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Solutions of nonlinear systems by reproducing kernel method

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Abstract

A novel approximate solution is obtained for viscoelastic fluid model by reproducing kernel method (RKM). The resulting equation for viscoelastic fluid with magneto-hydrodynamic flow is transformed to the nonlinear system by introducing the dimensionless variables. Results are presented graphically to study the efficiency and accuracy of the reproducing kernel method. Results show that this method namely RKM is an efficient method for solving nonlinear system in any engineering field.

Keywords: Reproducing kernel method, series solutions, nonlinear systems.
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1. Introduction

We conceive the following nonlinear system in this work [29]:

$$\begin{cases} z^{(\nu)} + a_0(\varpi)z^{(i\nu)} + a_1(\varpi)z''' + a_2(\varpi)z'' + a_3(\varpi)z' + a_4(\tau)z \\ \qquad +s^{(\nu)} + a_5(\varpi)s^{(i\nu)} + a_6(\varpi)s''' + a_7(\varpi)s'' + a_8(\varpi)s' + a_9(\varpi)s + M_1(z,s) = 0, \\ \\ z^{(\nu)} + b_0(\varpi)z^{(i\nu)} + b_1(\varpi)z''' + b_2(\varpi)z'' + b_3(\varpi)z' + b_4(\varpi)z \\ \qquad +s^{(\nu)} + b_5(\varpi)s^{(i\nu)} + b_6(\varpi)s''' + b_7(\varpi)s'' + b_8(\varpi)s' + b_9(\varpi)s + M_2(z,s) = 0, \\ \\ z = 0, \quad z' = 0, \quad s = 0, \quad \text{at} \quad \varpi = -1, \\ z = 0, \quad z' = 0, \quad s = 0, \quad \text{at} \quad \varpi = 1, \end{cases} \tag{1.1}$$

where M_1 and M_2 are nonlinear functions of z, s, and $a_j(\varpi), b_j(\varpi)$ are continuous, $j = 0, 1, \dots, 9$. Ordinary differential systems are significant for real–world problems. These systems have been implemented to many problems [8–10, 17]. Biswas et al. studied systems by many different methods [11, 15, 20].

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In this work, we demonstrate the solutions of (1.1). We suppose that (1.1) has a unique solution. Put

$$\begin{cases} B_{11}z = z^{(\nu)} + a_0(\varpi)z^{(i\nu)} + a_1(\varpi)z''' + a_2(\varpi)z'' + a_3(\varpi)z' + a_4(\varpi)z, \\ B_{12}s = s^{(\nu)} + a_5(\varpi)s^{(i\nu)} + a_6(\varpi)s''' + a_7(\varpi)s'' + a_8(\varpi)s' + a_9(\varpi)s, \\ B_{21}z = z^{(\nu)} + b_0(\varpi)z^{(i\nu)} + b_1(\varpi)z''' + b_2(\varpi)z'' + b_3(\varpi)z' + b_4(\varpi)z, \\ B_{22}s = s^{(\nu)} + b_5(\varpi)s^{(i\nu)} + b_6(\varpi)s''' + b_7(\varpi)s'' + b_8(\varpi)s' + b_9(\varpi)s, \\ B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \quad \text{and} \quad \nu = (z,s)^T, \end{cases}$$

then (1.1) can be written as:

$$\begin{cases} L\nu = M(z,s), \quad -1 \leqslant \varpi \leqslant 1, \\ \nu(-1) = 0 = \nu(1), \end{cases} \tag{1.2}$$

where $M = (M_1, M_2)^T$, $\nu \in \widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$, $M \in \widehat{W}_2^1[-1,1] \oplus \widehat{W}_2^1[-1,1]$. The space

$$\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1],$$

is defined as

$$\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1] = \{\nu = (z,s)^T \mid h, \nu \in \widehat{W}_2^6[-1,1]\}.$$

The inner product and norm are presented as

$$\langle \nu, u \rangle = \sum_{i=1}^{2} \langle \nu_i, u_i \rangle_{\widehat{W}_2^6}, \quad \|\nu\| = \left(\sum_{i=1}^{2} \|\nu_i\|^2 \right)^{\frac{1}{2}}, \quad \nu, u \in \widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1].$$

$\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$ is a reproducing kernel space. $\widehat{W}_2^1[-1,1] \oplus \widehat{W}_2^1[-1,1]$ may be described in a similar way.

