

Research Article

New Exact Solutions and Modulation Instability for the Nonlinear (2+1)-Dimensional Davey-Stewartson System of Equation

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The Davey-Stewartson Equation (DSE) is an equation system that reflects the evolution in finite depth of soft nonlinear packets of water waves that move in one direction but in which the waves' amplitude is modulated in spatial directions. This paper uses the Generalized Elliptic Equation Rational Expansion (GEERE) technique to extract fresh exact solutions for the DSE. As a consequence, solutions with parameters of trigonometric, hyperbolic, and rational function are achieved. To display the physical characteristics of this model, the solutions obtained are graphically displayed. Modulation instability assessment of the outcomes acquired is also discussed and it demonstrates that all the solutions built are accurate and stable.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are used in multiple study areas to define significant phenomena. Exact NLPDEs solutions play an important part in the research of physics, applied mathematics, and engineering, including solid state physics, fluid mechanics, population ecology, plasma physics, plasma waves, biology, optical fibres, propagation of shallow waves, heat flow, quantum mechanics, and wave propagation phenomena. Current studies are underway to find new techniques to extract traveling wave solution for NLPDEs. These techniques include the tanh-sech method [1], the Jacobi elliptic function method [2], the homotopy perturbation method [3], the homogeneous balance method [4], sine-cosine method [5, 6], the extended Weierstrass transformation method [7–10], the expansion method [11, 12], the Bäcklund transformation method [13], the inverse scattering method [14], modified extended mapping method [15], Darboux transformation [16], the extended tanh-coth method [17], Hirota bilinear method [18], modified extended direct algebraic method [19], lumped Galerkin method [20], and auxiliary equation method [21]. A lot of these methods are dependent on the type of problem and may or may not be suitable for other different problems [22–31].

The Davey-Stewartson Equation (DSE) [32] is an equation system that reflects the evolution in finite depth of soft nonlinear packets of water waves traveling in one direction but in which the amplitude of waves is modulated in spatial directions. This equation also defines long wave–short wave resonances and the development of a 3-dimensional wave packet on finite depth water [33, 34]. A few solutions have been acquired for this equation [35–38]. The DSE, a two-dimensional system that coexists with both short waves and long waves, and a precise representation of two-dimensional modulation of nonlinear waves should require both short wave and long wave modes. A pair of coupled nonlinear DSE in two dependent variables can be decreased to the (1+1)-dimensional nonlinear Schrödinger (NLS) equation by carrying out a suitable dimensional reduction. A number of analytical methods have been created to be used for NLPDEs such as the DSE, which in recent years have specific types of solutions, such as growing and decaying solutions [39–41], dromions, breathers, instantaneous, propagating, and regular wave patterns, and fluid flow and heat transfer over a stretching or shrinking sheet in a porous medium [34, 42, 43].

The Generalized Elliptic Equation Rational Expansion (GEERE) technique developed by Wan, Song, Yin, and

Zhang [44] is applied in this paper. This technique is a very strong technique that can be used to obtain several particular solutions including rational formal regular triangular, Weierstrass doubly regular, rational formal solitary wave, and rational formal Jacobi solutions [44]. In addition, the modulation instability assessment of the solutions obtained will be explored. Modulation instability is universal, and the basis for the formation of soliton solutions results from the interaction between dispersion and nonlinearity in the spatial and time domain [45–47]. It is therefore of interest to derive explicit DSE equation solutions using the GEERE method. This paper is structured as follows: in Section 2 new soliton solutions are built for DSE. DSE assessment of modulation instability is shown in Section 3. Section 4 also presents the outcomes and discussion. Finally, in Section 5, the conclusion is provided.

2. Nonlinear (2+1)-Dimensional Davey-Stewartson Equation

The Davey-Stewartson system [32] is of the form

$$\begin{aligned} i\frac{\partial Z}{\partial t} + \frac{1}{2}\alpha^2\left(\frac{\partial^2 Z}{\partial x^2} + \alpha^2\frac{\partial^2 Z}{\partial y^2}\right) - \nu|Z|^2 Z + Z\frac{\partial Q}{\partial x} &= 0, \\ \frac{\partial^2 Q}{\partial x^2} - \alpha^2\frac{\partial^2 Q}{\partial y^2} - 2\nu\frac{\partial |Z|^2}{\partial x} &= 0. \end{aligned} \quad (1)$$

Here $z(x, y, t)$ is a complex value function, x is the dimensionless variable, y is the propagation coordinate, t is the time, $\nu = \pm 1$, and $\alpha^2 = \pm 1$. The case where $\alpha = 1$ is called the DSE I equation, while that where $\alpha = i$ is the DSE II. The ν parameter denotes the focusing or defocusing case. The DSE has the following types of soliton solutions: conventional line, algebraic, periodic, and lattice solution. Conventional line soliton has a structure that is essentially one-dimensional. The solitons of periodic, algebraic, and lattice have a structure of two dimensions. The DSE I and II are common examples of two-dimensional integrable equations that emerge as a higher-dimensional generalization of the nonlinear Schrödinger equation (NLSE). DSE has several applications that include the description of gravity-capillarity surface wave packets within the shallow water boundary. Several strong techniques have been created to obtain explicit solutions for (1), such as the technique of homotopy analysis [48], the sine-cosine method [49], and the technique of variational iteration [50]. To obtain fresh soliton solutions for (1) using the following traveling wave equation, we apply the widely discussed GEERE technique in [44]:

$$\begin{aligned} z(x, y, t) &= Z(\xi)e^{i\gamma}, \\ q(x, y, t) &= Q(\xi), \\ \xi &= x + y - ct, \\ \gamma &= k_1x + k_2y + k_3t, \end{aligned} \quad (2)$$

where c, k_1, k_2 , and k_3 are real constants. Substituting (2) into (1) gives the real and imaginary part:

$$c = \alpha^2(k_1 + \alpha^2k_2), \quad (3)$$

$$\begin{aligned} \alpha^2(1 + \alpha^2)Z''(\xi) + 2\nu Z^3(\xi) \\ - 2[(Q'(\xi) + k_3) + \alpha^2(k_1^2 + \alpha^2k_2^2)]Z(\xi), \end{aligned} \quad (4)$$

$$(1 - \alpha^2)Q''(\xi) - 2\nu(Z^2(\xi))' = 0. \quad (5)$$

Integrating (5) with respect to ξ and setting the constant of integration to zero, we have

$$Q'(\xi) = \frac{2\nu}{1 - \alpha^2}Z^2(\xi). \quad (6)$$

Putting (6) into (4) gives

$$\begin{aligned} \alpha^2(\alpha^4 - 1)Z''(\xi) + 2\nu(\alpha^2 + 1)Z^3(\xi) \\ - (\alpha^2 - 1)[2k_3 + \alpha^2(k_1^2 + \alpha^2k_2^2)]Z(\xi) = 0. \end{aligned} \quad (7)$$

Introducing a new ansatz of finite rational expansion in the form:

$$Z(\xi) = a_0 + \sum_{i=1}^N \frac{a_i\phi^i(\xi) + b_i\phi^{i-1}(\xi)\phi'(\xi)}{(\mu\phi(\xi) + 1)^i}, \quad (8)$$

the new parameter $\phi = \phi(\xi)$ and

$$\phi' = \sqrt{\sum_{j=0}^4 l_j\phi^j}, \quad (9)$$

where l_j, a_0, a_i , and b_i ($j = 0, 1, 2, \dots; i = 1, 2, 3, \dots, N$) are constant real values to be computed. Using homogeneous balance principle by balancing the highest nonlinear terms and the highest order partial derivative terms in (7) we have $N = 1$; hence

$$Z(\xi) = a_0 + \frac{a_1\phi(\xi) + b_1\phi'(\xi)}{\mu\phi(\xi) + 1}, \quad (10)$$

putting (10) and (9) in (7) we have a system of algebraic equations in parameters, $l_1, l_2, l_3, l_4, a_0, a_1, b_1, \mu$. The system of algebraic equations after solving gives the following family of solutions with respect to different cases.

Case 1. $l_4 = 0, l_3 = 0$.

