frontiers in Physics

ORIGINAL RESEARCH
published: 04 September 2019
doi: 10.3389/fphy.2019.00126

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Optical Solitons With M-Truncated and Beta Derivatives in Nonlinear Optics

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This paper studies optical solitons with M-truncated and beta derivatives (BD) for the Complex Ginzburg-Landau equation (CGLE) with Kerr Law nonlinearity. Two well-known integration schemes which are generalized tanh method (GTM) and generalized Bernoulli sub-ODE method (GBM) are utilized to extract such optical soliton solutions. For the successful existence of the solutions, the constraints conditions have been presented. The discussion for the physical features of the obtained solutions is reported.

Keywords: complex Ginzburg-Landau equation, generalized tanh method, generalized Bernoulli sub-ODE method, beta derivative, optical solitons

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Specialty section:
This article was submitted to Mathematical Physics, a section of the journal Frontiers in Physics

Received: *15 April 2019*
Accepted: *21 August 2019*
Published: *04 September 2019*

Citation:
Yusuf A, Inc M and Baleanu D (2019) Optical Solitons With M-Truncated and Beta Derivatives in Nonlinear Optics. Front. Phys. 7:126. doi: 10.3389/fphy.2019.00126

1. INTRODUCTION

Nonlinearity has been potent field of research and its vitality is thought of through a sweer-amplitude wave oscillation analyzed in numerous fields from plasmas and fluids to biological and chemical phenomenon, solid state, to mention a few. Therefore, the most captivating viewpoint in nonlinear physical phenomenon are solitons. The availability of solitonic concepts are due to the philosophical balance of dispersion and nonlinearity [1]. A lot of researches on solitons and associated aspects of solitary wave (SW) solutions for example monopulse water wave which depict the first soliton can be found in Miller and Ross [2], Podlubny [3], Oldham [4], Kiryakova [5], and El-Sayed and Gaber [6]. Moreover, various mathematical insight and modeling can be interpreted through optical solitons for their numerical and analytical structures of the numerous mechanism. These stimulated many engineers and scientists to focus on the establishments of solitons with optical structures with the help of various integration schemes [7–38].

For quite a long time, the effect of memory is an idea that has been of great concern in the locality of modeling. Genuinely, the integer systems are not conveniently addressing this memory effect [39–41]. Many researchers have presented that, one can get to know more on the memory effect through non-integer operators [42–45]. An extension to integer order systems such as conformable [46], beta [47], and M-derivatives [48] have also been introduced and they play a vital role in modeling physical systems. These extension to integer order systems satisfy a lot of characteristics that were not satisfied before and it can be employed to model several physical phenomenon. In this study, we establish new optical solitons for the governing equation with M-truncated and beta derivatives with the aid of two well-known method. The M-truncated and beta derivatives are defined in the following subsections, respectively.

1.1. Truncated M-Fractional Derivative

We define the truncated Mittag-Leffler function of one parameter by

$$ {}_iE_\beta(z)=\sum_{k=0}^{i}\frac{z^k}{\Gamma(\beta k+1)}. \tag{1}$$

Truncated M-fractional derivative (TMD) is a fractional derivative that has been introduced in Sousa and de Oliveira [48]. This derivative has expunged the obstacles with the existing derivatives. It is defined in the following definition.

Definition 1.1. *Assume that* $f:(0,\infty)\to\mathbb{R}$*, the TMD of* f *with order* γ *exhibited* ${}_iT_M^{\gamma,\beta}$ *is given by*

$$ {}_iT_M^{\gamma,\beta}f(\tau)=\lim_{\epsilon\to0}\frac{f(\tau+\,{}_iE_\beta(\epsilon\tau^{-\gamma}))-f(\tau)}{\epsilon}, \tag{2}$$

for $\tau>0$, and ${}_iE_\beta\gamma\in(0,1)$, $\beta>0$ is a truncated Mittag-Leffler function of one parameter, as defined in (1). Note that, if f is γ-differentiable in some open interval $(0,a), a>0$, and $\lim_{\tau\to0^+}\left({}_iT_M^{\gamma,\beta}f(\tau)\right)$. Then we attain

$$ {}_iT_M^{\gamma,\beta}f(0)=\lim_{\tau\to0^+}\left({}_iT_M^{\gamma,\beta}f(\tau)\right) \tag{3}$$

Theorem 1.1. *Surmise that* $f:(0,\infty)\to\mathbb{R}$ *is* $\gamma-$*differentiable for* $\tau_0>0$*, with* $\gamma\in(0,1],\beta>0$*, then* f *is continuous at* τ_0.

