Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 358473, 3 pages
http://dx.doi.org/10.1155/2013/358473

Research Article

One-Phase Problems for Discontinuous Heat Transfer in Fractal Media

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Received 7 November 2012; Accepted 20 December 2012

Academic Editor: József Kázmér Tar

We first propose the fractal models for the one-phase problems of discontinuous transient heat transfer. The models are taken in sense of local fractional differential operator and used to describe the (dimensionless) melting of fractal solid semi-infinite materials initially at their melt temperatures.

1. Introduction

We know that the local fractional calculus is set up on fractals. Fractal media is complex, and it appears in different fields of engineering and physics. Fractal physical parameters are considered as local fractional continuous functions, which is fractal characteristics of local fractional functional analysis from fractal geometry point of view. Moreover, the local fractional calculus is a powerful tool to model Fourier law of heat conductions in discontinuous heat transfer in fractal media. Local fractional heat-conduction equations may be applied to describe the fractal behaviors of discontinuous heat transfer in fractal media.

As it is known the Goodman's heat balance integral method represents an approximate technique for generating functional solutions to thermal problems that were described by differential equations [1–3]. Based on theory of fractional calculus [4, 5], both the Stefan problem and the heat-balance integral method governed by a fractional diffusion equation were investigated [6–8]. However, we mention that the above problems are considered in the smooth condition.

On the other hand the heat transfer with nonsmooth condition (fractal space) is an interesting topic. The various phenomena in nanoscale heat (e.g., a charged jet in electrospinning process) can produce both continuous nanofibers and discontinuous nanoporous material. For continuous case, the classical Fourier law is valid. However, for nanoporous material, the fractal Fourier law should be used. For examples, the generalized transfer equation in a medium with fractal geometry was considered in [9], the Fourier's law heat conduction in the discontinuous media was investigated in [10], and the heat transfer from discontinuous media was discussed in [11, 12].

Maybe, there are one-phase problems of fractal heat transfer in nanoporous materials. The aim of this paper is to study the fractal models for one-phase problems. The organization of the paper is organized as follows. In Section 2, we introduce the concept of local fractional derivative and give some results on local fractional chain rule and the fractal complex transform. Section 3 is devoted to the fractal models for the one-phase problems of discontinuous transient heat transfer. Finally, conclusions are given in Section 4.

2. Preliminaries

In this section, we give some basic definitions and properties of the local fractional differential operator theory which are used further in this paper. In order to discuss the fractal behaviors of materials, we start with the fractal result derived from the fractal geometry.

Lemma 1 (see [11, 12]). *Let F be a subset of the real line and be a fractal. If $f:(F,d)\rightarrow(\Omega',d')$ is a bi-Lipschitz mapping, then there is for constants $\rho,\tau>0$ and $F\subset R$,*

$$\rho^s H^s(F)\le H^s(f(F))\le\tau^s H^s(F) \tag{1}$$

such that for all $x_1, x_2\in F$,

$$\rho^\alpha|x_1-x_2|^\alpha\le|f(x_1)-f(x_2)|\le\tau^\alpha|x_1-x_2|^\alpha. \tag{2}$$

For the convenience of the reader, we represent here the following results.

Following Lemma 1, we have [11]

$$|f(x_1)-f(x_2)|\le\tau^\alpha|x_1-x_2|^\alpha \tag{3}$$

such that

$$|f(x_1)-f(x_2)|<\varepsilon^\alpha, \tag{4}$$

where α is fractal dimension of F.

Definition 2. If

$$|f(x)-f(x_0)|<\varepsilon^\alpha \tag{5}$$

with $|x-x_0|<\delta$, for $\varepsilon,\delta>0$ and $\varepsilon,\delta\in R$, then $f(x)$ is called local fractional continuous at $x=x_0$, and it is denoted by

$$\lim_{x\rightarrow x_0}f(x)=f(x_0). \tag{6}$$

$f(x)$ is local fractional continuous on the interval (a,b), denoted through [11–14]

$$f(x)\in C_\alpha(a,b) \tag{7}$$

if (5) is valid for $x\in(a,b)$.

