# NONUNIFORM CURRENTS FLOWING ON A PERFECTLY CONDUCTING CYLINDER 

A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES OF ÇANKAYA UNIVERSITY BY HÜSNÜ DENİZ BAŞDEMİR

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE
IN
THE DEPARTMENT OF
ELECTRONIC AND COMMUNICATION ENGINEERING

Title of the Thesis : Nonuniform Currents Flowing on a Perfectly Conducting Cylinder

Submitted by Hüsnü Deniz Başdemir

Approval of the Graduate School of Natural and Applied Sciences, Cankaya University


I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science


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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.


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## STATEMENT OF NON-PLAGIARISM PAGE

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# ABSTRACT <br> NONUNIFORM CURRENTS FLOWING ON A PERFECTLY CONDUCTING CYLINDER 

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September 2010, 42 pages

In this thesis, we applied physical theory of diffraction method to the perfectly electric conducting cylinder for investigation of the scattering fields. Physical optics method was used for obtain the uniform scattering fields which is produced by the induced current from the incident wave on the tangential plane of the perfectly electric conducting cylinder. Also with the help of the series expansion of the cylindric wave functions, physical optics integral was converted to series form. To be a last step of solution, nonuniform currents and nonuniform scattering fields were obtained with the assistance of the exact solution of the scattering waves from the cylinder. In addition a Matlab code was developed for investigation of the scattering electric fields and induced currents. All this mentioned currents and fields were plotted.

Keywords: Scattering Fields from Cylinder, Fringe Waves, Creeping Waves.

# MÜKEMMEL İLETKEN SİLİNDİR ÜZERİNDE DOLANAN SÜREKSİZ AKIMLAR 

BAŞDEMİR, Hüsnü Deniz<br>Yüksek Lisans, Elektronik ve Haberleşme Mühendisliği Anabilim Dalı<br>Tez Yöneticisi: Doç. Dr. Yusuf Ziya UMUL<br>Eylül 2010, 42 sayfa

Bu tezde, mükemmel elektrik iletken bir silindirden saçılan alanların araştırılması için kırınımın fiziksel teorisi metodunu uyguladık. Mükemmel elektrik iletken silindirin teğetsel düzleminde gelen alan tarafından indüklenen akım tarafından üretilen sürekli saçılan alanın hesaplanmasında fiziksel optik metodu kullanıldı. Ayrıca silindirik dalga fonksiyonunun seri açılımı yardımıyla fiziksel optik integrali seri formuna dönüştürüldü. Ç̧̈zümün son adımı olarak süreksiz akımlar ve süreksiz saçılan alanlar, silindirden saçılan alanların kesin çözümü yardımıyla elde edildi. Ek olarak saçılan alanların ve indüklenen akımların incelenmesi için bir Matlab kodu geliştirildi. Tüm bahsedilen akımlar ve alanlar çizdirildi.

Anahtar Kelimeler: Silindirden Saçılan Alanlar, Saçak Alanları, Sürünen Alanlar.

## ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Assoc. Prof. Dr. Yusuf Ziya UMUL for his supervision, special guidance, suggestions, and encouragement through the development of this thesis.

It is a pleasure to express my special thanks to my friends for their constant support.

## TABLE OF CONTENT

STATEMENT OF NON PLAGIARIZM. ..... iii
ABSTRACT ..... iv
ÖZ. ..... v
ACKNOWLEDGEMENTS ..... vi
TABLE OF CONTENTS ..... vii
LIST OF FIGURES ..... ix
LIST OF ABBREVIATIONS. ..... X
CHAPTERS:

1. INTRODUCTION. ..... 1
1.1. Background ..... 1
1.2. Objectives. ..... 3
1.3. Organization of the Thesis ..... 3
2. CURRENT BASED TECHNIQUE ..... 4
2.1. Basic Idea of The Current Based Techniques. ..... 4
2.2. Physical Optics ..... 5
2.3. Physical Theory of Diffraction. ..... 6
3. INTRODUCTION AND SOLUTION OF THE PROBLEM. ..... 9
3.1. Introduction and Solution of the Problem ..... 9
4. SERIES EXPANSION OF THE PO INTEGRAL AND CORRECTION OF THE TOTAL CURRENT ..... 15
4.1. Series Expansion of the PO Integral. ..... 15
4.2. Correction of the Total Current Flowing on the Cylinder. ..... 17
5. NUMERICAL ANALYSIS ..... 21
6. CONCLUSION ..... 24
REFERENCES ..... R1
APPENDIX A MATLAB CODES ..... A1
APPENDIX B CURRICULUM VITALE ..... A5

## LIST OF FIGURES

## FIGURES

Figure 1 Induction of the Current by the Incident Wave. ..... 4
Figure 2 Physical Optic Lit region. ..... 6
Figure 3 Different Shapes Where the Incident Field Generates the ..... 8 Nonuniform Source
Figure 4 The Geometry of the Problem. ..... 9
Figure 5 Cylindric, Cartesian and Spheric Coordinate System. ..... 10
Figure 6 Ray Path of the Scattering Field ..... 14
Figure 7 Shows the Known Length a, b, Angle and Unknown Length c.. ..... 14
Figure 8 Total Corrected Surface Current ..... 20
Figure 9 Total Surface Current According to Harrington. ..... 20
Figure 10 Total Surface Current. ..... 21
Figure 11 Uniform Surface Current ..... 21
Figure 12 Nonuniform Surface Current ..... 22
Figure 13 Total Scattered Electric Field ..... 22
Figure 14 Uniform Scattered Electric Field. ..... 23
Figure 15 Nonuniform Scattered Electric Field ..... 23

# LIST OF ABBREVIATIONS 

| PTD | Physical Theory of Diffraction |
| :--- | :--- |
| PO | Physical Optics |
| MTPO | Modified Theory of Physical Optics |
| GO | Geometrical Optics |
| GTD | Geometrical Theory of Diffraction |
| UTD | Uniform Theory of Diffraction |
| PEC | Perfect Electric Conductor |

## CHAPTER 1

## INTRODUCTION

### 1.1 Background

In general wave radiation has to be analysed taking into account scatterers. Scatterers are finite structures. These structures reflect waves from uniform tangent planes or diffract waves from its nonuniform parts [1]. In a medium total field consist of incident field ( $E_{i}, H_{i}$ ) and scattering field ( $E_{s}, H_{s}$ ). Incident fields are produced by the source when the absence of any scatterer. Total electric and magnetic fields can be written

