Modified theory of physical optics approach to wedge diffraction problems

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Abstract: The problem of diffraction from a perfectly conducting wedge is examined with the modified theory of physical optics (MTPO). The exact wedge diffraction coefficient is compared with the asymptotic edge waves of MTPO integral and related surface currents are evaluated. The scattered electric fields are expressed by using these current components. The total, incident and reflected diffracted fields are compared with the exact series solution of the wedge problem, numerically.

OCIS codes: (260.0260) Physical Optics; (260.1960) Diffraction theory; (260.2110) Electromagnetic theory.

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#5798 - $15.00 US Received 19 November 2004; revised 29 December 2004; accepted 29 December 2004

1. Introduction

Wedge diffraction is an important canonical problem in the geometrical theory of diffraction (GTD), which is an extension of geometrical optics [1]. A ray based solution for wedge diffraction problem is developed by Keller for GTD [2, 3]. He compared his analysis with an asymptotic expansion of Sommerfeld's solution [4] and defined a scalar diffraction coefficient for electric and magnetic polarized waves. However this coefficient gives infinite field values at the transition regions. Uniform theories are developed in order to overcome this defect [5, 6].

Physical optics (PO) approach to diffraction problems are commonly used in literature [7]. The surface currents, induced on the illuminated part of the scatterer, are considered in the classical PO theory. This procedure gives no information about the geometry of the object and the currents in the shadow region. For example, the diffracted fields can be evaluated only by adding the fringe currents to the PO surface current as in physical theory of diffraction (PTD) [8, 9] for wedge diffraction problems. The PO solution of the diffraction by a dielectric wedge is presented by Kim et al. [10] from the formulation of dual integral equation. A correction to this solution is also presented by calculating the non-uniform current component along the dielectric interfaces [11]. A new method is developed by Taket and Burge by considering the asymptotic PO scattering integral with geometrical optics field in order to find approximate solutions to perfectly conducting wedge and vertex problems [12]. This method is applied to the problem of scattering from dielectric wedges with planar surfaces and a diffraction coefficient, having a good agreement with experimental results, is calculated [13]. The three-dimensional scattering from a dipole fed wedge with imperfectly conducting faces, is examined by Papadopoulos and Chrissoulidis with PO method [14, 15]. Geometrical optics (GO) fields and lateral waves are taken into account and a correction factor is developed for the corner region.

The modified theory of physical optics (MTPO), which was introduced recently by Umul [14], is an improved version of PO. The method is developed in order to find exact edge diffracted fields of the perfectly conducting half plane problem, directly from the asymptotic evaluation of the PO integral. MTPO considers the surface current, induced on the scatterer, and the equivalent fields on the aperture part in order to construct the solution. The integral, written for the surface current of the scatterer, gives the reflected and reflected edge diffracted fields and the integral, obtained by considering the aperture fields, represents the incident and incident diffracted fields. This model enables one to consider the geometry of the scatterer and the fields in the shadow region. When the wedge geometry is considered, the geometrical optics fields remain the same but the edge diffracted waves are affected from the perfectly conducting surface at the shadow boundary. This effect can be considered in the MTPO integral by turning the aperture by the interior angle of the wedge, but the calculated diffracted field will not satisfy the boundary conditions for the shadow surface, because the integral for the illuminated surface remains the same. For this reason, the MTPO currents will be determined by using the exact wedge diffraction coefficient in this work.

It is the aim of this paper to integrate the boundary conditions on the two surfaces of the perfectly conducting wedge to the Modified Theory of Physical Optics (MTPO) integral and construct the surface currents and the exact solution. The exact wedge diffraction coefficient for an electric polarized wave is compared with the current terms in the MTPO integral and a new surface current is evaluated for a perfectly conducting wedge. This component can also be used for curved wedges. It is supposed that only one face of the wedge is illuminated and the other one is at the shadow region, because in its present form, MTPO considers the problems with only one face illuminated, in order to obtain an aperture integral. If two of the faces are illuminated, there must be a second surface integral instead of the aperture integral.

A time factor e^{jwt} is assumed and suppressed throughout the paper.

Received 19 November 2004; revised 29 December 2004; accepted 29 December 2004

2. MTPO current for perfectly conducting wedge: electric polarization

The MTPO currents will be evaluated by considering the exact wedge diffraction coefficient. The asymptotic evaluation of MTPO integrals by edge point technique gives the edge diffracted fields. The sinusoidal term in the integrals will be replaced with an unknown function of $I(\phi_0,\beta)$, which will be determined by equating the asymptotically evaluated field to the wedge diffraction coefficient.