Definition 3. Let $f(x)\in C_\alpha(a,b)$. Local fractional derivative of $f(x)$ of order α at $x=x_0$ is defined as [11–14]

$$f^{(\alpha)}(x_0)=\left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0}=\lim_{x\rightarrow x_0}\frac{\Delta^\alpha(f(x)-f(x_0))}{(x-x_0)^\alpha}, \tag{8}$$

where $\Delta^\alpha(f(x)-f(x_0))\cong\Gamma(1+\alpha)\Delta(f(x)-f(x_0))$.

If $y(x)=(f\circ u)(x)$ where $u(x)=g(x)$, then we have [11, 14]

$$\frac{d^\alpha y(x)}{dx^\alpha}=f^{(\alpha)}(g(x))\left(g^{(1)}(x)\right)^\alpha, \tag{9}$$

where $f^{(\alpha)}(g(x))$ and $g^{(1)}(x)$ exist.

If $y(x)=(f\circ u)(x)$ where $u(x)=g(x)$, then we have [11, 14]

$$\frac{d^\alpha y(x)}{dx^\alpha}=f^{(1)}(g(x))\,g^{(\alpha)}(x), \tag{10}$$

where we assume that $f^{(1)}(g(x))$ and $g^{(\alpha)}(x)$ exist.

Let us suppose that there is a relation as given below [14]

$$X=\frac{(px)^\alpha}{\Gamma(1+\alpha)},\qquad Y=\frac{(qy)^\alpha}{\Gamma(1+\alpha)}, \tag{11}$$

where q and p are constants and $0<\alpha\le1$, then there exists an equation transformation pair, namely,

$$p^\alpha\frac{dU_1(X)}{dX}+q^\alpha\frac{dU_2(Y)}{dY}=0\Longleftrightarrow\frac{d^\alpha U_1(x)}{dx^\alpha}+\frac{d^\alpha U_2(y)}{dy^\alpha}=0. \tag{12}$$

We stress on the fact that the above method is different from fractional complex transform method discussed in [15, 16]. The fractional complex transform method is proposed in [15, 16], while fractal complex transform method is based on the local fractional calculus theory [14].

3. Fractal Models for One-Phase Problems

We propose a one-phase fractal problem that describes the (dimensionless) melting of a fractal solid semi-infinite material initially at its melt temperature. The corresponding equations are given by the following expressions:

$$\frac{\partial^\alpha u}{\partial t^\alpha}=\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}},\quad 0<x<s,\ t>0, \tag{13}$$

$$\frac{\partial^\alpha u}{\partial x^\alpha}=\beta^\alpha\frac{d^\alpha s}{dt^\alpha},\quad x=s(t),\ t>0, \tag{14}$$

$$u=0,\qquad x>0,\qquad t=0, \tag{15}$$

$$u=0,\qquad x=s(t),\qquad t>0, \tag{16}$$

$$u=1,\qquad x=0,\qquad t\ge0. \tag{17}$$

We mention that (13) governs the flow of heat in the fractal liquid region [11, 12], the fractal Stefan condition (14) describes the absorption of heat at the melt front where the fractal Stefan number β^α [11] (it is also derived from fractal complex transform [14]). Equations (15) and (16) prescribe the temperature at the fractal fixed boundary $x=0$ and on the moving melt front $x=s(t)$, and (16) gives the initial temperature of the fractal semi-infinite solution domain. We notice that (13) is derived from the local fractional one-dimensional heat conduction equation with fractal media, which can be written in the form [11]

$$\rho^\alpha c^\alpha\frac{\partial^\alpha u}{\partial t^\alpha}=K^{2\alpha}\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}, \tag{18}$$

where $K^{2\alpha}$ denotes the thermal conductivity of the fractal material, which is related to fractal dimensions of materials. It is shown that the fractal dimensions of materials are an important characteristic value. Here, we consider the fractal Fourier flow, which is discontinuous; however, it is found that it is local fractional continuous. Like classical Fourier flow, its thermal conductivity is an approximate value for fractal one when $\alpha=1$ [11].