Theorem 1.2. *Let* $0<\gamma\le1,\beta>0,a,b\in\mathbb{R}, f,g,\gamma$-*differentiable, at a point* $\tau>0$*. Then*

- ${}_iT_M^{\gamma,\beta}(af+bg)=a_iT_M^{\gamma,\beta}(f)+b_iT_M^{\gamma,\beta}(f),\ a,b\in\mathbb{R}$
- ${}_iT_M^{\gamma,\beta}(t^\mu)=\mu\tau^{\mu-\gamma},\ \mu\in\mathbb{R}$
- ${}_iT_M^{\gamma,\beta}(fg)=f_iT_M^{\gamma,\beta}(g)+g_iT_M^{\gamma,\beta}(f),$
- ${}_iT_M^{\gamma,\beta}(\frac{f}{g})=\frac{g_iT_M^{\gamma,\beta}(f)-f_iT_M^{\gamma,\beta}(g)}{g^2},$
- *If* f *is differentiable, then* ${}_iT_M^{\gamma,\beta}(f)(\tau)=\frac{\tau^{1-\gamma}}{\Gamma(\beta+1)}\frac{df}{d\tau},$
- ${}_iT_M^{\gamma,\beta}(fog)(\tau)=f'(g(\tau))_iT_M^{\gamma,\beta}g(\tau)$*, for* f *differentiable at* g.

1.2. Beta Derivative

The beta derivative can be stated by [49]

$$ {}_0^AT_\eta^\gamma(F(\eta))=\lim_{\epsilon\to0}\frac{F\left(\eta+\epsilon(\eta+\frac{1}{\Gamma(\gamma)})\right)-F(\eta)}{\epsilon}. \tag{4}$$

along with the properties as comes next

1.

$$ {}_0^AT_\eta^\gamma(aF(\eta)+bG(\eta))=a{}_0^AT_\eta^\gamma F(\eta)+b{}_0^AF_\eta^\gamma G(\eta){}_0^A \tag{5}$$

2.

$$ T_\eta^\gamma(c)=0, \tag{6}$$

for any c depicting a constant,

3.

$$ {}_0^AT_\eta^\gamma(F(\eta).G(\eta))=G(\eta){}_0^AT_\eta^\gamma F(\eta)+F(\eta){}_0^AT_\eta^\gamma G(\eta) \tag{7}$$

4.

$$ {}_0^AT_\eta^\gamma\left(\frac{F(\eta)}{G(\eta)}\right)=\frac{G(\eta){}_0^AT_\eta^\gamma F(\eta)-F(\eta){}_0^AT_\eta^\gamma G(\eta)}{G^2(\eta)}. \tag{8}$$

Considering $\epsilon=\left(\eta+\frac{1}{\Gamma(\gamma)}\right)^{\gamma-1}h,\ h\to0$ when $\epsilon\to0$, therefore we have

$$ {}_0^AT_\eta^\gamma F(\eta)=\left(\eta+\frac{1}{\Gamma(\gamma)}\right)^{1-\gamma}\frac{dF(\eta)}{d\eta}, \tag{9}$$

with

$$ \xi=\frac{l}{\gamma}\left(\eta+\frac{1}{\Gamma(\gamma)}\right)^{\gamma} \tag{10}$$

where l is a constant.

5.

$$ {}_0^AT_\eta^\gamma\left(\frac{F(\tau)}{G(\eta)}\right)=l\frac{dF(\tau)}{d\tau}. \tag{11}$$

The arrangements of the paper is as follows: In section 2 the governing equation has been presented. In section 3, applications have been reported, whereas section 4 provides the discussion of the obtained results along with their physical features. Finally, concluding remark is given in section 5.