The exact wedge diffraction coefficient can be written as

$$D_w = D_{wi} - D_{wr} \tag{1}$$

where

$$D_{wi} = \frac{A(n)}{\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\phi-\phi_0}{n}\right)} \tag{2}$$

and

$$D_{wr} = \frac{B(n)}{\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\phi+\phi_0}{n}\right)} \tag{3}$$

for a wedge which is illuminated by an electric polarized wave [15, 16]. D_{wi} and D_{wr} are the coefficients, related to the incident diffracted and reflected diffracted waves, respectively. *A(n)* and *B(n)* are functions, which will be determined by the stationary phase evaluation of the incident and reflected part of the MTPO integral. The internal angle of the wedge is equal to $2\pi-\psi$. *n* has the value of ψ/π.

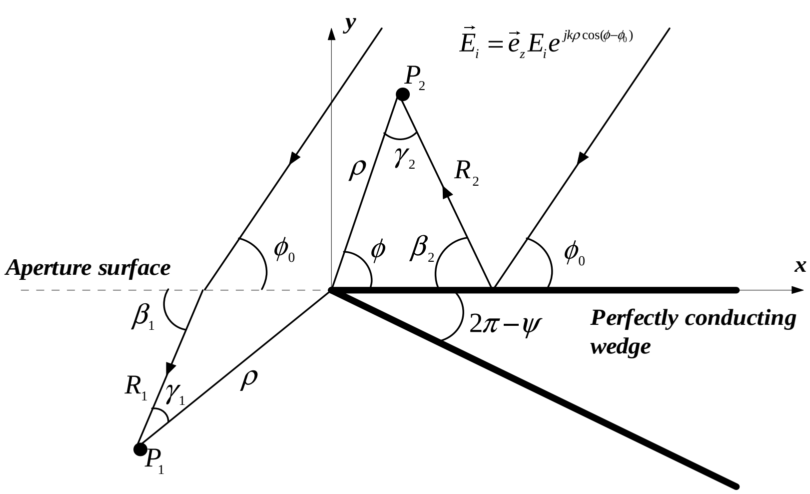

Fig. 1. Perfectly conducting wedge geometry

The MTPO integrals can be written as

$$\vec{E}_t \approx \vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \left(\int_{x'=-\infty}^{0} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} I_1(\phi_0, \beta_1) dx' - \int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR_2}}{\sqrt{kR_2}} I_2(\phi_0, \beta_2) dx' \right) \quad (4)$$

by considering the geometry in Fig. 1 [14]. The first integral in Eq. (4) represents the incident and incident diffracted fields and is written for the aperture. The surface is taken into account for $x' \in (-\infty, 0]$, because the integration of the field will give the incident waves for $\phi \in (\pi, \pi + \phi_0)$. If the aperture was considered for $x' \in [0, \infty)$, there would be incident field in the region of $\phi \in (-\phi_0, 0)$, which is actually the shadow region. For this reason, the boundary in Eq.4 is the most appropriate choice for the geometry of the problem. The second integral of Eq. (4) gives the reflected and reflected diffracted waves. I_1 and I_2 are weight functions, coming from the surface currents. The surface current, which is flowing on the illuminated part of the wedge, can be defined as

$$\vec{J}_S = \frac{2E_i}{Z_0} I_2(\phi_0, \beta_2) e^{jkx'\cos\phi_0} \vec{e}_z \quad (5)$$

and the equivalent current on the aperture can be written as

$$\vec{J}_A = -\frac{2E_i}{Z_0} I_1(\phi_0, \beta_1) e^{jkx'\cos\phi_0} \vec{e}_z \quad (6)$$

for a plane wave incidence. The aim of the following analysis is to determine the unknown functions of I_1 and I_2 by using the exact wedge diffraction coefficient, given in Eq. (1). The asymptotic evaluation of Eq. (4) by the edge point technique, gives the wedge diffracted field. I_1 and I_2 can be found by equating the asymptotic waves to the exact wedge diffracted fields. The wedge diffracted fields can be evaluated as

$$\vec{E}_{wd} = \vec{e}_z \frac{E_i}{\sqrt{2\pi}} e^{-j\frac{\pi}{4}} \frac{\left[I_1(\phi_0, \phi - \pi) + I_2(\phi_0, \pi - \phi) \right]}{\cos\phi + \cos\phi_0} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (7)$$

by using the edge point method, given in Ref. [14]. At the corner of the wedge, β_1 and β_2 are equal to $\phi - \pi$ and $\pi - \phi$, respectively. The exact wedge diffracted field can be expressed as

$$E_{dz} = \frac{E_i}{\sqrt{2\pi}} e^{-j\frac{\pi}{4}} (D_{wi} - D_{wr}) \frac{e^{-jk\rho}}{\sqrt{k\rho}} \quad (8)$$

