Equivalent functions for the Fresnel integral

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Abstract: Fresnel integral is modeled with three equivalent functions. The first function is derived by considering the sum of the first term of the Fresnel integral's asymptotic expansion $\{\hat{F}(x)\}$ and an exponential function which approaches to infinity at the zero of the Fresnel function's argument and has the properties of a unit step function. The second one is the sum of a unit step function and the transition function defined for the simplified uniform theory of diffraction. The third function considers directly eliminating the infinity coming from $\hat{F}(x)$. The amplitude and the phase of Fresnel integral and its equivalent functions are compared numerically. The result is applied to the modified theory of physical optics solution of the diffraction of edge waves from a half plane problem.

OCIS codes: (050.0000) General; (000.3860) Mathematical methods in physics; (080.0080) Geometrical optics; (080.2720) Geometrical optics, mathematical methods.

References and links

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1. Introduction

Fresnel integral has a wide range of application in the electromagnetic diffraction theory. The exact solution of the half plane problem is represented with the Fresnel integral by Sommerfeld [1]. Lewis, Boersma and Ahluwalia [2, 3] developed the uniform asymptotic theory of diffraction (UAT) by changing the geometrical optics fields with the related integral. Fresnel integral is also used in the uniform theory of diffraction (UTD), which is introduced by Kouyoumjian and Pathak [4]. There are many programs in order to calculate the Fresnel integrals, but it requires much time for complex scattering problems. For this reason a new transition function is defined by Umul [5] recently in order to express the related function of UTD with simple exponential expressions for the simplified theory of diffraction (SUTD).

It is the aim of this paper to define three equivalent functions instead of the Fresnel integral by using a function which will cancel the infinity coming from the pole of the first term in Fresnel function's asymptotic expansion. The amplitude and the phase of the related functions will be compared numerically. The result will be applied to the modified theory of diffraction (MTPO) solution of the diffraction of edge waves from a half plane problem. This solution contains the integration of Fresnel function.

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A time factor e^{jwt} is assumed and suppressed throughout the paper.

2. Derivation of the methods

Three equivalent functions for the Fresnel integral will be derived by considering the asymptotic expansion of

$$F(x) \approx u(-x) + \hat{F}(x) + 0\left(x^{-3/2}\right) \quad (1)$$

for $|x| \to \infty$. $F(x)$ is the Fresnel integral, defined by

$$F(x) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int_x^{\infty} e^{-jt^2} dt \quad (2)$$

and $\hat{F}(x)$ is the first term in its asymptotic expansion, which can be given as

$$\hat{F}(x) = \frac{e^{-j\left(x^2+\frac{\pi}{4}\right)}}{2\sqrt{\pi}x} \quad (3)$$

for $|x| \to \infty$. $u(-x)$ is the unit step function, which is equal to 1 for x<0 and 0 for x>0.

2.1 First method

The method relies on canceling the pole of $\hat{F}(x)$ by defining a unit step function which approaches to infinity for $x = 0$. This is a similar approach with the transition function of UAT, which can be given as

$$F(x) - \hat{F}(x) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \int_x^{\infty} e^{-jt^2} dt - \frac{e^{-j\left(x^2+\frac{\pi}{4}\right)}}{2\sqrt{\pi}x} \quad (4)$$

for $|x| \to \infty$. Equation (4) has a pole at $x = 0$ and is equal to $u(-x)$ everywhere except $x = 0$ according to Eq. (1). Equation (1) can be written as

$$F(x) - \hat{F}(x) = u(-x) + g(x) \quad (5)$$

where $g(x)$ is an unknown function which goes to infinity at $x = 0$ and is equal to zero, otherwise. A function can be introduced as

$$\vartheta(x) = \frac{1}{1 - e^{e^{j\frac{\pi}{4}} 2\sqrt{\pi}x}} \quad (6)$$

which has the same properties with Eq. (4) and represents $u(-x) + g(x)$. An equivalent function can be defined for the Fresnel integral as

$$E_{F1}(x) = \vartheta(x) + \hat{F}(x) \quad (7)$$

by considering Eqs. (5) and (6). The infinity of $\vartheta(x)$ is canceled by adding $\hat{F}(x)$. A $\infty-\infty$ indeterminacy is created as a result of this summation. As a result, the equivalent function can be defined by the equation of

$$E_F(x) = \frac{1}{1-e^{e^{j\frac{\pi}{4}}2\sqrt{\pi}x}} + \frac{e^{-j\left(x^2+\frac{\pi}{4}\right)}}{2\sqrt{\pi}x} \tag{8}$$

and can be used instead of the Fresnel integral.

It is important to note that the equivalent functions are correlated with the uniform theory of diffraction (UAT) physically. In a diffraction problem, unit step function represents the geometrical optics (GO) fields and $\hat{F}(x)$ has relation with the geometrical theory of diffraction (GTD) coefficients. The equivalent function uses a modified form of unit step function in order to cancel the infinity coming from $\hat{F}(x)$. This is the same approach with the modified GO terms of UAT. This is obvious from Eqs. (5) and (7).

