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Numerical solution of linear integral equations system using the Bernstein collocation method

Ahmad Jafarian[1], Safa A Measoomy Nia[1], Alireza K Golmankhaneh[2] and Dumitru Baleanu[3,4,5*]

*Correspondence:
dumitru@cankaya.edu.tr
[3]Department of Mathematics and Computer Sciences, Faculty of Art and Sciences, Çankaya University, Balgat, Ankara, 0630, Turkey
[4]Department of Chemical and Materials Engineering, Faculty of Engineering, King Abdulaziz University, P.O. Box 80204, Jeddah, 21589, Saudi Arabia
Full list of author information is available at the end of the article

Abstract

Since in some application mathematical problems finding the analytical solution is too complicated, in recent years a lot of attention has been devoted by researchers to find the numerical solution of this equations. In this paper, an application of the Bernstein polynomials expansion method is applied to solve linear second kind Fredholm and Volterra integral equations systems. This work reduces the integral equations system to a linear system in generalized case such that the solution of the resulting system yields the unknown Bernstein coefficients of the solutions. Illustrative examples are provided to demonstrate the preciseness and effectiveness of the proposed technique. The results are compared with the exact solution by using computer simulations.

1 Introduction

As a matter of fact, it might be said that many phenomena of almost all practical engineering and applied science problems like physical applications, potential theory and electrostatics are reduced to solving integral equations. Since these equations usually cannot be solved explicitly, so it is required to obtain approximate solutions. There are numerous numerical methods which have been focusing on the solution of integral equations. For example, Tricomi in his book [1], introduced the classical method of successive approximations for integral equations. Variational iteration method [2] and Adomian decomposition method [3] were effective and convenient for solving integral equations. Also, the Homotopy analysis method (HAM) was proposed by Liao [4] and then has been applied in [5]. Taylor expansion approach was presented for solving integral equations by Kanwal and Liu in [6] and then has been extended in [7, 8]. In addition, Babolian *et al.* [9] solved some integral equations systems by using the orthogonal triangular basis functions. Jafari *et al.* [10] applied Legendre wavelets method to find numerical solution system of linear integral equations. Moreover, some different valid methods for solving this kind of equations have been developed. First time, the Bernstein polynomials have been used for the solution of some linear and nonlinear differential equations in [11–14]. Mandal and Bhattacharya [15] obtained approximate numerical solutions of some classes of integral equations by using Bernstein polynomials. Also, they used these polynomials to approximate solution of linear Volterra integral equations [16]. In addition, Maleknejad *et al.* [17] has applied the polynomials for solving Volterra integral equations of the second kind. Furthermore, in [18] an architecture of artificial neural networks (ANNs) was suggested to approximate

solution of linear integral equations systems. For this aim, first the truncations of the Taylor expansions for unknown functions were substituted in the original system. Then the proposed neural network has been applied for adjusting the real coefficients of the given expansions in the resulting system.

In this paper, we are going to propose a new numerical approach to approximate the solutions of linear Fredholm and Volterra integral equations systems of the second kind. This method converts the given systems with unique solutions, into a system of linear algebraic equations in generalized case. To do this, first the Bernstein polynomials of certain degree n of unknown functions are substituted in the given integral equations system. Suppose that the given closed interval $[a,b]$ is partitioned into uniform spacing $(b-a)/n$ and nodes $t_i = a + ih$ (for $i = 0,\dots,n$). If we put $t = t_i$ (for $i = 0,\dots,n$), the given system of integral equations, yields a linear algebraic system. The solution of the resulting system yields the unknown Bernstein coefficients of the solution functions.

Here is an outline of the paper. Section 2 describes how to find approximate solutions of the given linear integral equations systems by using the proposed approach. In Section 3, the convergence of the method is established for each class of integral equations systems. Finally in Section 4, two numerical examples are provided and the results are compared with the analytical solutions to demonstrate the validity and applicability of the method. Also, a comparison is made with other numerical approaches that were proposed recently for solving the given systems.

2 The general method

The basic definition of integral equation is given in [15, 17, 19]. In this section, we intend to use the Bernstein polynomials to get a new numerical method for solving the linear Fredholm and Volterra integral equations systems of the second kind. In other words, it will be described how to apply these polynomials for approximating solutions of the unknowns in the systems.

2.1 System of the Fredholm integral equations

In this subdivision, we want to obtain a numerical solution of the linear Fredholm integral equations system of the second kind in the form

$$\sum_{j=1}^{m}\big(A_{ij}(t)F_j(t)\big) = f_i(t) + \sum_{j=1}^{m}\left(\lambda_{ij}\int_a^t k_{ij}(s,t)F_j(s)\,ds\right), \quad i = 1,\dots,m \tag{1}$$

or

$$\begin{cases} \sum_{j=1}^{m}(A_{1j}(t)F_j(t)) = f_1(t) + \sum_{j=1}^{m}(\lambda_{1j}\int_a^b k_{1j}(s,t)F_j(s)\,ds), \\ \vdots \\ \sum_{j=1}^{m}(A_{ij}(t)F_j(t)) = f_i(t) + \sum_{j=1}^{m}(\lambda_{ij}\int_a^b k_{ij}(s,t)F_j(s)\,ds), \\ \vdots \\ \sum_{j=1}^{m}(A_{mj}(t)F_j(t)) = f_m(t) + \sum_{j=1}^{m}(\lambda_{mj}\int_a^b k_{mj}(s,t)F_j(s)\,ds). \end{cases} \tag{2}$$

Let us consider

$$F_{j,n}(t) = \sum_{p=0}^{n} a_{j,p}B_{n,p}(t), \tag{3}$$

where

$$a_{j,p} = F_j(a + ph) \quad \text{and} \quad h = (b - a)/n,$$

which are the Bernstein expansions of degree n for the unknown functions $F_j(t)$ for $j = 1, \dots, m$. After substituting these polynomials instead of the unknowns in the system (2), we have:

$$\begin{cases} \sum_{j=1}^{m} (A_{1,j}(t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(t)) \\ \quad = f_1(t) + \sum_{j=1}^{m} (\lambda_{1,j} \int_a^b k_{1,j}(s,t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(s)\, ds), \\ \vdots \\ \sum_{j=1}^{m} (A_{i,j}(t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(t)) \\ \quad = f_i(t) + \sum_{j=1}^{m} (\lambda_{i,j} \int_a^b k_{i,j}(s,t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(s)\, ds), \\ \vdots \\ \sum_{j=1}^{m} (A_{m,j}(t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(t)) \\ \quad = f_m(t) + \sum_{j=1}^{m} (\lambda_{m,j} \int_a^b k_{m,j}(s,t) \sum_{p=0}^{n} a_{j,p} B_{p,n}(s)\, ds). \end{cases} \tag{4}$$