[15] and the functions of I_1 and I_2 can be found as

$$I_1(\phi_0, \phi - \pi) = \frac{A(n)(\cos\phi + \cos\phi_0)}{\cos\frac{\pi}{n} - \cos\frac{\phi - \phi_0}{n}}, \quad I_2(\phi_0, \pi - \phi) = -\frac{B(n)(\cos\phi + \cos\phi_0)}{\cos\frac{\pi}{n} - \cos\frac{\phi + \phi_0}{n}} \quad (9)$$

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by equating Eq. (7) to Eq. (8). ϕ must be replaced with $\pi+\beta_1$ in I_1 and $\pi-\beta_2$ in I_2. One obtains

$$I_1(\phi_0,\beta_1)=\frac{A(n)(\cos\phi_0-\cos\beta_1)}{\cos\frac{\pi}{n}-\cos\frac{\pi+\beta_1-\phi_0}{n}} \quad , \quad I_2(\phi_0,\beta_2)=-\frac{B(n)(\cos\phi_0-\cos\beta_2)}{\cos\frac{\pi}{n}-\cos\frac{\pi-\beta_2+\phi_0}{n}} \tag{10}$$

for the weight functions of incident and reflected diffracted fields, respectively. Eq. (4) can be written as

$$E_{tz}=E_{iz}+E_{rz} \tag{11}$$

where the incident scattered field is equal to

$$E_{iz}=\frac{kE_i A(n)}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\int_{-\infty}^{0}\frac{\cos\phi_0-\cos\beta_1}{\cos\frac{\pi}{n}-\cos\frac{\pi+\beta_1-\phi_0}{n}}e^{jkx'\cos\phi_0}\frac{e^{-jkR}}{\sqrt{kR}}dx' \tag{12}$$

and the reflected scattered field can be given as

$$E_{rz}=\frac{kE_i B(n)}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\int_{0}^{\infty}\frac{\cos\phi_0-\cos\beta_2}{\cos\frac{\pi}{n}-\cos\frac{\pi-\beta_2+\phi_0}{n}}e^{jkx'\cos\phi_0}\frac{e^{-jkR}}{\sqrt{kR}}dx' \tag{13}$$

by using Eq. (11) in Eq. (4). The stationary phase evaluation of Eq. (12) is equal to $E_i e^{jk\rho\cos(\phi-\phi_0)}$ and that of Eq. (13) gives the reflected field as $-E_i e^{jk\rho\cos(\phi+\phi_0)}$. In order to determine the constants of *A(n)* and *B(n)*, the stationary phase evaluation of Eqs. (12) and (13) must be used. The stationary phase points for these integrals can be found as

$$\beta_s=\mp\phi_0 \tag{14}$$

by equating the phase function's first derivative to zero. $\beta_s=\phi_0$ gives the reflected field for $\phi\le\pi-\phi_0$ and the incident field for $\pi\le\phi\le\pi+\phi_0$ which is the actual wave. $\beta_s=-\phi_0$ represents the reflected field for $\phi\ge\pi+\phi_0$ and the incident field for $\pi-\phi_0\le\phi\le\pi$ which is equal to zero because of the I_1 and I_2 functions. The details of this analysis can be found in Ref. [14]. The amplitude function is equal to

$$f(x'_s)\approx\frac{kE_i Q(n)}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\lim_{\beta\to\phi_0}\frac{\cos\phi_0-\cos\beta}{\cos\frac{\pi}{n}-\cos\frac{\pi\mp(\beta-\phi_0)}{n}} \tag{15}$$

at the stationary point of $\beta=\phi_0$. *Q(n)* represents *A(n)* or *B(n)*. There is an uncertainty at the limit operation and the amplitude function can be evaluated by applying the L'Hopital's rule. Here, *(+)* and *(-)* signs give the incident and reflected fields, respectively. One obtains