The phase and amplitude of the Fresnel function will be compared with $E_{F1}(x)$. The Fresnel integral can be written as

$$F(x) = |F(x)| e^{j\angle F(x)} \tag{9}$$

where $|F(x)|$ is defined as $\sqrt{F(x)F^*(x)}$. The amplitude and the phase functions are equal to

$$|F(x)| = \sqrt{\left[\frac{1}{2} - \frac{1}{\sqrt{\pi}}\int_0^x \cos\left(t^2 - \frac{\pi}{4}\right)dt\right]^2 + \left[\frac{1}{\sqrt{\pi}}\int_0^x \sin\left(t^2 - \frac{\pi}{4}\right)dt\right]^2} \tag{10}$$

and

$$\angle F(x) = tg^{-1}\frac{\int_0^x \sin\left(t^2 - \frac{\pi}{4}\right)dt}{\frac{\sqrt{\pi}}{2} - \int_0^x \cos\left(t^2 - \frac{\pi}{4}\right)dt} \tag{11}$$

respectively. The Matlab codes of the Fresnel function and the integrals, written in Eq. (10) is given in the Appendix. It is also possible to express the equivalent function as

$$E_{F1}(x) = |E_{F1}(x)| e^{j\angle E_{F1}(x)} \tag{12}$$

for the amplitude and the phase functions can be written as

$$|E_{F1}(x)| = \frac{\sqrt{\left[2\sqrt{\pi}xf_1(x) + f_3(x)\cos\left(x^2 + \frac{\pi}{4}\right)\right]^2 + \left[2\sqrt{\pi}xf_2(x) - f_3(x)\sin\left(x^2 + \frac{\pi}{4}\right)\right]^2}}{2\sqrt{\pi}|xf_3(x)|} \tag{13}$$

and

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$$\angle E_{F1}(x) = tg^{-1}\frac{2\sqrt{\pi}xf_2(x) - f_3(x)\sin\left(x^2+\frac{\pi}{4}\right)}{2\sqrt{\pi}xf_1(x) + f_3(x)\cos\left(x^2+\frac{\pi}{4}\right)} \tag{14}$$

respectively. The functions, defined in Eqs. (13) and (14) can be expressed as

$$f_1(x) = 1 - e^{\sqrt{2\pi}x}\cos\left(\sqrt{2\pi}x\right) \tag{15.a}$$

$$f_2(x) = e^{\sqrt{2\pi}x}\sin\left(\sqrt{2\pi}x\right) \tag{15.b}$$

$$f_3(x) = 1 - 2e^{\sqrt{2\pi}x}\cos\left(\sqrt{2\pi}x\right) + e^{2\sqrt{2\pi}x} \tag{15.c}$$

by considering Eq. (8). The amplitude and phase functions of $F(x)$ and $E_F(x)$ will be compared in order to test their identity.

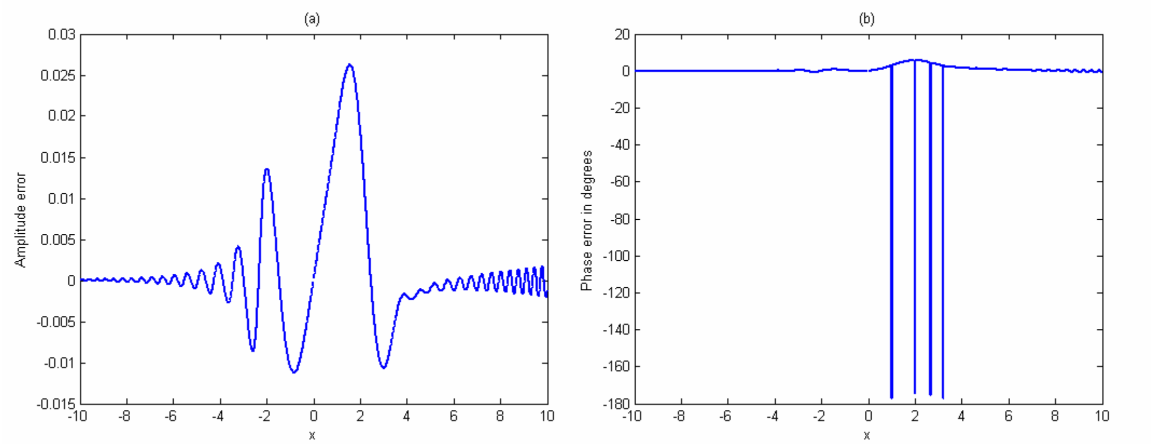

Fig. 1. The amplitude and phase errors

Figure 1 shows the variation of the amplitude and phase error functions versus the variable of x. The natural logarithm of the proportion of the Fresnel integral and the equivalent function can be written as

$$\ln\frac{F(x)}{E_{F1}(x)} = e_A(x) + je_P(x) \tag{16}$$

where $e_A(x)$ shows the amplitude error of

$$e_A(x) = \ln\frac{|F(x)|}{|E_{F1}(x)|} \tag{17}$$

and $e_P(x)$ represents the phase error as

$$e_P(x) = \angle F(x) - \angle E_{F1}(x). \tag{18}$$

Such a representation gives a physical understanding of the error. Equation (17) shows the deviation in amplitude. Equation (18) expresses the phase error directly in degrees and puts

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forward the amount of the point to point phase error. This representation is more meaningful from a logarithmic expression.