2. GOVERNING EQUATION

The CGLE equation [50, 51] in the sense of M-truncated derivative is given by:

$$\begin{aligned} &i{}_0^E\mathcal{D}_{M,\tau}^{\gamma,\beta}u+a{}_0^E\mathcal{D}_{M,\eta}^{2\gamma,\beta}u+b\mathbb{F}(|u|^2)u\\ &=\frac{1}{|u|^2u^*}\left\{\delta{}_0^E\mathcal{D}_{M,\eta}^{2\gamma,\beta}(|u|^2)|u|^2-B({}_0^E\mathcal{D}_{M,\eta}^{\gamma,\beta}u)^2\right\}+Au, \end{aligned} \tag{12}$$

whereas in the sense of beta derivative is given by

$$\begin{aligned} &i{}_0^E\mathcal{D}_t^{\gamma}u+a{}_0^E\mathcal{D}_\eta^{2\gamma}u+b\mathbb{F}(|u|^2)u\\ &=\frac{1}{|u|^2u^*}\left\{\delta{}_0^E\mathcal{D}_\eta^{2\gamma}(|u|^2)|u|^2-B({}_0^E\mathcal{D}_\eta^{\gamma}u)^2\right\}+Au, \end{aligned} \tag{13}$$

where ${}_o^E\mathcal{D}_{M,\tau}^{\gamma,\beta}$, ${}_o^E\mathcal{D}_{M,\eta}^{\gamma,\beta}$, and ${}_o^E\mathcal{D}_\tau^{\gamma}$, ${}_o^E\mathcal{D}_\tau^{\gamma}$ depicts M-truncated and beta derivatives, respectively. $0<\gamma\le1$, describing the order of the fractional derivatives and a,b,δ,B, and A are real constants.

In Equations (12) and (13), $\mathbb{F}\in\mathbb{R}$, and the complex function and its smoothness is necessary to be possessed $\mathbb{F}(|u|^2)u:\mathbb{C}\to\mathbb{C}$. Consider $\mathbb{C}$ to be a two-dimensional linear space $\mathbb{R}^2$, and that $\mathbb{F}(|u|^2)u$ is k times continuously differentiable, so that

$$ \mathbb{F}(|u|^2)u\in\bigcup_{m,n=1}^{\infty}c^k\left((-n,n)\times(-m,m);\mathbb{R}^2\right). $$

2.1. Mathematical Analysis

To solve Equations (12) and (13), the beginning step is as come next

$$u(\eta,\tau) = u(\xi)e^{i\phi(\eta,\tau)}, \tag{14}$$

the shape of the pulse is represented by $u(\eta,\tau)$ so that in the sense of M-truncated derivatives we have

$$\xi = \frac{\Gamma(\beta+1)}{\gamma}\left(\eta^{\gamma} - \upsilon\tau^{\gamma}\right) \tag{15}$$

and

$$\phi(\eta,\tau) = -\frac{\Gamma(\beta+1)}{\gamma}\left(k\eta^{\gamma} - w\upsilon\tau^{\gamma}\right) + \theta_0(\xi), \tag{16}$$

and in the sense of beta derivative we have

$$\xi = \frac{1}{\gamma}\left(\eta + \frac{1}{\Gamma(\gamma)}\right)^{\gamma} - \frac{\upsilon}{\gamma}\left(\tau + \frac{1}{\Gamma(\gamma)}\right)^{\gamma} \tag{17}$$

and

$$\phi(\eta,\tau) = -\frac{k}{\gamma}\left(\eta + \frac{1}{\Gamma(\gamma)}\right)^{\gamma} + \frac{w}{\gamma}\left(\tau + \frac{1}{\Gamma(\gamma)}\right)^{\gamma} + \theta_0(\xi), \tag{18}$$

where w is the wave number of the soliton, k denotes the soliton frequency, υ indicates the speed of the soliton, $\phi(\eta,\tau)$ is the phase component, $\theta_0(\xi)$ depicts an additional phase function depending on ξ. Plugging (15) and (17) into (12) and (13), respectively, and decomposing the real and imaginary parts, one attains

$$wu + a(u'' - k^2u) + b\mathcal{F}(u^2)u = 2(\delta - 2B)\frac{u'^2}{u} + 2\delta u'' + Au, \tag{19}$$

and

$$\upsilon = -2ak. \tag{20}$$

Equation (20) denotes the soliton velocity. Setting $\delta = 2B$ in Equation (19) yields.

$$(a - 4B)u'' - (w + ak^2 + A)u + b\mathcal{F}(u^2)u = 0. \tag{21}$$

2.2. Kerr Law

This law has got its origin through the reality that a light wave in an optical fiber heads to responses by a nonlinear patterns from non-harmonic motion of electrons bound in molecules, brought externally by an electric field. Although the responses by nonlinear terms are seriously low, over a long distance of propagation, the effects standstill in numerous patterns measuring in terms of light wavelength. This law is given by $\mathcal{F}(u) = u$, therefore Equation (21) becomes.