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$$\mp \frac{E_i Q(n)}{\frac{1}{n}\sin\frac{\pi}{n}} e^{jk\rho\cos(\phi\pm\phi_0)} = \mp E_i e^{jk\rho\cos(\phi\pm\phi_0)} \tag{16}$$

from the asymptotic evaluation of Eqs. (12) and (13). The solution of Eq. (16) gives the *Q(n)* function as $\frac{1}{n}\sin\frac{\pi}{n}$ which is the constant amplitude term of the exact wedge diffraction coefficient. As a result MTPO integrals of the reflected and incident scattered fields can be written as

$$E_{rz} = \frac{kE_i \sin\left(\pi/n\right)}{n\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_0^{\infty} \frac{\cos\phi_0 - \cos\beta_2}{\cos\frac{\pi}{n} - \cos\frac{\pi-\beta_2+\phi_0}{n}} e^{jkx'\cos\phi_0} \frac{e^{-jkR}}{\sqrt{kR}} dx' \tag{17}$$

and

$$E_{iz} = \frac{kE_i \sin\left(\pi/n\right)}{n\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{-\infty}^{0} \frac{\cos\phi_0 - \cos\beta_1}{\cos\frac{\pi}{n} - \cos\frac{\pi+\beta_1-\phi_0}{n}} e^{jkx'\cos\phi_0} \frac{e^{-jkR}}{\sqrt{kR}} dx' \tag{18}$$

respectively.

3. Numerical results

The MTPO integrals, evaluated for wedge currents, will be compared with the exact series solution of the related problem. Eq. (17) represents the reflected and reflected wedge diffracted fields while Eq. (18) gives the incident and incident diffracted waves. The incident and reflected edge diffracted fields of MTPO integral will be compared with the exact solution. Eqs. (17), (18) and (19) are considered with this purpose. The incident edge diffracted waves can be plotted by subtracting the incident field of $E_i e^{jk\rho\cos(\phi-\phi_0)}$ from Eq. (18) for $\phi \le \pi + \phi_0$. The reflected diffracted field can be found by subtracting the reflected field from Eq. (17) for $\phi \le \pi - \phi_0$.The exact diffracted waves can also be obtained by applying the same subtraction method of the geometrical optics fields to Eq. (19). The exact solution of Helmholtz equation for the perfectly conducting wedge problem can be given as

$$E_z = \frac{E_i 4\pi}{\psi} \sum_{m=1}^{\infty} e^{j\vartheta_m\frac{\pi}{2}} J_{\vartheta_m}(k\rho) \sin\vartheta_m\phi \sin\vartheta_m\phi_0 \tag{19}$$

where ϑ_m is equal to $m\pi/\psi$ [15]. The angle of incidence will be taken as $\phi_0 = \pi/3$. The interior angle of the wedge is equal to $\pi/6$ and observation distance from the origin is considered as $\rho = 6\lambda$, where λ is the wavelength.

Figure 2 shows the variation of the normalized electric field intensity versus the observation angle for the incident diffracted fields obtained from Eqs. (18) and (19). There is a deviation near $\phi = \pi$ because of the Hankel function (Debye asymptotic expansions of the Hankel functions are used in the integrals) in Eq. (18). The related function increases rapidly

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when its argument approaches to zero. This behavior affects the plot near the stated point. It is important to note that the asymptotic evaluation of Eq. (18) gives the exact asymptotic incident diffracted fields.

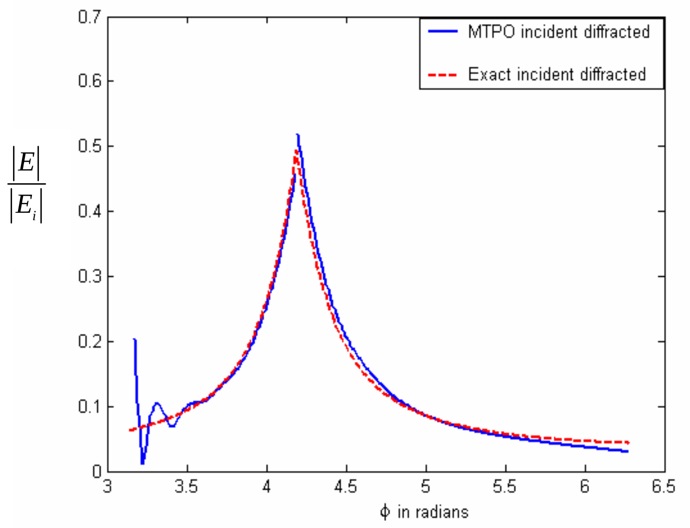

Fig. 2. Incident diffracted fields at the perfectly conducting wedge (MTPO and exact solution)