Family 1.1. $a_0 = \pm\sqrt{-(\alpha^2(\alpha^2 - 1)l_2)/\kappa/\sqrt{2}}, a_1 = a_1, b_1 = a_1, k_3 = -(1/4)\alpha^2(2k_2^2\sigma^2 + 2k_1^2 + \alpha^2l_2 + l_2), \mu = 0$

$$\begin{aligned}
Z_{11}(\xi) = & \pm \frac{\sqrt{-(\alpha^2(\alpha^2-1)l_2)/\kappa}}{\sqrt{2}} + a_1 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right) \\
& \pm a_1 \sqrt{l_2 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right)^2 + l_1 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right) + l_0} \\
& \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right],
\end{aligned} \tag{11}$$

$$\begin{aligned}
Q_{11}(\xi) &= \frac{2\nu}{3(1-\alpha^2)} (Z_{11}(\xi))^3, \\
l_0 &= \frac{l_1^2}{4l_2},
\end{aligned} \tag{12}$$

$$l_2 > 0.$$

$$\begin{aligned}
Z_{12}(\xi) = & \pm \frac{\sqrt{-(\alpha^2(\alpha^2-1)l_2)/\kappa}}{\sqrt{2}} \\
& + \frac{a_1 \left(\left(l_2 \sqrt{l_1^2 \left(\sin^2 \left(\sqrt{l_2(-ct+x+y)} \right) \right) - 1} \right) / l_2 \right) + l_1 \left(\sin \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right)}{2l_2} \\
& \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right],
\end{aligned} \tag{13}$$

$$\begin{aligned}
Q_{12}(\xi) &= \frac{2\nu}{3(1-\alpha^2)} (Z_{12}(\xi))^3, \\
l_0 &= 0, \\
l_2 &< 0.
\end{aligned} \tag{14}$$

$$\begin{aligned}
Z_{13}(\xi) = & \pm \frac{\sqrt{-\alpha^2(\alpha^2-1)l_2/\kappa}}{\sqrt{2}} \\
& + \frac{a_1 \left(l_2 \sqrt{l_1^2 \left(\sin^2 h \left(\sqrt{l_2(-ct+x+y)} \right) \right) - 1} \right) / l_2 + 4l_0 + l_1 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right)}{2l_2} \\
& \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right],
\end{aligned} \tag{15}$$

$$\begin{aligned}
Q_{13}(\xi) &= \frac{2\nu}{3(1-\alpha^2)} (Z_{13}(\xi))^3 \\
l_0 &= 0, \\
l_2 &> 0.
\end{aligned} \tag{16}$$

Family 1.2. $a_0 = 0$, $a_1 = a_1$, $b_1 = 2\sqrt{(\alpha^2(\alpha^2-1)(-\kappa)l_2^2(l_1^2-4l_0l_2)^2)/(\kappa l_1^3-4\kappa l_0 l_1 l_2)} \pm a_1$,
 $k_3 = (1/2)\alpha^2(2l_2\alpha^2+2l_2+k_2^2\sigma^2+k_1^2)$, $\mu = 0$

$$\begin{aligned}
Z_{14}(\xi) &= a_1 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right) \pm a_1 + \left(\frac{l_1}{2l_2 \left(e^{\sqrt{l_2(-ct+x+y)}} - l_1/2l_2 \right) + l_1} \right) \frac{2}{\kappa l_1^3 - 4\kappa l_0 l_1 l_2} \\
&\times \left[\sqrt{l_2 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right)^2 + l_1 \left(e^{\sqrt{l_2(-ct+x+y)}} - \frac{l_1}{2l_2} \right) + l_0 \sqrt{-\alpha^2 (\alpha^2 - 1) \kappa l_2^2 (l_1^2 - 4l_0 l_2)^2}} \right] \\
&\times \exp \left[i \left(k_1 x + k_2 y + \frac{1}{2} \alpha^2 (2l_2 \alpha^2 + 2l_2 + k_2^2 \sigma^2 + k_1^2) \right) \right],
\end{aligned} \tag{17}$$

$$Q_{14}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{14}(\xi))^3,$$

$$l_0 = \frac{l_1^2}{4l_2}, \tag{18}$$

$$l_2 > 0.$$

$$\begin{aligned}
Z_{15}(\xi) &= \frac{\csc \left(\sqrt{l_2(ct-x-y)} \right)}{2\kappa l_1^3 l_2} \left(a_1 \kappa l_1^3 \left(l_1 \left(\sin \left(\sqrt{l_2(ct-x-y)} \right) - 1 \right) - 2l_2 \right) + 2l_2 \sqrt{-\alpha^2 (\alpha^2 - 1) \kappa l_1^4 l_2^2} \right. \\
&\times \left. \sqrt{-\frac{l_1^2 \cos^2 \left(\sqrt{l_2(ct-x-y)} \right)}{l_2}} \right) \times \exp \left[i \left(k_1 x + k_2 y + \frac{1}{2} \alpha^2 (2l_2 \alpha^2 + 2l_2 + k_2^2 \sigma^2 + k_1^2) \right) \right],
\end{aligned} \tag{19}$$

$$Q_{15}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{15}(\xi))^3,$$

$$l_0 = 0, \tag{20}$$

$$l_2 < 0.$$

$$\begin{aligned}
Z_{16}(\xi) &= \frac{\operatorname{csch} \left(\sqrt{-l_2(ct-x-y)} \right)}{2\kappa l_1^3 l_2} \left[a_1 \kappa l_1^3 \left(l_1 \left(\sinh \left(\sqrt{-l_2(ct-x-y)} \right) - 1 \right) - 2l_2 \right) + 2l_2 \sqrt{-\alpha^2 (\alpha^2 - 1) \kappa l_1^4 l_2^2} \right. \\
&\times \left. \sqrt{-\frac{l_1^2 \cos^2 h \left(\sqrt{-l_2(ct-x-y)} \right)}{l_2}} \right] \times \exp \left[i \left(k_1 x + k_2 y + \frac{1}{2} \alpha^2 (2l_2 \alpha^2 + 2l_2 + k_2^2 \sigma^2 + k_1^2) \right) \right],
\end{aligned} \tag{21}$$

$$Q_{16}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{16}(\xi))^3,$$

$$l_0 = 0, \tag{22}$$

$$l_2 < 0.$$

Case 2. $l_1 = 0, l_3 = 0.$

Family 2.1. $a_0 = -\sqrt{-\alpha^2(\alpha^2 - 1)l_2/\kappa/\sqrt{2}}$, $a_1 = a_1$, $b_1 = a_1$,
 $k_3 = -(1/4)\alpha^2 t(2k_2^2\sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2)$, $\mu = 0$

$$\begin{aligned} Z_{21}(\xi) &= a_1 \sqrt{\kappa} \sqrt{l_0} \sqrt{l_4^2 l_2^3 \tan^2 \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) \left(l_2 l_4^3 \tan^2 \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) + 4 \right) + 16l_0} \\ &+ 2l_2 l_4 \sqrt{l_0} a_1 \sqrt{\kappa} \tan \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) - 2\sqrt{l_0} \sqrt{-\alpha^2(\alpha^2 - 1)l_2^2} \\ &\times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right], \end{aligned} \quad (23)$$

$$Q_{21}(\xi) = \frac{2\nu}{3(1-\alpha^2)} (Z_{21}^3(\xi))^3,$$

$$l_0 = \frac{l_1^2}{4l_2}, \quad (24)$$

$$l_2 > 0,$$

$$l_4 < 0.$$

$$\begin{aligned} Z_{22}(\xi) &= a_1 \sqrt{\kappa} \sqrt{l_0} \sqrt{l_4^2 l_2^3 \tan^2 h \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) \left(l_2 l_4^3 \tan^2 h \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) + 4 \right) + 16l_0} \\ &+ 2l_2 l_4 \sqrt{l_0} a_1 \sqrt{\kappa} \tanh \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) - 2\sqrt{l_0} \sqrt{-\alpha^2(\alpha^2 - 1)l_2^2} \\ &\times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right], \end{aligned} \quad (25)$$

$$Q_{22}(\xi) = \frac{2\nu}{3(1-\alpha^2)} (Z_{22}^3(\xi))^3,$$

$$l_0 = \frac{l_1^2}{4l_2}, \quad (26)$$

$$l_2 > 0,$$

$$l_4 > 0.$$

Family 2.2. $a_0 = 0$, $a_1 = a_1$, $b_1 = ((\sqrt{\alpha^4 \kappa l_0 l_2} - \alpha^2 \kappa l_0 l_2 - 2a_1 \kappa l_0)/2\kappa l_0)$, $k_3 = -(1/4)\alpha^2 t(2k_2^2\sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2)$,
 $\mu = -i\sqrt{l_2}/\sqrt{l_0}$