$$(a - 4B)u'' - (w + ak^2 + A)u + bu^3 = 0. \tag{22}$$

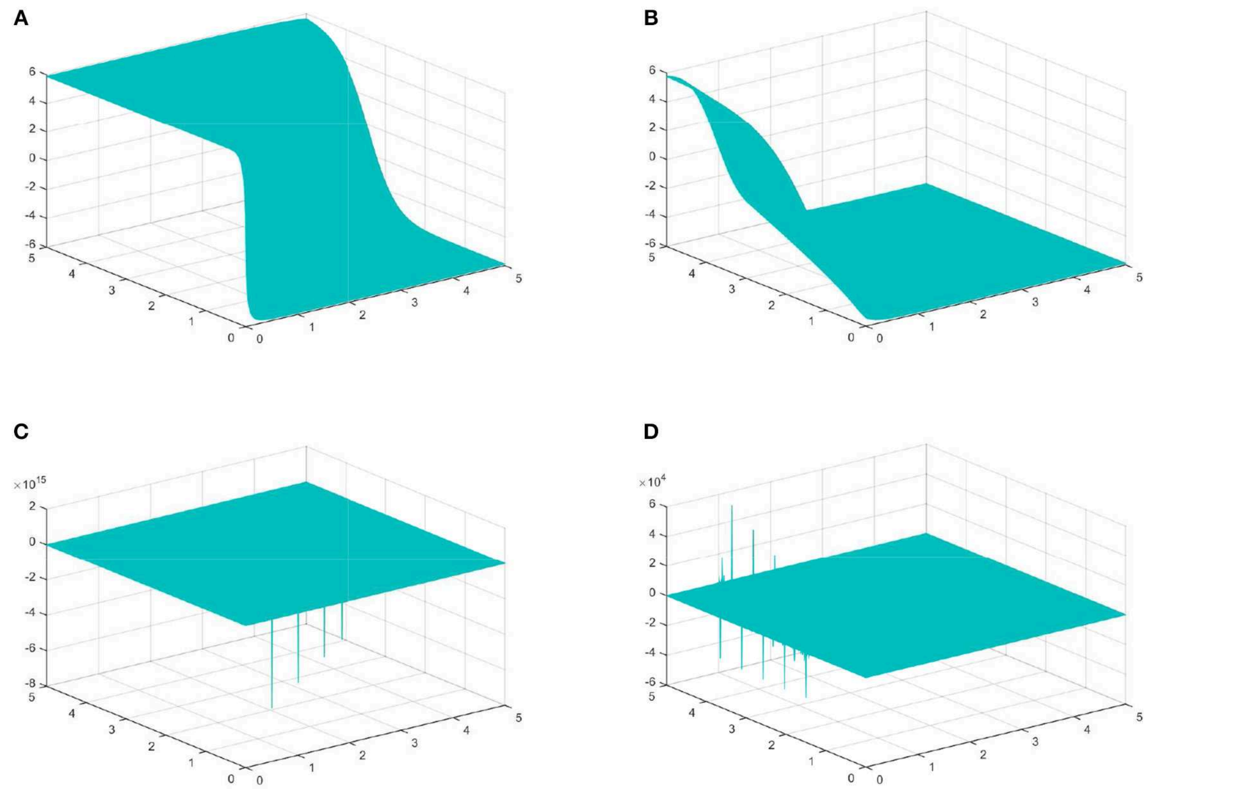

FIGURE 1 | Physical features with suitable values of the parameters. **(A)** M-truncated with $\gamma = 0.5$, $\beta = 0.9$ for (27). **(B)** Beta with $\gamma = 0.5$ for (27). **(C)** M-truncated with $\gamma = 0.5$, $\beta = 0.9$ for (28). **(D)** Beta with $\gamma = 0.5$, $\beta = 0.9$ for (27).

3. APPLICATIONS

This section will utilize the GT and GB sub-ODE methods to provide optical solitons for the governing equation with beta-derivative.