In order to find $a_{j,n}$ in Eq. (3), the system (4) is converted to an algebraic system of linear equations by replacing t with $t_p = a + p(b - a)/n$ (for $p = 0, \dots, n$). Notice that in this way we can skip the singularity problem. After this work, the system is transformed to the following form:

$$\sum_{j=1}^{m} \sum_{p=0}^{n} \big(A_{i,j}(t_q) B_{p,n}(t_q)\big) \cdot a_{j,p} = f_i(t_q) + \sum_{j=1}^{m} \sum_{p=0}^{n} \left(\lambda_{i,j} \int_a^b k_{i,j}(s, t_q) B_{p,n}(s)\, ds\right) \cdot a_{j,p}, \quad i = 1, \dots, m; q = 0, \dots, n. \tag{5}$$

For brevity, we define below symbols as:

$$T_{q,p}^{(i,j)} = A_{i,j}(t_q) B_{p,n}(t_q), \qquad R_{q,p}^{(i,j)} = \lambda_{ij} \int_a^b k_{ij}(s, t_q) B_{p,n}(s)\, ds.$$

Now we can write the system (5) in the form

$$\sum_{j=1}^{m} \sum_{p=0}^{n} T_{q,p}^{(i,j)} \cdot a_{j,p} = f_i(t_q) + \sum_{j=1}^{m} \sum_{p=0}^{n} R_{q,p}^{(i,j)} \cdot a_{j,p}, \quad i = 1, \dots, m; q = 0, \dots, n. \tag{6}$$

Consequently, the expression (6) can be summarized in a matrix form as follows:

$$WY = E + VY, \tag{7}$$

where

$$Y = [a_{1,0}, \dots, a_{1,n}, \dots, a_{i,0}, \dots, a_{i,n}, \dots, a_{m,0}, \dots, a_{m,n}]^T,$$

$$E = \big[f_1(t_0), \dots, f_1(t_n), \dots, f_i(t_0), \dots, f_i(t_n), \dots, f_m(t_0), \dots, f_m(t_n)\big]^T,$$

$$W = \begin{bmatrix} W^{(1,1)} & \cdots & W^{(1,m)} \\ \vdots & \ddots & \vdots \\ W^{(m,1)} & \cdots & W^{(m,m)} \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} V^{(1,1)} & \cdots & V^{(1,m)} \\ \vdots & \ddots & \vdots \\ V^{(m,1)} & \cdots & V^{(m,m)} \end{bmatrix}.$$

Parochial matrices $W^{(i,j)}$, $V^{(i,j)}$ for $(i,j = 1,\dots,m)$ are defined with following elements:

$$W^{(i,j)} = \begin{bmatrix} W_{0,0}^{(i,j)} & \cdots & W_{0,n}^{(i,j)} \\ \vdots & \ddots & \vdots \\ W_{n,0}^{(i,j)} & \cdots & W_{n,n}^{(i,j)} \end{bmatrix}, \qquad V^{(i,j)} = \begin{bmatrix} V_{0,0}^{(i,j)} & \cdots & V_{0,n}^{(i,j)} \\ \vdots & \ddots & \vdots \\ V_{n,0}^{(i,j)} & \cdots & V_{n,n}^{(i,j)} \end{bmatrix},$$

where

$$W_{q,p}^{(i,j)} = T_{q,p}^{(i,j)} \quad \text{and} \quad V_{q,p}^{(i,j)} = R_{q,p}^{(i,j)}.$$

The resulting generalized linear system can be solved for $a_{j,p}$ for $j = 1,\dots,m$; $p = 0,\dots,n$ by a standard method, and hence $F_j(t)$ is obtained.

2.2 System of Volterra integral equations

At first, consider again the system of linear Volterra integral equations (1). For numerical solving of the present system, the unknown function $F_j(t)$ is approximated by its Bernstein approximation (3). Now we have the following system:

$$\begin{cases} \sum_{j=1}^{m}\sum_{p=0}^{n}(A_{1,j}(t)B_{p,n}(t))\cdot a_{j,p} \\ \quad = f_1(t) + \sum_{j=1}^{m}\sum_{p=0}^{n}(\lambda_{1,j}\int_a^t k_{1,j}(s,t)B_{p,n}(s)\,ds)\cdot a_{j,p}, \\ \vdots \\ \sum_{j=1}^{m}\sum_{p=0}^{n}(A_{i,j}(t)B_{p,n}(t))\cdot a_{j,p} \\ \quad = f_i(t) + \sum_{j=1}^{m}\sum_{p=0}^{n}(\lambda_{i,j}\int_a^t k_{i,j}(s,t)B_{p,n}(s)\,ds)\cdot a_{j,p}, \\ \vdots \\ \sum_{j=1}^{m}\sum_{p=0}^{n}(A_{m,j}(t)B_{p,n}(t))\cdot a_{j,p} \\ \quad = f_m(t) + \sum_{j=1}^{m}\sum_{p=0}^{n}(\lambda_{m,j}\int_a^t k_{m,j}(s,t)B_{p,n}(s)\,ds)\cdot a_{j,p}. \end{cases} \tag{8}$$

Similarly, by replacing the variable t with $t_p = a + p(b-a)/n$ for $p = 0,\dots,n$, we obtain the generalized linear system

$$\sum_{j=1}^{m}\sum_{p=0}^{n} T_{q,p}^{(i,j)}\cdot a_{j,p} = f_i(t_q) + \sum_{j=1}^{m}\sum_{p=0}^{n} R_{q,p}^{(i,j)}\cdot a_{j,p}, \quad i = 1,\dots,m; q = 0,\dots,n, \tag{9}$$

where

$$T_{q,p}^{(i,j)} = A_{i,j}(t_q)B_{p,n}(t_q), \qquad R_{q,p}^{(i,j)} = \lambda_{ij}\int_a^t k_{ij}(s,t_q)B_{p,n}(s)\,ds.$$

Consequently, the expression (6) can be summarized in a matrix form as follows:

$$\begin{bmatrix} W^{(1,1)} & \cdots & W^{(1,m)} \\ \vdots & \ddots & \vdots \\ W^{(m,1)} & \cdots & W^{(m,m)} \end{bmatrix}\begin{bmatrix} Y_1 \\ \vdots \\ Y_m \end{bmatrix} = \begin{bmatrix} E_1 \\ \vdots \\ E_m \end{bmatrix} + \begin{bmatrix} V^{(1,1)} & \cdots & V^{(1,m)} \\ \vdots & \ddots & \vdots \\ V^{(m,1)} & \cdots & V^{(m,m)} \end{bmatrix}\begin{bmatrix} Y_1 \\ \vdots \\ Y_m \end{bmatrix},$$

where

$$Y_i = \begin{bmatrix} a_{i,0} \\ \vdots \\ a_{i,n} \end{bmatrix}, \qquad E_i = \begin{bmatrix} f_i(t_0) \\ \vdots \\ f_i(t_n) \end{bmatrix}, \qquad W^{(i,j)} = \begin{bmatrix} W_{0,0}^{(i,j)} & \cdots & W_{0,n}^{(i,j)} \\ \vdots & \ddots & \vdots \\ W_{n,0}^{(i,j)} & \cdots & W_{n,n}^{(i,j)} \end{bmatrix} \tag{10}$$

and

$$V^{(i,j)} = \begin{bmatrix} V_{0,0}^{(i,j)} & \cdots & V_{0,n}^{(i,j)} \\ \vdots & \ddots & \vdots \\ V_{n,0}^{(i,j)} & \cdots & V_{n,n}^{(i,j)} \end{bmatrix}.$$

In the above generalized linear system, the symbols are defined as follows:

$$W_{q,p}^{(i,j)} = T_{q,p}^{(i,j)} \quad \text{and} \quad V_{q,p}^{(i,j)} = R_{q,p}^{(i,j)}.$$

3 Convergence analysis

In this section, we prove that the present numerical method converges to the exact solutions of the systems (2) and (1).