$$\begin{aligned} Z_{23}(\xi) &= \frac{\kappa l_0 16 \sqrt{l_0}}{16 \sqrt{l_0} - 8i \sqrt{l_2} l_2 l_4 \tan \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) / \sqrt{2}} \left[\left(\sqrt{\alpha^2(\alpha^2 - 1) \kappa l_0 l_2} - 2a_1 \kappa l_0 \right) \right. \\ &\times \sqrt{l_4^2 l_2^3 \tan^2 \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) \left(l_2 l_4^3 \tan^2 \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) + 4 \right) + 16l_0} \\ &\left. + 4a_1 l_2 l_4 \kappa l_0 \tan \left(\frac{\sqrt{l_2}(-ct+x+y)}{\sqrt{2}} \right) \right] \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right], \end{aligned} \quad (27)$$

$$Q_{23}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{23}^3(\xi))^3,$$

$$l_0 = \frac{l_1^2}{4l_2}, \quad (28)$$

$$l_2 > 0,$$

$$l_4 > 0.$$

$$Z_{24}(\xi) = \frac{\kappa l_0 16 \sqrt{l_0}}{16 \sqrt{l_0} - 8i \sqrt{l_2} l_2 l_4 \tanh(\sqrt{-l_2}(-ct+x+y)/\sqrt{2})} \left[\left(\sqrt{\alpha^2(\alpha^2-1)\kappa l_0 l_2 - 2a_1 \kappa l_0} \right) \right. \\ \times \sqrt{l_4^2 l_2^3 \tan^2\left(\frac{\sqrt{-l_2}(-ct+x+y)}{\sqrt{2}}\right) \left(l_2 l_4^3 \tanh^2\left(\frac{\sqrt{-l_2}(-ct+x+y)}{\sqrt{2}}\right) + 4 \right) + 16l_0} \\ \left. + 4a_1 l_2 l_4 \kappa l_0 \tan\left(\frac{\sqrt{-l_2}(-ct+x+y)}{\sqrt{2}}\right) \right] \times \exp\left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right], \quad (29)$$

$$Q_{24}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{24}^3(\xi))^3,$$

$$l_0 = \frac{l_1^2}{4l_2}, \quad (30)$$

$$l_2 < 0,$$

$$l_4 > 0.$$

Case 3. $l_3 = 0, l_4 = 0, l_0 = 0.$

Family 3.1. $a_0 = 0, a_1 = a_1, b_1 = (\sqrt{\kappa h_2^2 \alpha^2 - \kappa l_2^2 \alpha^4} - 2a_1 \kappa l_1) / 2\kappa l_1, k_3 = -(1/4)\sigma^2(l_2 \alpha^2 + l_2 + 2k_2^2 \alpha^2 + 2k_1^2), \mu = l_2 / l_1$

$$Z_{31}(\xi) = \frac{2a_1 \kappa l_1 \left(l_1 \left(\sin\left(\sqrt{l_2}(-(-ct+x+y))\right) - 1 \right) - l_2 \sqrt{\left(l_1^2 \left(\sin^2\left(\sqrt{l_2}(-(-ct+x+y))\right) - 1 \right) \right) / l_2} \right)}{2\kappa l_1 l_2 \left(\sin\left(\sqrt{l_2}(-(-ct+x+y))\right) + 1 \right)} \\ + \frac{2\kappa l_1 l_2 \left(\sin\left(\sqrt{l_2}(-(-ct+x+y))\right) + 1 \right) + l_2 \sqrt{-\alpha^2(\alpha^2-1)\kappa l_2^2} \sqrt{l_1^2 \left(\sin^2\left(\sqrt{l_2}(-(-ct+x+y))\right) - 1 \right) / l_2}}{2\kappa l_1 l_2 \left(\sin\left(\sqrt{l_2}(-(-ct+x+y))\right) + 1 \right)} \quad (31)$$

$$\times \exp\left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right],$$

$$Q_{31}(\xi) = \frac{2\gamma}{3(1-\alpha^2)} (Z_{31}^3(\xi))^3, \quad (32)$$

$$l_2 < 0.$$

$$\begin{aligned}
 Z_{32}(\xi) = & \frac{2a_1\kappa l_1 \left(l_1 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right) - l_2 \sqrt{l_1^2 \left(\sin^2 h \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right) / l_2} \right)}{2\kappa l_1 l_2 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) + 1 \right)} \\
 & + \frac{2\kappa l_1 l_2 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) + 1 \right) + l_2 \sqrt{-\alpha^2(\alpha^2-1)\kappa l_2^2} \sqrt{l_1^2 \left(\sin^2 h \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right) / l_2}}{2\kappa l_1 l_2 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) + 1 \right)} \\
 & \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{4} \alpha^2 t (2k_2^2 \sigma^2 + 2k_1^2 + \alpha^2 l_2 + l_2) \right) \right],
 \end{aligned} \tag{33}$$

$$\begin{aligned}
 Q_{32}(\xi) = & \frac{2\nu}{3(1-\alpha^2)} (Z_{32}(\xi))^3, \\
 & l_2 > 0.
 \end{aligned} \tag{34}$$

Family 3.2. $a_0 = 0, a_1 = a_1, b_1 = (2\sqrt{\kappa h_2^2 \alpha^2 - \kappa l_2^2 \alpha^4} - a_1 \kappa l_1) / \kappa l_1, k_3 = -(1/2)\alpha^2(2l_2\alpha^2 + 2l_2 + k_2^2\alpha^2 + k_1^2), \mu = 2l_2/l_1$

$$\begin{aligned}
 Z_{33}(\xi) = & \frac{1}{2\kappa l_1 l_2} \left(\csc \left(\sqrt{l_2(-(-ct+x+y))} \right) \right. \\
 & \cdot \left[a_1 \kappa l_1 \left[l_1 \left(\sin \left(\sqrt{l_2(-(-ct+x+y))} \right) - 1 \right) \right. \right. \\
 & \left. \left. - l_2 \sqrt{\frac{l_1^2 \left(\sin^2 \left(\sqrt{l_2(-(-ct+x+y))} \right) - 1 \right)}{l_2}} \right] \right. \\
 & \left. + 2l_2 \sqrt{-\alpha^2(\alpha^2-1)\kappa l_2^2} \right. \\
 & \left. \times \sqrt{\frac{l_1^2 \left(\sin^2 \left(\sqrt{l_2(-(-ct+x+y))} \right) - 1 \right)}{l_2}} \right] \right) \\
 & \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{2} \alpha^2 (2l_2\alpha^2 + 2l_2 + k_2^2\alpha^2 + k_1^2) \right) \right],
 \end{aligned} \tag{35}$$

$$\begin{aligned}
 Q_{33}(\xi) = & \frac{2\nu}{3(1-\alpha^2)} (Z_{33}(\xi))^3, \\
 & l_2 < 0.
 \end{aligned}$$

$$\begin{aligned}
 Z_{34}(\xi) = & \frac{1}{2\kappa l_1 l_2} \left(\operatorname{csch} \left(\sqrt{l_2(-ct+x+y)} \right) \right. \\
 & \cdot \left[a_1 \kappa l_1 \left[l_1 \left(\sinh \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right) \right. \right. \\
 & \left. \left. - l_2 \sqrt{\frac{l_1^2 \left(\sin^2 h \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right)}{l_2}} \right] \right. \\
 & \left. + 2l_2 \sqrt{-\alpha^2(\alpha^2-1)\kappa l_2^2} \right. \\
 & \left. \times \sqrt{\frac{l_1^2 \left(\sin^2 h \left(\sqrt{l_2(-ct+x+y)} \right) - 1 \right)}{l_2}} \right] \right) \\
 & \times \exp \left[i \left(k_1 x + k_2 y - \frac{1}{2} \alpha^2 (2l_2\alpha^2 + 2l_2 + k_2^2\alpha^2 + k_1^2) \right) \right], \\
 Q_{34}(\xi) = & \frac{2\nu}{3(1-\alpha^2)} (Z_{34}(\xi))^3, \\
 & l_2 > 0.
 \end{aligned} \tag{37}$$

