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ORIGINAL ARTICLE

A novel method to detect almost cyclostationary structure

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Abstract This paper is devoted to establish a computational approach to investigate that a discrete-time almost cyclostationary model is a suitable choice to fit on an observed dataset. The main idea is estimating the support of spectra and applying multiple testing. The simulated and real datasets are applied to study the performance of the introduced approach. The results confirm that the presented method acts efficiently in view of power study.

1. Introduction

Classical time series analysis is based on stationarity assumption. But in real world problems, many processes have periodic rhythms and consequently the theories and applications of stationary processes are not suitable for these processes. Periodically correlated and almost periodically correlated processers (PC and APC, in abbreviation), that may be respectively called cyclostationary and almost cyclostationary processes (CS and ACS, in abbreviation), are two candidates to fit on these rhythmic processes. Unlike the stationary processes, the mean, autocovariance (ACVF, in abbreviation) and autocovariance (ACF, in abbreviation) of CS and ACS are respectively periodic and almost periodic. The supports of spectral square $[0, 2\pi) \times [0, 2\pi)$ of ACS processes consist of the main diagonal and the parallel lines to it, i. e., $T_j(x) = x \pm \alpha_j, j = 1, 2, \cdots,$. The CS and ACS processes have been nicely considered by Gladyshev [7,8], Gardner [4], Hurd [10], Hurd and Gerr [11], Hurd and Leskow [12], Leskow and Weron [21], Gardner [5], Leskow [20], Lii and Rosenblatt [22,23], Gardner et al. [6], Hurd and Miamee [13], Lenart [14,15], Napolitano [30], Lenart [16], Lenart and Pipien [17,18], Mahmoudi et al. [29], Napolitano [31,32], Mahmoudi and Maleki [28], Nematollahi et al.

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[33], Lenart and Pipien [19], Mahmoudi et al. [24,25,27], and Mahmoudi et al. [26].

Although there are huge studies about the ACS processes, but the goodness of fit test problem about these processes has been less considered in literature. Mahmoudi et al. [26] introduced an approach to address that a known ACS time series is a good candidate to fit on an observed time series. They used the periodogram, as an estimator of the spectral density of ACS processes, to estimate the supports of observed datasets. The given approach was based on the linear regression to find the slopes and the constants of the support lines. The aim of this paper is to establish a computational approach to investigate that a discrete-time ACS model is a suitable choice to fit on an observed dataset. We will introduce a consistent estimator for the *joint spectral coherency*. Then, by using this estimator, we find the support lines for observed dataset. Finally, the appropriateness of ACS model is investigated by using multiple tests strategy. The given approach is more powerful than the periodogram's asymptotic distribution (PAD, in abbreviation) technique given by Mahmoudi et al. [26].

The rest of the paper is organized as follows. In Section 2, a new test to detect ACS time series structure is established. The performance of the introduced approach is also studied using simulation study and real data analysis, in Section 3.

2. Spectral support estimation (SSE, in abbreviation) method

A second order process $\{X_t : t \in Z\}$ is an ACS process if its mean and ACVF, i.e., $\mu(t) = E(X_t)$, and $B(t,\tau) = cov(X_t, X_{t+\tau})$, are almost periodic at t, for every integer τ.

Assume that the following assumptions are satisfied.

(A1) $\{X_t : t \in Z\}$ is a zero-mean and real-valued time series.
(A2) X_t is an ACS time series.

Corduneanu [2] and Hurd [10] proved that

$$B(t,\tau) \sum_{\omega \in W_t} a(\omega,\tau) e^{i\omega t},$$

where

$$a(\omega,\tau) = \lim_{n\to\infty}\left(\frac{1}{n}\sum_{j=1}^{n} B(j,\tau)e^{-i\omega t}\right),$$

and the set $W_\tau = \{\omega \in [0,2\pi) : a(\omega,\tau) \neq 0\}$ is a countable set of frequencies.

(A3) $W = \bigcup_{\tau\in Z} W_\tau$, is a finite set.

Consequently,

$$B(t,\tau) = \sum_{\omega\in W} a(\omega,\tau)e^{i\omega t},$$

and the set

$$S = \bigcup_{\omega\in W}\left\{(\nu,\gamma) \in [0,2\pi)^2 : \gamma = \nu - \omega\right\},$$

Supports the spectral measure of X_t.