Theorem 1 *Let $F_{j,n}(t)$ for $j = 1, \dots, m$ be the Bernstein polynomials of degree n such that their coefficients have been produced by solving the generalized linear system* (7). *Then the given polynomials converge to the exact solution of the Fredholm integral equations system* (2), *when $n \to +\infty$.*

Proof Consider the system (2). Since the series (3) converge to $F_j(t)$ for $j = 1, \dots, m$, respectively, then we conclude that:

$$\sum_{j=1}^{m} \big(A_{ij}(t)F_{j,n}(t)\big) = f_i(t) + \sum_{j=1}^{m} \left(\lambda_{ij} \int_a^b k_{ij}(s,t)F_{j,n}(s)\,ds \right), \quad i = 1, \dots, m, \tag{11}$$

and it holds that

$$F_j(t) = \lim_{n \to \infty} F_{j,n}(t).$$

We defined the error function $e_n(t)$ by subtracting Eqs. (2) and (11) as follows:

$$e_n(t) = \sum_{i=1}^{m} e_{i,n}(t), \tag{12}$$

where

$$e_{i,n}(t) = \sum_{j=1}^{m} A_{i,j}(t)\big(F_j(t) - F_{j,n}(t,r)\big) - \sum_{j=1}^{m} \lambda_{ij} \left(\int_a^b k_{i,j}(s,t)\big(F_j(s) - F_{j,n}(s)\big)\,ds \right).$$

We must prove that when $n \to +\infty$, the error function $e_n(t)$ tends to zero. Hence, we proceed as follows:

$$\begin{aligned}\|e_n\| &\leq \sum_{i=1}^{m} \|e_{i,n}\| \\ &\leq \sum_{i=1}^{m}\sum_{j=1}^{m}\left(\left\|A_{i,j}(t)\right\| \cdot \left\|\left(F_i(t) - F_{i,n}(t)\right)\right\|\right) \\ &\quad + \sum_{i=1}^{m}\sum_{j=1}^{m}\left(|\lambda_{i,j}| \int_a^b \|k_{i,j}\| \cdot \left\|F_j(s) - F_{j,n}(s)\right\| ds\right).\end{aligned}$$

Since $\|k_{i,j}\|$ and $\|A_j\|$ are bounded, therefore, $\|F_j(s) - F_{j,n}(s)\| \to 0$ implies that $\|e_n\| \to 0$ and the proof is completed. □

Theorem 2 *Suppose that $F_{j,n}(t)$ for $j = 1, \ldots, m$ are the Bernstein polynomials of degree n such that their coefficients have been produced by solving the generalized linear system* (10). *Then the given polynomials converge to the exact solution of the Volterra integral equations system* (1), *when $n \to +\infty$.*

Proof Consider the system (1). Using a similar procedure as an outline in mentioned Theorem 1, we have the following corollary in which we are refrained from going through proof details. Now the error of the approximation method can be written as

$$e_n(t) = \sum_{i=1}^{m} e_{i,n}(t), \tag{13}$$

where

$$e_{i,n}(t) = \sum_{j=1}^{m} A_{i,j}(t)\left(F_j(t) - F_{j,n}(t,r)\right) - \sum_{j=1}^{m} \lambda_{ij}\left(\int_a^t k_{i,j}(s,t)\left(F_j(s) - F_{j,n}(s)\right) ds\right).$$

Due to Theorem 1, the error function $e_n(t)$ must tend to zero, when $n \to +\infty$. Hence, we proceed as follows:

$$\begin{aligned}\|e_n\| &\leq \sum_{i=1}^{m} \|e_{i,n}\| \\ &\leq \sum_{i=1}^{m}\sum_{j=1}^{m}\left(\left\|\left(F_i(t) - F_{i,n}(t)\right)\right\|\right) \\ &\quad + \sum_{i=1}^{m}\sum_{j=1}^{m}\left(|\lambda_{i,j}| \int_a^t \|k_{i,j}\|\left(\left\|F_j(s) - F_{j,n}(s)\right\|\right) ds\right).\end{aligned}$$

Since $\|k_{i,j}\|$ and $\|A_j\|$ are bounded, therefore, $\|(F_j(s) - F_{j,n}(s))\| \to 0$ implies that $\|e_n\| \to 0$ and the proof follows immediately. The stability and the convergence of Bernstein polynomials is studied in [12, 13]. □

4 Numerical examples

In this section, in order to investigate the accuracy of the proposed method, we have chosen three examples of linear integral equations systems of the second kind. Also, to show the efficiency of the present method for our problem, results will be compared with the exact solution. Moreover, the present method is compared with artificial neural network (ANN) method and the trapezoidal quadrature rule (TQR) [18].

Example 1 Consider the system of linear Fredholm integral equations

$$\begin{cases} 2F_1(t) + 3F_2(t) = f_1(t) + \int_0^1 (t+s)F_1(s)\,ds + \int_0^1 tsF_2(s)\,ds, \\ 3F_1(t) - 4F_2(t) = f_2(t) + \int_0^1 (2ts-1)F_1(s)\,ds + \int_0^1 (t-s)F_2(s)\,ds \end{cases}$$

with

$$f_1(t) = 2e^t - \frac{3}{t-2} - t(e-1) - t\big(\ln(4)-1\big) - 1,$$
$$f_2(t) = -2t + \ln(4) + 3e^t + \frac{4}{t-2} - t\ln(2) - 2 + e,$$

where the exact solution is $F_1(t) = e^t$ and $F_2(t) = \frac{1}{2-t}$. In this example, we illustrate the use of the present technique to approximate solution of this integral equations system. Using Eq. (7), the coefficients matrices W, V and E are calculated for $n = 3$ as following:

$$W = \begin{bmatrix} W^{(1,1)} & W^{(1,2)} \\ W^{(2,1)} & W^{(2,2)} \end{bmatrix}, \qquad V = \begin{bmatrix} V^{(1,1)} & V^{(1,2)} \\ V^{(2,1)} & V^{(2,2)} \end{bmatrix}, \qquad E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},$$