Figure 1(a) is the plot of the amplitude error. It can be observed that The maximum error change occurs in the interval of $x \in [-4,4]$ and maximum error is nearly 0.027.

Figure 1(b) shows the graph of the phase error function. The maximum errors occur in the interval of $x \in [0,4]$ and has four maxima values, nearly equal to -177^0.

It is important to note that the ripples, seen in Fig. 1(a) for $x \geq 4$ are the result of the step size N, given in the Appendix. When N increases, the amplitude of the ripples will decrease, but it is apparent that computing time will increase. N is taken as 20.000 for the plots, given in Fig. 1.

2.2 Second method

The property of

$$F(x) = u(-x) + \operatorname{sgn}(x) F(|x|) \tag{19}$$

will be taken into account in order to derive the second equivalent function. $\operatorname{sgn}(x)$ is the signum function, which is defined as

$$\operatorname{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x \leq 0 \end{cases}. \tag{20}$$

The transition function of SUTD can be written as

$$f(x) = p(x)\left(1 - e^{-\sqrt{\pi}|x|}\right) \tag{21}$$

where

$$p(x) = e^{j\frac{\pi}{4}e^{-|x|}} \tag{22}$$

which is given in Ref. [5]. The transition function of UTD can be expressed as

$$T(x) = \frac{F(|x|)}{\hat{F}(|x|)} \tag{23}$$

and the term of $F(|x|)$ can be evaluated as

$$F(|x|) = f(x)\hat{F}(|x|) \tag{24}$$

by equating Eq. (23) to Eq. (21). As a result one obtains

$$E_{F2}(x) = u(-x) + \operatorname{sgn}(x) f(x) \hat{F}(|x|) \tag{25}$$

when Eq. (24) is combined with Eq. (19).

The relation of the equivalent function with uniform theory of diffraction (UTD) was discussed in Ref. [5]. Eq. (25) is derived from Eq. (19), which is the general expression of a UTD scattered field for $\operatorname{sgn}(x) = \hat{F}(x)/\hat{F}(|x|)$. SUTD transition function is used instead of UTD transition function in order to model the Fresnel integral.

The equivalent function in Eq. (25) can be written as

$$E_{F2}(x) = |E_{F2}(x)| e^{j\angle E_{F2}(x)} \tag{26}$$

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where $\left|E_{F2}(x)\right|$ and $\angle E_{F2}(x)$ are the amplitude and phase functions of $E_{F2}(x)$, respectively. $\left|E_{F2}(x)\right|$ can be represented as

$$\left|E_{F2}(x)\right| = \sqrt{[q_1(x)]^2 + [q_2(x)]^2} \tag{27}$$

and the phase function can be written as

$$\angle E_{F2}(x) = tg^{-1}\frac{q_1(x)}{q_2(x)} \tag{28}$$

where

$$\begin{aligned} q_1(x) &= \operatorname{sgn}(x)\frac{1-e^{-\sqrt{\pi}|x|}}{2\sqrt{\pi}|x|}\sin\left(\frac{\pi}{4}e^{-|x|} - \frac{\pi}{4} - x^2\right) \\ q_2(x) &= u(-x) + \operatorname{sgn}(x)\frac{1-e^{-\sqrt{\pi}|x|}}{2\sqrt{\pi}|x|}\cos\left(\frac{\pi}{4}e^{-|x|} - \frac{\pi}{4} - x^2\right) \end{aligned} \tag{29}$$

which can be evaluated from Eq. (25). The formula of

$$\ln\frac{F(x)}{E_{F2}(x)} = e_A(x) + je_P(x) \tag{30}$$

will be used in determining the error comparison of the equivalent function and the Fresnel integral. $e_A(x)$ and $e_P(x)$ were defined by Eqs. (17) and (18) and are valid for the present case when the Eqs. (27) and (28) are considered.