Figure 3 depicts the variation of the reflected diffracted fields at the wedge versus the observation angle. It is observed that the diffracted waves, obtained from Eqs. (17) and (19) are harmonious except the neighborhood of $\phi = 0$ and $\phi = 2\pi$. The argument of the Hankel function in Eq. (17) approaches to zero for these values of the observation angle.

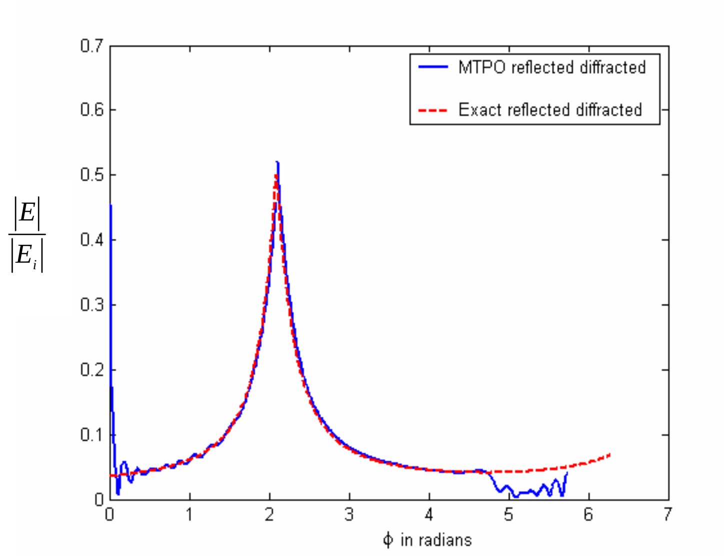

Fig. 3. Reflected diffracted fields at the perfectly conducting wedge (MTPO and exact solution)

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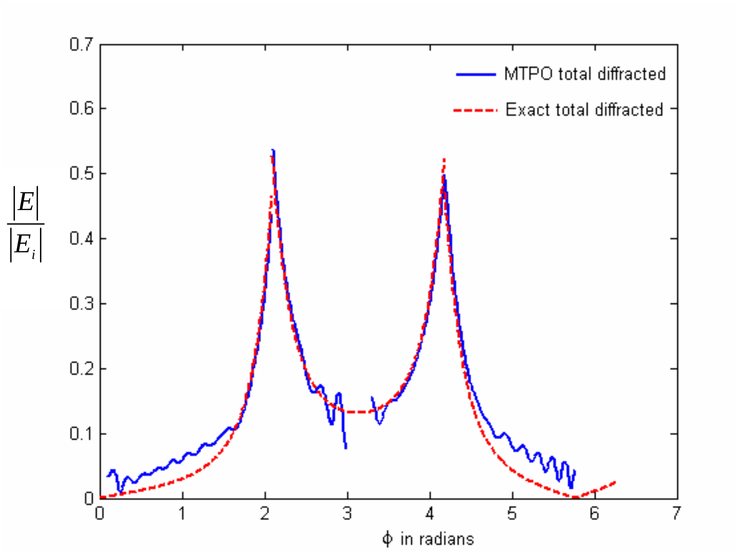

Fig. 4. Total diffracted fields at the perfectly conducting wedge (MTPO and exact solution)

In Fig. 4, the total diffracted field at a perfectly conducting wedge is compared for Eqs. (17), (18) and (19). It is seen that MTPO diffracted waves are harmonious especially in the transition regions (reflection and shadow boundaries) with the exact edge diffracted waves except for nearby of the observation angle values, given above.