$$
\begin{equation*}
E_{t}=E_{i}+E_{s} \tag{1.1}
\end{equation*}
$$

and

$$
\begin{equation*}
H_{t}=H_{i}+H_{s} \tag{1.2}
\end{equation*}
$$

in the Eq. (1.1) and (1.2). Diffraction problems are the most popular scattering problems. Solving of the scattering problems is obtained with using different methods. There are three major group of solving methods. First one is analytic and the second one is numerical and the last one is asymptotic. We are interested in high frequency (HF) asymptotic technique. This technique is deviates two major group which are ray based and current based. Geometrical optics (GO) which is one of the ray based techniques tells us in the high frequency, electromagnetic waves travels like rays in the vacuum. This method shows us incident and reflected fields, these fields help us to determination of good and bad zones for the communication but this technique does not include diffraction phenomena. Geometrical theory of diffraction (GTD) [2] and its uniform version of uniform theory of diffraction (UTD) [3] have been most popular techniques for investigating the diffraction problems. These techniques have a problem at the caustic regions. They give infinite field at those
regions. Physical optics (PO) and equivalent current method [4] can be used for fix that problem. Although these techniques are fixed the problem in the lit region they give wrong answer in the shadow regions. This obstacle can be overcome with using physical theory of diffraction (PTD) [5]. This technique is widely used in the shadow diffraction problems. Unfortunately, this technique is applicable just only for the solution known problem. The new and interesting method for obtaining the exact solution of diffraction is modified theory of physical optics (MTPO) [6]. This technique eliminates all this difficulties with defining three axioms. In the analysis of the scattering plane waves from the cylinder with using GTD was investigated by Pathak in 1979 [7]. Asymptotic expression for the scattering by a perfectly conducting cylinder was investigated by Franz [8]. A new method for investigation of the plane wave scattering by a perfectly conducting circular cylinder near a plane surface was presented by Borghi F., Santarsiero M., Frezza F., Schettini G [9]. Asymptotic expansions of exact solutions of the scattering fields from perfectly conducting cylinder was investigated on the complex plane and critical discussions of geometrical optics, physical optics and the geometrical theory of diffraction was presented by Kouyoumjian [10]. Diffracted and reflected fields by any convex cylinder were constructed by Keller [11]. The eigen function solution for electromagnetic scattering by the cylinder was published in 1881 [12], the parabolic cylinder in 1914 [13]. Debye obtained asymptotic approximation for the current on the illumination side of the cylinder by using the saddle point method [14]. Riblet gave the first two term of the asymptotic expansion of the current on the illuminated part of the cylinder [15]. Wetzel published a study about the high frequency currents on the all parts of the cylinder [16]. Wait obtained the current on a parabolic cylinder in the vicinity of the shadow boundary [17]. An analytical solution was presented for the electromagnetic scattering from a dielectric cylinder by Lawrence and Sarabandi [18]. Scattering from a perfectly conducting cylindrical reflector was examined by U. Yalçın [19]. Physical optic integral was obtained for a cylinder fed by Umul, Yengel and Aydin [20]. Plane wave scattering by a perfectly conducting circular cylinder near a plane surface was investigated by Borghi, Gori, Santarsiero, Frezza, Schettini [21]. Diffraction of waves generated by magnetic line source by the edges of a cylindrically curved surface with different phase impedance was presented by

Büyükaksoy and Uzgören [22]. The time factor of $e^{j o t}$ is assumed and suppressed throughout the thesis.

### 1.2 Objectives

It is aim of this work to obtain nonuniform current and investigation of the contribution to the scattering field from the perfectly electric conducting cylinder and plot the scattered electric field and currents. Used method for this aim is physical theory of diffraction (PTD). Second part of this thesis obtains uniform currents and uniform scattering electric field with using integral technique. Third part of this thesis conversion of uniform scattering integral to the series expression. When this conversion is applied series expansion of the Hankel functions is taken in to account. Final step of this thesis is obtained exact nonuniform electric field and plotting the results.

### 1.3 Organization of the Thesis

This thesis contains six chapters. Chapter 1 is an introduction of short history of scattering theory and objectives of this thesis.

In Chapter 2 current based techniques which are used in this thesis are introduced.

In Chapter 3 our problem and its solution is introduced.

In Chapter 4 includes series conversion of the PO integrals for obtaining the nonuniform scattering fields is given.

Chapter 5 includes the numerical analysis of the fields and currents.

Chapter 6 includes conclusion and finalization part of this thesis.

## CHAPTER 2

## CURRENT BASED TECHNIQUES

### 2.1 Basic Idea of the Current Based Techniques

The basic idea for this approximation is creating current on the tangential plane of the scatterer object by the incident field. Figure 1 shows the induced current on the object.


Figure 1 Induction of the Current by the Incident Wave

The scattering field is radiated by this current means that ${ }^{\prime}$ is the induced current; this current can be thought the source of the scattering field. When this current is modeling with respect to mathematical formulations boundary conditions have to be consideration. Scattering field includes geometrical optical fields which are incident $\left(E_{i}\right)$, reflected $\left(E_{r}\right)$ and includes diffracted fields $\left(E_{d}\right)$. Examples of the current
based techniques are the modified theory of physical optics (MTPO) [6], physical optics (PO) [23] and physical theory of diffraction (PTD) [24].

### 2.2 Physical Optics (PO)

Physical optics is a high frequency integrative technique that was suggested by Mcdonald in 1912 [25]. It is used for the investigation of the scattering from large metallic objects. It is suitable for acoustic and electromagnetic waves. In the acoustic this technique is known extended Kirchoff approximation. According to this technique induced field on the object is determined by the geometrical optics. Geometrical optics tells us in the high frequencies electromagnetic waves travels like rays so this technique often called ray optics. In a homogeneous medium energy moves along ray paths that are straight lines. We can be thought the surface of the scattering object is an infinite tangential plane with respect to small wavelength according to obstacle. This surface is called geometric optic surface. Central idea is to achieve current on the scatterer. Current induced by the incident field can be written,

$$
\begin{equation*}
\dot{J}_{P O}=2 \stackrel{r}{n} \times \stackrel{1}{H}_{i} \quad \text {, on the lit region } \tag{2.1}
\end{equation*}
$$

and

$$
\begin{equation*}
J_{P O}=0 \quad, \text { on the shadow region } \tag{2.2}
\end{equation*}
$$

where $\dot{n}$ is the unit normal vector outward from the illuminated part of the cylinder as shown in Figure 2. $\stackrel{1}{H}_{i}$ is the tangential component of the incident plane wave. It is defined on the scatterer's surface. From the equivalent current on the free space it takes twice time. As can be seen from the equations, PO just defined on the lit region of the scatterer so does not include diffraction phenomena. According to physical optics, the fields equal to zero in the shadow region. This means that PO rejects the shadow fields. Figure 2 shows the PO lit region which is from $90^{\circ}$ to $270^{\circ}$. PO lit region is bounded with the cutoff of the GO's rays. This surface is determined between the reflection boundaries so this surface is called geometric optic surface.

This surface can be determined various objects. Scatterer's tangential surface which is the illuminated by the incident wave is the example of the geometric optic surface.


Figure 2 Physical Optic Lit Region

In the far field total scattering PO field can be found

$$
\begin{equation*}
\dot{E}_{s}^{P O} \cong-j \omega ' \tag{2.3}
\end{equation*}
$$

from the Eq. (2.3). $\dot{A}$ shows the magnetic vector potential. For finding the magnetic vector potential physical optic integral

$$
\begin{equation*}
\stackrel{r}{A}=\frac{\mu_{0}}{4 \pi} \iint_{S^{\prime}}^{r} J_{P O}^{r} \frac{\exp (-j k R)}{R} d s^{\prime} \tag{2.4}
\end{equation*}
$$

from the Eq. (2.4) is used over the scatterers surface. $G=\frac{\exp (-j k R)}{R}$ is the free space Green's function, minus sign on the exponential term correspond to waves propagating to outward direction. $R$ is the distance between the source and observation point. Green's function shows the phase and magnitude variations away from the source. PO is a good approximation for large structures so the larger plate the better approximation. Because of the contribution of the nonuniform parts are rejected, this technique has defects [26]. This defect was fixed by the physical theory of diffraction (PTD).