The theory of reproducing kernels [5] was utilized for the first time by Zaremba. Reproducing kernel theory has significant implementations in numerical analysis, differential equations, probability and statistics [18, 35, 37, 40]. Recently, utilizing the reproducing kernel method (RKM), some authors investigated fractional differential equations, nonlinear oscillators with discontinuity, singular and nonlinear partial differential equations [1–4, 6, 7, 12–14, 16, 19, 21–28, 30–34, 36, 38, 39, 41].

This paper is prepared as follows. Section 2 presents several useful reproducing kernel Hilbert spaces and reproducing kernel functions. The representation in $\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$ and a related linear operator are given in Section 3. This section gives the main results. Numerical examples are shown in Section 4. The final section contains some conclusions.

2. Reproducing kernel spaces and their reproducing kernel functions

Definition 2.1. We give the space $\widehat{W}_2^6[-1,1]$ as

$$\widehat{W}_2^6[-1,1] = \{\nu \in AC[-1,1] : \nu', \nu'', \nu^{(3)}, \nu^{(4)}, \nu^{(5)} \in AC[-1,1],\ \nu^{(6)} \in L^2[-1,1], \\ \nu(-1) = \nu'(1) = \nu'(-1) = \nu'(1) = 0\}.$$

The inner product and the norm in $\widehat{W}_2^5[-1,1]$ are given as

$$\langle v,u\rangle_{\widehat{W}_2^6} = \sum_{i=0}^{5} v^{(i)}(0)u^{(i)}(0) + \int_0^1 v^{(6)}(x)u^{(6)}(x)dx, \quad v,u \in \widehat{W}_2^6[-1,1],$$

and

$$\|v\|_{\widehat{W}_2^6} = \sqrt{\langle v,v\rangle_{\widehat{W}_2^6}}, \quad v \in \widehat{W}_2^6[-1,1].$$

Theorem 2.2. *Reproducing kernel function $\widetilde{R_y}$ of $\widehat{W}_2^6[-1,1]$ is obtained as:*

$$\widetilde{R_y}(x) = \begin{cases} \sum_{i=1}^{12} c_i(y)x^{i-1}, & x \leqslant y, \\ \\ \sum_{i=1}^{12} d_i(y)x^{i-1}, & x > y. \end{cases}$$

Proof. By Definition 2.1, we have

$$\left\langle v,\widetilde{R_y}\right\rangle_{\widehat{W}_2^6} = \sum_{i=0}^{5} v^{(i)}(-1)\widetilde{R_y}^{(i)}(-1) + \int_{-1}^1 v^{(6)}(x)\widetilde{R_y}^{(6)}(x)dx, \quad v,\widetilde{R_y} \in \widehat{W}_2^6[-1,1].$$

Integrating this equation by parts, we have

$$\begin{aligned}\left\langle v,\widetilde{R_y}\right\rangle_{\widehat{W}_2^6} =& v(-1)\widetilde{R_y}(-1) + v'(-1)\widetilde{R_y}'(-1) + v''(-1)\widetilde{R_y}''(-1) \\ &+ v^{(3)}(-1)\widetilde{R_y}^{(3)}(-1) + v^{(4)}(-1)\widetilde{R_y}^{(4)}(-1) + v^{(5)}(-1)\widetilde{R_y}^{(5)}(-1) \\ &+ v^{(5)}(1)\widetilde{R_y}^{(6)}(1) - v^{(5)}(-1)\widetilde{R_y}^{(6)}(-1) - v^{(4)}(1)\widetilde{R_y}^{(7)}(1) \\ &+ v^{(4)}(-1)\widetilde{R_y}^{(7)}(-1) + v'''(1)\widetilde{R_y}^{(8)}(1) - v'''(-1)\widetilde{R_y}^{(8)}(-1) \\ &- u''(1)\widetilde{R_y}^{(9)}(1) + v''(-1)\widetilde{R_y}^{(9)}(-1) + v'(1)\widetilde{R_y}^{(10)}(1) \\ &- v'(-1)\widetilde{R_y}^{(10)}(-1) - v(1)\widetilde{R_y}^{(11)}(1) \\ &+ v(-1)\widetilde{R_y}^{(11)}(-1) + \int_{-1}^1 v(x)\widetilde{R_y}^{(12)}(x)dx.\end{aligned} \tag{2.1}$$

