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ABSTRACT

The aim of this paper is to investigate heat and mass transfer of Jeffery fluid on a stretching sheet. Moreover, the influence of magnetic field with mixed convection, convective boundary condition and Soret and Dufour effects is also brought into the consideration along with chemical reaction and thermophoresis condition. The problem is modeled by system of partial differential equations and solutions are obtained by optimal homotopy analysis method. In addition, for comprehensive interpretation of the influence of the system parameters results are shown by graphs and tables.

Mostly non-Newtonian fluid fields have applications in industries and chemical engineering. Moreover, non-Newtonian behavior has great importance while studying heat and momentum transfer. The novelty of the present study is that we have analyzed a non-Newtonian fluid called Jeffery fluid on stretching sheet in the presence of magnetic field using mixed convection and convective boundary conditions. The solutions are obtained by optimal homotopy analysis method and useful results are discussed with the aid of graphs and tables.

NOMENCLATURE

σ^*	Electrical conductivity
υ	Kinematic viscosity
T	Fluid temperature
T_f	Surface temperature
C	Concentration field
C_p	Specific heat
ρ	Fluid density
γ	Biot number
Sr	Soret number
Df	Dufour number
De	Mass diffusivity
β_1	Deborah number
λ_2	Retardation time
M	Hartmann number
k_T	Thermal diffusion
Pr	Prandtl number
Sc	Schmidt number
C_∞	Ambient concentration
T_∞	Ambient temperature
T_r	Reference temperature
Gr_x	Local Grashof number
Re_x	Local Reynolds number

β Ratio of stretching rates
u Velocity along x-axis
v Velocity along y-axis
w Velocity along z-axis
k Chemical reaction parameter
μ Dynamic viscosity of fluid field
σ Thermal diffusivity of fluid
τ Thermophoretic parameter
V_T Thermophoretic velocity
g Gravitational acceleration
k_2 Thermophoretic coefficient
λ Local buoyancy parameter
C_s Concentration susceptibility
k^* Thermal conductivity of fluid
β_c Concentration expansion coefficient
N_1 Concentration buoyancy parameter
B_0 Magnitude of applied magnetic field
λ_1 Ratio of relaxation and retardation times

I. INTRODUCTION

Mostly fluids used at physical and industrial levels are "non-Newtonian". They play important role in chemical engineering, food engineering, plastic processing industries, bio-chemical engineering, petroleum production, oil exploration, power engineering and medical engineering. Non-Newtonian behaviour has great importance while studying heat and momentum transfer. Many physical phenomenon are modeled by constitutive equations based on non-Newtonian fluids. Solutions of such system of equations are highly complex. A lot of work has been done in the field of non-Newtonian fluids by many researchers.[1–6] Jeffery fluid is a non-Newtonian fluid. Some investigators highlighted the mixed convection, convective condition and MHD effects on Jeffery fluids on stretching surface.[7–12]

The boundary layer flow because of stretching sheet arises in different industrial manufacturing processes like glass-fiber, paper industry, metallurgy, textile industry, extrusion of plastic, rubber, metal and polymer sheets. Heat transfer plays an essential role in such flows that is explored in the attempts.[13–19] Rashidi *et al.*[20] discussed two-dimensional flow over a stretching surface in porous medium. Further thermal radiation and non-uniform magnetic field are taken into account. Heat transfer in Micropolar fluid over stretched sheet with Joule heating and convective boundary condition was studied by Waqas *et al.*[21] Some significant efforts regarding this can be shown by researchers.[22–26]

In thermophoresis, particles move from hot surface to cold surface with velocity called thermophoretic velocity. The force due to thermophoresis effect is known as thermophoretic force. Effects of thermophoresis in MHD flow of Maxwell and Oldroyd B fluid with joule heating was investigated by researchers (see Refs. 27, 28). Kandasamy *et al.*[29] explored heat source/sink and fluid viscosity with thermophoresis in porous medium. *et al.*[30] discussed viscoelastic fluid flow with thermophoresis and Soret and Dufour effects.

The Soret effect can be examined in mixture of moving particles which exhibit individual responses to the force of a temperature gradient. The Dufour effect is the energy flux due to a mass concentration gradient. The concentration gradient results in a temperature change. Ashraf *et al.*[31] studied Soret and Dufour effects on Oldroyd-B fluid. Heat and mass transfer with Soret and Dufour effects has been investigated by Srinivasacharya.[32–35] Erying-Powell fluid has been considered for Soret and Dufour effects by Qasim.[36]

Besthapu *et al.*[37] worked on stagnation point flow of non Newtonian nanofluid with thermal radiation and slip condition. Maxwell model has been considered in order to study entropy generation in methanol-based nanofluid by Qasim *et al.*[38] Numerical investigation is carried out for Newtonian fluid under the influence of buoyancy and entropy generation by Ganesh *et al.*[39] Marangoni boundary layer flow with nonlinear thermal radiations investigated by researchers.[40,41] Aman *et al.*[42] discussed fractional Maxwell fluid on a moving plate with second order slip.