3.1. Application for GTM

According to GTM [52], Equation (22) has possessed the solution as comes next

$$u(\eta,\tau)=a_0+a_1\Phi(\xi), \tag{23}$$

with a_0 and a_1 depicting an unknown constants and $\Phi(\xi)$ holds for the Ricatti equation

$$\Phi(\xi)'=C+\Phi(\xi)^2, \tag{24}$$

with μ a non-zero constant. Plugging Equation (23) together with Equation (24) in Equation (22), one reaches

$$\begin{aligned}&a_1A\Phi(\xi)+a_0A+a_0^3b-8a_1BC\Phi(\xi)+2aa_1C\Phi(\xi)\\&+aa_1k^2\Phi(\xi)+\ aa_0k^2+a_0wa_1^3b\Phi(\xi)^3+3a_0a_1^2b\Phi(\xi)^2\\&+3a_0^2a_1b\Phi(\xi)-8a_1B\Phi(\xi)^3+a_1+w\Phi(\xi)+2aa_1\Phi(\xi)^3=0\end{aligned} \tag{25}$$

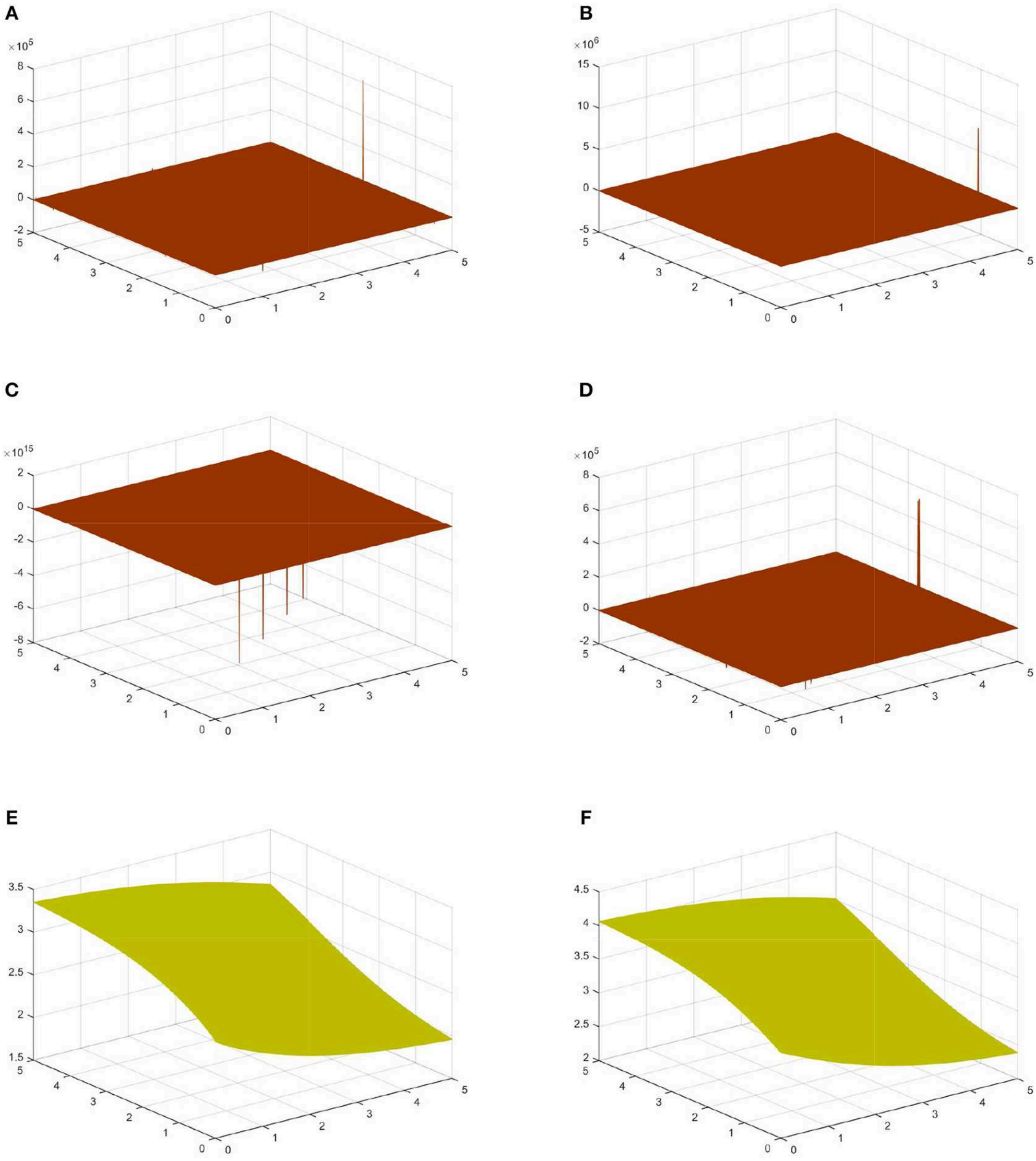

FIGURE 2 | Physical features with suitable values of the parameters. **(A)** M-truncated with $\gamma=0.5$, $\beta=0.9$ for (29). **(B)** Beta with $\gamma=0.5$ for (29). **(C)** M-truncated with $\gamma=0.5$, $\beta=0.9$ for (30). **(D)** Beta with $\gamma=0.5$, $\beta=0.9$ for (30). **(E)** M-truncated with $\gamma=0.85$, $\beta=0.76$ for (39). **(F)** Beta with $\gamma=0.85$ for (39).

