\big[d(fx_{j+1}, Tx_{n+1}) + d(gx_{n+1}, Sx_{j+1})\big]\Big\} \\ &= \max\Big\{ d(y_j, y_n), d(y_{j+1}, y_j), d(y_{n+1}, y_n), \frac{1}{2}\big[d(y_{j+1}, y_n) + d(y_{n+1}, y_j)\big]\Big\} \\ &< d(y_j, y_n) + \frac{\delta}{6} < \epsilon + \delta, \end{aligned}$$

so that by (12) and assumption (iv), we get

$$d(fx_{j+1}, gx_{n+1}) \leq \alpha(Sx_{j+1}, Tx_{n+1}) d(fx_{j+1}, gx_{n+1}) < \epsilon,$$

i.e., $d(y_{n+1}, y_{j+1}) < \epsilon$. But then

$$\begin{aligned} d(y_n, y_j) &\le d(y_n, y_{n+1}) + d(y_{n+1}, y_{j+1}) + d(y_{j+1}, y_j) \\ &< \left(\frac{\delta}{6}\right) + \epsilon + \left(\frac{\delta}{6}\right) \\ &= \epsilon + \left(\frac{\delta}{3}\right), \end{aligned}$$

which contradicts (19). Therefore, $\{y_n\}$ is a Cauchy sequence. By the completeness of X, there exists a $z \in X$ such that $y_n \to z$ as $n \to \infty$ and, consequentially, fx_{2n}, Tx_{2n+1}, gx_{2n+1} and $Sx_{2n+2} \to z$ as $n \to \infty$.

Since the pair (f, S) is reciprocal continuous and absorbing, so by reciprocal continuity, we have $fSx_{2n} \to fz$ and $Sfx_{2n} \to Sz$ as $n \to \infty$. By absorbing property, there is an $R > 0$ such that $d(fx_{2n}, fSx_{2n}) \le Rd(fx_{2n}, Sx_{2n})$ and $d(Sx_{2n}, Sfx_{2n}) \le Rd(fx_{2n}, Sx_{2n})$, which letting $n \to \infty$ gives $fSx_{2n} \to z$ and $Sfx_{2n} \to z$. Thus, we have $z = fz = Sz$. Similarly, the absorbing and reciprocal continuity of the pair (g, T) provides us $z = gz = Tz$. Thus, z is a common fixed point of f, g, S, and T. □

Theorem 2.6 *Adding the condition* (H-2): *For all common fixed points x and y of f, g, S, and T*, $\alpha(Sx, Ty) \ge 1$, *to the hypotheses of Theorem* 2.5, *the uniqueness of the fixed point is obtained*.

Remark 2.4 Theorem 2.6 generalizes, extends and improves the results of Jungck (Theorem 3.1, [8]), Cho *et al.* (Theorem 3.2, [4]) and Rao and Rao [22].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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