In present paper, the 3D flow of Jeffery fluid over a stretched surface is discussed. The flow exhibits Soret and Dufour effects with convective boundary condition. Chemical reaction and thermophoresis effects are also taken into account. Mathematical modeling of the problem is analyzed and solutions are obtained by optimal homotopy analysis method. Graphical and tabular form of different parameters are examined.

II. MATHEMATICAL ANALYSIS

Consider three-dimensional incompressible magnetohydrodynamic mixed convection boundary layer flow of Jeffery fluid over a stretching surface. Heat and mass transfer analysis is done along with chemical reaction, thermophoresis and Soret-Dufour effects. A magnetic field of strength B_0 is transversely applied to the plate (see Figure 1). Moreover, gravitational force is neglected, external electric and induced magnetic field effect is negligible because magnetic Reynolds number is very small. Mathematical modeling of the problem is given below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

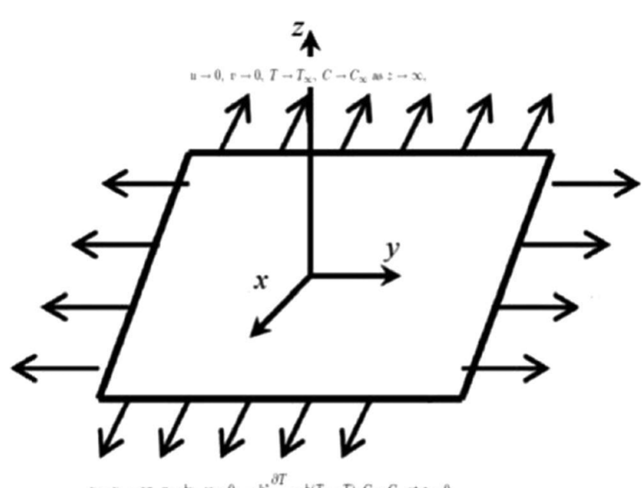

FIG. 1. Physical model.

$$uu_x + vu_y + wu_z = \left(\frac{\nu}{1+\lambda_1}\right)[u_{zz} + \lambda_2(uu_{xzz} + vu_{yzz} + wu_{zzz} + u_z u_{xz} + v_z u_{yz} + w_z u_{zz})] - \frac{\sigma^* B_0^2}{\rho} u + g[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)], \tag{2}$$

$$uv_x + vv_y + wv_z = \left(\frac{\nu}{1+\lambda_1}\right)[v_{zz} + \lambda_2(uv_{xzz} + vv_{yzz} + wv_{zzz} + u_z v_{xz} + v_z v_{yz} + w_z v_{zz})] - \frac{\sigma^* B_0^2}{\rho} v, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \sigma\frac{\partial^2 T}{\partial z^2} + \frac{D_e k_T}{C_s C_p}\frac{\partial^2 C}{\partial z^2}, \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_e\frac{\partial^2 C}{\partial z^2} + \frac{D_e k_T}{T_m}\frac{\partial^2 T}{\partial z^2} - k(C - C_\infty) - \frac{\partial}{\partial z}[V_T(C - C_\infty)]. \tag{5}$$

Thermophoretic velocity given by:

$$V_T = -k_2\frac{V_T}{T_r}\frac{\partial T}{\partial z}. \tag{6}$$

τ is given by:

$$\tau = -\frac{k_2(T_f - T_\infty)}{T_r}. \tag{7}$$

The boundary conditions are considered as:

$$u = u_e = ax,\ \ v = by\ \ w = 0,\ \ -k^*\frac{\partial T}{\partial z} = h(T_f - T),\ \ C = C_w,\ \ at\ \ z = 0, \tag{8}$$

$$u \to 0,\ \ v \to 0,\ \ T \to T_\infty,\ \ C \to C_\infty,\ \ as\ \ z \to \infty, \tag{9}$$

where a and b have dimension reciprocal of time. Similarity transformations are:

$$u = axf'(\eta),\ \ v = ayg'(\eta),\ \ w = -\sqrt{a\nu}[f(\eta) + g(\eta)],\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ \ \eta = \sqrt{\frac{a}{\nu}}z. \tag{10}$$