Case 4. $l_0 = 0, l_1 = 0, l_4 = 0.$

(36) Family 4.1. $a_0 = 0, a_1 = a_1, b_1 = ((\sqrt{\alpha^2 \kappa l_3^2 - \alpha^4 \kappa l_3^2} / l_2) - 2a_1 \kappa) / 2\kappa, k_3 = -(1/2)\alpha^2 t (k_2 \alpha^2 + l_2 - k_2^2 \alpha^2 - l_1^2), \mu = l_3 / l_2$

$$\begin{aligned}
Z_{41}(\xi) &= \frac{a_1 l_2 \csc^2((1/2)l_2(-ct+x+y))}{l_3} \\
&- \frac{1}{2\kappa l_2} \left[\left(\sqrt{-\alpha^2(\alpha^2-1)} \kappa l_2^2 \right. \right. \\
&- 2a_1 \kappa l_2 \cot^2\left(\frac{1}{2}l_2(-ct+x+y)\right) \\
&\left. \left. \cdot \sqrt{\frac{l_2^2 \sec^4((1/2)l_2(-ct+x+y))(l_3 - l_2^2 \sec^2((1/2)l_2(-ct+x+y)))}{l_3^3}} \right) \right] \\
&\times \exp \left[i \left(k_1 x \right. \right. \\
&\left. \left. + k_2 y - \frac{1}{2} \alpha^2 t (k_3 \alpha^2 + l_2 - k_3^2 \alpha^2 - l_1^2) \right) \right],
\end{aligned} \tag{39}$$

$$\begin{aligned}
Q_{41}(\xi) &= \frac{2\gamma}{3(1-\alpha^2)} (Z_{41}^3(\xi))^3, \\
l_2 &< 0.
\end{aligned} \tag{40}$$

$$\begin{aligned}
Z_{42}(\xi) &= \frac{a_1 l_2 \csc^2((1/2)l_2(-ct+x+y))}{l_3} \\
&- \frac{1}{2\kappa l_2} \left[\left(\sqrt{-\alpha^2(\alpha^2-1)} \kappa l_2^2 \right. \right. \\
&- 2a_1 \kappa l_2 \cot^2 h\left(\frac{1}{2}l_2(-ct+x+y)\right) \\
&\left. \left. \cdot \sqrt{\frac{l_2^2 \sec^4 h((1/2)l_2(-ct+x+y))(l_3 - l_2^2 \sec^2 h((1/2)l_2(-ct+x+y)))}{l_3^3}} \right) \right] \\
&\times \exp \left[i \left(k_1 x \right. \right. \\
&\left. \left. + k_2 y - \frac{1}{2} \alpha^2 t (k_3 \alpha^2 + l_2 - k_3^2 \alpha^2 - l_1^2) \right) \right],
\end{aligned} \tag{41}$$

$$\begin{aligned}
Q_{42}(\xi) &= \frac{2\gamma}{3(1-\alpha^2)} (Z_{42}^3(\xi))^3, \\
l_2 &> 0.
\end{aligned} \tag{42}$$

Family 4.2. $a_0 = 0$, $a_1 = a_1$, $b_1 = (-2a_1 \sqrt{\kappa} + i\sqrt{2}\mu)/2\sqrt{\kappa}$, $k_3 = (1/2)(k_1^2 - k_2^2)$, $\mu = 0$

$$\begin{aligned}
Z_{43}(\xi) &= \frac{1}{2\sqrt{\kappa}} \left[i(\sqrt{2}\mu \right.
\end{aligned}$$

$$\begin{aligned}
&+ 2ia_1 \sqrt{\kappa} \\
&\left. \cdot \sqrt{\frac{l_2^2 \sec^4((1/2)l_2(-ct+x+y))(l_3 - l_2^2 \sec^2((1/2)l_2(-ct+x+y)))}{l_3^3}} \right] \\
&+ \frac{a_1 l_2 \sec^2((1/2)l_2(-ct+x+y))}{l_3} \\
&\times \exp \left[i \left(k_1 x \right. \right. \\
&\left. \left. + k_2 y + \frac{1}{2} t (k_1^2 - k_2^2) \right) \right],
\end{aligned} \tag{43}$$

$$\begin{aligned}
Q_{43}(\xi) &= \frac{2\gamma}{3(1-\alpha^2)} (Z_{43}^3(\xi))^3, \\
l_2 &< 0.
\end{aligned} \tag{44}$$

$$\begin{aligned}
Z_{44}(\xi) &= \frac{1}{2\sqrt{\kappa}} \left[i(\sqrt{2}\mu \right. \\
&+ 2ia_1 \sqrt{\kappa} \\
&\left. \cdot \sqrt{\frac{l_2^2 \operatorname{sech}^4((1/2)l_2(-ct+x+y))(l_3 - l_2^2 \operatorname{sech}^2((1/2)l_2(-ct+x+y)))}{l_3^3}} \right] \\
&+ \frac{a_1 l_2 \sec^2 h((1/2)l_2(-ct+x+y))}{l_3} \\
&\times \exp \left[i \left(k_1 x \right. \right. \\
&\left. \left. + k_2 y + (1/2) t (k_1^2 - k_2^2) \right) \right],
\end{aligned} \tag{45}$$

$$\begin{aligned}
Q_{44}(\xi) &= \frac{2\gamma}{3(1-\alpha^2)} (Z_{44}^3(\xi))^3, \\
l_2 &> 0.
\end{aligned} \tag{46}$$

Case 5. $l_0 = 0, l_1 = 0$.

Family 5.1. $a_0 = 0$, $a_1 = a_1$, $b_1 = (a_1(-\sqrt{\kappa})l_3 - 4i\sqrt{2}l_4)/\sqrt{\kappa}l_3$, $k_3 = (1/2)(k_1^2 - k_2^2)$, $\mu = 4l_4/l_3$

$$\begin{aligned}
Z_{51}(\xi) &= \frac{1}{(4l_2 l_4 \sec^2(\sqrt{-l_2}\xi/2)/l_3 (2\sqrt{-l_2}l_4 \tan((1/2)\sqrt{-l_2}\xi) + l_3) + 1} \left[\frac{a_1 l_2 \sec^2(\sqrt{-l_2}\xi/2)}{2\sqrt{-l_2}l_4 \tan((1/2)\sqrt{-l_2}\xi) + l_3} + \left(\frac{(a_1(-\sqrt{\kappa})l_3 - 4i\sqrt{2}l_4)}{\sqrt{\kappa}l_3} \times \left[\frac{l_4^4 \sec^8(\sqrt{-l_2}\xi/2)}{(2\sqrt{-l_2}l_4 \tan((1/2)\sqrt{-l_2}\xi) + l_3)^4} \right. \right. \right. \\
&+ \frac{l_3^3 \sec^6(\sqrt{-l_2}\xi/2)}{(2\sqrt{-l_2}l_4 \tan((1/2)\sqrt{-l_2}\xi) + l_3)^3} \\
&\left. \left. \left. + \frac{l_2^3 \sec^4(\sqrt{-l_2}\xi/2)}{(2\sqrt{-l_2}l_4 \tan((1/2)\sqrt{-l_2}\xi) + l_3)^2} \right]^{1/2} \right) \right] \\
&\times \exp \left[i \left(k_1 x \right. \right. \\
&\left. \left. + k_2 y + \frac{1}{2} t (k_1^2 - k_2^2) \right) \right],
\end{aligned} \tag{47}$$

$$\begin{aligned}
Q_{51}(\xi) &= \frac{2\gamma}{3(1-\alpha^2)} (Z_{51}^3(\xi))^3, \\
l_2 &< 0.
\end{aligned} \tag{48}$$

$$\begin{aligned}
 Z_{52}(\xi) &= \frac{1}{(4l_2l_4 \sec^2(\sqrt{l_2}\xi/2)/l_3(2\sqrt{l_2}l_4 \tan((1/2)\sqrt{l_2}\xi) + l_3)) + 1} \left[\frac{a_1l_2 \sec^2(\sqrt{l_2}\xi/2)}{2\sqrt{l_2}l_4 \tan((1/2)\sqrt{l_2}\xi) + l_3} + \left(\frac{(a_1(-\sqrt{\kappa})l_3 - 4i\sqrt{2}l_4)}{\sqrt{\kappa}l_3} \times \left[\frac{l_4l_2^4 \sec^8(\sqrt{l_2}\xi/2)}{(2\sqrt{l_2}l_4 \tan((1/2)\sqrt{l_2}\xi) + l_3)^4} \right. \right. \right. \\
 &+ \left. \left. \frac{l_3l_2^3 \sec^6(\sqrt{l_2}\xi/2)}{(2\sqrt{l_2}l_4 \tan((1/2)\sqrt{l_2}\xi) + l_3)^3} \right. \right. \\
 &+ \left. \left. \frac{l_2^3 \sec^4(\sqrt{l_2}\xi/2)}{(2\sqrt{l_2}l_4 \tan((1/2)\sqrt{l_2}\xi) + l_3)^2} \right]^{1/2} \right) \\
 &\times \exp \left[i \left(k_1x \right. \right. \\
 &+ \left. \left. k_2y + \frac{1}{2}t(k_1^2 - k_2^2) \right) \right],
 \end{aligned} \tag{49}$$

$$Q_{52}(\xi) = \frac{2\nu}{3(1-\alpha^2)} (Z_{52}^3(\xi))^3, \tag{50}$$

$l_2 > 0$.