Collecting the terms in $\Phi^i (i = 0, 1, 2, 3)$, one attains

$$\begin{aligned} a_0\left(a_0^2 b + ak^2 + A + w\right) &= 0, \\ a_1\left(3a_0^2 b + 2aC + ak^2 + A - 8BC + w\right) &= 0, \\ 3a_0 a_1^2 b &= 0, \\ a_1\left(a_1^2 b + 2a - 8B\right) &= 0. \end{aligned} \tag{26}$$

Solving Equation (26), we obtain

Result 1. $C = -\frac{k^2}{2}$, $A = -4Bk^2 - w$, $a_0 = 0$, $b \neq 0$, $a_1 = \pm\sqrt{\frac{2(4B-a)}{b}}$. If $C < 0$, we attain

$$u(\eta, \tau) = -\sqrt{\frac{2(4B-a)}{b}}\sqrt{-C}tanh(\sqrt{-C}\xi) \times e^{i\phi(\eta,\tau)}, \tag{27}$$

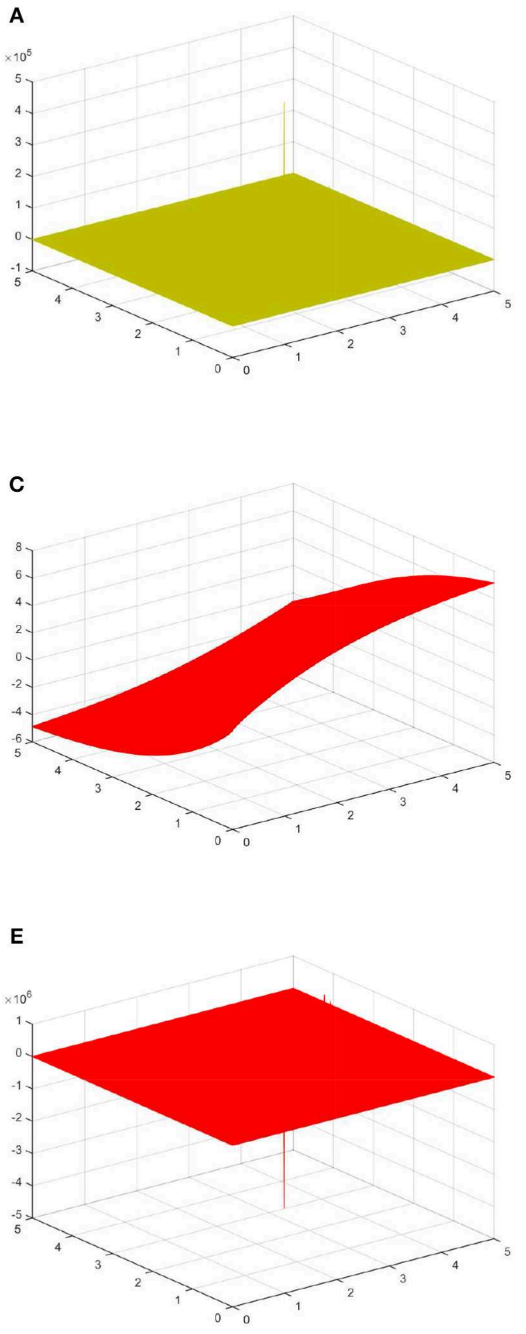

FIGURE 3 | Physical features with suitable values of the parameters. **(A)** M-truncated with $\gamma = 0.85$, $\beta = 0.76$ for (40). **(B)** Beta with $\gamma = 0.85$, $\beta = 0.9$ for (40). **(C)** M-truncated with $\gamma = 0.85$, $\beta = 0.76$ for (41). **(D)** Beta with $\gamma = 0.85$ for (41). **(E)** M-truncated with $\gamma = 0.85$, $\beta = 0.76$ for (42). **(F)** Beta with $\gamma = 0.85$, $\beta = 0.9$ for (42).