where

$$W^{(1,1)} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ \frac{29,630}{50,000} & \frac{8,889}{10,000} & \frac{11,110}{25,000} & \frac{2,120}{28,610} \\ \frac{212}{2,861} & \frac{1,111}{2,500} & \frac{8,889}{10,000} & \frac{2,963}{5,000} \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

$$W^{(1,2)} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ \frac{8,889}{10,000} & \frac{13,333}{10,000} & \frac{6,667}{10,000} & \frac{1,111}{10,000} \\ \frac{1,111}{10,000} & \frac{6,667}{10,000} & \frac{13,333}{10,000} & \frac{8,889}{10,000} \\ 0 & 0 & 0 & 3 \end{bmatrix},$$

$$W^{(2,1)} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ \frac{8,889}{10,000} & \frac{13,333}{10,000} & \frac{6,667}{10,000} & \frac{1,111}{10,000} \\ \frac{1,111}{10,000} & \frac{6,667}{10,000} & \frac{13,333}{10,000} & \frac{8,889}{10,000} \\ 0 & 0 & 0 & 3 \end{bmatrix},$$

$$W^{(2,2)} = \begin{bmatrix} -4 & 0 & 0 & 0 \\ \frac{-296}{250} & \frac{-8,889}{5,000} & \frac{-8,889}{10,000} & \frac{-456}{3,079} \\ \frac{-456}{3,079} & \frac{-8,889}{10,000} & \frac{-8,889}{5,000} & \frac{-2,963}{2,500} \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

$$V^{1,1}=\begin{bmatrix} \frac{21}{20} & \frac{1}{10} & \frac{3}{20} & \frac{1}{5} \\ \frac{2}{15} & \frac{11}{60} & \frac{7}{30} & \frac{17}{60} \\ \frac{13}{60} & \frac{4}{15} & \frac{19}{60} & \frac{11}{30} \\ \frac{3}{10} & \frac{7}{20} & \frac{2}{5} & \frac{9}{20} \end{bmatrix}, \quad V^{1,2}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{60} & \frac{1}{30} & \frac{1}{20} & \frac{1}{15} \\ \frac{1}{30} & \frac{1}{15} & \frac{1}{10} & \frac{2}{15} \\ \frac{1}{20} & \frac{1}{15} & \frac{3}{20} & \frac{1}{5} \end{bmatrix}, \quad E_1=\begin{bmatrix} \frac{5}{2} \\ \frac{3,563}{1,233} \\ \frac{2,339}{625} \\ \frac{1,333}{250} \end{bmatrix},$$

$$V^{2,1}=\begin{bmatrix} \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-13}{60} & \frac{-11}{60} & \frac{-3}{20} & \frac{-7}{60} \\ \frac{-11}{60} & \frac{-7}{60} & \frac{-1}{20} & \frac{1}{60} \\ \frac{-3}{20} & \frac{-1}{20} & \frac{1}{20} & \frac{3}{20} \end{bmatrix}, \quad V^{2,2}=\begin{bmatrix} \frac{-1}{20} & \frac{-1}{10} & \frac{-3}{20} & \frac{-1}{5} \\ \frac{1}{30} & \frac{-1}{60} & \frac{-1}{15} & \frac{-7}{60} \\ \frac{7}{60} & \frac{1}{15} & \frac{1}{60} & \frac{-1}{30} \\ \frac{1}{5} & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} \end{bmatrix}, \quad E_2=\begin{bmatrix} \frac{7,420}{2,390} \\ \frac{1,901}{635} \\ \frac{1,097}{348} \\ \frac{1,291}{362} \end{bmatrix}.$$

Now by using the above matrices, thevector solution of the generalized linear system (7) is obtained as follows:

$$Y=\begin{bmatrix} a_{1,0} \\ a_{1,1} \\ a_{1,2} \\ a_{1,3} \\ a_{2,0} \\ a_{2,1} \\ a_{2,2} \\ a_{2,3} \end{bmatrix}=\begin{bmatrix} 0.9999 \\ 1.3380 \\ 1.8181 \\ 2.7186 \\ 0.5001 \\ 0.5918 \\ 0.6835 \\ 1.0001 \end{bmatrix}.$$

Consequently, the approximate functions $F_{1,3}(t)$ and $F_{2,3}(t)$ can be written as follows:

$$F_{1,3}=\frac{2,353}{8,445}t^3+\frac{272}{639}t^2+\frac{2,185}{2,154}t+\frac{11,171}{11,172},$$

$$F_{2,3}=\frac{9}{40}t^3-\frac{1}{345,468,948,216,863}t^2+\frac{2,747}{9,989}t+\frac{1,846}{3,691}.$$

Figures 1 and 2 show the accuracy of the solution functions $F_{1,3}(t)$ and $F_{2,3}(t)$, respectively. As shown, the difference between the exact solutionand the computed solution is dispensable. Numerical results can be seen in Table 1 and also Table 2 illustrates the absolute values of the errors obtained here and the absolute errors of [18] for this example.

Trapezoidal rule Moreover, this example is going to show the difference between the present method and the trapezoidal quadrature rule.Consider again Example 1, let the region of integration be subdivided into 4 equal intervals of width $h = 0.25$, 5 integration nodes $t_i = 0.25i$ for $i = 0, \dots, 4$. Table 2 illustrates the absolute values of the errors obtained here and the absolute errors of [18] for this example. It should be noted that the Lagrange basis functions have been used for finding the 4th degree collocation polynomials through the points $(t_i, F_{i,j})$ for $i = 0, \dots, 4$; $j = 1, 2$.

Example 2 Let the system of linear Volterra integral equations

$$\begin{cases} (3t-8)F_1(t)+(-2t+5)F_2(t)=f_1(t)+\int_0^t (t+s)F_1(s)\,ds+\int_0^t tsF_2(s)\,ds, \\ 4tF_1(t)+(t-5)F_2(t)=f_2(t)+\int_0^t (2t-s)F_1(s)\,ds+\int_0^t (s+st)F_2(s)\,ds, \end{cases} \quad 0\le t\le 2,$$