3. Modulation Instability of Davey-Stewartson Equation

Equation (1) reveals a certain formal connection with the defocusing nonlinear Schrödinger equation in 1+1 dimension [51, 52], given below

$$i \frac{\partial Z}{\partial t} + \frac{1}{2} \alpha^2 \frac{\partial^2 Z}{\partial x^2} - \nu |Z|^2 Z = 0. \tag{51}$$

Most nonlinear higher-order NLPDEs demonstrate instability leading to the research of steady-state modulation as a result of interaction between nonlinear and dispersive impacts. The modulation instability analysis of (51) is carried out by using the standard linear stability analysis [53, 54] to check how weak and time dependent perturbations develop along the propagation distance [53, 54]. The 1+1 dimension defocusing nonlinear Schrödinger equation (51) of the Non-linear (2+1)-Dimensional Davey-Stewartson Equation (1) has a state solution of the form

$$\begin{aligned}
 Z(x, y, t) &= (\sqrt{P} + \phi(x, y, t)) e^{iy}, \\
 \gamma(t) &= P\beta t,
 \end{aligned} \tag{52}$$

where the optical power P is normalized and β is a constant. The perturbation $\phi(x, y, t)$ is examined by utilizing linear stability analysis. Putting (52) into (51) and linearizing, we obtain

$$i \frac{\partial \phi}{\partial t} + \frac{1}{2} \alpha^2 \frac{\partial^2 \phi}{\partial x^2} + \sqrt{P^3} (\beta + \nu) - 3P\nu\phi^* = 0, \tag{53}$$

where $*$ is a complex conjugate. The solution of (53) is considered in the form giving below

$$\phi(x, y, t) = \psi_1 e^{i(h_1x+h_2y-t\lambda)} + \psi_2 e^{-i(h_1x+h_2y-t\lambda)}, \tag{54}$$

where λ is the frequency of perturbation and h_1, h_2 normalized wave numbers. The dispersion relation determines how spatial oscillations e^{ih_1x}, e^{ih_2y} are linked to time oscillations

$e^{i\lambda t}$ of a wave number; substituting (54) into (53), we obtain the dispersive relation as

$$\lambda = \pm \frac{\alpha^2 \psi_1 h_1^2 + 6\psi_2 \nu P}{2\psi_1}, \quad \psi_1 \neq 0. \tag{55}$$

The dispersion relation in (55) above demonstrates that the steady-state stability depends on the self-phase modulation, stimulated scattering, group velocity dispersion, and self-phase modulation. The expression of (55) shows that the value of the frequency λ is real for all values of h except when $\psi = 0$, which means that the steady-state solution is stable.

4. Results and Discussion

Rational, periodic, and solitonic solutions for the DSE have been successfully obtained using the GEERE method. The constructed solutions by this method are novel and unique from other solutions obtained using different methods [48–50]. Different forms of solutions such as trigonometric, hyperbolic trigonometric, and rational functions are obtained because the assumed solution of GEERE is different from other methods. Physical properties of some obtained results are demonstrated graphically using suitable parameters. Figure 1 evaluates the rational solitary wave solution (11), periodic solitary wave solution (13), and soliton solitary wave solution (15) plotted at $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = 0.1; y = 0.8)$, $(a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = -30; \kappa = -0.7; c = 0.1; y = 0.8)$, and $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively. The rational solitary wave solution (17), periodic solitary wave solution (19), and soliton solitary wave solution (21) are evaluated in Figure 2 plotted at $(a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = 20; \kappa = 0.7; c = -0.1; y = 0.8)$, $(a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = -30; \kappa = -0.7; c = -0.1; y = 0.8)$, and $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = -0.1; y = 0.8)$, respectively. Figure 3 shows a periodic solitary wave solution (23) and a dark solitary wave solution (25) plotted at $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = 30; \kappa = -0.7; c = 0.1; y = 0.8)$ and $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 =$

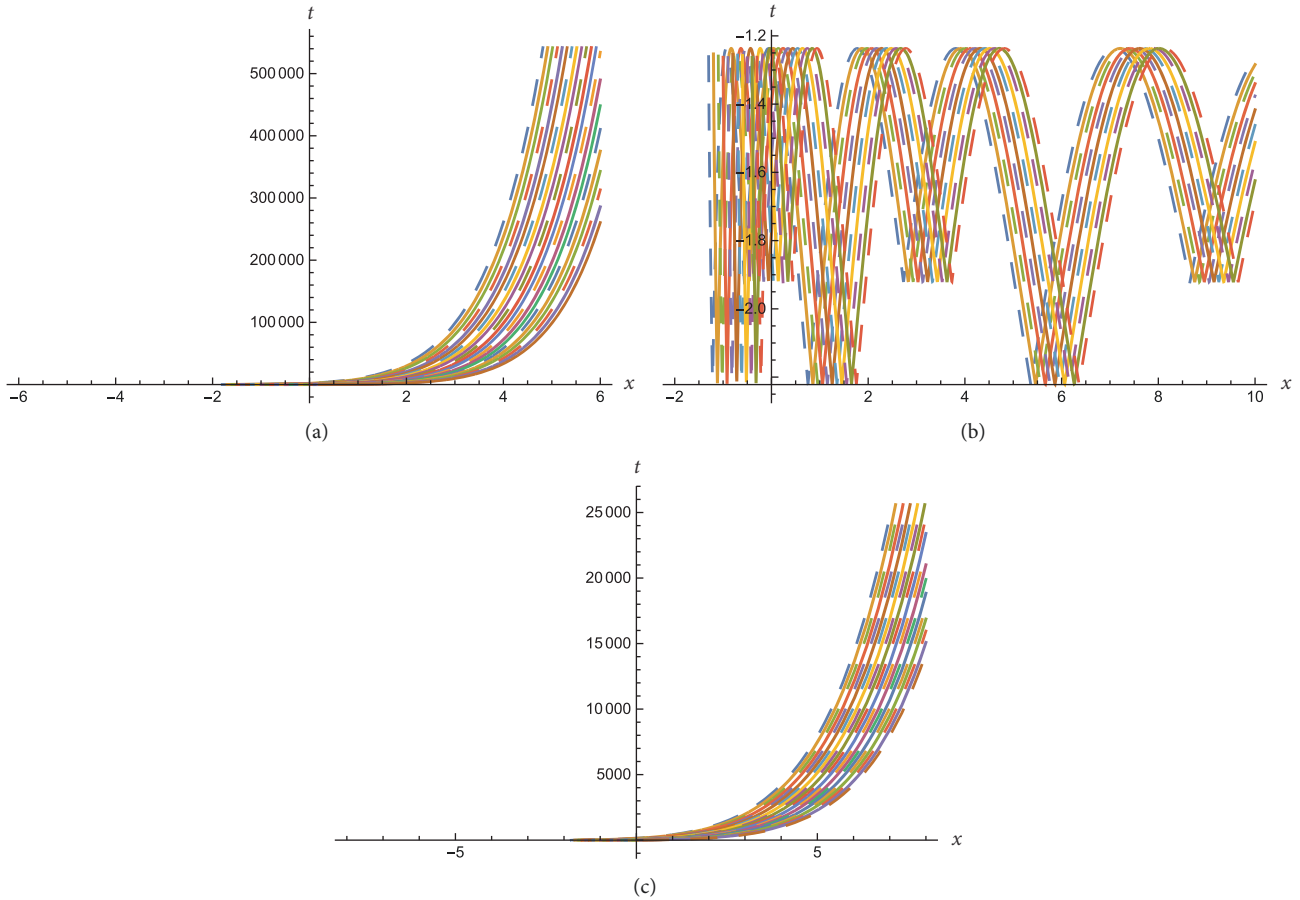


FIGURE 1: Plot of the exact traveling wave solutions of (a) rational solitary wave solution (11) with parameters $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = 0.1; y = 0.8$; (b) periodic solitary solution (13) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = -30; \kappa = -0.7; c = 0.1; y = 0.8$; and (c) soliton solitary solution (15) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = 0.1; y = 0.8$.