By reproducing property, we have

$$\langle v,\widetilde{R_y}\rangle_{\widehat{W}_2^6} = v(y).$$

Since $\widetilde{R_y} \in \widehat{W}_2^6[-1,1]$, we get

$$\widetilde{R_y}(-1) = R_y'(-1) = \widetilde{R_y}(1) = R_y'(1) = 0. \tag{2.2}$$

If

$$\begin{cases} \widetilde{R_y}''(-1) + \widetilde{R_y}^{(9)}(-1) = 0, \\ \widetilde{R_y}^{(3)}(-1) - \widetilde{R_y}^{(8)}(-1) = 0, \\ \widetilde{R_y}^{(4)}(-1) + \widetilde{R_y}^{(7)}(-1) = 0, \\ \widetilde{R_y}^{(5)}(-1) - \widetilde{R_y}^{(6)}(-1) = 0, \\ \widetilde{R_y}^{(6)}(1) = 0, \\ \widetilde{R_y}^{(7)}(1) = 0, \\ \widetilde{R_y}^{(8)}(1) = 0, \\ \widetilde{R_y}^{(9)}(1) = 0, \end{cases} \tag{2.3}$$

then (2.1) gives

$$\widetilde{R_y}^{(12)}(x) = \delta(x-y).$$

When $x \neq y$,

$$\widetilde{R_y}^{(12)}(x) = 0,$$

therefore

$$\widetilde{R_y}(x) = \begin{cases} \sum_{i=1}^{12} c_i(y)x^{i-1}, & x \leqslant y, \\ \\ \sum_{i=1}^{12} d_i(y)x^{i-1}, & x > y. \end{cases} \tag{2.4}$$

Since

$$\widetilde{R_y}^{(12)}(x) = \delta(x-y),$$

we get

$$\partial^k \widetilde{R}_{y^+}(y) = \partial^k \widetilde{R}_{y^-}(y), \quad k = 0, 1, \cdots, 10, \tag{2.5}$$

and

$$\partial^{11} \widetilde{R}_{y^+}(y) - \partial^{11} \widetilde{R}_{y^-}(y) = 1. \tag{2.6}$$

The unknown coefficients $c_i(y)$ and $d_i(y)$ $(i = 1, 2, \cdots, 10)$ can be obtained from (2.2), (2.3), (2.4), (2.5), (2.6). This completes the proof. □

Definition 2.3. The space $\widehat{W}_2^1[-1,1]$ is defined by

$$\widehat{W}_2^1[-1,1] = \{u \in AC[-1,1] : \ u' \in L^2[-1,1]\}.$$

The inner product and the norm in $\widehat{W}_2^1[-1,1]$ are obtained as

$$\langle v, u \rangle_{\widehat{W}_2^1} = v(-1)u(-1) + \int_{-1}^{1} v'(x)u'(x)dx, \quad v, u \in \widehat{W}_2^1[-1,1],$$

and

$$\|v\|_{\widehat{W}_2^1} = \sqrt{\langle v, v \rangle_{\widehat{W}_2^1}}, \quad v \in \widehat{W}_2^1[-1,1].$$

Theorem 2.4. *The space $\widehat{W}_2^1[-1,1]$ is a reproducing kernel space, and its reproducing kernel function $\widetilde{Q_y}$ is obtained*

$$\widetilde{Q_y}(x) = \begin{cases} 2+x, & -1 \leqslant x \leqslant y \leqslant 1, \\ 2+y, & -1 \leqslant y < x \leqslant 1. \end{cases}$$

Proof. The proof of Theorem 2.4 is similar to the proof of Theorem 2.2. Therefore the proof is omitted. □

3. Solution representation in $\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$

Lemma 3.1. *If $B_{ij} : \widehat{W}_2^6[-1,1] \to \widehat{W}_2^1[-1,1]$, $i, j = 1, 2$, are bounded linear operators, then*

$$B : \widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1] \to \widehat{W}_2^1[-1,1] \oplus W_2^1[-1,1],$$

is a bounded linear operator.