Clearly above similarity transformations satisfying equation (1). Moreover, substituting these similarity transformations into Eqs. (2), (3), (4) and (5), we get

$$f''' + (1+\lambda_1)[(f+g)f'' - (f')^2] + \beta_1[(f'')^2 - (f+g)f'''' - g'f'''] - (1+\lambda_1)M^2 f' + \lambda[\theta + N_1\phi] = 0, \tag{11}$$

$$g''' + (1+\lambda_1)[(f+g)g'' - (g')^2] + \beta_1[(g'')^2 - (f+g)g'''' - f'g'''] - (1+\lambda_1)M^2 g' = 0, \tag{12}$$

$$\theta'' + Pr(f+g)\theta' + PrDf\phi'' = 0, \tag{13}$$

$$\phi'' + Sc(f+g)\phi' - Sck\phi + ScSr\theta'' - Sc\tau(\phi'\theta' - \phi\theta'') = 0, \tag{14}$$

$$f(0) = 0,\ \ g(0) = 0,\ \ f'(0) = 1,\ \ g'(0) = \beta,\ \ \theta'(0) = -\gamma[1 - \theta(0)]\ \ \phi(0) = 1,\ \ f'(\infty) = 0,\ \ g'(\infty) = 0,\ \ \theta(\infty) = 0,\ \ \phi(\infty) = 0, \tag{15}$$

where prime shows the differentiation with respect to η. The parameters and dimensionless numbers are as follow:

$$\beta_1 = \lambda_2 a,\ \ M^2 = \frac{\sigma^* B_0^2}{\rho a},\ \ \beta = \frac{b}{a},\ \ \gamma = \frac{h}{k^*}\sqrt{\frac{\nu}{a}},\ \ Pr = \frac{\nu}{\sigma},\ \ \lambda = \frac{Gr_x}{Re_x^2},$$
$$Gr_x = \frac{g\beta_T(T_f - T_\infty)x^3}{\nu^2},\ \ N_1 = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_f - T_\infty)},\ \ Df = \frac{D_e K_T(C_w - C_\infty)}{C_s C_p(T_f - T_\infty)\nu},$$
$$Sr = \frac{D_e K_T(T_f) - T_\infty)}{T_m \nu(C_w - C_\infty)},\ \ \tau = \frac{-k_2(T_f - T_\infty)}{T_r},\ \ Sc = \frac{\nu}{D},\ \ k = \frac{k_1}{a}. \tag{16}$$

Local Nusselt number is as follow:

$$Nu/Re_x^{\frac{1}{2}} = -\theta'(0), \tag{17}$$

Local Sherwood number is as follow:

$$Sh/Re_x^{\frac{1}{2}} = -\phi'(0), \tag{18}$$

where $Re_x = \frac{u_e x}{\nu}$. Let initial approximations and auxiliary linear operators

$$f_0(\eta) = 1 - e^{-\eta},\ \ g_0(\eta) = \beta(1 - e^{-\eta}),\ \ \theta_0(\eta) = \left(\frac{\gamma}{1+\gamma}\right)e^{-\eta},\ \ \phi_0(\eta) = e^{-\eta},$$

$$L_f = f''' - f',\ \ L_g = g''' - g',\ \ L_\theta = \theta'' - \theta,\ \ L_\phi = \phi'' - \phi. \tag{19}$$

III. CONVERGENCE ANALYSIS

Homotopy solutions contain parameters which can control convergence. These parameters are c_0^f, c_0^g, c_0^θ and c_0^ϕ. By minimizing residual errors the optimum values of c_0^f, c_0^g, c_0^θ and c_0^ϕ are obtained.

BVPh2.0 is used so that minimum error can be obtained. Three arrays of total optimum convergence control parameters are attained at 2^{nd}, 4^{th} and 6^{th} iterations. Table II indicates values of optimum convergence-control parameter at 6^{th} iteration and singular averaged squared residual errors.

IV. ANALYSIS OF RESULTS

In this section, we have analyzed the graphs for different parameters. Figure 2 depicted the behaviour of $f'(\eta)$ due to β_1. As β_1 depends on λ_2 and λ_2 increases the velocity of fluid. Figure 3 shows that as β increases there is decrease in $f'(\eta)$. Figure 4 shows that for $\beta = 0$ velocity of fluid is zero. By increasing β the $g'(\eta)$ increases and $g'(\eta)$ decreases. Physically when β increases from zero then the lateral surface starts moving towards y-direction that is why $f'(\eta)$ decreases and $g'(\eta)$ increases. Behaviour of $f'(\eta)$ is opposite for λ and λ_1 (see Figures 5 and 6). $f'(\eta)$ and $g'(\eta)$ decrease as M increases because Lorentz force is produced by magnetic field whose direction is opposite to the direction of flow due to which velocity decreases as shown in Figures 7 and 8. Behaviour of velocity component $f'(\eta)$ for $N1$ is shown in Figures 9. With the increase in N_1, $f'(\eta)$ increases.