$9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8$), respectively. Periodic solitary wave solution (27) and dark solitary wave solution (29) plotted at $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = 30; \kappa = -0.7; c = 0.1; y = 0.8)$ and $(a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively, are evaluated in Figure 4. Figure 5 evaluates bright solitary wave solution (31) and soliton solitary wave solution (33) plotted at $(a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8)$ and $(a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = 10; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively. Periodic solitary wave solution (35) and soliton solitary wave solution (37) plotted at $(a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8)$ and $(a_1 = 9; \alpha = 0.6; l_1 = 9; l_2 = 10; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively, are evaluated in Figure 6. Figure 7 shows periodic solitary wave solution (39) and dark solitary wave solution (41) plotted at $(a_1 = 9; \alpha = 0.6; l_2 = -1; l_3 = -1; \kappa = 0.7; c = 0.1; y = 0.8)$ and $(a_1 = 9; \alpha = 0.6; l_2 = 9; l_3 = 1; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively. Figure 8 shows periodic solitary wave solution (43) and dark solitary wave solution (45) plotted at $(a_1 = 9; \alpha = 0.6; l_2 = -1; l_3 = -1; \kappa = 0.7; c = 0.1; y = 0.8)$ and $(a_1 = 1; \alpha = 0.6; l_2 = 1; l_3 = 1; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively. Also periodic solitary wave solution (47) and dark solitary wave solution (49) plotted at $(a_1 = 9; l_2 = -3; l_3 = -3; l_4 = 1; \kappa = 0.7; c = 0.1; y = 0.8)$ and

$(a_1 = 1; l_2 = 5; l_3 = 5; l_4 = 0; \kappa = 0.7; c = 0.1; y = 0.8)$, respectively, are evaluated in Figure 9. The dispersion relation among frequency (λ) and wave numbers (h_1, h_2) of (55) is plotted in Figure 10. This relation determines how spatial oscillations e^{ih_2x}, e^{ih_2y} are linked to time oscillations $e^{i\lambda t}$ of a wave number of obtained solutions for the DSE.

5. Conclusion

By implementing the strong GEERE technique, this article has effectively built fresh solitonic and reasonable regular solutions for the DSE. Some freshly obtained solitary wave solutions are provided graphically to display their physical characteristics and also provide information on the circumstances for the creation of solitons of light and kink. The solutions extracted have theoretical and experimental applications that can be anticipated to occur in other associated revolutionary equations such as the nonlinear Schrödinger equation to explain the Bose-Einstein condensation. We contrasted our solution to current existing solution and claimed that there are many new solutions. The equation recognize the huge variety of possible solutions for only values of a small subset of parameters, which helps to comprehend the physical phenomena of this equation. Modulation instability

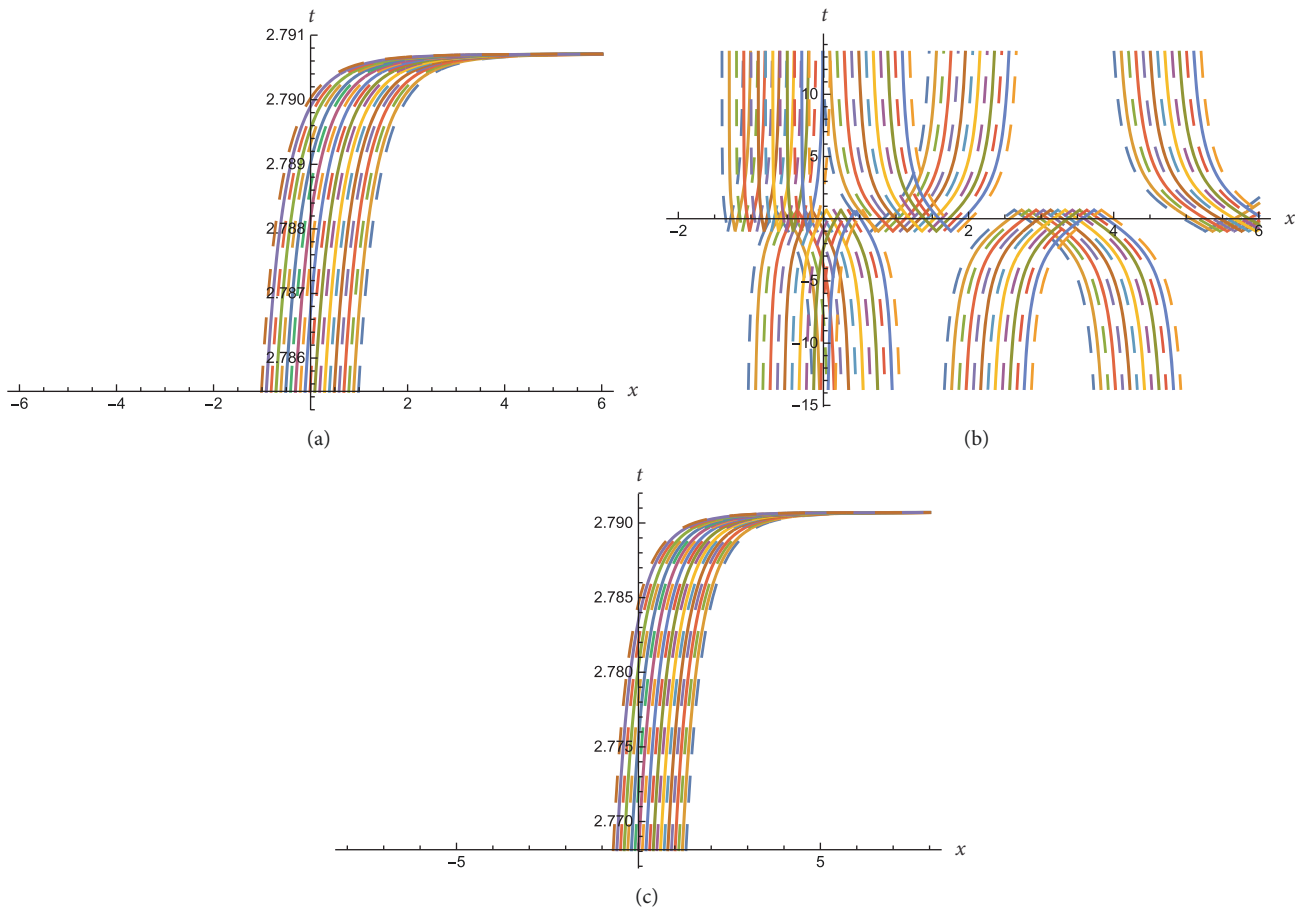


FIGURE 2: Plot of the exact traveling wave solutions of (a) rational solitary wave solution (17) with parameters $a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = 20; \kappa = 0.7; c = -0.1; y = 0.8$; (b) periodic solitary wave solution (19) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0; l_1 = 9; l_2 = -30; \kappa = -0.7; c = -0.1; y = 0.8$; and (c) soliton solitary wave solution (21) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_1 = 9; l_2 = 20; \kappa = 0.7; c = -0.1; y = 0.8$.