Proof. We get

$$\begin{aligned}
\|Bu\| &= \left(\sum_{i=1}^{2} \|\sum_{j=1}^{2} B_{ij}v_j\|^2\right)^{\frac{1}{2}} \\
&\leqslant \left[\sum_{i=1}^{2}\left(\sum_{j=1}^{2}\|B_{ij}\|\|v_j\|\right)^2\right]^{\frac{1}{2}} \\
&\leqslant \left[\sum_{i=1}^{2}\left(\sum_{j=1}^{2}\|B_{ij}\|^2\right)\left(\sum_{j=1}^{2}\|v_j\|^2\right)\right]^{\frac{1}{2}} \\
&= \left(\sum_{i=1}^{2}\sum_{j=1}^{2}\|B_{ij}\|^2\right)^{\frac{1}{2}}\|v\|.
\end{aligned}$$

B is bounded by the boundedness of B_{ij}. This completes the proof. □

Now, put

$$\varphi_{ij}(x) = \widetilde{Q}_{x_i}(x)\overrightarrow{e_j} = \begin{cases} (\widetilde{Q}_{x_i}(x), 0)^T, & j = 1, \\ (0, \widetilde{Q}_{x_i}(x))^T, & j = 2, \end{cases}$$

and $\psi_{ij}(x) = B^*\varphi_{ij}(x)$, $i = 1, 2, \cdots$, $j = 1, 2$, where B^* is the conjugate operator of B. The orthonormal system of $\{\widehat{\psi}_{ij}(x)\}_{(1,1)}^{(\infty,2)}$ of $\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$ can be obtained as

$$\widehat{\psi}_{ij}(x) = \sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij}\psi_{zq}(x), \quad i = 1, 2, \cdots, \quad j = 1, 2.$$

Theorem 3.2. *Assume $\{x_i\}_{i=1}^{\infty}$ is dense in $[-1,1]$. Then $\{\psi_{ij}(x)\}_{(1,1)}^{(\infty,2)}$ is a complete system in*

$$\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1].$$

Proof. Let $\langle v(x), \psi_{ij}(x)\rangle = 0$ $(i = 1, 2, \cdots)$, for each fixed $v(x) \in \widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$. We have

$$\langle Bv(x), \varphi_{ij}(x)\rangle = 0. \tag{3.1}$$

Note that

$$v(x) = \sum_{j=1}^{2} v_j(x)\overrightarrow{e_j} = \sum_{j=1}^{2}\langle v(.), R_x(.)\overrightarrow{e_j}\rangle\overrightarrow{e_j}.$$

Thus,

$$Bv(x_i) = \sum_{j=1}^{2}\langle Bv(y), \varphi_{ij}(y)\rangle\overrightarrow{e_j} = 0, \quad (i = 1, 2, \cdots),$$

by (3.1). We obtain $(Bv)(x) = 0$. We acquire $v \equiv 0$ by B^{-1}. As a result, $\{\psi_{ij}(x)\}_{(1,1)}^{(\infty,2)}$ is a complete system in $\widehat{W}_2^6[-1,1] \oplus \widehat{W}_2^6[-1,1]$. This completes the proof. □

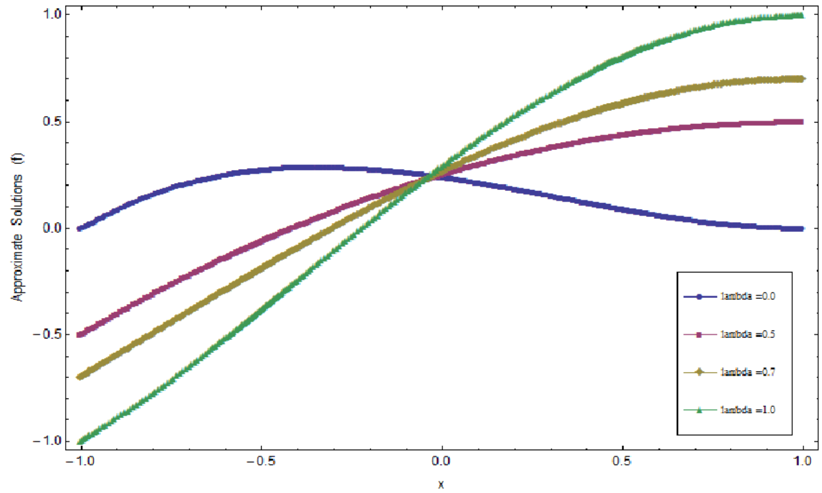

Figure 1: Approximate solutions of f for $M = 0.1, \alpha = 0.2, K = 0.1,$ and $R = 0.2$.