Moreover, the coefficients

$$a(\omega,\tau) = \int_0^{2\pi} e^{i\xi\tau} r_\omega(d\xi),$$

are the Fourier transforms of the measures $r_\omega(\cdot)$.

We note that the $r_\omega$will be identified if the spectral measure of X_t be restricted on the line $\gamma = \nu - \omega$, modulo 2π, where $\omega \in W$.

(A4) The measure r_0 is absolutely continuous with respect to the Lebesgue measure.

Dehay and Hurd [3] showed by considering this assumption and $\sum_{\tau=-\infty}^{\infty} |a(\omega,\tau)| < \infty$, for any $\omega \in W$, result in a spectral density function $f_\omega(\cdot)$ exists such that

$$f_\omega(\nu) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty} a(\omega,\tau)e^{-i\nu\tau}.$$

Therefore, an ACS time series X_t can be represented by

$$X_t = \int_0^{2\pi} e^{-itx}\zeta(dx), t \in \mathbb{Z},$$

where ζ is a random spectral measure on $[0,2\pi)$ such that $E\left(\zeta(d\theta)\overline{\zeta(d\theta')}\right) = 0, \left(\theta,\theta'\right) \notin S.$

The spectral distribution matrix and the spectral density matrix of ζ, are respectively presented by

$$\boldsymbol{F}(d\lambda) = \left[F_{k,j}(d\lambda)\right]_{j,k=1,\cdots,m},$$

and

$$\boldsymbol{f}(\lambda) = \frac{d\boldsymbol{F}}{d\lambda} = \left[f_{k,j}(\lambda)\right]_{j,k=1,\cdots,m},$$

where

$$F_{k,j}(d\lambda) = E\left(\zeta(d\lambda+\alpha_k)\overline{\zeta(d\lambda+\alpha_j)}\right), k,j = 1,\cdots,m,$$

and $f_{k,j}$ is the corresponding spectral density of $F_{k,j}$.

Let $\{X_t, t \in \mathbb{Z}\}$ be ACS with spectral density $\mathbf{f}(\lambda), \lambda \in [0,2\pi)$. The supports of the spectra for ACS processes are the lines $T_j\left(T_k^{-1}(x)\right)$, where $T_j(x) : B_1 \to B_j$, is defined by $T_j(x) = x + \alpha_j$, for $j = 1,\cdots,m$. For an observed time series $X_0,\cdots,X_{N-1}$, of $\{X_t : t \in Z\}$, assume $d_X(\lambda)$ and $I_X(\lambda)$ are respectively the discrete Fourier transform (DFT, in abbreviation) and periodogram of the this observed time series defined by

$$d_X(\lambda) = N^{-1/2}\sum_{t=0}^{N-1} X_t e^{-it\lambda}, \lambda \in [0,2\pi),$$

and

$$I_X(\lambda) = |d_X(\lambda)|^2, \lambda \in [0,2\pi).$$

Lenart [16], Lenart and Pipien [17,19] and Mahmoudi et al. [24] have been studied the distribution of DFT and periodogram for the ACS time series.

Since X_t is harmonizable, $\zeta(d\lambda)$ can be well estimated by $d_X(\lambda)$.Also regarding to ACS property, $E\left(\zeta(d\lambda)\overline{\zeta(d\lambda')}\right) = 0$, for $\lambda' \neq T_j(\lambda), \lambda \in B_1, j = 1,\cdots,m.$

Subsequently, the *joint spectral coherency* can be defined as follows:

$$C\left(\lambda,\lambda'\right)=\frac{\left|f\left(\lambda,\lambda'\right)\right|}{\sqrt{\left|f(\lambda,\lambda)f\left(\lambda',\lambda'\right)\right|}},\lambda,\lambda'\in[0,2\pi),$$

where $f\left(\lambda,\lambda'\right)=E\left(\xi(d\lambda)\overline{\xi\left(d\lambda'\right)}\right)$.

Hurd and Gerr [11] introduced the joint spectral coherency for PC time series and called it *coherence statistic*.