### 2.3 Physical Theory of Diffraction (PTD)

This technique is suggested by Ufimtsev in 1950's [5]. It is used for compute the scattering waves from the objects in the high frequencies. It is the natural extension of the physical optic technique. When Ufimtsev was improving PO technique, he
was aware of the Sommerfeld's exact wedge solution [27]. Physical optic current does not include the contribution of the nonuniform current. The fundamental idea of Ufimtsev's concept is divide the induced current into two components. This total current can be thought the source of the scattering field. It can be written

$$
\begin{equation*}
'_{t}^{\prime}=\dot{J}_{\text {uniform }}+\dot{J}_{\text {nonuniform }} \tag{2.3}
\end{equation*}
$$

in the Eq. (2.3). First part is called uniform part of the current. This part is written from the PO. PO current is described on the lit region of the scatterer's tangential surface which is called geometric optic surface. PO current on the shadow part of the obstacle is equal to zero so physical optic method does not include the diffractions. The incident wave shows the equal distribution over the tangential plane. Its amplitude is constant and its phase is a linear function of the plane coordinates. This is the reason of the called uniform component. When PO current is described boundary values are have to take into consideration. This value comes from the acoustic surface. On the acoustically soft plane, total field on the scatterer is equal to zero but its normal derivative is equal to

$$
\begin{equation*}
\frac{\partial u_{\text {soft }}}{\partial n}=2 \frac{\partial u_{i}}{\partial n} \tag{2.4}
\end{equation*}
$$

the Eq. (2.4). On the acoustically hard plane, normal derivative of the incident field is equal to zero but total field is equal to

$$
\begin{equation*}
\frac{\partial u_{\text {hard }}}{\partial n}=2 u_{i} \tag{2.5}
\end{equation*}
$$

the Eq. (2.5). These conditions tells us for electromagnetic in the acoustically soft plane, tangential polarization of the incident electric field is different than zero but incident magnetic field equal to zero and in the acoustically hard plane tangential polarization of the incident electric field is equal to zero but incident magnetic polarization is different than zero. Acoustically hard plane is described perfectly electric conductor surface in the electromagnetic. The main contribution of PTD to the diffraction phenomena is the nonuniform part which is introduced by Ufimtsev. Nonuniform parts are the discontinuity of the scattering objects. Examples of discontinuities are edges, smooth bending, discontinuity of curvature are given in the Figure 3.


Figure 3 Different Shapes Where the Incident Field Generates the Nonuniform Source

As seen in the Figure 3, a is the sharp edge, b is the smooth bending and c is the discontinuity of curvature. These discontinuities are the reason of diffraction. Current induced on this discontinuities especially on the edges was called fringe currents and the fields are radiating from this discontinuities was called fringe fields by the Ufimtsev [24]. Actually Ufimtsev did not find the nonuniform currents instead of this he found the nonuniform field due to the nonuniform current. Ufimtsev obtained this fringe fields subtracting PO scattered waves from the Sommerfeld's exact solution. Special feature of this technique allows us calculate of the fields which are in the shadow and the caustic regions. Although this technique is just applicable for the solution known problem, its serious contributions to the technology can't be disregarded. With the contribution of this technique to the modern low radar systems, Lockheed Martin firms produced F-117 Stelth Fighter airplane. PTDs defect was fixed with describing the nonuniform current by the modified theory of physical optics.

## CHAPTER 3

## INTRODUCTION and SOLUTION of the PROBLEM

### 3.1 Introduction and Solution of the Problem

Our problem is investigation of the nonuniform currents which is induced by the incident field and scattering fields from the perfectly electric conducting (PEC) cylinder. PEC means that total electric field on the tangential plane is equal to zero. This determination comes from the acoustic. On the acoustically this plane called hard surface because of the boundary conditions. The problem's geometry is given in the Figure 4.


Figure 4 The Geometry of the Problem

PTD method will be used for analysing of the problem. First of all we will use the PO method for obtain the uniform component of the scattering field.

Incident wave is given

$$
\begin{equation*}
\dot{E}_{i}=\stackrel{r}{e_{z}} E_{0} e^{-j k x} \tag{3.1}
\end{equation*}
$$

in the Eq. (3.1). Incident wave is $z$ polarized, propagates in the positive $x$ direction and $E_{0}$ is the constant amplitude factor.

Normal vector is shown in the Figure 4 which is the negative $x$ direction. PO or uniform current is written

$$
\begin{equation*}
\dot{J}_{P O}=2 \stackrel{r}{n} \times \dot{H}_{i} \tag{3.2}
\end{equation*}
$$

in the Eq. (3.2). $\stackrel{1}{n}$ is the normal vector and equal to negative $x$ direction. $\stackrel{1}{J}_{P O}$ is equal to ${ }^{\prime}{ }_{\text {es }}$ from the equivalent source theorem. Induced electric current can be written from the Eq. (3.2) so that incident magnetic field has to be found. Incident magnetic field component is found from the Maxwell equation

$$
\begin{equation*}
\stackrel{\stackrel{r}{H}}{H_{i}}=-\frac{1}{j \omega \mu_{0}} \nabla \times \stackrel{r}{E_{i}} . \tag{3.3}
\end{equation*}
$$

This equation shows that to be a vectorial electric and magnetic fields can be changed to each other. From starting to this point incident magnetic field component of the electromagnetic field in the cartesian coordinate system is found to be

$$
\begin{equation*}
\stackrel{r}{H_{i}}=\frac{E_{0} k}{\omega \mu_{0}} e^{-j k x} e_{y}^{r} \tag{3.4}
\end{equation*}
$$

Eq. (3.4) shows the incident magnetic field which is y polarized and propagates positive $x$ direction. Because of the problem in the cylindric coordinate system, incident magnetic fields coordinate have to be changed from cartesian to cylindric coordinate system. Figure 5 shows the three different coordinate systems.


Figure 5 Cylindric, Cartesian and Spheric Coordinate System

With using the Figure 5, $x=\rho \cos \phi, y=\rho \sin \phi$ and ${ }^{\prime} n^{\prime} \dot{e}_{\rho}$ can be written. Coordinate system changing matrices can be constructed

$$
\left[\begin{array}{l}
e_{e_{p}}  \tag{3.5}\\
e^{\mathbf{j}} \\
e_{\mathrm{\delta}} \\
e_{z}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
e_{e_{r}} \\
e^{r} \\
e_{y} \\
e_{z}
\end{array}\right]
$$

from the Ref. [28]. From the matrices unit vector to the $y$ direction is written as

$$
\begin{equation*}
\stackrel{1}{e}_{y}=\stackrel{1}{e}_{\rho} \sin \phi+\dot{e}_{\phi} \cos \phi . \tag{3.6}
\end{equation*}
$$

in the Eq. (3.6). Using this changing parameters incident magnetic field component is written as

$$
\begin{equation*}
\stackrel{r}{H_{i}}=\stackrel{r}{e_{\rho}} \frac{E_{0} k}{\omega \mu_{0}} \sin \phi e^{-j k \rho \cos \phi}+\stackrel{r}{e_{\phi}} \frac{E_{0} k}{\omega \mu_{0}} \cos \phi e^{-j k p \cos \phi} \tag{3.7}
\end{equation*}
$$

the Eq. (3.7). Using the Eq. (3.2) under the surface condition,

$$
\begin{equation*}
\stackrel{r}{J}_{P O}=2 \stackrel{r}{n} \times\left.\stackrel{r}{H}_{i}\right|_{\rho=a}=\stackrel{r}{e_{z}} \frac{2 E_{0} k}{\omega \mu_{0}} \cos \phi e^{-j k a \cos \phi} \tag{3.8}
\end{equation*}
$$