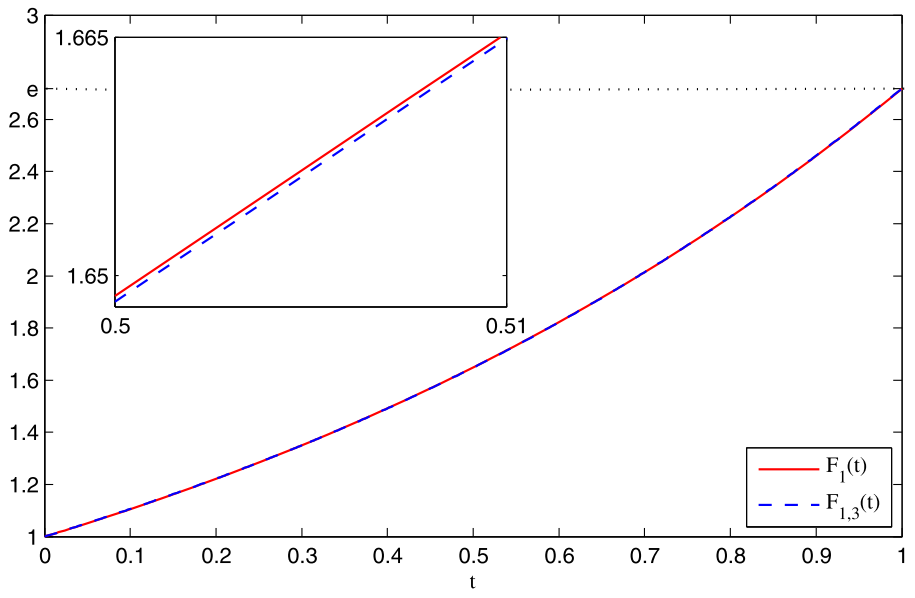

Figure 1 The comparison between $F_1(t)$ and $F_{1,3}(t)$ for Example 1. Show the accuracy of the solution functions $F_1(t)$ and $F_{1,3}(t)$, respectively with exact solutions. As shown, the difference between the exact solution and the computed solution is dispensable.

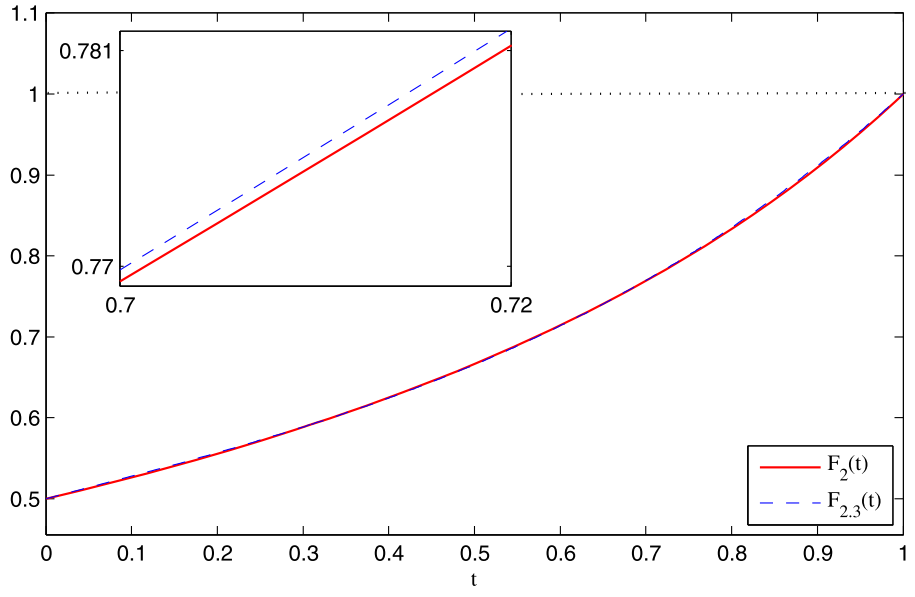

Figure 2 The comparison between $F_2(t)$ and $F_{2,3}(t)$ for Example 1. The comparison between solution of Example 1.

Table 1 Numerical results with error analysis for Example 1

$t = \frac{1}{2^r}$	Exact solution		Approximate solution		Error	
	$F_1(t)$	$F_2(t)$	$F_{1,3}(t)$	$F_{2,3}(t)$	$e_{1,3}(t)$	$e_{2,3}(t)$
$r = 1$	1.6487	0.6667	1.6484	0.6658	0.0004	0.0009
$r = 2$	1.2840	0.5714	1.2845	0.5724	0.0004	0.0010
$r = 3$	1.1331	0.5333	1.1339	0.5350	0.0008	0.0016
$r = 4$	1.0645	0.5161	1.0650	0.5174	0.0005	0.0012
$r = 5$	1.0317	0.5079	1.0320	0.5087	0.0003	0.0008
$r = 6$	1.0157	0.5039	1.0159	0.5044	0.0001	0.0005
$r = 7$	1.0078	0.5020	1.0079	0.5023	0.0001	0.0003
$r = 8$	1.0039	0.5010	1.0039	0.5012	0.0000	0.0002
$r = 9$	1.0020	0.5005	1.0019	0.5007	0.0001	0.0002

Table 2 Absolute errors for Example 1

$t = \frac{1}{2^r}$	Presented method		ANN method		TQR	
	$e_{1,4}(t)$	$e_{2,4}(t)$	$e_{1,4}(t)$	$e_{2,4}(t)$	$e_{1,4}(t)$	$e_{2,4}(t)$
$r = 1$	3.50×10^{-6}	1.17×10^{-5}	7.32×10^{-4}	3.84×10^{-4}	4.11×10^{-4}	7.35×10^{-4}
$r = 2$	2.90×10^{-6}	1.06×10^{-5}	1.15×10^{-4}	5.62×10^{-4}	3.81×10^{-4}	5.14×10^{-4}
$r = 3$	4.78×10^{-5}	2.50×10^{-4}	5.34×10^{-4}	6.11×10^{-4}	1.29×10^{-2}	8.41×10^{-4}
$r = 4$	5.03×10^{-5}	2.50×10^{-4}	6.71×10^{-4}	6.30×10^{-4}	4.17×10^{-3}	3.28×10^{-3}
$r = 5$	3.71×10^{-5}	1.63×10^{-4}	9.80×10^{-4}	3.27×10^{-4}	2.08×10^{-3}	8.20×10^{-4}
$r = 6$	2.53×10^{-5}	8.90×10^{-5}	1.17×10^{-4}	4.83×10^{-4}	3.27×10^{-3}	3.53×10^{-3}
$r = 7$	1.79×10^{-5}	4.30×10^{-5}	4.15×10^{-4}	4.88×10^{-4}	1.08×10^{-2}	2.71×10^{-2}
$r = 8$	1.37×10^{-5}	1.77×10^{-5}	3.12×10^{-4}	4.87×10^{-4}	6.31×10^{-3}	3.84×10^{-4}
$r = 9$	1.16×10^{-5}	4.30×10^{-6}	1.16×10^{-4}	4.82×10^{-4}	2.10×10^{-4}	3.84×10^{-3}

with

$$f_1(t) = 5e^t - 2t - 9\sin(t) - t^2e^t + 2t\cos(t) - te^t + 3t\sin(t),$$
$$f_2(t) = -3t - 4e^t + \sin(t) - t^2e^t + t\cos(t) + te^t + 4t\sin(t) - 1.$$