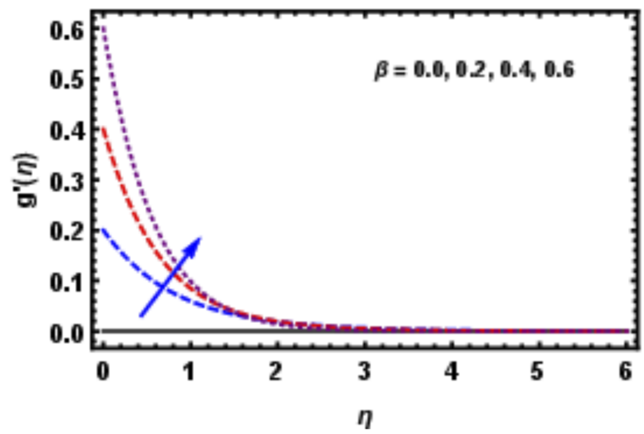

FIG. 4. Influence of β on velocity $g'(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

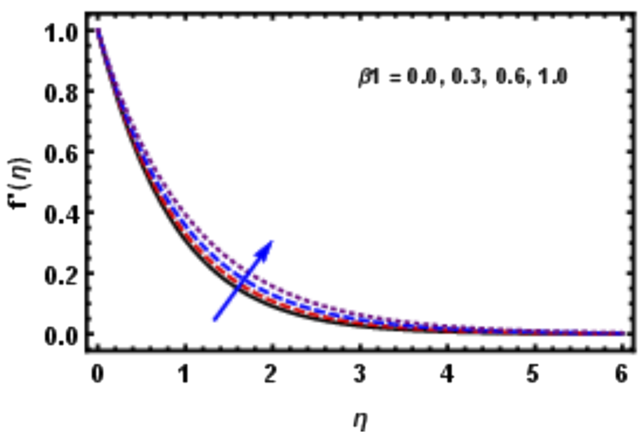

FIG. 2. Influence of β_1 on velocity $f'(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

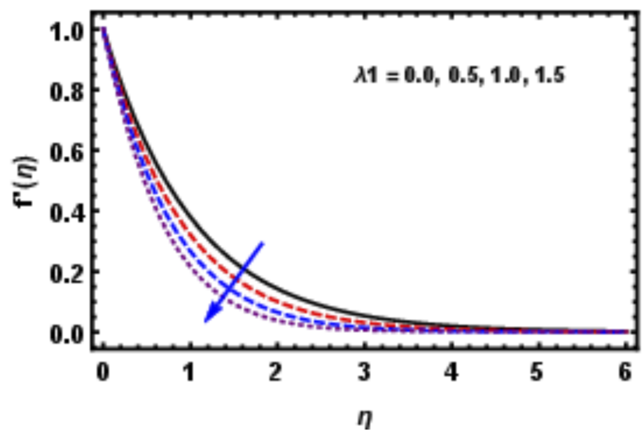

FIG. 5. Influence of λ_1 on velocity $f'(\eta)$ when $\lambda=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

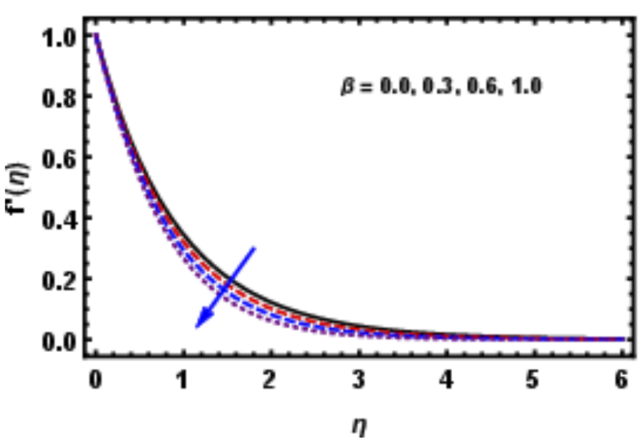

FIG. 3. Influence of β on velocity $f'(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

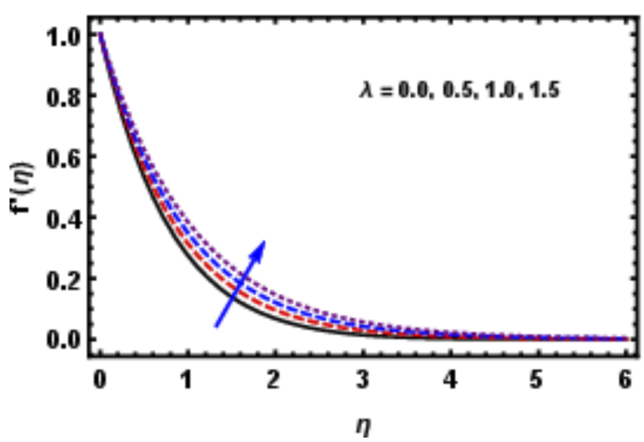

FIG. 6. Influence of λ on velocity $f'(\eta)$ when $\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