PO current is written. For simplification of the current, speed of light $c=\frac{1}{\sqrt{\varepsilon_{0} \mu_{0}}}$ and wave number $k=\frac{\omega}{c}$ is used. Reconstruction of the current is given

$$
\begin{equation*}
\stackrel{r}{J}_{P O}=\stackrel{r}{e} z_{z} \frac{2 E_{0}}{Z_{0}} \cos \phi e^{-j k a \cos \phi} \tag{3.9}
\end{equation*}
$$

in the Eq. (3.9). $Z_{0}$ is the impedance of the vacuum. After finding the current, scattering field must be calculated. For calculation of the scattering field PO integral have to be constructed. PO integral is written from

$$
\begin{equation*}
\stackrel{r}{A}=\frac{\mu_{0}}{4 \pi} \iint_{S^{\prime}}^{r} J_{P O} \frac{e^{-j k R}}{R} d S^{\prime} \tag{3.10}
\end{equation*}
$$

the Eq. (3.10) which is called physical optic integral and $A^{\prime}$ is the magnetic vector potential. If PO current is inserted in the Eq. (3.10)

$$
\begin{equation*}
\stackrel{r}{A}=\frac{\mu_{0}}{2 \pi} \int_{z=-\infty}^{\infty} \int_{\phi^{\prime}=\frac{\pi}{2}}^{\frac{3 \pi}{2}} e_{z} \frac{E_{0}}{Z_{0}} \cos \phi e^{-j k \cos \phi} \frac{e^{-j k R}}{R} a d \phi^{\prime} d z^{\prime} \tag{3.11}
\end{equation*}
$$

magnetic vector potential takes the form Eq. (3.11). Evaluation of this integral gives magnetic vector potential then it gives scattering fields from the cylinders lit region. First step for evaluation of this integral to eliminate the $z^{\prime}$ part of the integral.

The $z$ ' part of the integral can be evaluated as

$$
\begin{equation*}
\int_{C} e^{-j k \cosh \alpha} d \alpha=\frac{\pi}{j} H_{0}^{(2)}(k R) \tag{3.12}
\end{equation*}
$$

the Eq. (3.12) means that $z$ part of the integral gives the Hankel function. Detailed explanation of the evaluation of $z$ part was given in the Ref. [6].

As a result Eq. (3.11) takes the form

$$
\begin{equation*}
\stackrel{r}{A}=\stackrel{r}{e_{z}} \frac{\mu_{0} E_{0} a}{2 Z_{0} j} \int_{\phi^{\prime}=\frac{\pi}{2}}^{\frac{3 \pi}{2}} \cos \phi^{\prime} e^{-j k a \cos \phi^{\prime}} H_{0}^{(2)}\left(k R_{1}\right) d \phi^{\prime} \tag{3.13}
\end{equation*}
$$

for the magnetic vector potential. Uniform scattering electric field can be written from

$$
\begin{equation*}
\dot{E}_{s} \cong-j \omega '_{A}^{\prime} \tag{3.14}
\end{equation*}
$$

the Eq. (3.14). At the end uniform scattering electric field is written with using Eq. (3.14)

Total exact scattering field from the cylinder can be written

$$
\begin{equation*}
\stackrel{\mathrm{E}}{s}_{\text {total }}=\stackrel{\mathrm{r}}{e_{z}} E_{0} \sum_{n=-\infty}^{\infty} j^{-n} a_{n} H_{n}^{(2)}(k \rho) e^{j n \phi} \tag{3.16}
\end{equation*}
$$

and

$$
\begin{equation*}
a_{n}=\frac{-J_{n}(k a)}{H_{n}^{(2)}(k a)} \tag{3.17}
\end{equation*}
$$

from the Ref. [25]. $J_{n}$ is the zeroth order Bessel function, $H_{n}$ is the third order Bessel function which is called Hankel function and $a$ is radius of the cylinder. For finding the nonuniform scattering field from the cylinder according to PTD

$$
\begin{equation*}
\dot{E}_{s}^{N U}=\dot{E}_{s}^{\text {total }}-\dot{E}_{s}^{P O} \tag{3.18}
\end{equation*}
$$

Eq. (3.18) can be used. Using Eq. (3.18) nonuniform scattering electric field is written as
in the Eq. (3.19) which is the exact scattering nonuniform field.
Another way to find the nonuniform field is evaluating the nonuniform current over the surface of the cylinder so that nonuniform current have to be written. Surface current on the cylinder is written

$$
\begin{equation*}
{\underset{J}{J_{\text {exact }}^{\text {suface }}}}=\stackrel{r}{e_{z}} \frac{-2 E_{0}}{\omega \mu \pi a} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{j n \phi}}{H_{n}^{(2)}(k a)} \tag{3.20}
\end{equation*}
$$

from Ref. [25]. From the Eq. (3.9) PO current is written

$$
\begin{equation*}
\stackrel{r}{J}_{P O}=\stackrel{r}{e} z_{z} \frac{2 E_{0}}{Z_{0}} \cos \phi e^{-j k a \cos \phi} \tag{3.21}
\end{equation*}
$$

Nonuniform current is equal to

$$
\begin{equation*}
\dot{J}_{N U}=\dot{J}_{\text {exact }}^{\text {surface }}-\dot{J}_{P O} \tag{3.22}
\end{equation*}
$$

according to PTD [5]. Nonuniform current takes the form

$$
\begin{equation*}
\stackrel{r}{J}_{N U}=e_{z} \frac{-2 E_{0}}{\omega \mu \pi a} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{j n \phi}}{H_{n}^{(2)}(k a)}-r_{z} \frac{2 E_{0}}{Z_{0}} \cos \phi e^{-j k a \cos \phi} \tag{3.23}
\end{equation*}
$$

when Eq. (3.22) is used. When Eq. (3.10) is used

$$
\begin{equation*}
\stackrel{r}{A_{N U}}=\stackrel{r}{e} e_{z} \frac{\mu_{0}}{2 \pi} \int_{z=-\infty}^{\infty} \int_{\phi=0}^{2 \pi}\left[\frac{-E_{0}}{\omega \mu \pi a} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{j n \phi^{\prime}}}{H_{n}^{(2)}(k a)}-\frac{E_{0}}{Z_{0}} \cos \phi^{\prime} e^{-j k a \cos \phi}\right] \frac{e^{-j k R}}{R} a d \phi^{\prime} d z^{\prime} \tag{3.24}
\end{equation*}
$$

Eq. (3.24) is the nonuniform scattering field. $z^{\prime}$ part of the integral is evaluated from the Eq. (3.12) so the nonuniform scattering field takes the form

$$
\begin{equation*}
\stackrel{r}{A_{N U}}=\stackrel{r}{e_{z}} \frac{-\mu_{0} E_{0}}{2 j \varrho \mu \pi} \sum_{n=-\infty}^{\infty} \frac{j^{-n}}{H_{n}^{(2)}(k a)} \int_{\phi=0}^{2 \pi} e^{j n \phi} H_{0}^{(2)}\left(k R_{1}\right) d \phi^{\prime}-\stackrel{r}{e}_{z} \frac{\mu_{0} a E_{0}}{2 j Z_{0}} \int_{\phi=0}^{2 \pi} \cos \phi^{\prime} H_{0}^{(2)}\left(k R_{1}\right) d \phi^{\prime} \tag{3.25}
\end{equation*}
$$

$R_{1}$ is the ray path and written from the Figure 5 with using cosine theorem. If this integral takes it gives exact scattering nonuniform electric field. Because of finding the exact scattering field, PO integral will be transformed to the series solution after this transformation with using known exact scattering total field from the cylinder the nonuniform exact solution will be obtained with respect to series form.


Figure 6 Ray Path of the Scattering Field
In the Figure 6 shows the scattering field ray path $R_{1}$ with respect to observation point P and angles. $a$ is the radius of the cylinder is equal to $\rho^{\prime}$ which is the distance from the origin to the surface of the scatterer and $\rho$ is the distance from the origin to observation point. Ray path $R_{1}$ is written from the cosine theorem. Cosine theorem shows that if you know any two different lengths with the angle between to each outer unknown part of the plotted from one cusp to another length can be find. Figure 6 shows this theorem.