In addition, an estimation of the joint spectral coherency $C\left(\lambda,\lambda'\right)$ can be defined as

$$\widehat{C}\left(\lambda,\lambda'\right)=Correlation\left(|d_X(\lambda)|,\left|d_X\left(\lambda'\right)\right|\right).$$

Mahmoudi et al. [24] showed that for ACS processes, $C\left(\lambda,\lambda'\right)$, is equal to zero, for $\lambda'\neq T_j(\lambda),\lambda\in B_1,j=1,\cdots,m$.

In practical situations, the bootstrap estimation methods will be used to generate more samples from the DFT. For$\lambda\in B_1$, the estimation for $T_j(\lambda)$ is $\lambda_j^*\in B_j$ such that $\widehat{C}\left(\lambda,\lambda'\right)$ reaches its maximum on $B_1\times B_j$ at $\left(\lambda,\lambda'\right)$.

The summary of estimation procedure of T_j's is as follows:

(i) To produce a sample of $d_X(\lambda)$, using $X_0,\cdots,X_{N-1}$, the moving block bootstrap (MBB) methodology with parameters $M=20,n=100$ and $\alpha=0.05$ [37] is applied for given $\lambda\in[0,2\pi)$.

(ii) $\widehat{C}\left(\lambda,\lambda'\right)$ is calculated for $\lambda\in B_1$ and $\lambda'\in B_j,j=1,\cdots,T$,using n samples of DFT in λ and $\lambda',\{d_1(\lambda),\cdots,d_n(\lambda)\}$ and $\left\{d_1\left(\lambda'\right),\cdots,d_n\left(\lambda'\right)\right\}$.

(iii) $\lambda\in B_1$ is fixed to obtain $\lambda^*\in B_j$ such that (λ,λ^*) maximizes $\widehat{C}\left(\lambda,\lambda'\right)$.

(iv) Repeat Step (iii) until finding $\lambda_1^*,\cdots,\lambda_J^*\in B_j$ corresponding to $\lambda_1,\cdots,\lambda_J\in B_1$.

(v) Assign $\lambda_k^*=\widehat{T}_j(\lambda_k),k=1,\cdots,m;$ which estimate $T_j,j=1,\cdots,m$.

Theorem 2.1.. *$\widehat{T}_j$ is consistent estimator for T_j.*

Proof.. This is the consequence of the following facts. Since the functions $T_j,j=1,\cdots,m$ are monotone and $\widehat{C}\xrightarrow{p}C$, then the proof is completed by using Theorem 20.5 in Bilingsley [38]. It should be noted that the convergence of $\widehat{C}$ to C is in the sense that each element of $\widehat{C}\left(\widehat{C}_{ij},i,j=1,\cdots,m\right)$ is converged to the corresponding element of $C(C_{ij},i,j=1,\cdots,m)$.□

The desired null test is$H_0:X_t$ is ACS with spectra on the lines $T_j(x)=x+\alpha_j$, for $j=1,\cdots,m$. To investigate that whether ACS model is a suitable choice to fit, the applicable technique is implementing the multiple testing. Assume the spectra of X_t is supported on the lines $T_j(x)=\beta_jx+\gamma_j,j=1,\cdots,m$. As can be seen, the hypothesis test H_0 can be rewritten by the multiple hypothesis tests

$H_0':\ \gamma_j=\alpha_j,\ and\ \beta_j=1$, for $j=1,\cdots,m$.

Case I. The cycle m is known

To carry out these multiple tests, Benjamini and Hochberg [1], Holm [9], Storey [34], Storey and Tibshirani [35], Strimmer [36] gave several multiple testing techniques such as the Bonferroni correction, locally false discovery rate (LFDR) and qvalue methods.

Case II. The cycle m is unknown

In this case, the number of cycle m can be estimated using the *ciphered spectral support* (CSS) graph that will be applied to visualize the spectral coherency. Note that, the points $\left(\lambda,\lambda'\right)\in[0,2\pi)^2$ are highlighted by the values of $\widehat{C}\left(\lambda,\lambda'\right)$.

Now, we can continue as the case I, by replacing m by $\widehat{m}$.

3. Numerical results

This section is divided into two subsections. Section 3.1 is related to simulation study including two examples. A real example is also presented in Section 3.2.