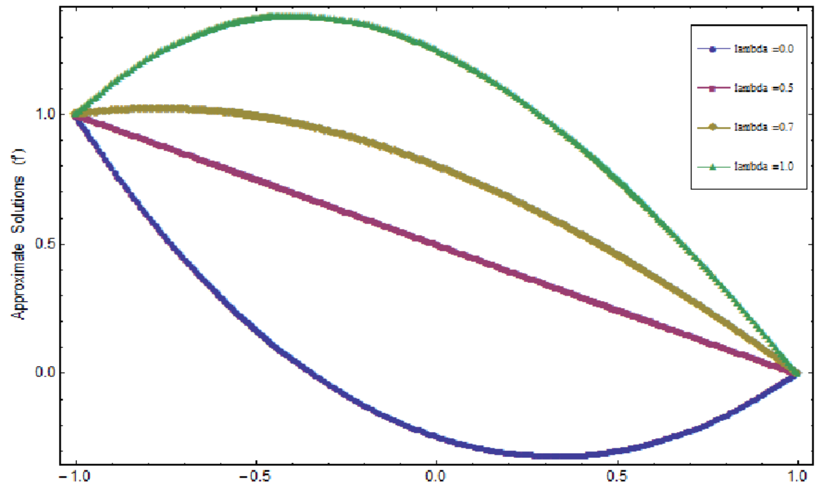

Figure 2: Approximate solutions of f' for $M = 0.1, \alpha = 0.2, K = 0.1,$ and $R = 0.2$.

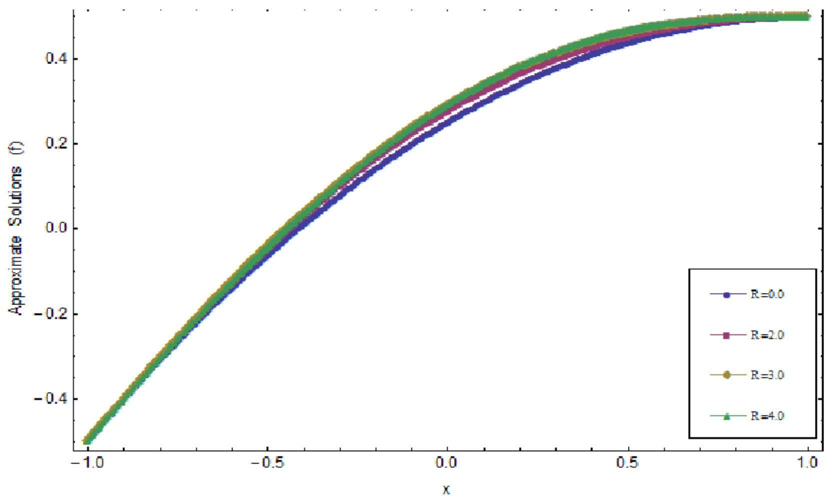

Figure 3: Approximate solutions of f for $M = 0.1, K = 0.2, \lambda = 0.5,$ and $\alpha = 0.2$.