Figure 7 Shows the Known Length a, b, Angle and Unknown Length c The unknown part c can be found from $c^{2}=a^{2}+b^{2}-2 a b \cos \theta$ this equation is called cosine theorem. From using to this theorem ray path $R_{1}$ is written as

$$
\begin{equation*}
R_{1}=\left[a^{2}+\rho^{2}-2 a \rho \cos \left(\phi-\phi^{\prime}\right)\right]^{\frac{1}{2}} \tag{3.26}
\end{equation*}
$$

in the Eq. (3.26).

## CHAPTER 4

## SERIES EXPANSION of the PO INTEGRAL and CORRECTION of the TOTAL CURRENT

### 4.1 Series Expansion of the PO Integral

PO integral was given in the Eq. (3.13). The solution of the homogeneous Helmholtz wave equation gives differential equations. If these differential equations are solved cylindric wave functions are obtained which is called Bessel functions. This functions are harmonic. Hankel functions are the special combination of Bessel functions. Hankel functions show the propagation directions of waves. Hankel functions of first kind show inward propagation and Hankel functions of second kind show outward propagation of the waves. When transforming to PO integral to series form firstly Hankel function is transformed. For this operation Hankel functions series expansion is given

$$
H_{0}^{(2)}\left(\left|\rho-\rho^{\prime}\right|\right)=\left\{\begin{array}{l}
\sum_{n=-\infty}^{\infty} H_{n}^{(2)}\left(\rho^{\prime}\right) J_{n}(\rho) e^{j n\left(\phi-\phi^{\prime}\right)}  \tag{4.1}\\
\sum_{n=-\infty}^{\infty} J_{n}\left(\rho^{\prime}\right) H_{n}^{(2)}(\rho) e^{j n\left(\phi-\phi^{\prime}\right)}
\end{array}\right.
$$

in the Eq. (4.1) from Ref. [25]. $J_{n}$ is the Bessel function of first kind, $H_{n}^{(2)}$ is the Hankel function of second kind, $\rho$ is the distance from origin to observation point and $\rho$ is the radius of the cylinder. The plane wave can be represented by an infinite sum of cylindrical wave functions of the form

$$
\begin{equation*}
e^{-j k x}=e^{-j k a \cos \phi}=\sum_{n=-\infty}^{\infty} j^{-n} J_{n}(k a) e^{j n \phi^{\prime}} \tag{4.2}
\end{equation*}
$$

from the Ref. [1]. From the trigonometric relation cosine function can be written as

$$
\begin{equation*}
\cos \phi^{\prime}=\frac{1}{2}\left[e^{j \phi^{\prime}}+e^{-j \phi^{\prime}}\right] \tag{4.3}
\end{equation*}
$$

in the Eq. (4.3). Combination of the Eq. (4.2) and Eq. (4.3) PO integrals excepting part of Hankel function is written as

$$
\begin{equation*}
\cos \phi^{\prime} e^{-j k a \cos \phi^{\prime}}=\sum_{n=-\infty}^{\infty} \frac{j^{-n} J_{n}(k a)}{2}\left[e^{j \phi^{\prime}(1+n)}+e^{-j \phi^{\prime}(1-n)}\right] \tag{4.4}
\end{equation*}
$$

in the Eq. (4.4).
Hankel function part of the PO integral is written as

$$
\begin{equation*}
H_{0}^{(2)}\left(k R_{1}\right)=\sum_{n=-\infty}^{\infty} J_{n}(k a) H_{n}^{(2)}(k \rho) e^{j n\left(\phi-\phi^{\prime}\right)} \tag{4.5}
\end{equation*}
$$

in the Eq. (4.5) with respect to Eq. (4.1). In this equation $R_{1}$ is the ray path. PO integral takes the form

$$
\begin{equation*}
\stackrel{r}{A}=\stackrel{r}{e_{z}} \frac{\mu_{0} E_{0} a}{2 Z_{0} j} \sum_{n=-\infty}^{\infty} \int_{\phi^{\prime}=\frac{\pi}{2}}^{\frac{3 \pi}{2}} \frac{j^{-n} J_{n}(k a)}{2}\left[e^{j \phi^{\prime}(1+n)}+e^{-j \phi^{\prime}(1-n)}\right] J_{n}(k a) H_{n}^{(2)}(k \rho) e^{j n\left(\phi-\phi^{\prime}\right)} d \phi^{\prime} \tag{4.6}
\end{equation*}
$$

If the integral part is imposed absence of the coefficient, integral takes the form

$$
\begin{equation*}
I_{1}=\sum_{n=-\infty}^{\infty} j^{-n} J_{n}(k a) J_{n}(k a) H_{n}^{(2)}(k \rho) \frac{1}{2} \int_{\phi^{\prime}=\frac{\pi}{2}}^{\frac{3 \pi}{2}}\left[e^{j \phi^{\prime}(1+n)}+e^{-j \phi^{\prime}(1-n)}\right] e^{j n\left(\phi-\phi^{\prime}\right)} d \phi^{\prime} \tag{4.7}
\end{equation*}
$$

in the Eq. (4.7). From this point integral was changed to evaluation known exponential integral. Just the integral part is given

$$
\begin{equation*}
I_{2}=\frac{e^{j n \phi}}{2} \int_{\phi^{\prime}=\frac{\pi}{2}}^{\frac{3 \pi}{2}}\left[e^{j \phi^{\prime}(1+n-n)}+e^{-j \phi^{\prime}(1-n+n)}\right] d \phi^{\prime} \tag{4.8}
\end{equation*}
$$

in the Eq. (4.8). The solution of this integral is given

$$
\begin{equation*}
I_{2}=-\left.e^{j n \phi} \sin \phi^{\prime}\right|_{\phi^{\prime}=\frac{\pi}{2}} ^{\frac{3 \pi}{2}}=2 e^{j n \phi} \tag{4.9}
\end{equation*}
$$

in the Eq. (4.9). Scattering electric field which is the series form is written from (3.14) with inserting (4.9) to (4.6). It comes to
the Eq. (4.10). Exact scattered field from the shadow region can be found from the Eq. (3.16) and (4.10). The equations are inserted in the Eq. (3.18) and the solution is $\stackrel{{ }_{E}^{N U}}{E_{s}}=E_{0} \sum_{n=-\infty}^{\infty} j^{-n} a_{n} H_{n}^{(2)}(k \rho) e^{j n \phi}+\frac{\omega \mu_{0} E_{0} a}{Z_{0}} \sum_{n=-\infty}^{\infty} j^{-n} J_{n}(k a) J_{n}(k a) H_{n}^{(2)}(k \rho) e^{j n \phi}$
to be founded with respect to series solution in the Eq. (4.11).