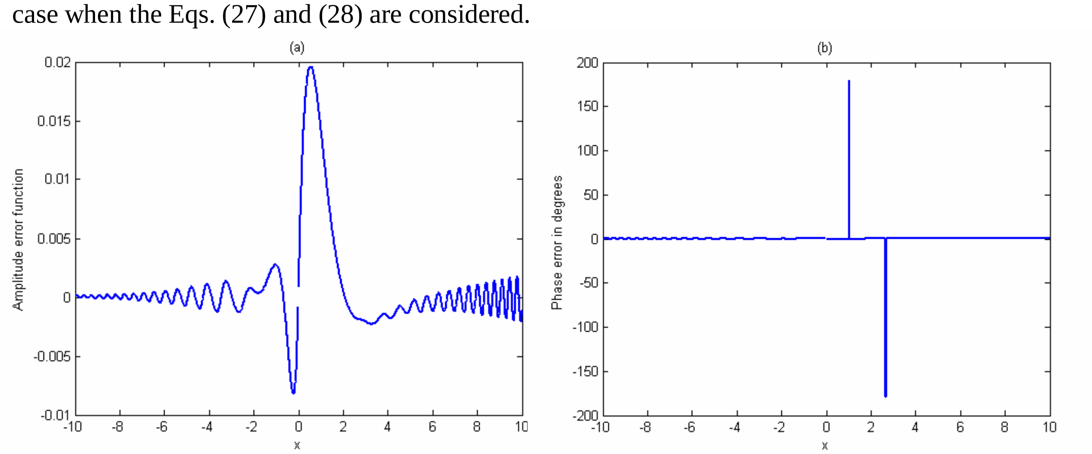

Fig. 2. The amplitude and phase errors of $F(x)$ and $E_{F2}(x)$

Figure 2 shows the variation of amplitude and phase errors versus *x*. The maximum amplitude error does not exceed 0,02 and exists in the neighborhood of x=0 according to Fig. 2(a). It is observed from Fig. 2(b) that there are two error maxima, which occur at x=1,03 and x=2,67. The phase error values at these points are nearly equal to $\mp 180^0$. N is taken as 50.000 for the related plots.

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2.3 Third method

An equivalent representation for the function *g(x)*, given in Eq. (5), will be defined as a third step. As mentioned before, *g(x)* is equal to zero everywhere except *x=0*, where it approaches to infinity. Such a function can be defined by

$$g_1(x) = \mathrm{sgn}(-x)\frac{e^{-e^{j\frac{\pi}{4}}2\sqrt{\pi}|x|}}{1-e^{-e^{j\frac{\pi}{4}}2\sqrt{\pi}|x|}} \tag{31}$$

and the equivalent Fresnel function can be written as

$$E_{F3}(x) = u(-x) + \hat{F}(x) + g_1(x) \tag{32}$$

by using Eq. (31) in Eq. (5).

There is no uniform theory in literature that has the same approach with the third equivalent function. A function of *g(x)*, which gives the exact Fresnel function when added to the term of $u(-x)+\hat{F}(x)$, is described. Since its properties are known (although the function itself is unknown), a function $g_1(x)$ which shows a similar variation is defined. This function eliminates the infinity of $\hat{F}(x)$ and when added to $u(-x)$, gives an equivalent function for the Fresnel integral. A new uniform approach for diffraction can be derived by using this concept.

$E_{F3}(x)$ can be expressed as

$$E_{F3}(x) = |E_{F3}(x)| e^{j\angle E_{F3}(x)} \tag{33}$$

for the amplitude function is equal to

$$|E_{F3}(x)| = \sqrt{g_r(x) + g_i(x)} \tag{34}$$

and the phase function can be obtained as

$$\angle E_{F3}(x) = tg^{-1}\frac{g_i(x)}{g_r(x)}. \tag{35}$$

The functions in Eqs.(34) and (35) can be defined as

$$g_r(x) = u(-x) + \frac{\cos\left(x^2 + \frac{\pi}{4}\right)}{2\sqrt{\pi}x} - \mathrm{sgn}(-x)e^{-\sqrt{2\pi}|x|}\frac{e^{-\sqrt{2\pi}|x|} - \cos\sqrt{2\pi}|x|}{f_3(|x|)} \tag{36}$$

and

$$g_i(x) = -\frac{\sin\left(x^2 + \frac{\pi}{4}\right)}{2\sqrt{\pi}x} - \mathrm{sgn}(-x)e^{-\sqrt{2\pi}|x|}\frac{\sin\sqrt{2\pi}|x|}{f_3(|x|)} \tag{37}$$

where $f_3(x)$ was given in Eq. (15.c). The error functions can be derived by taking the natural logarithm of the proportion of the Fresnel integral and the equivalent function which can be written as