$$u(\eta,\tau) = -\sqrt{\frac{2(4B-a)}{b}}\sqrt{-C}coth(\sqrt{-C}\xi)\times e^{i\phi(\eta,\tau)}. \quad (28)$$

If $C > 0$, we acquire

$$u(\eta,\tau) = \pm\sqrt{\frac{2(4B-a)}{b}}\sqrt{C}tan(\sqrt{C}\xi)\times e^{i\phi(\eta,\tau)}, \quad (29)$$

$$u(\eta,\tau) = -\sqrt{\frac{2(4B-a)}{b}}\sqrt{C}cot(\sqrt{C}\xi)\times e^{i\phi(\eta,\tau)}. \quad (30)$$

Result 2. $2C + k^2 \neq 0$, $a = \frac{-A+8BC-w}{2C+k^2}$, $a_0 = 0$, $b \neq 0$, $a_1 = \pm\sqrt{\frac{2(A+4Bk^2+w)}{2bC+bk^2}}$. If $C < 0$, we have

$$u(\eta,\tau) = -\sqrt{\frac{2\left(A+4Bk^2+w\right)}{2bC+bk^2}}\sqrt{-C}tanh(\sqrt{-C}\xi)\times e^{i\phi(\eta,\tau)}, \quad (31)$$

$$u(\eta,\tau) = -\sqrt{\frac{2\left(A+4Bk^2+w\right)}{2bC+bk^2}}\sqrt{-C}coth(\sqrt{-C}\xi)\times e^{i\phi(\eta,\tau)}. \quad (32)$$

If $C > 0$, we attain

$$u(\eta,\tau) = \sqrt{\frac{2\left(A+4Bk^2+w\right)}{2bC+bk^2}}\sqrt{C}tan(\sqrt{C}\xi)\times e^{i\phi(\eta,\tau)}, \quad (33)$$

$$u(\eta,\tau) = -\sqrt{\frac{2\left(A+4Bk^2+w\right)}{2bC+bk^2}}\sqrt{C}cot(\sqrt{C}\xi)\times e^{i\phi(\eta,\tau)}, \quad (34)$$

where ξ and $\phi(\eta,\tau)$ are defined by (15) and (16) for M-truncated derivative solutions and by (17) and (18) for beta derivative solutions.

3.2. Application for GBM

This section will apply GBM for Equations (12) and (13). According to GB sub-ODE method [53], Equation (22) has possessed the solution as comes next

$$u(\eta,\tau) = a_0 + a_1\Phi(\xi), \quad (35)$$

with a_0 and a_1 representing an unknown constants and $\Phi(\xi)$ holds for the Ricatti equation

$$\Phi(\xi)' + \lambda\Phi(\xi) = \mu\Phi(\xi)^2, \quad (36)$$

with μ a non-zero constant. Plugging Equation (36) together with Equation (35) in Equation (22), one attains

$$\begin{aligned} a_1A\Phi(\xi) + a_0A + a_0^3b + aa_1k^2\Phi(\xi) + aa_1\lambda^2\Phi(\xi) + \\ a_1w\Phi(\xi) + aa_0k^2 + a_0w - 4a_1B\lambda^2\Phi(\xi) + 12a_1B\lambda\mu\Phi(\xi)^2 \\ - 8a_1B\mu^2\Phi(\xi)^3 - 3aa_1\lambda\mu\Phi(\xi)^2 + 2aa_1\mu^2\Phi(\xi)^3 \\ a_1^3b\Phi(\xi)^3 + 3a_0a_1^2b\Phi(\xi)^2 + 3a_0^2a_1b\Phi(\xi) = 0. \end{aligned} \quad (37)$$

Collecting the terms in $\Phi^i (i = 0, 1, 2, 3)$, one obtains

$$\begin{aligned} a_0\left(a_0^2b + ak^2 + A + w\right) = 0, \\ a_1\left(3a_0^2b + ak^2 + a\lambda^2 + A - 4B\lambda^2 + w\right) = 0, \\ 3a_1\left(a_0a_1b - \lambda\mu(a-4B)\right) = 0, \\ a_1\left(a_1^2b + 2\mu^2(a-4B)\right) = 0. \end{aligned} \quad (38)$$