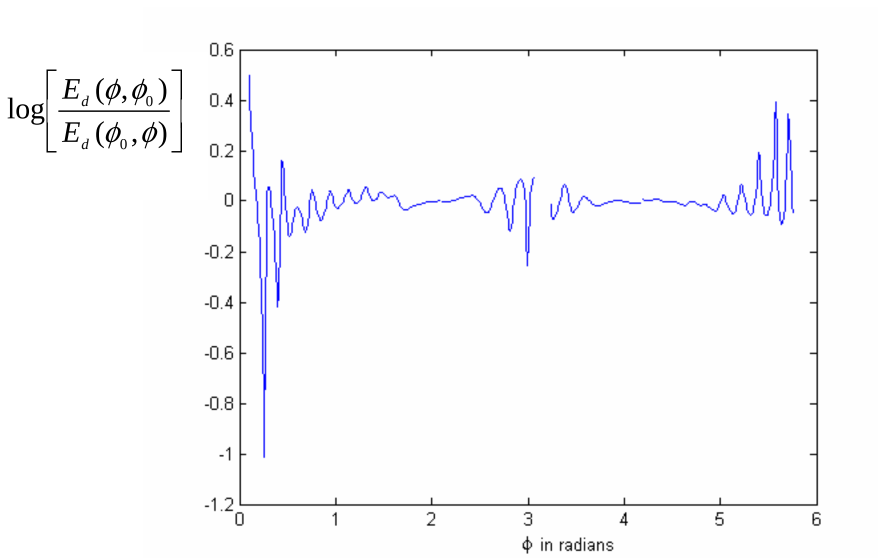

Fig. 5. Reciprocity check of the total diffracted MTPO integral

Reciprocity analysis of the MTPO integrals is given in Fig. 5. The total diffracted field is considered. According to the reciprocity theorem, the field must not be affected from the change of the places of the source and the observation point. The total diffracted field is divided its interchanged version as $|E_d(\phi,\phi_0)| \big/ |E_d(\phi_0,\phi)|$. The logarithm of the overall function is taken so that the expression will be equal to zero if the two functions are equal. The asymptotic evaluation of MTPO integrals gives the exact diffracted and Geometric Optic waves. For this reason, the only deviation in the reciprocity plot is a result of the discussed behavior of the integrals which also affects the graphics in Figs. 2, 3 and 4. It is important to note that the major deviations are observed in the neighborhoods of $\phi = 0$, $\phi = \pi$ and $\phi = 2\pi$.

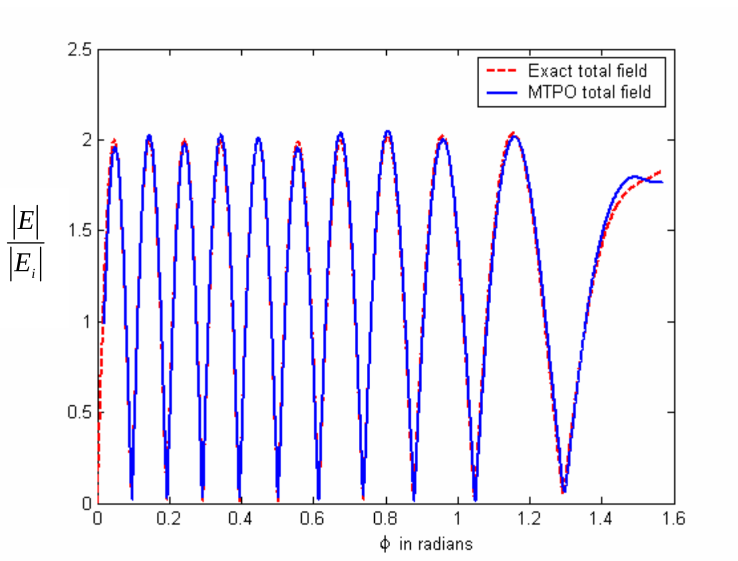

Fig. 6. Total scattered fields from a perfectly conducting wedge (MTPO and exact solution)

Figure 6 shows the variation of the exact and MTPO total scattered fields between 0 and 1.6 radians. It can be observed that the two plots are harmonious. The minima of the field are closer to 0.

4. Conclusion

In this work, the diffraction of an electric polarized plane wave from a perfectly conducting wedge has been examined. The MTPO integrals for an edge problem are considered by utilizing their weight functions, which represents the MTPO surface currents, as unknown functions. The surface currents are evaluated by equating exact wedge diffracted fields to the asymptotic edge expansion of MTPO integrals. The calculated current components are used in the MTPO integrals in order to determine the scattered fields. The numerical comparison of MTPO integral and exact solution for wedge problem shows that there are considerable deviations when ϕ approaches to 0, π and 2π, but at the other regions, the plots are harmonious. If the asymptotic evaluated fields of the MTPO integrals are used in the figures, there will be no deviation.