$$\ln \frac{F(x)}{E_{F3}(x)} = e_A(x) + je_P(x) \tag{38}$$

where $e_A(x)$ shows the amplitude error of

$$e_A(x) = \ln \frac{|F(x)|}{|E_{F3}(x)|} \tag{39}$$

and $e_P(x)$ represents the phase error as

$$e_P(x) = \angle F(x) - \angle E_{F3}(x). \tag{40}$$

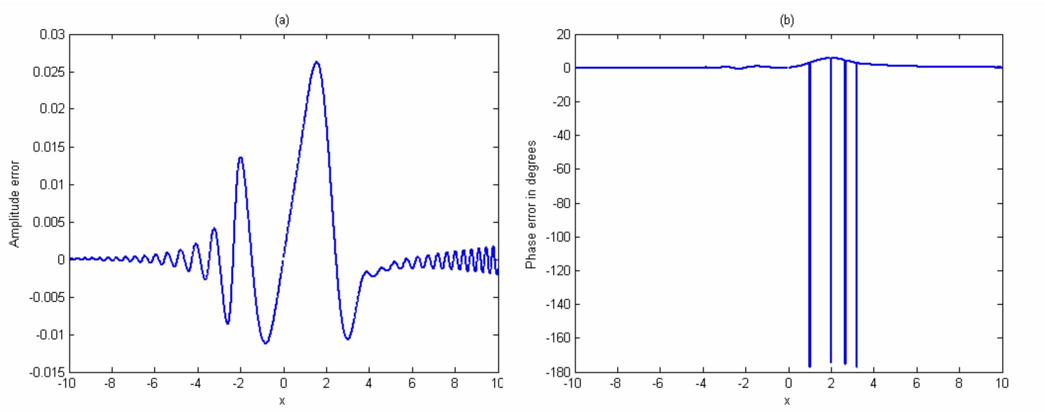

Fig. 3. The amplitude and phase errors of $F(x)$ and $E_{F3}(x)$

Figure 3 shows the variation of the amplitude and phase errors, given in Eqs.(39) and (40), with respect to x. It can be observed that the maxima of amplitude error are assembled in the neighborhood of the value where the argument of the function is zero. This is the result of the fact that equivalent functions are constructed by using the large argument expansion of the Fresnel integral.

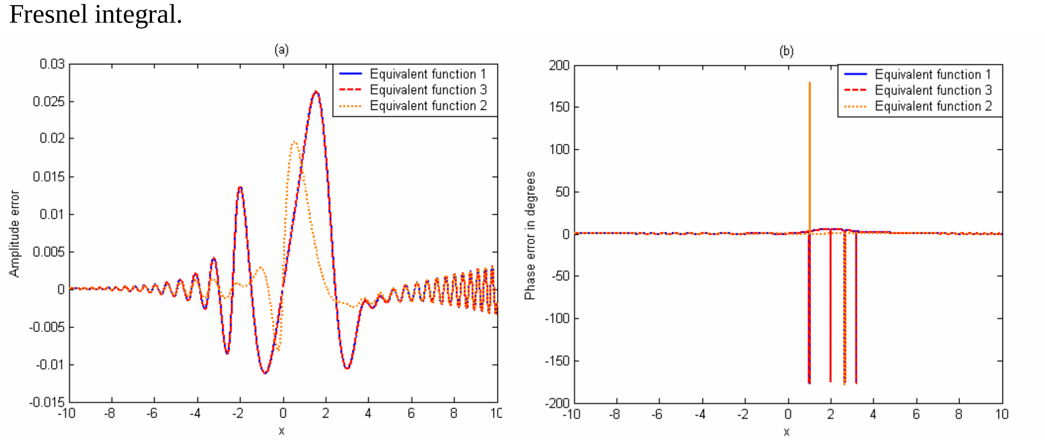

Fig. 4. Comparison of the amplitude and phase errors of the equivalent functions

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Figure 4 shows the variation of Eqs. (16), (30) and (38) versus x. It can be observed that $E_{F2}(x)$ shows a better error performance against the other functions. The phase error of $E_{F2}(x)$ is also more acceptable than the others. In terms of computation cost, all of the equivalent functions require the same computing time, approximately 2 seconds. It requires one minute to compute the Fresnel integral given in the Appendix for N=50.000. The computation time will decrease if lesser terms are considered for N, but this time the amplitude of the ripples, seen in the amplitude plots for x>5, increases. This causes more error in evaluations.

It can be seen from Fig. 4 that the error variation of $E_{F1}(x)$ and $E_{F3}(x)$ are about the same. In order to examine the reason of this result, the difference of the equivalent functions can be considered as

$$E_{F2}(x) - E_{F3}(x) = \vartheta(x) - u(-x) - g_1(x) \tag{41}$$

from their definitions. The equation of

$$E_{F2}(x) - E_{F3}(x) = \vartheta(x) - \begin{cases} \vartheta(x) & x \le 0 \\ -\dfrac{e^{-Kx}}{1-e^{-Kx}} & x \ge 0 \end{cases} \tag{42}$$

can be obtained for $K = \exp(j\pi/4)2\sqrt{\pi}$. It is apparent that Eq. (42) is equal to zero for x<0 and approaches to zero for x>0.

The equivalent functions are compared with the Fresnel integral for real argument up to here. A comparison for complex argument will be performed in order to test their validity. The formula derived in Eq. (16) will be taken into account for the first equivalent function. The argument will be taken as $x\exp(j\pi/3)$.