The exact solution of the present problem is, $F_1(t) = \sin(t)$ and $F_2(t) = e^t$. Similarly, the present method is applied to approximate solution of the integral equations system. We calculate the coefficients matrices W, V and E by using Eq. (10) for $n = 4$ as following:

$$W = \begin{bmatrix} W^{(1,1)} & W^{(1,2)} \\ W^{(2,1)} & W^{(2,2)} \end{bmatrix}, \qquad V = \begin{bmatrix} V^{(1,1)} & V^{(1,2)} \\ V^{(2,1)} & V^{(2,2)} \end{bmatrix}, \qquad E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},$$

where

$$W^{(1,1)} = \begin{bmatrix} -8 & 0 & 0 & 0 & 0 \\ -\frac{10,283}{5,000} & -\frac{1,670}{609} & -\frac{835}{609} & -\frac{188}{617} & -\frac{127}{5,000} \\ -\frac{5}{16} & -\frac{5}{4} & -\frac{15}{8} & -\frac{5}{4} & -\frac{5}{16} \\ -\frac{137}{10,000} & -\frac{341}{2,078} & -\frac{773}{1,047} & -\frac{1,546}{1,047} & -\frac{1,691}{1,527} \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 5 \\ \frac{1,286}{391} \\ \frac{3,492}{1,597} \\ -\frac{2,093}{1,250} \\ -\frac{1,878}{119} \end{bmatrix},$$

$$W^{(1,2)} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ \frac{791}{625} & \frac{27}{16} & \frac{524}{621} & \frac{3}{16} & \frac{39}{2,500} \\ \frac{3}{16} & \frac{3}{4} & \frac{9}{8} & \frac{3}{4} & \frac{3}{16} \\ \frac{39}{5,000} & \frac{469}{5,000} & \frac{262}{621} & \frac{524}{621} & \frac{791}{1,250} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$W^{(2,1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{791}{1,250} & \frac{524}{621} & \frac{262}{621} & \frac{469}{5,000} & \frac{39}{5,000} \\ \frac{1}{4} & 1 & \frac{3}{2} & 1 & \frac{1}{4} \\ \frac{117}{5,000} & \frac{528}{1,877} & \frac{791}{625} & \frac{1,577}{623} & \frac{2,373}{1,250} \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix},$$

$$W^{(2,2)} = \begin{bmatrix} -5 & 0 & 0 & 0 & 0 \\ -\frac{682}{479} & -\frac{2,373}{1,250} & -\frac{1,009}{1,063} & -\frac{1,099}{5,211} & -\frac{11}{625} \\ -\frac{1}{4} & -1 & -\frac{3}{2} & -1 & -\frac{1}{4} \\ -\frac{137}{10,000} & -\frac{341}{2,078} & -\frac{773}{1,047} & -\frac{1,546}{1,047} & -\frac{1,691}{1,527} \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} -5 \\ -\frac{4,131}{607} \\ -\frac{2,501}{247} \\ -\frac{3,999}{203} \\ -\frac{15,526}{353} \end{bmatrix},$$

$$V^{1,1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{737}{3,433} & \frac{165}{1,391} & \frac{183}{5,120} & \frac{43}{7,680} & \frac{11}{30,720} \\ \frac{81}{160} & \frac{1}{2} & \frac{27}{80} & \frac{2}{15} & \frac{11}{480} \\ \frac{932}{1,273} & \frac{821}{969} & \frac{891}{1,024} & \frac{1701}{2,560} & \frac{343}{1,314} \\ \frac{14}{15} & \frac{16}{15} & \frac{6}{5} & \frac{4}{3} & \frac{22}{15} \end{bmatrix},$$

$$V^{1,2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{141}{4,538} & \frac{132}{5,843} & \frac{77}{10,240} & \frac{19}{15,360} & \frac{1}{12,288} \\ \frac{19}{160} & \frac{7}{40} & \frac{11}{80} & \frac{7}{120} & \frac{1}{96} \\ \frac{515}{2,587} & \frac{343}{891} & \frac{1,651}{3,313} & \frac{343}{803} & \frac{729}{4,096} \\ \frac{4}{15} & \frac{8}{15} & \frac{4}{5} & \frac{16}{15} & \frac{4}{3} \end{bmatrix},$$

$$V^{2,1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{834}{3,433} & \frac{781}{7,680} & \frac{27}{1,024} & \frac{29}{7,680} & \frac{7}{30,720} \\ \frac{21}{32} & \frac{19}{40} & \frac{21}{80} & \frac{11}{120} & \frac{7}{480} \\ \frac{3,596}{3,373} & \frac{650}{703} & \frac{548}{737} & \frac{243}{512} & \frac{1,701}{10,240} \\ \frac{22}{15} & \frac{4}{3} & \frac{6}{5} & \frac{16}{15} & \frac{14}{15} \end{bmatrix},$$

$$V^{2,2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{423}{4,538} & \frac{347}{5,120} & \frac{231}{10,240} & \frac{19}{5,120} & \frac{1}{4,096} \\ \frac{19}{80} & \frac{7}{20} & \frac{11}{40} & \frac{7}{60} & \frac{1}{48} \\ \frac{1,359}{4,096} & \frac{657}{1,024} & \frac{1,701}{2,048} & \frac{729}{1,024} & \frac{361}{1,217} \\ \frac{2}{5} & \frac{4}{5} & \frac{6}{5} & \frac{8}{5} & 2 \end{bmatrix}.$$

Now by using the above matrices, the vector solution of the generalized linear system (10) is obtained as follows:

$$Y = \begin{bmatrix} a_{1,0} \\ a_{1,1} \\ a_{1,2} \\ a_{1,3} \\ a_{1,1} \\ a_{2,0} \\ a_{2,1} \\ a_{2,2} \\ a_{2,3} \\ a_{2,4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.4976 \\ 1.0113 \\ 1.1247 \\ 0.9062 \\ 1.0000 \\ 1.4883 \\ 2.3751 \\ 3.7268 \\ 7.3813 \end{bmatrix}.$$