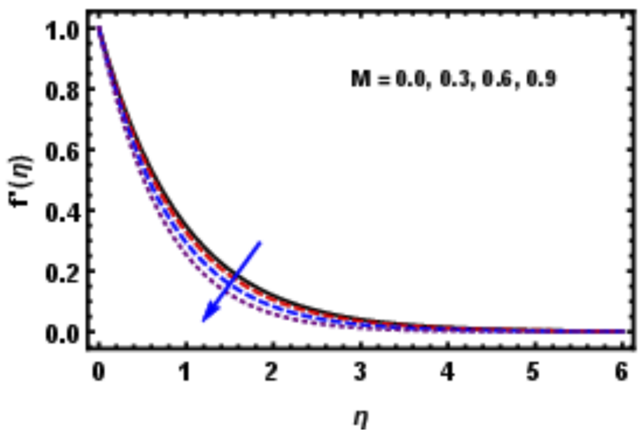

FIG. 7. Influence of M on velocity $f'(\eta)$ when $\lambda=\lambda_1=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

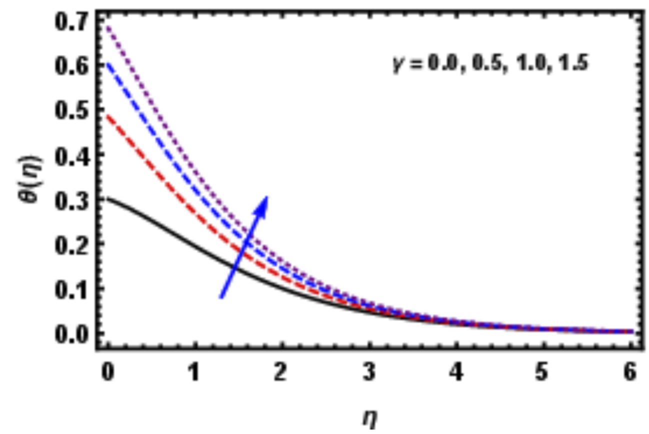

FIG. 10. Influence of γ on temperature $\theta(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$ and $Pr=1.0$.

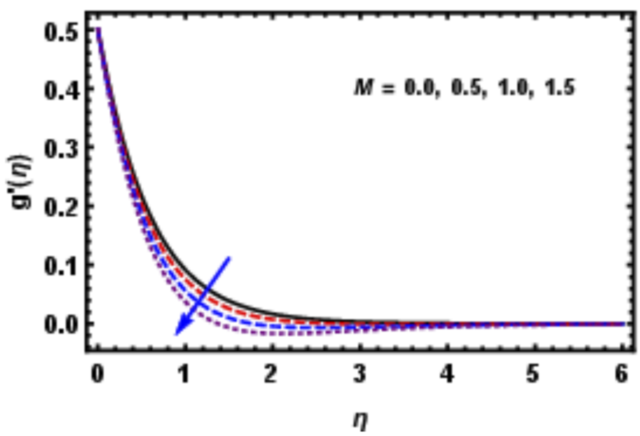

FIG. 8. Influence of M on velocity $g'(\eta)$ when $\lambda=\lambda_1=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

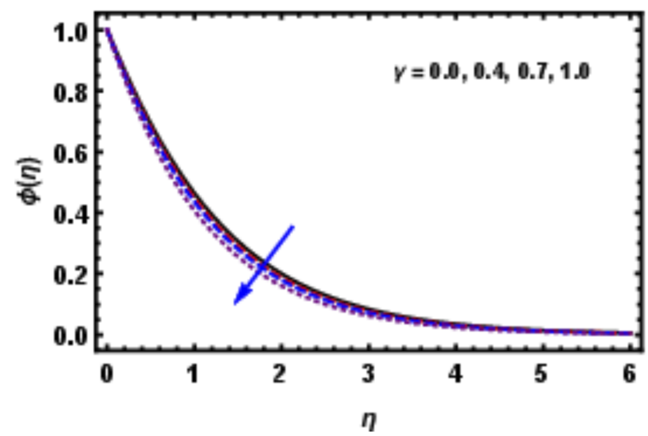

FIG. 11. Influence of γ on temperature $\phi(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$ and $Pr=1.0$.

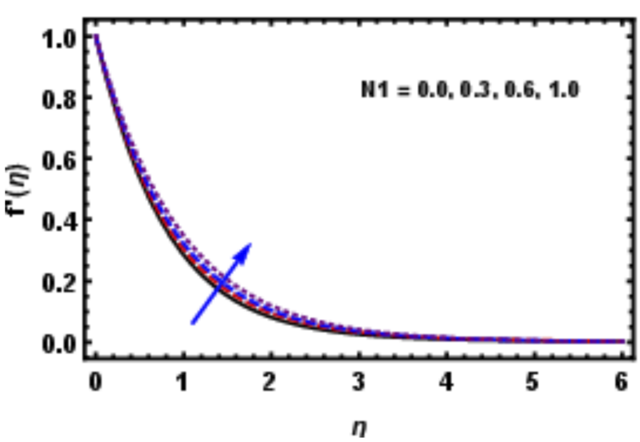

FIG. 9. Influence of N_1 on velocity $f'(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=0.3$, $\gamma=0.6$ and $Pr=1.0$.