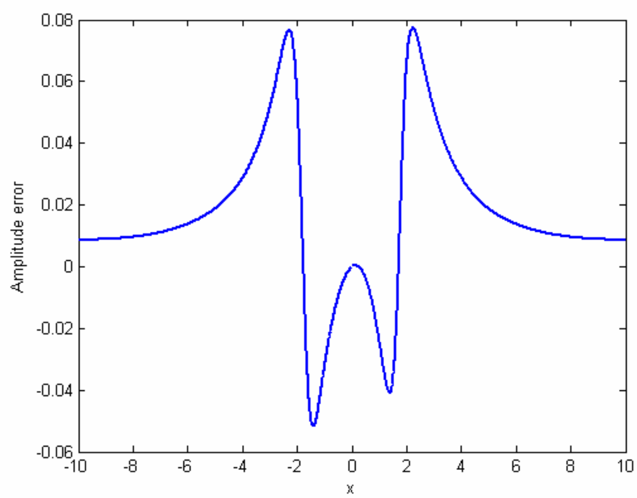

Fig. 5. Error plot for the complex argument Fresnel function

Figure 5 shows the variation of the amplitude error in a logarithmic scale versus x. It can be observed that the error has increased, compared to the real argument case. This is not a physical state especially that can be met in diffraction problems but it is important to know the behavior of the equivalent functions for a mathematical point of view.

The first equivalent function will be compared with the Fresnel integral for pure imaginary argument as a second step.

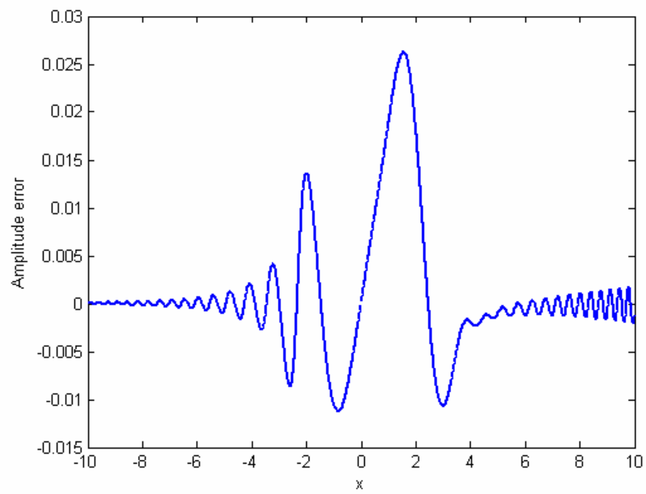

Fig. 6. Amplitude error for pure imaginary argument

Figure 6 depicts the plot of amplitude error according to an imaginary argument $\left[E_{F1}(jx)\right]$ with respect to x. It can be seen that the variation of the amplitude error is the same with the one, plotted in Fig. 1.

3. Numerical example: Scattering of edge waves from a PEC half plane

A physical optics scattering problem will be observed in this section in order to the scattering of edge diffracted waves from a half plane will be examined by using the method of modified theory of diffraction. The geometry of the problem is given in Fig. 7.

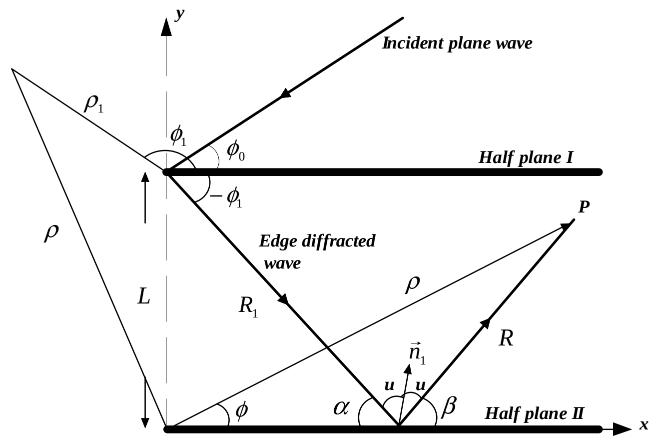

Fig. 7. Geometry of the two half plane problem.

A magnetic polarized plane wave with unit amplitude is illuminating the first half plane. The second half plane is lying in the shadow region of the first half plane. It is obvious that only the edge diffracted waves reach the second plane. The main objective of this analysis is to obtain an integral which contains Fresnel function. The computation time of the integral is expected to increase, but it will be shown that this defeat can be eliminated by using equivalent functions instead of Fresnel integral.

The MTPO integral of reflected fields will be considered. The edge diffracted waves of the first half plane can be written as

$$\vec{H}_{di} = \vec{e}_z \left[e^{jk\rho_1 \cos(\phi_1 - \phi_0)} \operatorname{sgn}(\xi_1) F(|\xi_1|) + e^{jk\rho_1 \cos(\phi_1 + \phi_0)} \operatorname{sgn}(\xi_2) F(|\xi_2|) \right] \quad (43)$$

where

$$\begin{aligned} \xi_1 &= -\sqrt{2k\rho_1} \cos\frac{\phi_1 - \phi_0}{2} \\ \xi_2 &= -\sqrt{2k\rho_1} \cos\frac{\phi_1 + \phi_0}{2} \end{aligned} \quad (44)$$

which are the detour parameters of UTD. ρ_1 and ϕ_1 are the cylindrical coordinate quantities of the first half plane and are related to the second half plane coordinates as