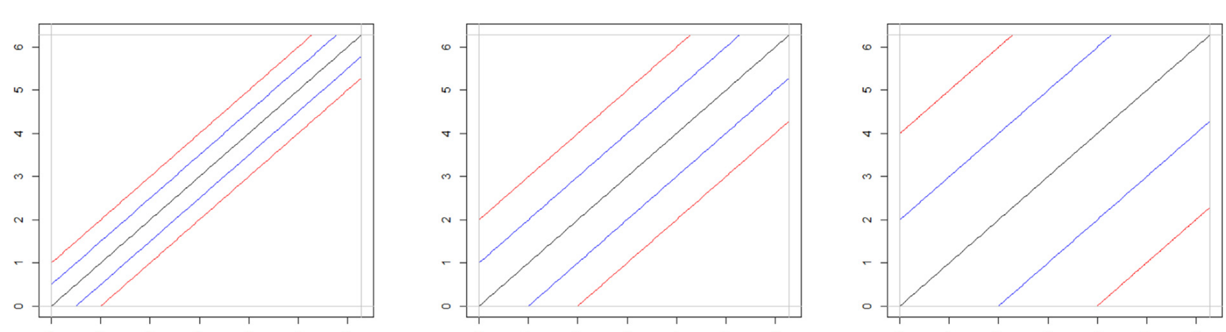

Fig. 1 Spectral square, Left: $\omega=0.5$, Middle: $\omega=1$, and Right: $\omega=2$.

3.1. Simulation study

In this subsection, two examples are provided to study the ability of the introduced approach. For Example 1, at first, the simulation results of using the SSE method in spectral support estimation will be demonstrated. Then, for two examples, the power and the accuracy of proposed method are evaluated and compared with the PAD method that recently has been given by Mahmoudi et al. [26]. All simulations and computations are done by using the R 3.3.2. software with 1000 Monte Carlo runs.

Example 1. To study the performance of SSE method in estimating spectral functions $T_j, j = 1, \cdots, m$, different datasets are generated from the process

$$X_t = (1 + \cos(\omega t))Y_t, \omega \in (0, \infty),$$

such that

$$Y_t = 0.6Y_{t-1} - 0.5Y_{t-2} + Z_t,$$

and Z_t is a sequence of IIDN(0,1).

The spectral square of the considered X_t is supported on the lines

Table 1 Spectral support estimation results based SSE method.