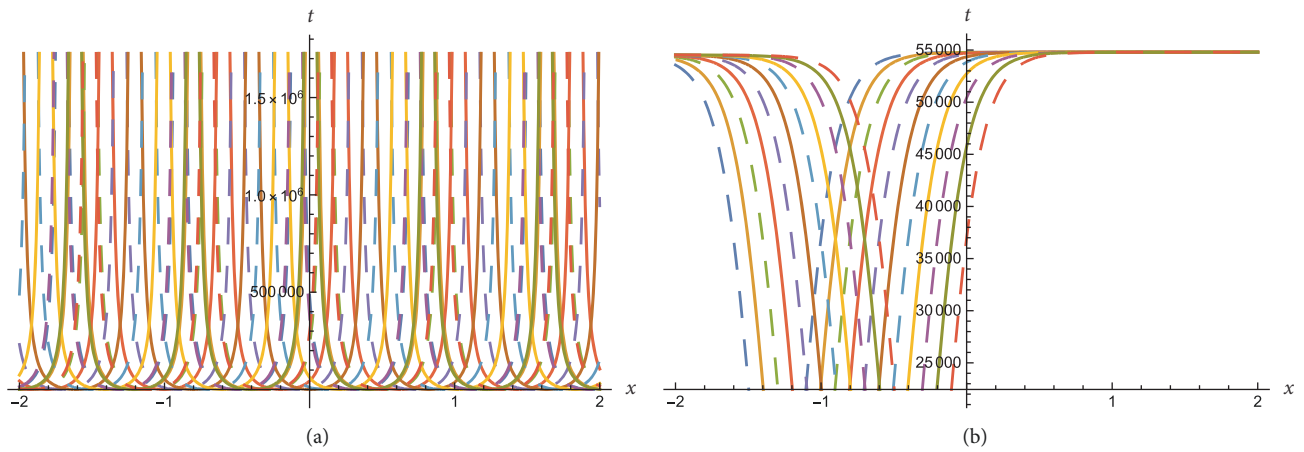


FIGURE 3: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (23) with parameters $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = 20; \kappa = 0.7; c = 0.1; y = 0.8$; (b) dark solitary wave solution (25) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0; l_4 = 9; l_2 = -30; \kappa = 0.7; c = 0.1; y = 0.8$.

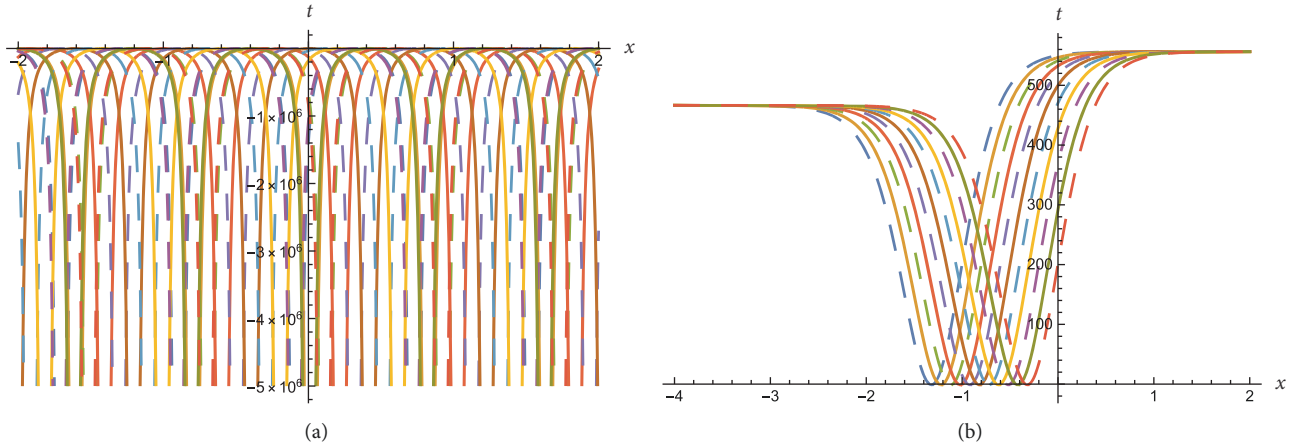


FIGURE 4: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (27) with parameters $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = 30; \kappa = -0.7; c = 0.1; y = 0.8$; (b) dark solitary wave solution (29) plotted at $a_1 = 1; \alpha = 0.6; l_0 = 0.7; l_4 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8$.

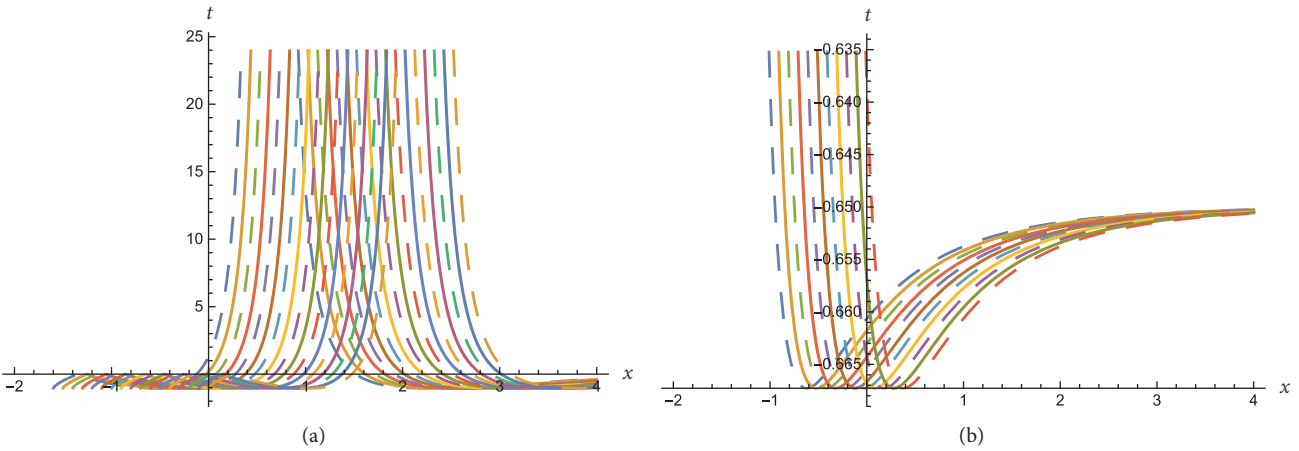


FIGURE 5: Plot of the exact traveling wave solutions of (a) bright solitary wave solution (31) with parameters $a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8$; (b) soliton solitary wave solution (33) plotted at $a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = 10; \kappa = 0.7; c = 0.1; y = 0.8$.

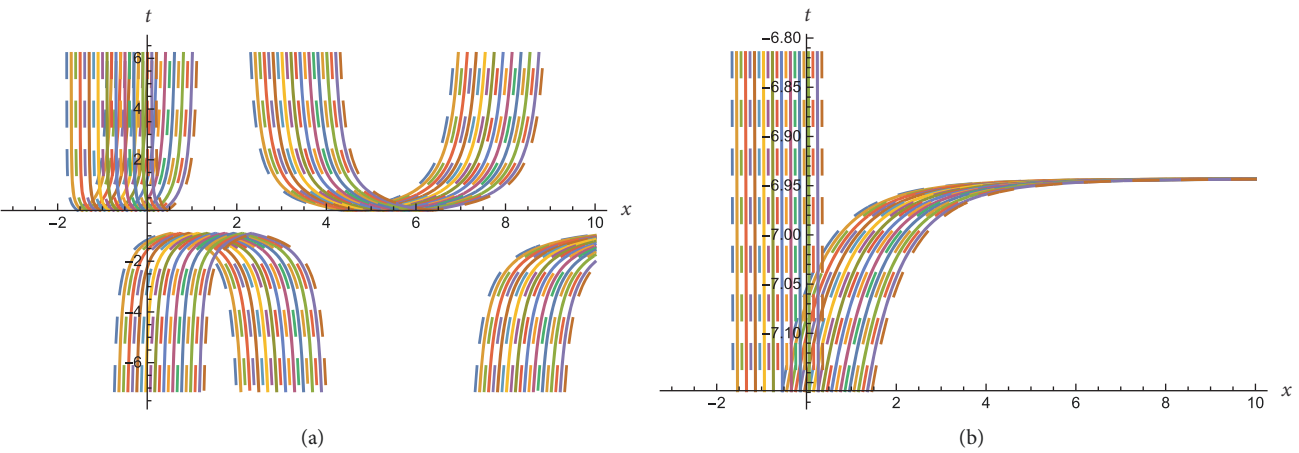


FIGURE 6: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (35) with parameters $a_1 = 1; \alpha = 0.6; l_1 = 9; l_2 = -10; \kappa = 0.7; c = 0.1; y = 0.8$; (b) soliton solitary wave solution (37) plotted at $a_1 = 9; \alpha = 0.6; l_1 = 9; l_2 = 10; \kappa = 0.7; c = 0.1; y = 0.8$.