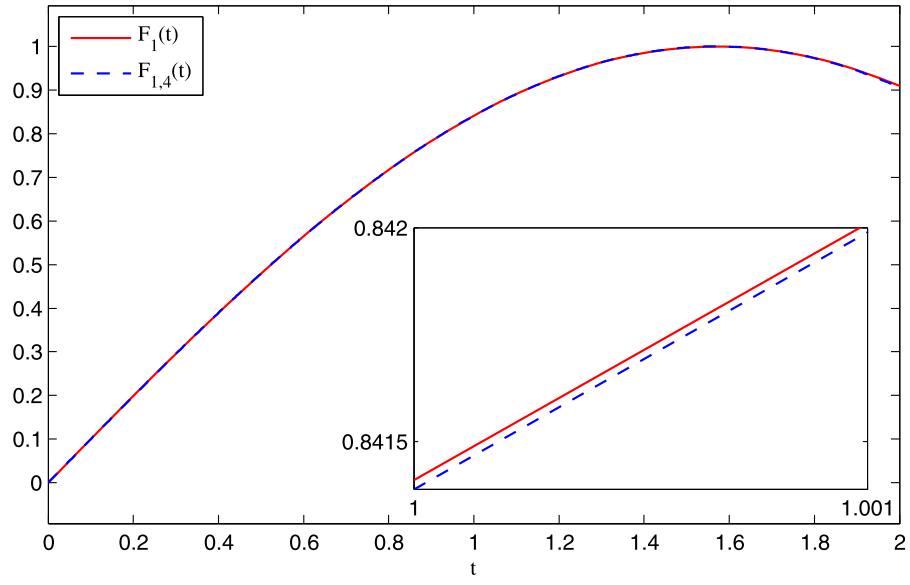

Figure 3 The comparison between $F_1(t)$ and $F_{1,4}(t)$ for Example 2. Show the accuracy of the solution functions $F_1(t)$ and $F_{1,4}(t)$, respectively with exact solutions. As shown, the difference between the exact solution and the computed solution is dispensable.

Consequently, the approximate functions $F_{1,4}(t)$ and $F_{2,4}(t)$ can be written as follows:

$$F_{1,4} = \frac{108}{3{,}565}t^4 - \frac{319}{1{,}532}t^3 + \frac{142}{5{,}855}t^2 + \frac{613}{616}t,$$

$$F_{2,4} = \frac{197}{1{,}779}t^4 + \frac{352}{10{,}621}t^3 + \frac{489}{818}t^2 + \frac{1{,}793}{1{,}836}t + 1.$$

Similarly, Figures 3 and 4 show the accuracy of the solution functions $F_{1,4}(t)$ and $F_{2,4}(t)$, respectively. As shown, the difference betweenthe exact solution and the computed solution is dispensable. Similarly, numerical results can be seen in Table 3.

Trapezoidal rule Suppose that the region of integration is subdivided into 5 equal intervals of width $h = 0.4$, 6 integration nodes $t_i = 0.4i$ for $i = 0, \dots, 5$. Table 4 illustrates the absolute values of the errors obtained here and the absolute errors of [18] for this example. Furthermore, the Lagrange interpolation method has been used to design the interpolation polynomials.

Example 3 Consider

$$\begin{cases} (2t^2+3)F_1(t) = f_1(t) + \int_0^t (t^2-2s)F_1(s)\,ds + \int_0^t (s^2-t)F_2(s)\,ds \\ \qquad\qquad + \int_0^t 2sF_3(s)\,ds, \\ (1-3t^2)F_2(t) = f_2(t) + \int_0^t s(t+1)F_1(s)\,ds + \int_0^t st(t^2+1)F_2(s)\,ds \\ \qquad\qquad + \int_0^t (2s^2+t^3)F_3(s)\,ds, \\ (3t^2+6)F_3(t) = f_3(t) + \int_0^t (s-t)F_1(s)\,ds + \int_0^t (s^2-t^3)F_2(s)\,ds \\ \qquad\qquad + \int_0^t (2ts+s^2)F_3(s)\,ds, \end{cases}$$

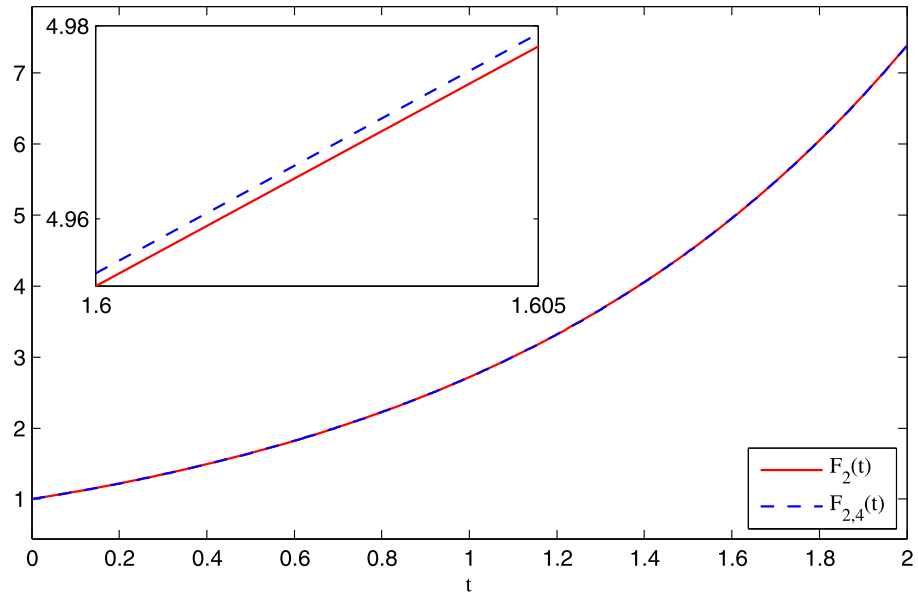

Figure 4 The comparison between $F_2(t)$ and $F_{2,4}(t)$ for Example 2. The comparison between solution of Example 2.

Table 3 Numerical results with error analysis for Example 2

$t = \frac{1}{2^r}$	Exact solution		Approximate solution		Error	
	$F_1(t)$	$F_2(t)$	$F_{1,4}(t)$	$F_{2,4}(t)$	$e_{1,4}(t)$	$e_{2,4}(t)$
$r = 1$	0.4794	1.6487	0.4795	1.6488	0.0001	0.0001
$r = 2$	0.2474	1.2840	0.2472	1.2825	0.0002	0.0016
$r = 3$	0.1247	1.1331	0.1244	1.1315	0.0003	0.0016
$r = 4$	0.0625	1.0645	0.0622	1.0634	0.0002	0.0011
$r = 5$	0.0312	1.0317	0.0311	1.0311	0.0001	0.0006
$r = 6$	0.0156	1.0157	0.0156	1.0154	0.0001	0.0003
$r = 7$	0.0078	1.0078	0.0078	1.0077	0.0001	0.0002
$r = 8$	0.0039	1.0039	0.0039	1.0038	0.0001	0.0001
$r = 9$	0.0020	1.0020	0.0019	1.0019	0.0001	0.0001