$$\rho_1 = \sqrt{\rho^2 + L^2 - 2L\rho \sin\phi} \quad (45.a)$$

$$\phi_1 = \cos^{-1} \frac{\rho \cos\phi}{\rho_1} \quad (45.b)$$

for L is the distance between the half planes. MTPO surface current can be found by considering the boundary condition of $\vec{J}_{es} = \vec{n} \times \vec{H}_t \big|_{S'}$ as

$$\vec{J}_{MTPO} = 2H_{diz}\big|_{S'} \left(\vec{e}_x \cos\frac{\beta - \alpha}{2} - \vec{e}_y \sin\frac{\beta - \alpha}{2} \right) \quad (46)$$

where the modified unit vector of the second surface is equal to $\vec{n}_1 = \cos(u + \alpha)\vec{e}_x + \sin(u + \alpha)\vec{e}_y$ for $u = \frac{\pi}{2} - \frac{\alpha + \beta}{2}$. The integral of the reflected magnetic field can be written as

$$\vec{H}_r = \frac{e^{j\frac{\pi}{4}}}{2\sqrt{2\pi}\, j} \int_0^\infty \nabla \times \left(\vec{J}_{MTPO} \frac{e^{-jkR}}{\sqrt{kR}} \right) dx' \quad (47)$$

where the curl operation will be applied according to the observation point coordinates [6]. The resultant integral can be expressed as

$$\vec{H}_r = \vec{e}_z \frac{k e^{j\frac{\pi}{4}}}{\sqrt{2\pi}} \int_0^\infty H_{diz}\big|_{S'} \sin\frac{\beta + \alpha}{2} \frac{e^{-jkR}}{\sqrt{kR}} dx' \quad (48)$$

where $H_{diz}\big|_{S'}$ represents the value of the incident field on the reflection surface and is equal to

$$H_{diz}\big|_{S'} = e^{jkR_1 \cos(\phi_1 - \phi_0)} \operatorname{sgn}\left(-\sqrt{2kR_1} \cos\frac{\phi_1 - \phi_0}{2} \right) F\left(\left| \sqrt{2kR_1} \cos\frac{\phi_1 - \phi_0}{2} \right| \right) + (\phi_0 \to -\phi_0) \quad (49)$$

for R_1 is the value of ρ_1 on the surface. The related quantities are equal to

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$$R_1 = \sqrt{(x')^2 + L^2} \tag{50.a}$$

$$\phi_1 = 2\pi - \alpha \tag{50.b}$$

on the surface of reflection. α is equal to $tg^{-1}(L/x')$. Equation (48) will be plotted for the Fresnel integral and the first equivalent function, used in Eq. (49).

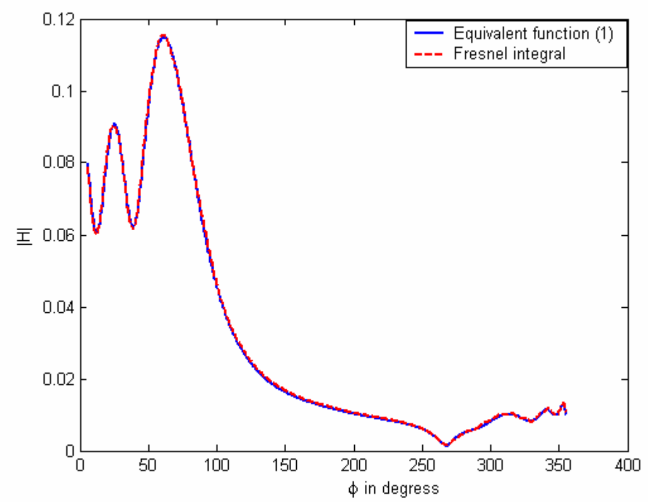

Fig. 8. Reflected magnetic field from the second half plane

Figure 8 depicts the variation of the reflected magnetic field from the second half plane versus observation angle. L is taken as 3λ and ϕ_0 is equal to 30^0. It can be seen that using the equivalent functions in Eqs. (48) and (49) instead of Fresnel integrals, gives the same result by requiring lesser computation time. N is equal to 20000 for this computation.