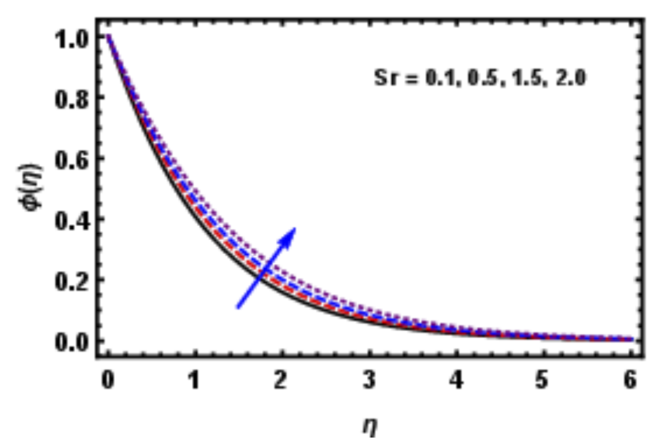

FIG. 12. Influence of Sr on concentration $\phi(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

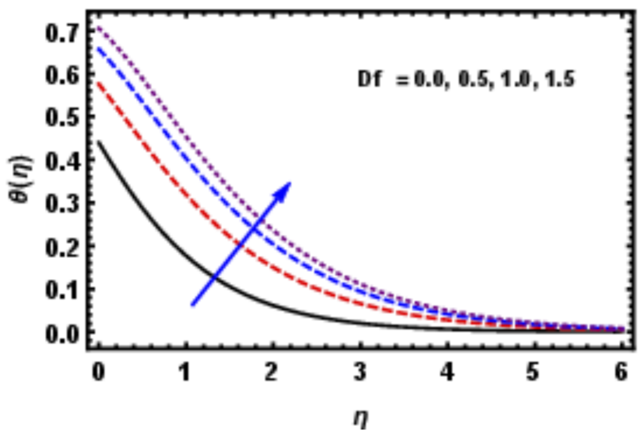

FIG. 13. Influence of Df on temperature $\theta(\eta)$ when $\lambda=\lambda_1=M=Sc=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

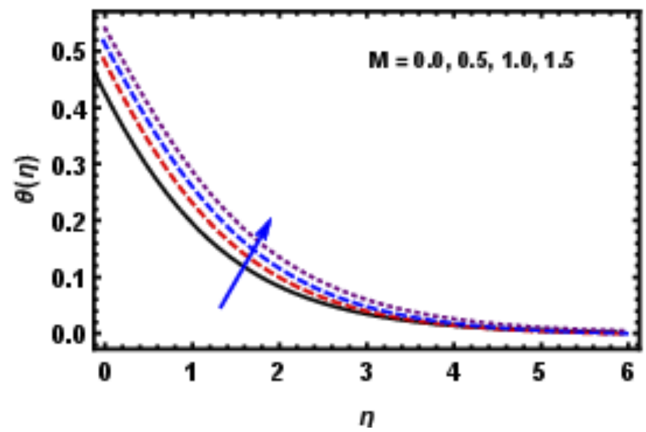

FIG. 16. Influence of M on concentration $\theta(\eta)$ when $\lambda=\lambda_1=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

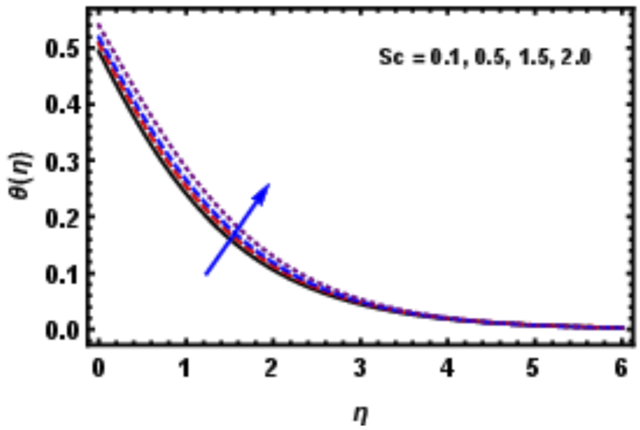

FIG. 14. Influence of Sc on concentration $\theta(\eta)$ when $\lambda=\lambda_1=M=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