The alternative form of the condition (14) can be derived from the fact that the total local fractional derivative of the temperature at $x = s(t)$ is zero, that is, $D^{\alpha}u(s(t),t)/Dt^{\alpha} = 0$, which leads us to the following expression:

$$\frac{\partial u}{\partial x}\frac{d^{\alpha}s}{dt^{\alpha}} + \frac{d^{\alpha}u}{dt^{\alpha}} = 0. \qquad (19)$$

Then, by using (13) and (19) we conclude that

$$\frac{d^{\alpha}s}{dt^{\alpha}} = -\frac{d^{\alpha}u/dt^{\alpha}}{\partial u/\partial x} = -\frac{\partial^{2\alpha}u/\partial x^{2\alpha}}{\partial u/\partial x}. \qquad (20)$$

As a result, it leads us to the following final equation:

$$\left(\frac{\partial^{\alpha}u}{\partial x^{\alpha}}\right)\left(\frac{\partial u}{\partial x}\right) = \beta^{\alpha}\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}, \quad x = s(t),\ t > 0. \qquad (21)$$

This result is no sense because fractal flow is local fractional continuous at x. If u is local fractional continuous, and u is continuous, we deduce that fractal dimension is $\alpha = 1$. Hence, we can obtain the classical results [2, 3].

Another alternative form of the condition (14) is derived from the fact that the total local fractional derivative of the temperature at $x = s(t)$ is zero, that is,

$$\frac{D^{\alpha}u(s(t),t)}{Dt^{\alpha}} = 0, \qquad (22)$$

which implies in our case that

$$\frac{\partial^{\alpha}u}{\partial x^{\alpha}}\left(\frac{ds}{dt}\right)^{\alpha} + \frac{d^{\alpha}u}{dt^{\alpha}} = 0. \qquad (23)$$

By using (13) and (23), we finally obtain

$$\left(\frac{ds}{dt}\right)^{\alpha} = -\frac{1}{\Gamma(1-\alpha)}\frac{d^{\alpha}s}{dt^{\alpha}} = -\frac{1}{\Gamma(1-\alpha)}\frac{\partial^{2\alpha}u/\partial x^{2\alpha}}{\partial^{\alpha}u/\partial x^{\alpha}}, \qquad (24)$$

which leads us to the final form as given below

$$\left(\frac{\partial^{\alpha}u}{\partial x^{\alpha}}\right)^{2} = \frac{\beta^{\alpha}}{\Gamma(1-\alpha)}\frac{\partial^{2\alpha}u}{\partial x^{2\alpha}}, \quad x = s(t),\ t > 0. \qquad (25)$$

4. Conclusions

In this paper we have proposed alternative fractal models for the one-phase problems of discontinuous transient heat transfer in fractal media. By applying the fractal complex transform and the chain rule within local fractional derivative, we have derived the one-phase problems of discontinuous transient heat transfer in fractal media, which describe the (dimensionless) melting of fractal solid semi-infinite materials initially at their melt temperatures. We consider the fractal models for the one-phase problems of discontinuous transient heat transfer. The fractal models for one-phase problems are classical examples when the fractional dimension is equal to 1. The discontinuous transient heat transfer in fractal media can serve as a good starting point for experimental investigations and further discussions.

Acknowledgments

This paper is sponsored by the National Natural Science Foundation of China (NSFC, Grant U1204703), the Key Scientific and Technological Project of Henan Province (122102310004), the Fundamental Research Funds for the Central Universities (HUST: 2012QN087, 2012QN088), and the Innovation Scientists and Technicians Troop Construction Projects of Zhengzhou City (10LJRC190, 121PRKXF658-4).

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