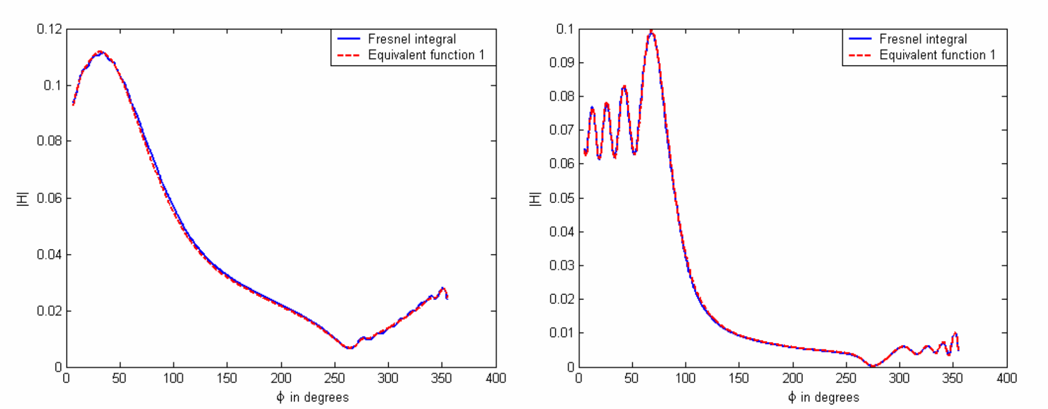

Fig. 9. Scattering integral for L=λ/2 and L=10λ

Figure 9 shows the variation of the scattering integral in Eq. (48) with respect to the observation angle for L=λ/2 and L=10λ. ϕ_0 is equal to $\pi/3$. It can be seen that the plots, obtained for the Fresnel integral and $E_{F1}(x)$ are in harmony. The ripples, observed in the first plot of Fig.9, are the result of the step size. Fresnel integral is plotted for 5000 terms in these graphs. If the value of N is increased, these ripples will become smaller and vanish. A computation cost analysis will be given below.

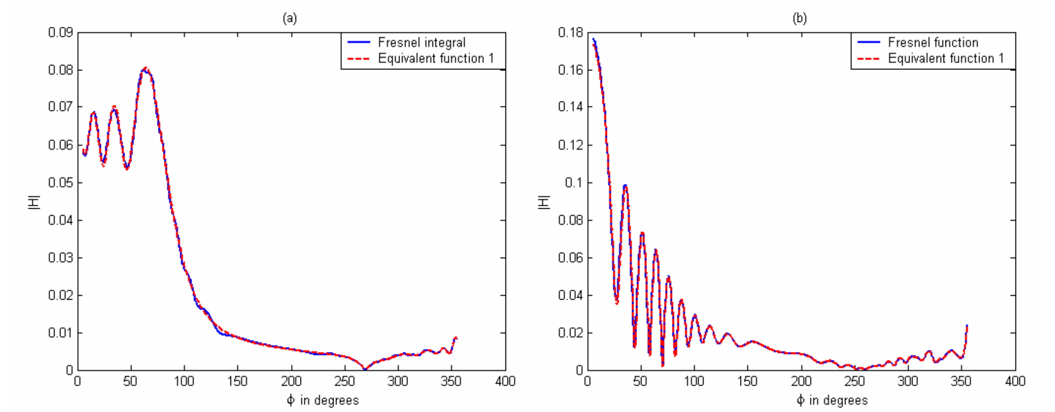

Fig. 10. Scattering integral for a) $\phi_0 = \pi/6$, b) $\phi_0 = 5\pi/6$

Figure 10 shows the variation of the scattering integral of Eq. (48) versus the observation angle for $\phi_0 = \pi/6$, $\phi_0 = 5\pi/6$ and L=5λ. It can be observed that the two integrals, evaluated for the Fresnel function and the first equivalent function are in harmony.

It is important to give information about the computation cost. Equation (48) contains the sum of two Fresnel integrals and this means that there are two “for loops”, each containing another “for loop”. The integral in Eq. (48) is computed by a “for loop” which consists of the sum of 700 terms. The time, required for the evaluation of Eq. (48), is nearly 90 seconds when N has the value of 5000 at the Fresnel integral code, given in the Appendix. This value causes some cripples at the plots as can be seen from Figs. 9 and 10. If the value of N is increased, the time of computation will also increase. It takes only 4 seconds to evaluate the integral, given in Eq. (48), when the equivalent functions are used instead of the Fresnel function in the integral.

4. Conclusion

In this work, three equivalent functions are derived for the Fresnel integral. It is stated that the first two functions are related with UAT and UTD approaches. The third equivalent function attempts to eliminate the infinity coming from $\hat{F}(x)$ and can be thought as a variation of UTD since UTD makes uniform the diffraction field by multiplying the diffraction coefficient by a transition function which has a zero at the transition region. This approach creates a $0/0$ indeterminacy. The concept of the third equivalent function is to eliminate the related infinity by creating a $\infty - \infty$ indeterminacy.

The numerical comparisons show that the functions represent the Fresnel integral with a very good degree of correctness. The equivalent functions consist of the sum of two or three basic functions. It is always easy to deal with these functions for numerical or analytic evaluations. Fresnel integrals can be found in many simulation programs, but there is always a need of integral evaluation and this creates a problem when dealing with the diffraction of complex bodies.

Appendix

The Matlab code, used for the plot of the Fresnel integral can be introduced as follows;

Fresnel integral

The Matlab code can be written as

```
x=-10:.01:10;
N=10000;
sum=0;
lwbound=0;
upbound=x;
delta=(upbound-lwbound)./N;
for i=0:N;
    t=lwbound+(i.*delta);
    f=exp(-j.*(t.^2));
    sum=sum+f;
end
fres=0.5-(exp(-j.*pi./4).*sum.*delta./sqrt(pi));
```

for the Fresnel integral, defined in Eq. (2).

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