Table 4 Absolute errors for Example 2

$t = \frac{1}{2^r}$	Presented method		ANN method		TQR	
	$e_{1,5}(t)$	$e_{2,5}(t)$	$e_{1,5}(t)$	$e_{2,5}(t)$	$e_{1,5}(t)$	$e_{2,5}(t)$
$r = 1$	2.66×10^{-5}	4.92×10^{-5}	1.16×10^{-2}	9.28×10^{-4}	8.32×10^{-3}	8.07×10^{-3}
$r = 2$	5.85×10^{-5}	1.37×10^{-5}	4.30×10^{-3}	1.48×10^{-3}	2.14×10^{-2}	4.81×10^{-3}
$r = 3$	8.55×10^{-5}	2.04×10^{-5}	2.04×10^{-3}	4.43×10^{-3}	3.01×10^{-2}	3.71×10^{-2}
$r = 4$	6.14×10^{-5}	4.11×10^{-5}	1.73×10^{-3}	5.91×10^{-3}	3.91×10^{-2}	3.16×10^{-2}
$r = 5$	3.89×10^{-5}	9.44×10^{-5}	8.01×10^{-4}	7.94×10^{-3}	1.12×10^{-1}	4.24×10^{-1}
$r = 6$	2.12×10^{-5}	5.17×10^{-5}	6.23×10^{-4}	1.94×10^{-2}	3.62×10^{-1}	2.80×10^{-1}
$r = 7$	1.11×10^{-5}	2.71×10^{-5}	5.37×10^{-4}	4.13×10^{-2}	8.54×10^{-2}	1.78×10^{-1}
$r = 8$	5.70×10^{-6}	1.38×10^{-5}	4.95×10^{-4}	6.89×10^{-2}	6.37×10^{-2}	7.07×10^{-2}
$r = 9$	2.90×10^{-6}	7.01×10^{-6}	4.75×10^{-4}	9.40×10^{-2}	8.64×10^{-2}	9.04×10^{-3}

Table 5 Numerical results with error analysis for Example 3

$t = \frac{1}{2^r}$	Exact solution			Approximate solution			Error
	$F_1(t)$	$F_2(t)$	$F_3(t)$	$F_{1,3}(t)$	$F_{2,3}(t)$	$F_{3,3}(t)$	$e_{1,3}(t), i = 1, \ldots, 3$
$r = 1$	10.500	–4.5000	1.7500	10.500	–4.5000	1.7500	0.0000
$r = 2$	9.2500	–4.8750	1.3125	9.2500	–4.8750	1.3125	0.0000
$r = 3$	8.6250	–4.9688	1.1406	8.6250	–4.9688	1.1406	0.0000
$r = 4$	8.3125	–4.9922	1.0664	8.3125	–4.9922	1.0664	0.0000
$r = 5$	8.1563	–4.9980	1.0322	8.1563	–4.9980	1.0322	0.0000
$r = 6$	8.0781	–4.9995	1.0159	8.0781	–4.9995	1.0159	0.0000
$r = 7$	8.0391	–4.9999	1.0079	8.0391	–4.9999	1.0079	0.0000
$r = 8$	8.0195	–5.0000	1.0039	8.0195	–5.0000	1.0039	0.0000
$r = 9$	8.0098	–5.0000	1.0020	8.0098	–5.0000	1.0020	0.0000

Table 6 Absolute errors for Example 3

$t = \frac{1}{2^r}$	ANN method			Trapezoidal quadrature rule		
	$e_{1,3}(t)$	$e_{2,3}(t)$	$e_{3,3}(t)$	$e_{1,3}(t)$	$e_{2,3}(t)$	$e_{3,3}(t)$
$r = 1$	2.4×10^{-2}	7.2×10^{-3}	7.8×10^{-4}	6.2×10^{-3}	5.8×10^{-3}	8.1×10^{-3}
$r = 2$	6.1×10^{-3}	4.9×10^{-3}	5.7×10^{-4}	2.0×10^{-2}	2.0×10^{-2}	2.1×10^{-2}
$r = 3$	1.3×10^{-2}	4.4×10^{-3}	5.1×10^{-4}	3.7×10^{-2}	2.1×10^{-2}	2.4×10^{-2}
$r = 4$	1.4×10^{-2}	4.2×10^{-3}	5.1×10^{-4}	2.8×10^{-2}	1.5×10^{-2}	3.2×10^{-2}
$r = 5$	1.5×10^{-2}	4.3×10^{-3}	5.1×10^{-4}	2.5×10^{-3}	4.8×10^{-3}	5.7×10^{-3}
$r = 6$	1.6×10^{-2}	4.2×10^{-3}	5.1×10^{-4}	2.1×10^{-2}	5.3×10^{-2}	6.7×10^{-3}
$r = 7$	1.6×10^{-2}	4.3×10^{-3}	5.0×10^{-4}	4.7×10^{-2}	6.9×10^{-3}	7.3×10^{-3}
$r = 8$	1.6×10^{-2}	4.3×10^{-3}	5.0×10^{-4}	4.3×10^{-2}	3.4×10^{-2}	2.2×10^{-2}
$r = 9$	1.6×10^{-2}	4.3×10^{-3}	5.1×10^{-4}	2.4×10^{-2}	3.3×10^{-4}	2.6×10^{-2}

where

$$f_1(t) = -\frac{1}{15}\left(6t^5 + 35t^4 - 95t^3 - 270t^2 - 225t - 360\right),$$

$$f_2(t) = -\frac{1}{30}\left(15t^7 + 10t^6 - 33t^5 + 275t^4 + 115t^3 - 390t^2 + 150\right),$$

$$f_3(t) = \frac{1}{60}\left(40t^6 - 66t^5 - 175t^4 + 250t^3 + 780t^2 + 360t + 360\right),$$

with the exact solution, $F_1(t) = 5t + 8$, $F_2(t) = 2t^2 - 5$ and $F_3(t) = t^2 + t + 1$. Again,we solved this example by this method and the results are given in Table 5. Table 6 illustrates the absolute errors of ANN method and TQR for this example.

As we can see this method will be useful when the exact solution is a polynomial. In other word, the proposed method give the analytical solution for the system, if the exact solution be polynomials of degree n or less than n.

5 Conclusions

In some cases, an analytical solution cannot be found for integral equations system, therefore, numerical methods have been applied. In this study, we have worked out a computational method to approximate solution of the Fredholm and Volterra integral equations systems of the second kind. The present course is a method for approximating unknown functions in terms of truncated sequences including Bernstein polynomials. It is clear that to get the best approximating solutions of the given systems, the truncation degree n must be chosen large enough. An interesting feature of this method is finding the analytical solution for given system, if the exact solution be polynomials of degree n or less than n.

Additionally, the proposed method has been compared with ANN method [18] and TQR. The analyzed examples illustrated the ability and reliability of the present method. The obtained solutions, in comparison with exact solutions admit a remarkable accuracy. Extensions to the case of more general systems of integral equations are left for future studies.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have equal contributions and they have approved the final version of the manuscript.

Author details
[1]Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran. [2]Department of Physics, Urmia Branch, Islamic Azad University, Urmia, Iran. [3]Department of Mathematics and Computer Sciences, Faculty of Art and Sciences, Çankaya University, Balgat, Ankara, 0630, Turkey. [4]Department of Chemical and Materials Engineering, Faculty of Engineering, King Abdulaziz University, P.O. Box 80204, Jeddah, 21589, Saudi Arabia. [5]Institute of Space Sciences, P.O. Box MG-23, Magurele, Bucharest 76900, Romania.

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