Solving Equation (38), we reaches

Result 1. $k = \pm\frac{\lambda}{\sqrt{2}}$, $A = -2B\lambda^2 - w$, $b \neq 0$, $a_0 = \pm\frac{\sqrt{4B\lambda^2-a\lambda^2}}{\sqrt{2}\sqrt{b}}$, $\lambda(a-4B) \neq 0$, $a_1 = \mu\sqrt{\frac{2(4B-a)}{b}}$. We obtain

$$u(\eta,\tau) = \left(\frac{\sqrt{4B\lambda^2-a\lambda^2}}{\sqrt{2}\sqrt{b}} - \frac{\lambda}{2}\sqrt{\frac{2(4B-a)}{b}}\left(tanh(\frac{\lambda}{2}\xi)-1\right)\right) \times e^{i\phi(\eta,\tau)}, \quad (39)$$

or

$$u(\eta,\tau) = \left(\frac{\sqrt{4B\lambda^2-a\lambda^2}}{\sqrt{2}\sqrt{b}} - \frac{\lambda}{2}\sqrt{\frac{2(4B-a)}{b}}\left(coth(\frac{\lambda}{2}\xi)-1\right)\right) \times e^{i\phi(\eta,\tau)}, \quad (40)$$

Result 2. $2k^2 - \lambda^2 \neq 0$, $a = -\frac{2\left(A+2B\lambda^2+w\right)}{2k^2-\lambda^2}$, $b \neq 0$, $a_0 = \pm\sqrt{\frac{\lambda^2\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}}$, $a_1 = -2\mu\sqrt{\frac{\lambda\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}}$. We acquire

$$u(\eta,\tau) = \left(\sqrt{\frac{\lambda^2\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}} + \lambda\sqrt{\frac{\lambda\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}} \times\left(tanh(\frac{\lambda}{2}\xi)-1\right)\right)e^{i\phi(\eta,\tau)}, \quad (41)$$

or

$$u(\eta,\tau) = \left(\sqrt{\frac{\lambda^2\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}} + \lambda\sqrt{\frac{\lambda\left(A+4Bk^2+w\right)}{b\left(2k^2-\lambda^2\right)}} \times\left(coth(\frac{\lambda}{2}\xi)-1\right)\right)e^{i\phi(\eta,\tau)}, \quad (42)$$

where ξ and $\phi(x,t)$ are defined by (15) and (16) for M-truncated derivative solutions and by (17) and (18) for beta derivative solutions.

4. DISCUSSION

The M-truncated and beta-derivatives have been successfully utilized to reach optical solitons for the underlying equation. This has been achieved by utilizing two potent integration schemes which are GTM and GBM. Singular-dark, dark and singular-periodic solutions have been reported. The GTM has provided dark optical soliton (DOS) (27) and (31), singular optical soliton (28) and (32), optical singular periodic (29), (30), (33), and (34). The GBM has provided optical dark solitons reported in (39) and (41), optical singular solitons reported in (40) and (42).

Solitary waves (SW) with mitigating intensity than the background can be interpreted by DOS [49]. SW with discontinuous derivative can be depicted by singular solitons [54, 55]. These sorts of SW are potent as a results of efficiency and applicability they possessed optical communications of a long distance. Optical fibers can be considered as a thin lengthy strands of pure-ultra glass so that an electromagnetic radiations can be communicated without any mitigation from one point to the next [56].

5. CONCLUSION

In this research, we have applied the well-known M-truncated and beta derivatives to reach the optical solitons for the governing equation with Kerr Law nonlinearity. Two techniques which GTM and GBM have been used to attain such solutions. For the successful existence of the solutions, the constraints conditions have been presented. The discussion for the physical features of the obtained solutions are have been reported. The explicit behavior for the obtained results by suitable choice of the parameter values have been presented in the presented **Figures 1–3**. The effects of the γ, β-M-truncated derivative and γ-beta derivative have influenced the behavior of the solutions. The obtained solutions are new and novel and can be of great potent in explaining physical systems in nonlinear optics.

DATA AVAILABILITY

All datasets generated for this study are included in the manuscript and the supplementary files.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

The handling editor is currently organizing a Research Topic with one of the authors DB, and confirms the absence of any other collaboration.