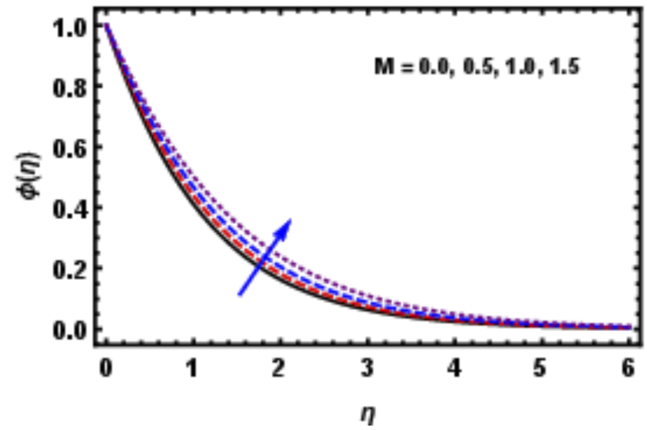

FIG. 17. Influence of on M concentration $\phi(\eta)$ when $\lambda=\lambda_1=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

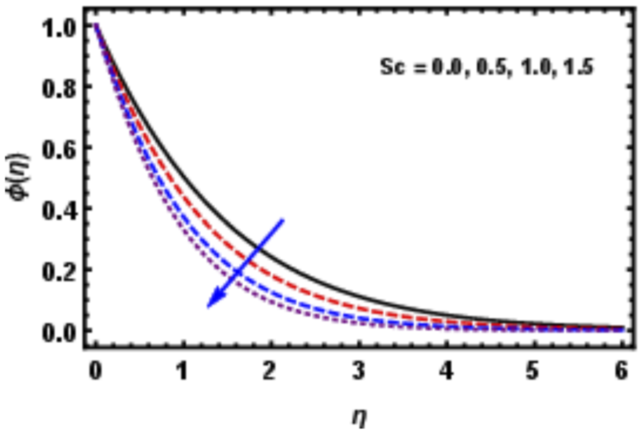

FIG. 15. Influence of Sc on concentration $\phi(\eta)$ when $\lambda=\lambda_1=M=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=N_1=0.3$, $\gamma=0.6$ and $Pr=1.0$.

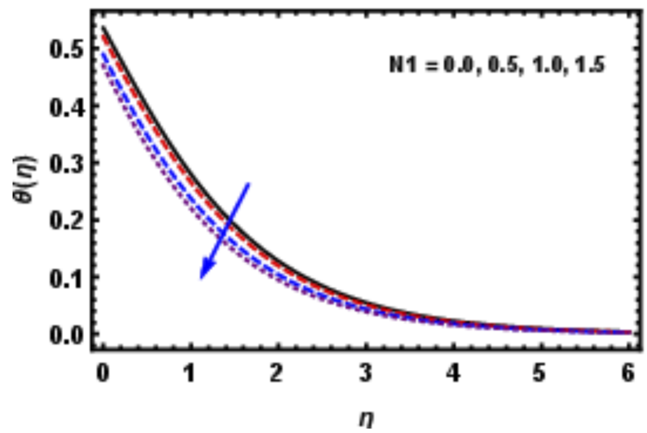

FIG. 18. Influence of $N1$ on concentration $\theta(\eta)$ when $\lambda=\lambda_1=M=Sc=Df=0.5$, $\beta_1=\tau=0.2$, $Sr=\beta=0.4$, $k=0.3$, $\gamma=0.6$ and $Pr=1.0$.

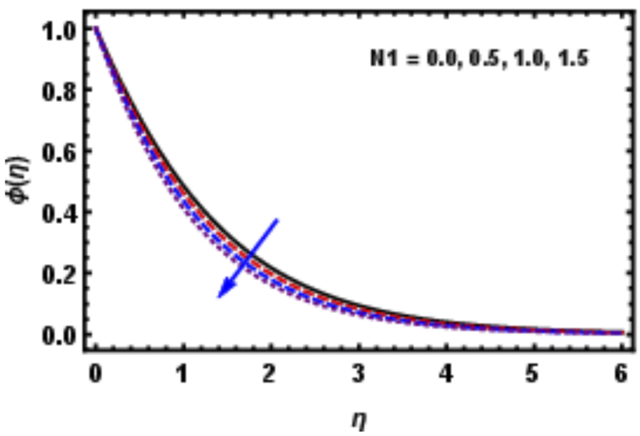

FIG. 19. Influence of $N1$ on concentration $\phi(\eta)$ when λ=λ_1=M=Sc=Df=0.5, β_1=τ=0.2, Sr=β=0.4, k=0.3, γ=0.6 and Pr=1.0.