### 4.2 Correction of the Total Current Flowing on the Cylinder

Although the solution of the Harrington was used derivation of the total current does not mach the same result. Derivation of the total current can be found from the total electric field which is given

$$
\begin{equation*}
\stackrel{\mathrm{r}}{E_{z}}=E_{0} \sum_{n=-\infty}^{\infty} j^{-n}\left[J_{n}(k \rho)+a_{n} H_{n}^{(2)}(k \rho)\right] e^{j n \phi} \tag{4.12}
\end{equation*}
$$

in the Eq. (4.12). $a_{n}$ which is complete the solution was given in (3.17).
The surface current on the cylinder can be obtained from

$$
\begin{equation*}
J_{z}=\left.H_{\phi}\right|_{\rho=a}=\left.\frac{1}{j \omega \mu} \frac{\partial E_{z}}{\partial \rho}\right|_{\rho=a} . \tag{4.13}
\end{equation*}
$$

If the derivative of the Eq. (4.13) is taken

$$
\begin{equation*}
J_{z}=\frac{E_{0}}{j \omega \mu} \sum_{n=-\infty}^{\infty} j^{-n} e^{j n \emptyset} \frac{\partial}{\partial \rho}\left[J_{n}(k \rho)+a_{n} H_{n}^{(2)}(k \rho)\right]_{\rho=a} \tag{4.14}
\end{equation*}
$$

Eq. (4.14) is produced with respect to Eq. (4.12) and (4.13). Derivative part of the equation can be shown

$$
\begin{equation*}
I=\left.J_{n}^{\prime}(k \rho)\right|_{\rho=a}+\left[a_{n}^{\prime} H_{n}^{(2)}(k \rho)+H_{n}^{\prime(2)}(k \rho) a_{n}\right]_{\rho=a} \tag{4.15}
\end{equation*}
$$

in the Eq. (4.15). $a_{n}$ part of these equation does not include the $\rho$ term so that the derivative of these part is equal to zero. Just Hankel part of these equation can be derivatived. Finally the equation takes the form

$$
\begin{equation*}
I=J_{n}^{\prime}(k a)-H_{n}^{\prime(2)} \frac{J_{n}(k a)}{H_{n}^{(2)}(k a)} \tag{4.16}
\end{equation*}
$$

and if the equation is regulated

$$
\begin{equation*}
I=\frac{J_{n}^{\prime}(k a) H_{n}^{(2)}(k a)-H_{n}^{\prime(2)}(k a) J_{n}(k a)}{H_{n}^{(2)}(k a)} \tag{4.17}
\end{equation*}
$$

Eq. (4.17) is obtained. From this point alternative representation of the Hankel functions with respect to first and second order Bessel functions can be used. First and second order Bessel functions are shown

$$
\begin{equation*}
H_{n}^{(1)}(k a)=J_{n}(k a)+j N_{n}(k a) \tag{4.18}
\end{equation*}
$$

and

$$
\begin{equation*}
H_{n}^{(2)}(k a)=J_{n}(k a)-j N_{n}(k a) \tag{4.19}
\end{equation*}
$$

in the Eq. (4.18) and (4.19). Help of this equations Bessel functions can be written

$$
\begin{equation*}
J_{n}(k a)=\frac{1}{2}\left[H_{n}^{(1)}(k a)+H_{n}^{(2)}(k a)\right] \tag{4.20}
\end{equation*}
$$

and

$$
\begin{equation*}
N_{n}(k a)=\frac{1}{2 j}\left[H_{n}^{(1)}(k a)-H_{n}^{(2)}(k a)\right] . \tag{4.21}
\end{equation*}
$$

Now all the transformations were obtained. The nominator of the Eq. (4.17) is written as

$$
\begin{equation*}
I I=J_{n}^{\prime}(k a) H_{n}^{(2)}(k a)-H_{n}^{\prime(2)}(k a) J_{n}(k a) \tag{4.22}
\end{equation*}
$$

Bessel function transformation which is in the Eq. (4.20) is inserted in the Eq. (4.22)

$$
\begin{equation*}
I I=\frac{1}{2}\left[H_{n}^{\prime 1}(k a)+H_{n}^{\prime(2)}(k a)\right] H_{n}^{(2)}(k a)-H_{n}^{\prime(2)}(k a) \frac{1}{2}\left[H_{n}^{(1)}(k a)+H_{n}^{(2)}(k a)\right] \tag{4.23}
\end{equation*}
$$

than Eq. (4.23) is obtained. These equations can be reducible to

$$
\begin{equation*}
I I=\frac{H_{n}^{\prime(1)}(k a) H_{n}^{(2)}(k a)-H_{n}^{\prime(2)}(k a) H_{n}^{(1)}(k a)}{2} \tag{4.24}
\end{equation*}
$$

the Eq. (4.24). Using the alternative representation of the Hankel functions which are given in the Eq. (4.18) and (4.19), Eq. (4.24) takes the form

$$
\begin{equation*}
I I=\frac{J_{n}^{\prime} J_{n}-j J_{n}^{\prime} N_{n}+j N_{n}^{\prime} J_{n}+N_{n}^{\prime} N_{n}-J_{n}^{\prime} J_{n}-j J_{n}^{\prime} N_{n}+j N_{n}^{\prime} J_{n}-N_{n}^{\prime} N_{n}}{2} . \tag{4.25}
\end{equation*}
$$

Simplification of the Eq. (4.25) gives the equation

$$
\begin{equation*}
I I=\frac{2 j\left(N_{n}^{\prime} J_{n}-N_{n} J_{n}^{\prime}\right)}{2}=j\left(N_{n}^{\prime} J_{n}-N_{n} J_{n}^{\prime}\right) . \tag{4.26}
\end{equation*}
$$

From the Ref. [25] combination of the Bessel functions are written

$$
\begin{equation*}
N_{n}^{\prime}(k a) J_{n}(k a)-N_{n}(k a) J_{n}^{\prime}(k a)=\frac{2}{\pi k a} \tag{4.27}
\end{equation*}
$$

so the Eq. (4.27) is obtained as equal to

$$
\begin{equation*}
I I=\frac{2 j}{\pi k a} . \tag{4.28}
\end{equation*}
$$

If the Eq. (4.28) is inserted in the Eq. (4.17) derivative part was obtained as

$$
\begin{equation*}
I=\frac{2}{\pi k a H_{n}^{(2)}(k a)} \tag{4.29}
\end{equation*}
$$

in the Eq. (4.29). Eq. (4.17) shows the derivation part of the Eq. (4.14). Finally the total current on the cylinder is obtained with inserting the derivation part of the Eq. (4.29) into (4.14).

These current is found as

$$
\begin{equation*}
J_{z}=\frac{2 E_{0}}{\omega \mu \pi k a} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{j n \phi}}{H_{n}^{(2)}(k a)} \tag{4.30}
\end{equation*}
$$

Eq. (4.30) shows the total current which is flowing on the cylinders surface. That is obtained from the total electric field which is including incident electric and scattered electric fields. According to Ref. [25] total surface current on the cylinder is given

$$
\begin{equation*}
J_{z}=\frac{-2 E_{0}}{\omega \mu \pi a} \sum_{n=-\infty}^{\infty} \frac{j^{-n} e^{j n \phi}}{H_{n}^{(2)}(k a)} \tag{4.31}
\end{equation*}
$$

in the Eq. (4.31). In the appearance there would not to seem the difference but our solution is denoted that there is a difference in the amplitudes of the two solutions. Our solution has positive amplitude and it is include wave number factor $k$ on the denominator part of the amplitude but according to Ref. [25] the solution does not include $k$ on the denominator part and that solution has negative amplitude.


Figure 8 Total Corrected Surface Current


Figure 9 Total Surface Current According to Harrington

To the Figure 8 and Figure 9 amplitude difference can be shown clearly. Impact of the wave factor, which is in the denominator, decreases the amplitude of the solution.

## CHAPTER 5

## NUMERICAL ANALYSIS

In this analysis part we will investigate the total current, PO current, fringe current, total scattered field, PO scattered field and fringe field. The distance between the observation point and the origin will be taken as $6 \lambda$ which is constant for all plots and $\lambda$ is the wavelength. Radius of the cylinder a will be taken $2 \lambda$ for all plots. Figure 10 plot shows the total current pattern with respect to observation angle. Amplitude of the current shows the equal distribution between the $90^{\circ}$ and the $270^{\circ}$. It takes the maximum value at the $180^{\circ}$. Figure 11 plot shows PO current on the cylinders lit region. Incident wave hits the left hand side of the cylinder means it propagates positive $x$ direction so PO current has equal distribution over the $180^{\circ}$. It includes GO and incident fields for lit region.