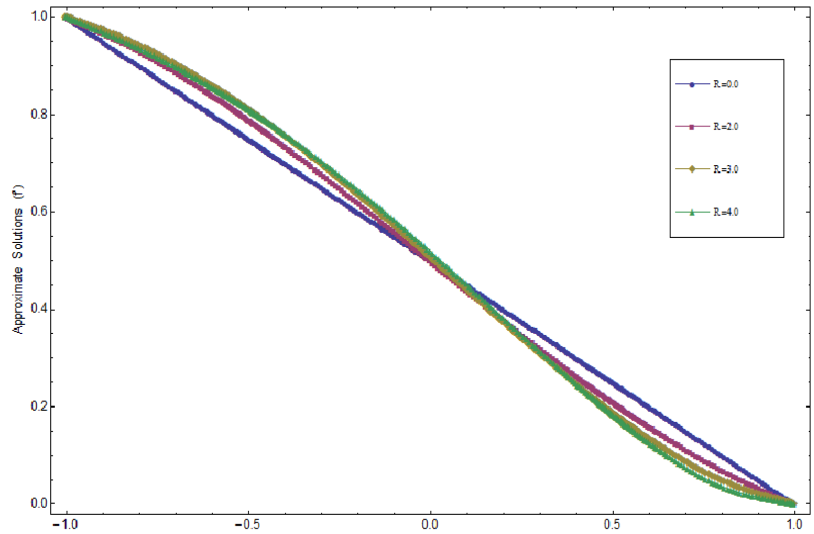

Figure 4: Approximate solutions of f' for $M = 0.1, K = 0.2, \lambda = 0.5,$ and $\alpha = 0.2$.

Theorem 3.3. *If* $\{x_i\}_{i=1}^{\infty}$ *is dense in* $[-1, 1]$, *then the solution of* (1.2) *satisfies*

$$v = \sum_{i=1}^{\infty}\sum_{j=1}^{2}\sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij} M(x_z, h(x_z), p(x_z)).$$

Proof. Let v be the solution of (1.2). By Theorem 3.2, $\{\psi_{ij}(x)\}_{(1,1)}^{(\infty,2)}$ is the complete orthonormal basis of $\widehat{W}_2^6[-1, 1] \oplus \widehat{W}_2^6[-1, 1]$. Therefore

$$\begin{aligned}
v &= \sum_{i=1}^{\infty}\sum_{j=1}^{2}\langle v(x), \widehat{\psi}_{ij}(x)\rangle\widehat{\psi}_{ij}(x)\\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{2}\langle v(x), \sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij}\widehat{\psi}_{zq}(x)\rangle\widehat{\psi}_{ij}(x)\\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{2}\sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij}\langle v(x), B^*\varphi_{zq}(x)\rangle\widehat{\psi}_{ij}(x)\\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{2}\sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij}\langle Bv(x), \varphi_{zq}(x)\rangle\widehat{\psi}_{ij}(x)\\
&= \sum_{i=1}^{\infty}\sum_{j=1}^{2}\sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij} M(x_z, h(x_z), p(x_z))\widehat{\psi}_{ij}(x).
\end{aligned}$$

□

The approximate solution v_n can be acquired from the n-term intercept of the exact solution v and

$$v_n = \sum_{i=1}^{n}\sum_{j=1}^{2}\sum_{z=1}^{i}\sum_{q=1}^{j}\beta_{zq}^{ij} M(x_z, h(x_z), p(x_z)).$$

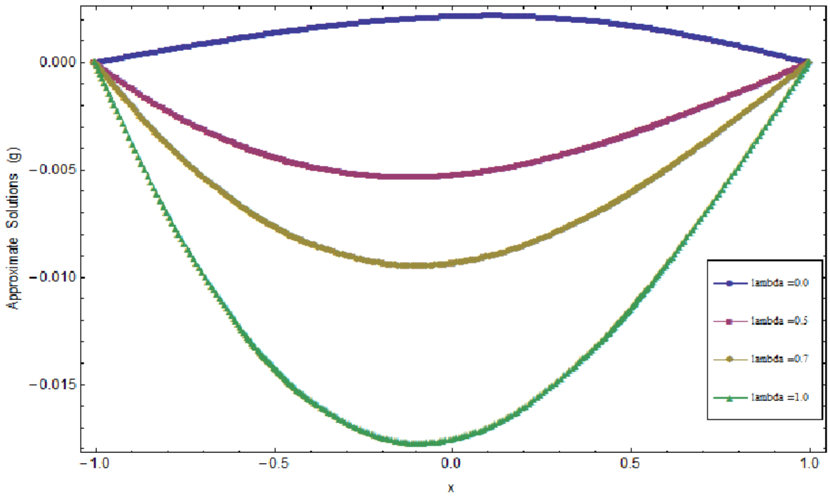

Figure 5: Approximate solutions of g for $M = 0.1, K = 0.1, \alpha = 0.2,$ and $R = 0.2$.