TABLE I. Optimal convergence control parameters and error analysis using **BVPh2.0**.

m	c_0^f	c_0^g	c_0^θ	c_0^ϕ	E_t^m	CPU TIMES [S]
2.0	−0.64	−0.60	−0.65	−1.28	9.01×10^{-3}	53.0139
4.0	−0.57	−0.51	−0.92	−1.36	1.99×10^{-3}	462.452
6.0	−0.57	−0.51	−1.20	−1.43	7.15×10^{-4}	3559.66

Figures 10–19 shows the variation of γ, Sr, Df, Sc, M and N_1 on temperature and concentration. By increasing γ, $\theta(\eta)$ increases while concentration decreases (see Figures 10 and 11). Figure 12 depicted the effect of Sr on $\phi(\eta)$. In Soret effect temperature gradient causes the mass flux that is why concentration increases. Effect of Df on $\theta(\eta)$ is presented by Figure 13. Dufour effect generate energy flux by composition gradient causes enhance in temperature. Temperature and concentration having opposite behavior for Sc. For Sc temperature is increasing and concentration is decreasing as shown in Figures 14 and 15. Figures 16 and 17 demonstrate that temperature and concentration increase with the enhance in M. Temperature and concentration decrease as N_1 increases (see Figures 18 and 19). Table I demonstrate values of convergence control parameter. Table II presents error analysis at 6th iteration. Error is decreasing with the increase in iterations. In Tables III and IV values of Nusselt and Sherwood numbers are presented for various parameters.

TABLE III. Local Nusselt numbers for certain noteworthy physical parameters.

β_1	β	λ_1	λ	N_1	Pr	Sc	Sr	k	M	$Nu/Re_x^{\frac{1}{2}}$
0.0										0.25402
0.2										0.25440
0.4										0.25478
	0.0									0.24284
	0.2									0.24869
	0.4									0.25440
		0.0								0.25585
		0.3								0.25498
		0.5								0.25411
			0.0							0.25375
			0.3							0.25414
			0.5							0.25440
				0.0						0.25411
				0.3						0.25440
				0.5						0.25459
					1.0					0.25440
					1.5					0.23911
					2.0					0.22605
						0.2				0.25970
						0.4				0.25617
						0.6				0.25263
							0.2			0.25311
							0.4			0.25440
							0.6			0.25569
								0.5		0.25095
								0.7		0.24751
								1.0		0.24234
									0.4	0.25466
									0.7	0.25307
									1.0	0.25223

For the validation of present analysis it is pertinent to notice that several results from the literature could be recovered from our present analysis for example if Soret and Dufour effects, MHD, mixed convection and thermophoresis conditions are eliminated then results of Shehzad *et al.*[43] could be recovered. Moreover, Ashraf *et al.*[30] considered Soret and Dufour effects in viscoelastic fluid with thermophoresis and mixed convection. In present work analysis of MHD mixed convection Jeffery fluid is considered with thermophoresis and Soret and Dufour effects which is comparable with the results of Ref. 30.

TABLE II. Error analysis taking optimal values using Table I at m = 6.

m	E_m^f	E_m^g	E_m^θ	E_m^ϕ	CPU TIMES [S]
6.0	1.16×10^{-5}	3.63×10^{-6}	3.42×10^{-4}	3.04×10^{-4}	20.0719
12.0	1.71×10^{-7}	4.05×10^{-8}	6.53×10^{-5}	7.06×10^{-5}	112.377
18.0	5.13×10^{-9}	1.44×10^{-9}	1.89×10^{-5}	3.82×10^{-5}	440.944

TABLE IV. Local Sherwood numbers for certain noteworthy physical parameters.

β_1	β	λ_1	λ	N_1	Pr	Sc	Sr	Df	k	M	τ	$Sh/Re_x^{\frac{1}{2}}$
0.0												0.67213
0.2												0.67347
0.4												0.67481
	0.0											0.64025
	0.2											0.65700
	0.4											0.67347
		0.2										0.67657
		0.4										0.67450
		0.6										0.67244
			0.0									0.67117
			0.3									0.67255
			0.5									0.67347
				0.0								0.67245
				0.3								0.67347
				0.5								0.674157
					1.0							0.67347
					1.5							0.67971
					2.0							0.68595
						0.2						0.55937
						0.4						0.63535
						0.6						0.71168
							0.2					0.68456
							0.4					0.67347
							0.6					0.66238
								0.2				0.66459
								0.4				0.67051
								0.6				0.676437
									0.5			0.72685
									0.7			0.77900
									1.0			0.85493
										0.6		0.67234
										0.8		0.66946
										1.0		0.66576
											0.4	0.67993
											0.7	0.68961
											1.0	0.69930

V. CONCLUSIONS

Heat and mass transfer of Jeffery fluid is analyzed on stretching sheet under the influence of magnetic field with mixed convection and convective boundary conditions. Moreover, Soret and Dufour effects, chemical reaction and thermophoresis conditions are considered and optimal homotopy analysis method is used to find solutions. Main observations are given below.

- $f'(\eta)$ and $g'(\eta)$ having opposite behaviour for β.
- M has same effect on velocity profiles.
- Features of Biot number and Sc on temperature and concentration are opposite. If one is increasing than other is decreasing.
- With the increase in M increases $\theta(\eta)$ and $\phi(\eta)$.
- N_1 decreases $\theta(\eta)$ and $\phi(\eta)$.

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