Figure 10 Total Surface Current


Figure 11 Uniform Surface Current

Figure 12 plots show the nonuniform current on the cylinder. Current amplitude consistency is between $120^{\circ}$ and $240^{\circ}$. It takes maximum amplitude at $180^{\circ}$. This current is difference total current and PO current. Figure 13 plots shows that total scattered electric field. It shows us field variations with respect to observation angle. It has one major lobe at the angel of $180^{\circ}$ and one minor lobe at the angle of $0^{\circ}$. This plot shows that there is a whole radiation from cylinders surface but the radiation is more powerful in the major lobs than minors.


Figure 12 Nonuniform Current


Figure 13 Total Scattered Electric Field

Figure 14 shows that PO (uniform) scattered electric field. Scattered field from the lit region of cylinder shows us equal distribution between the angles of $90^{\circ}$ and $270^{\circ}$. There is no radiation where the shadow region angles. All the radiation is bounded with the PO boundaries. Figure 15 shows that fringe (nonuniform) scattered field. Fringe scattered field takes the maximum values at the angel $90^{\circ}$ and $270^{\circ}$. This shows that fringe radiation is compensates the physical optics defects on the shadow part of the scatterer. PO radiation is just to cover lit region which is bounded the angle $90^{\circ}$ and $270^{\circ}$ but this plot shows that the radiation is exist to other angles.


Figure 14 Uniform Scattered Electric Field


Figure 15 Nonuniform Scattered Electric Field

## CHAPTER 6

## CONCLUSION

In this thesis nonuniform currents flowing on the cylinder and the nonuniform scattered electric field from this current investigated. Although diffraction of the electromagnetic waves from the cylinder is one of the fundamental scattering problem, literature did not include the application of the PTD for cylindric structure. Contribution of this thesis to the literature is the application of PTD to the PEC cylinder with the series expansion. The solution of the homogeneous Helmholtz wave equation for the cylindric coordinate system gives cylindrical wave functions. They can be represented with Bessel functions because Bessel functions behavior are harmonic. Uniform currents and uniform scattering field were obtained with using PO method. PO integral was converted to the series form using the Hankel functions which is the combination of the solution Bessel's equation than to take into account the known exact scattering field from the cylinder, the contribution of the nonuniform current to the scattering field obtained with using PTD method. In this thesis PO integral was converted to series solution because its conversion is much easier than conversion of the total exact scattering field from series to integral. In addition nonuniform scattering field can be found from the integration of the fringe current but it is too complex to evaluation of the exact current over the surface of the scatterer. In the solution of the Harrington [25] total field from obtained the total electric field has defect. This defect is the result of the solution of the total current. This defect effects on the amplitude of the total current so it fixed with the solving of the currents derivative according to boundary conditions. Alternative representations of the Bessel and Hankel functions was used when investigation of the correct solution of the total current on the cylinder. To be a last step of solving problem MatLab codes were performed for all the currents and the fields for discussion of the numerical results.

## REFERENCES

1. Balanis C. A., (1989), "Advanced Engineering Electromagnetics", Wiley, New York.
2. Keller J. B., (1962), "Geometrical Theory of Diffraction", J.Opt.Soc.Am., vol. 52, pp.116-130.
3. Kouyoumjian R. G. and Pathak P. H., (1974), "A Uniform Theory of Diffraction for Edge in a Perfectly Conducting Surface", Prog. IEEE, vol. 62 no. 11, pp. 1448-1461.
4. Knott E.F, Senior T.B.A, (1974), "Comparison of Three High-frequency Diffraction Techniques", Proc.IEEE, vol. 62, no. 11, pp. 1468-1474.
5. Ufimtsev P. Ya., (2007), "Fundamentals of the Physical Theory of Diffraction", Wiley, New Jersey.
6. Umul Y. Z., (2004), "Modified Theory of Physical Optics," Opt. Express, vol.12, no. 20, pp.4959-4972.
7. Pathak P., (1979), "An Asymptotic Analysis of the Scattering of Plane Waves by a Smooth Convex Cylinder", Radio.Sci.., vol.14, pp. 419-435.
8. Franz W., (1954)," Über die Green'she Funktion des Zylinders und der Kugel", Z. Naturforsch., vol. 9a, pp.705-716.
9. Borghi F., Santarsiero M., Frezza F., Schettini G., (1996), "Plane-wave Scattering by A Perfectly Conducting Circular Cylinder Near A Plane Wave Surface: Cylindrical-Wave Approach", J. Opt. Soc. Am., vol. 13, pp. 483493.
10. Kouyoumjian R. G., (1965), "Asymptotic High-Frequency Methods", Proc. IEEE, vol.53, pp.864-876.
11. Keller J. B., (1956), "Diffraction by a Convex Cylinder", IRE. Trans. on Antennas and Propagation, vol. AP-4, pp. 312-321.
12. Rayliigh L., (1881), "On the Electromagnetic Theory of Light", Phill. Mag. (GB), vol.12, pp. 81-101.
13. Epstein P. S., (1914), "Ph.D. dissertation", Munich, Germany.
14. Debye P., (1908), " Das Electromagnetisch Feld um einen Zylinder und Die Theorie des Regenbogens", Phys. Z., vol.9, pp.775-779.
15. Riblet H. J., (1959), "Second Order Geometric Optic Currents on a Cylinder", in Proc.McGill Symp. On Microwave Optics, pt. II Bedford, Mass.: Electronics Research Directorate, AF Cambridge Research Center, pp. 215-225.
16. Wetzel L., (1957), " High Frequency Current Distributions on Conducting Obstacles", Cruft Lab., Harvard University, Cambridge, Mass., Scientific Rept. 10, Contract AF 19, pp.604-786.
17. Waith J. R., (1959), "Electromagnetic Radiation from Cylindrical Structures", Pergamon, New York, Based on NBS Rept. 5553, 1958.
18. Lawrence D. E., Sarabandi K., (2002), "Electromagnetic Scattering from A Dielectric Cylinder Buried Beneath A Slightly Rough Surface", IEEE Trans Antennas and Propogation, vol. 50, no. 10.
19. Yalçın U., (2007), "Scattering from A Cylindrical Reflector: Modified Theory of Physical Optics Solution", J. Opt. Soc. Am. A., vol. 24, no.2, pp.502-506.
20. Umul Y. Z., Yengel E., Aydın A., (2003), " Comparision of Physical Optics Integral and Exact Solution for Cylinder Problem", in Proceedings of ELECO 2003, Third International Conference on Electrical and Electronics Engineering, Bursa, Turkey, pp.245-248, 31-34.
21. Borghi R., Gori F., Santarsiero M., Frezza F., Schettini G., (1996), " Plane Wave Scattering by A Perfectly Conducting Circular Cylinder Near A Plane Surface: Cylindrical-Wave Approach", J. Opt. Soc. Am. A., vol. 13, no.3, pp.483-493.
22. Büyükaksoy A., Uzgören G., (1988), "Diffraction of High Frequency Waves by A Cylindrically Curved Surface with Different Face Impedance", IEEE Transactions on Antennas and Propagation, vol. 36, no.5, pp.592-600.
23. Silver S., (1949), "Microwave Antenna Theory and Design", McGraw-Hill, New York.
24. Ufimtsev P.Ya., (1962), "Method of Edge Waves in the Physical Theory of Diffraction" Izd-Vo Sovyetskoye Radio, pp.1-243.
25. Harrington R.F., (2001), "Time-Harmonic Electromagnetic Fields", Wiley, New York, IEE Press Series on Electromagnetic Wave Theory.
26. Samii Y. R., (2004), "Lectures Notes on Advanced Engineering Dynamics," UCLA.
27. Sommerfeld A., (1896), "Mathematische Theorie der Diffraction," Math. Ann. 47, pp.317-374.
28. Bayrakçı H. E., (2000), "Elektromagnetik Alan Teorisi", Birsen Yaynnevi, İstanbul.