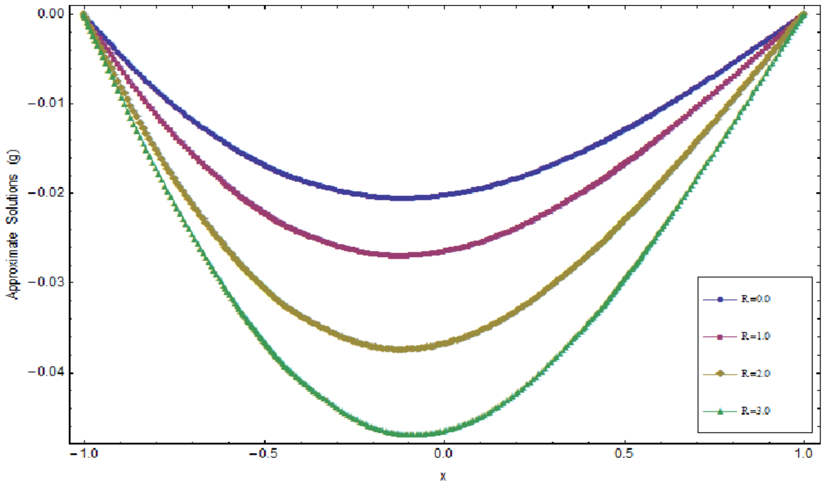

Figure 6: Approximate solutions of g for $\lambda = 0.5, K = 0.2, \alpha = 0.2,$ and $M = 0.1$.

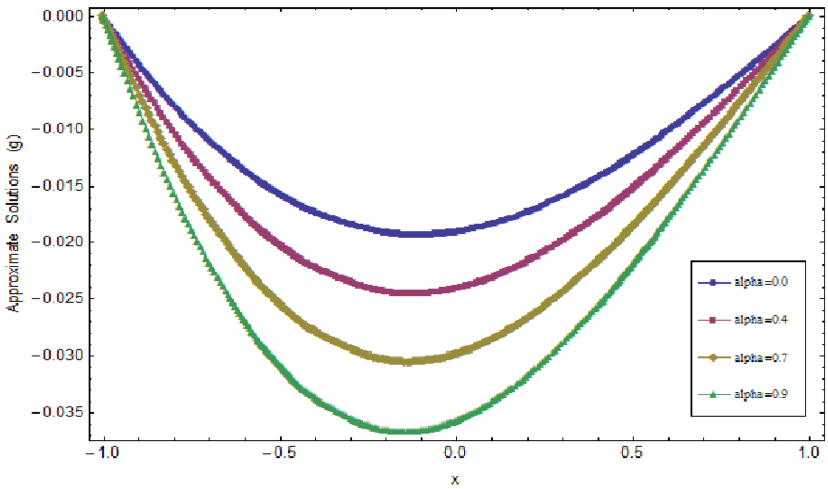

Figure 7: Plots approximate solutions of g for $\lambda = 0.5, K = 0.2, M = 0.1,$ and $R = 0.2$.

4. Numerical results

We conceive the following nonlinear system in the reproducing kernel space in this section.

$$\begin{cases} f^{(iv)} - M^2 f'' + 2Kg' + R(f'f'' - ff''') - \alpha f f^{(v)} \\ \qquad -3\alpha f'' f''' - \alpha f' f^{(iv)} - \alpha g g''' + \alpha (f'')^2 + \alpha g' g'' = 0, \\ \\ g'' - M^2 g - 2K^2 f' + R(f'g - fg') + \alpha f g''' - \alpha f' g'' = 0, \\ \\ f = -\lambda, \quad f' = 1, \quad g = 0, \quad \text{at} \quad \eta = -1, \\ f = \lambda, \quad f' = 0, \quad\ \ g = 0, \quad \text{at} \quad \eta = 1, \end{cases} \tag{4.1}$$

where M, K, α, R and λ are constants.

After homogenizing the conditions we acquired the numerical results of (4.1) for different values of M, K, α, R and λ and demonstrated them by Figures 1–7.

5. Conclusion

We investigated approximate solutions of nonlinear systems in the reproducing kernel space in this work. We presented our results with Figures 1–7. We verified that reproducing kernel method is very effective technique for solving nonlinear systems.

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