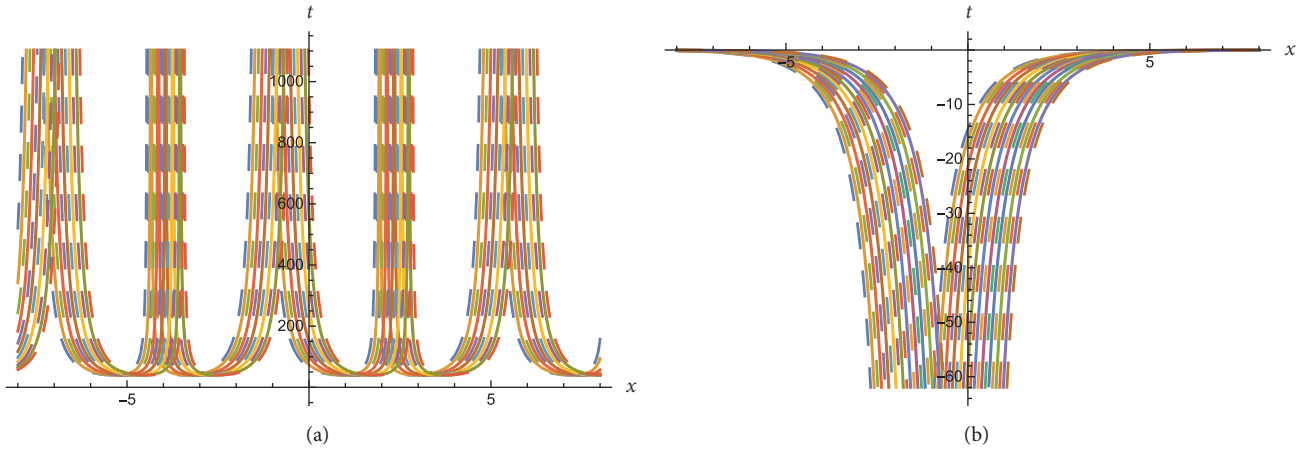


FIGURE 7: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (39) with parameters $a_1 = 9; \alpha = 0.6; l_2 = -1; l_3 = -1; \kappa = 0.7; c = 0.1; y = 0.8$; (b) dark solitary wave solution (41) plotted at $a_1 = 9; \alpha = 0.6; l_2 = 9; l_3 = 1; \kappa = 0.7; c = 0.1; y = 0.8$.

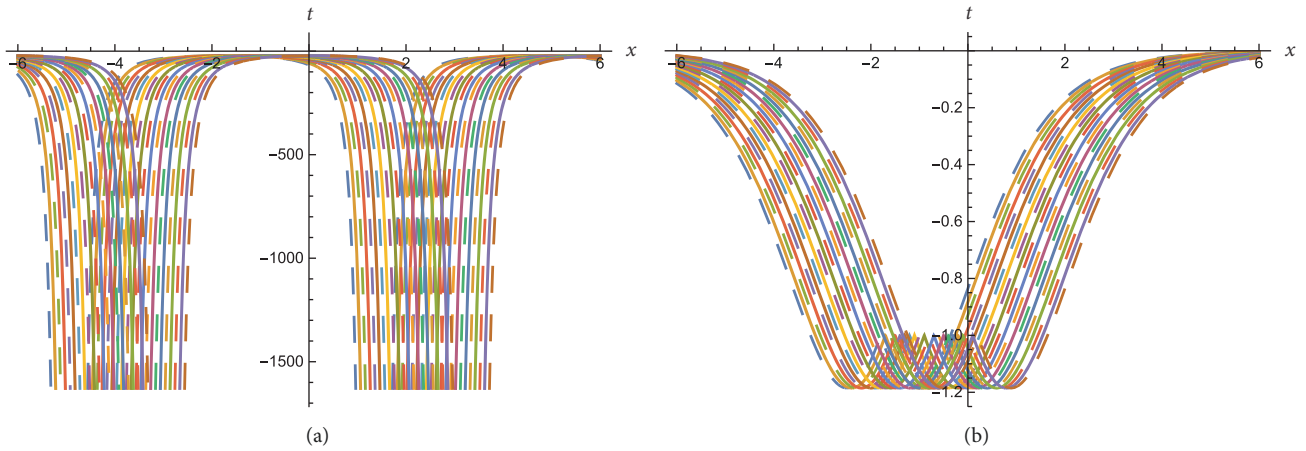


FIGURE 8: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (43) with parameters $a_1 = 9; \alpha = 0.6; l_2 = -1; l_3 = -1; \kappa = 0.7; c = 0.1; y = 0.8$; (b) dark solitary wave solution (45) plotted at $a_1 = 1; \alpha = 0.6; l_2 = 1; l_3 = 1; \kappa = 0.7; c = 0.1; y = 0.8$.

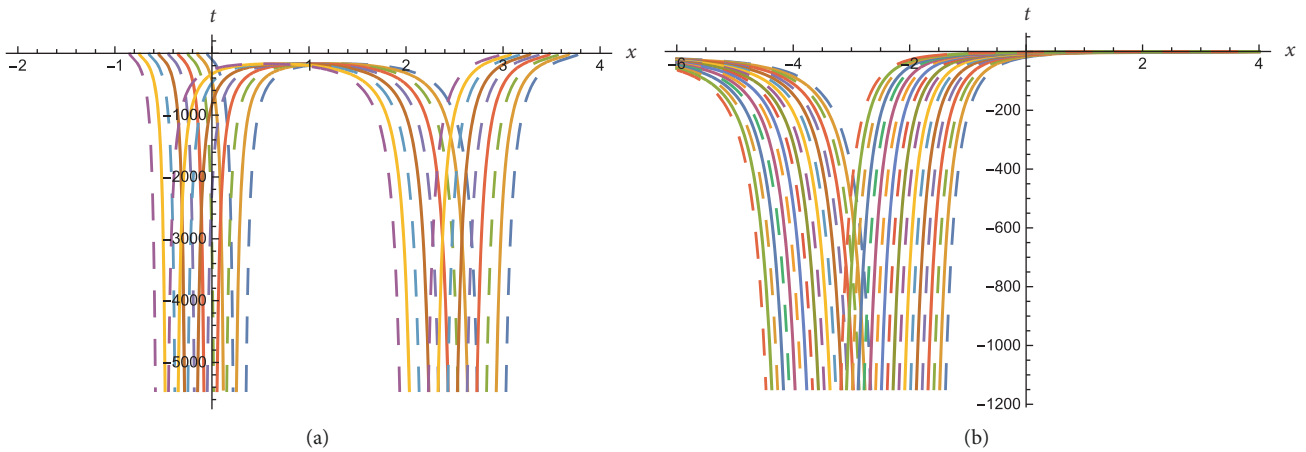


FIGURE 9: Plot of the exact traveling wave solutions of (a) periodic solitary wave solution (47) with parameters $a_1 = 9; l_2 = -3; l_3 = -3; l_4 = 1; \kappa = 0.7; c = 0.1; y = 0.8$; (b) dark solitary wave solution (49) plotted at $a_1 = 1; l_2 = 5; l_3 = 5; l_4 = 0; \kappa = 0.7; c = 0.1; y = 0.8$.

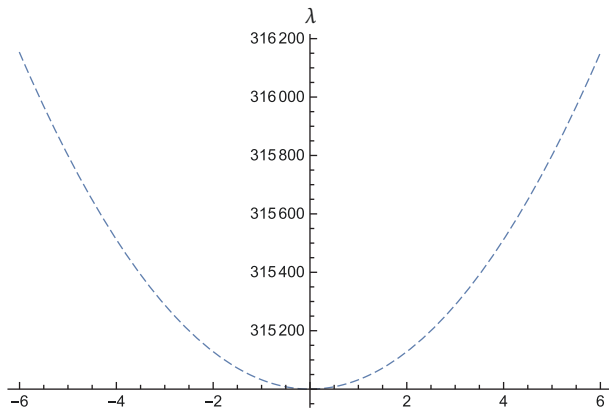


FIGURE 10: Diagram of dispersion relation among frequency (λ) and wave number (h_1).

assessment is used to verify the stability of the solutions acquired. We conclude that the modulation instability analytical expression is gained, which demonstrates that all built solutions are accurate and stable. The effectiveness and simplicity of the suggested GEERE technique show that it can be applied to distinct kinds of separate nonlinear models in different nonlinear science fields.

Data Availability

This manuscript is purely mathematical physics, and all computations were done using Mathematica software and did not involve any data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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