## APPENDIX A

## MATLAB CODES

## PO current

> l=0.1;
k=2.*pi/l;
fi=pi./2:0.01:3.*pi./2;
$a=2 . *$;
$\mathrm{Jpo}=\mathrm{k} . * \exp (-\mathrm{j} . * \mathrm{k} . * \mathrm{a} . * \cos (\mathrm{fi})) . * \cos (\mathrm{fi})$;
polar(fi,abs(Jpo))
\%title('PO current(uniform) on the Cylinder')

## Cylinder Nonuniform Current

$1=0.1$;
$\mathrm{k}=\left(2 .{ }^{*} \mathrm{pi}\right) . / \mathrm{l}$;
r=6.*1;
fi=0:0.01:2. *pi;
$\mathrm{a}=2 .{ }^{*} 1$;
$\mathrm{N}=1000$;
sum=0;
for $\mathrm{n}=-\mathrm{N}: \mathrm{N}$;
$\mathrm{Jz}=(1 . / \mathrm{pi} . * \mathrm{k} . * \mathrm{a}) . *(\mathrm{j} . \wedge(-\mathrm{n})) . * \exp (\mathrm{j} . * \mathrm{n} . * \mathrm{fi}) . / \mathrm{besselh}(\mathrm{n}, 2, \mathrm{k} . * \mathrm{a}) ;$
sum=sum+Jz;
end
fi2=pi./2:0.005:3.*pi./2;
Jpo=k.*exp(-j.*k.*a.* $\cos (f i 2)) . * \cos (f i 2)$;
Jnu=sum-Jpo;
polar(fi2,abs(Jnu))
\%title('Fringe (nonuniform) Current on the Cylinder')

## Total Surface Current on The Cylinder

$1=0.1$;

```
k=(2. *pi)./l;
r=6.*1;
fi=0:0.01:2.*pi;
a=2.*1;
N=1000;
sum=0;
for n=-N:N;
    Jz=(1./pi.*k.*a).*(((j.^(-n)).*exp(j.*n.*fi))./besselh(n,2,k.*a));
    sum=sum+Jz;
end
polar(fi,abs(sum))
hold on
%title('Total Surface Current on the Cylinder')
```


## PO Scattered Electric Field

```
l=0.1;
k=2.*pi./l;
r=6.*1;
a=2.*1;
fi=pi./2:0.01:3.*pi./2;
N=1000;
sum=0;
asinir=pi./2;
usinir=3.*pi./2;
delta=(usinir-asinir)./N;
for i=0:N
    fii=asinir+(i.*delta);
    R1=sqrt((r.^2)+(a.^2)-2.*r.*a.*\operatorname{cos(fi-fii));}
    Epo=cos(fii).*exp(-j.*k.*a.*cos(fii)).*besselh(0,2,k.*R1);
    sum=sum+Epo;
end
f=sum.*delta;
polar(fi,abs(f))
%title('PO Scattered E Field')
```


## Nonuniform Scattered Electric Field

$\mathrm{l}=0.1$;
$\mathrm{k}=\left(2 .{ }^{*} \mathrm{pi}\right) . / \mathrm{l}$;
r=6.*1;
fi=0:0.01:2.*pi;
$\mathrm{a}=2 . * 1$;
$\mathrm{N}=359$;

```
sum=0;
for n=-N:N;
    Ez=(j.^(-n)).*(-besselj(n,k.*a)./besselh(n,2,k.*a)).*besselh(n,2,k.*r).*exp(j.*n.*fi);
    sum=sum+Ez;
end
% polar(fi,sum)
% title('Total Scatered E field on the Cylinder')
```

$\mathrm{N} 1=359$;
sum2=0;
for $\mathrm{n}=-\mathrm{N}: \mathrm{N}$;
$\operatorname{Ez1} 1=(\mathrm{j} . \wedge-\mathrm{n}) . * \operatorname{bessel}(\mathrm{n}, \mathrm{k}, \mathrm{a}) . * \operatorname{besselj}(\mathrm{n}, \mathrm{k}, \mathrm{a}) . * \operatorname{besselh}(\mathrm{n}, 2, \mathrm{k} . * \mathrm{r}) . * \exp (\mathrm{j} . * \mathrm{n} . * \mathrm{fi}) ;$
sum2=sum2+Ez1;
end
$\mathrm{f}=$ sum-(-sum2);
polar(fi,abs(f))
\%title('Fringe(nonuniform) Scattered E Field')

## Total Scattered Electric Field From The Cylinder

```
l=0.1;
k=(2.*pi)./l;
r=6.*1;
fi=0:0.01:2.*pi;
a=2.*1;
N=359;%359
sum=0;
for n=-N:N;
    an=-besselj(n,k.*a)./besselh(n,2,k.*a);
    Ez=(j.^(-n)).*(an).*besselh(n,2,k.*r).*exp(j.*n.*fi);
    sum=sum+Ez;
end
polar(fi,abs(sum))
%title('Total Scattered E Field on the Cylinder')
```


## Corrected Surface Current

$\mathrm{l}=0.1$;
k=(2.*pi)./l;
r=6.*1;
fi=0:0.01:2.*pi;
$\mathrm{a}=2$.*1;
$\mathrm{N}=1000$;
sum=0;
for $\mathrm{n}=-\mathrm{N}: \mathrm{N}$;

```
    Jz=(-1).*(((j.^(-n)).*exp(j.*n.*fi))./besselh(n,2,k.*a));
    sum=sum+Jz;
end
polar(fi,abs(sum))
hold on
%title('Total Surface Current on the Cylinder According to Harrington')
N=1000;
sum1=0;
for n=-N:N;
    Jz1=(1./k).*((j.^(-n)).*exp(j.*n.*fi))./besselh(n,2,k.*a);
    sum1=sum1+Jz1;
end
polar(fi,abs(sum1),'-.r')
%title('Total Corrected Surface Current')
```


## APPENDIX B

## CURRICULUM VITALE

## PERSONAL INFORMATION

Surname, Name: Başdemir, Hüsnü Deniz
Date and Place of Birth: 1 April 1985, Doğanhisar
Marital Status: Single
Phone: +90 5433361635
Email: basdemir@cankaya.edu.tr
EDUCATION

| Degree | Institution | Year of Graduation |
| :--- | :--- | :--- |
| M.S. | Çankaya Univ., Elec. and <br> Comm. Engineering | 2010 |
| B.S. | Çankaya Univ., Elec. and <br> Comm. Engineering | 2008 |
| High School | Akşehir Anatolian High <br> School | 2003 |

WORK EXPERIENCE

| Year | Place | Enrollment |
| :--- | :--- | :--- |
| 2009- Present | Çankaya University | Specialist |
| 2007 July | Türk Telecom. | Trainer |
| 2006 July | Ortana Co. | Trainer |

## FOREIN LANGUAGES

Advanced English, Beginer French

## HOBBIES

Chess, Puzzle, Books, Swimming