ω	*Function*	*Frequency*	*N*			
			100	200	500	1000
0.5	T_1	$\lambda = \pi/4$	0.0044127	0.0039825	0.0028534	0.0014745
		$\lambda = \pi/2$	0.0041647	0.0035717	0.0021368	0.0019862
		$\lambda = 3\pi/4$	0.0049873	0.0030577	0.0021157	0.0013917
	T_2	$\lambda = \pi/4$	0.0041569	0.0039011	0.0028754	0.0010811
		$\lambda = \pi/2$	0.0041304	0.0034011	0.0023706	0.0019217
		$\lambda = 3\pi/4$	0.0041239	0.0036951	0.0028773	0.0012788
	T_3	$\lambda = \pi/4$	0.0046276	0.0036574	0.0023565	0.0012506
		$\lambda = \pi/2$	0.0048639	0.0034882	0.0024698	0.0017080
		$\lambda = 3\pi/4$	0.0041064	0.0038932	0.0020663	0.0016104
	T_4	$\lambda = \pi/4$	0.0046414	0.0039538	0.0025734	0.0018110
		$\lambda = \pi/2$	0.0041536	0.0038392	0.0024133	0.0013460
		$\lambda = 3\pi/4$	0.0044393	0.0032191	0.0023127	0.0014094
	T_5	$\lambda = \pi/4$	0.0048565	0.0030079	0.0024747	0.0013522
		$\lambda = \pi/2$	0.0042371	0.0031573	0.0025445	0.0018460
		$\lambda = 3\pi/4$	0.0049640	0.0035949	0.0028554	0.0014100
1	T_1	$\lambda = \pi/4$	0.0040159	0.0036779	0.0029991	0.0018807
		$\lambda = \pi/2$	0.0049093	0.0039136	0.0021341	0.0018746
		$\lambda = 3\pi/4$	0.0047868	0.0032730	0.0028301	0.0018643
	T_2	$\lambda = \pi/4$	0.0041872	0.0039824	0.0024475	0.0018823
		$\lambda = \pi/2$	0.0046542	0.0037860	0.0022851	0.0019406
		$\lambda = 3\pi/4$	0.0043848	0.0036136	0.0020596	0.0017315
	T_3	$\lambda = \pi/4$	0.0045563	0.0037864	0.0022836	0.0014196
		$\lambda = \pi/2$	0.0047335	0.0032964	0.0023455	0.0019098
		$\lambda = 3\pi/4$	0.0042127	0.0032329	0.0020641	0.0017861
	T_4	$\lambda = \pi/4$	0.0041624	0.0035495	0.0027126	0.0014757
		$\lambda = \pi/2$	0.0044934	0.0033073	0.0027398	0.0018732
		$\lambda = 3\pi/4$	0.0041748	0.0039776	0.0024566	0.0019231
	T_5	$\lambda = \pi/4$	0.0049482	0.0031503	0.0022185	0.0012939
		$\lambda = \pi/2$	0.0043922	0.0034259	0.0024945	0.0014217
		$\lambda = 3\pi/4$	0.0048948	0.0035623	0.0020412	0.0011188
2	T_1	$\lambda = \pi/4$	0.0047747	0.0030926	0.0027006	0.0012993
		$\lambda = \pi/2$	0.0044495	0.0035987	0.0021597	0.0011200
		$\lambda = 3\pi/4$	0.0044015	0.0035181	0.0022819	0.0016758
	T_2	$\lambda = \pi/4$	0.0044358	0.0037967	0.0029258	0.0015211
		$\lambda = \pi/2$	0.0049125	0.0039502	0.0020677	0.0013870
		$\lambda = 3\pi/4$	0.0046873	0.0038691	0.0020289	0.0018347
	T_3	$\lambda = \pi/4$	0.0045067	0.0037562	0.0027675	0.0017960
		$\lambda = \pi/2$	0.0046009	0.0035017	0.0021854	0.0018104
		$\lambda = 3\pi/4$	0.0045294	0.0033292	0.0024886	0.0018167
	T_4	$\lambda = \pi/4$	0.0042603	0.0034264	0.0020339	0.0019253
		$\lambda = \pi/2$	0.0046171	0.0030007	0.0024337	0.0011026
		$\lambda = 3\pi/4$	0.0041465	0.0036582	0.0027616	0.0012120
	T_5	$\lambda = \pi/4$	0.0044882	0.0031847	0.0028723	0.0013846
		$\lambda = \pi/2$	0.0045907	0.0038719	0.0028218	0.0011306
		$\lambda = 3\pi/4$	0.0043067	0.0033312	0.0025002	0.0015539

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Table 2 Power and size of SSE and PAD methods for Example 1.

Hypothesis		Method	N			
H_0	H_1		100	200	500	1000
$\omega = 0.5$	$\omega = 0.5$	Proposed Method	0.051	0.050	0.050	0.049
		PAD	0.051	0.051	0.050	0.049
	$\omega = 1$	Proposed Method	0.891	0.874	0.986	0.891
		PAD	0.797	0.812	0.907	0.976
	$\omega = 2$	Proposed Method	0.842	0.906	1.000	1.000
		PAD	0.763	0.854	0.950	0.979
$\omega = 1$	$\omega = 0.5$	Proposed Method	0.841	0.870	0.977	0.998
		PAD	0.755	0.817	0.903	0.993
	$\omega = 1$	Proposed Method	0.051	0.050	0.050	0.049
		PAD	0.052	0.050	0.050	0.050
	$\omega = 2$	Proposed Method	0.844	0.879	0.959	0.997
		PAD	0.776	0.809	0.905	0.990
$\omega = 2$	$\omega = 0.5$	Proposed Method	0.831	0.949	1.000	1.000
		PAD	0.766	0.875	0.966	0.987
	$\omega = 1$	Proposed Method	0.883	0.976	1.000	1.000
		PAD	0.793	0.883	0.957	0.994
	$\omega = 2$	Proposed Method	0.051	0.050	0.049	0.049
		PAD	0.051	0.050	0.049	0.048

Table 3 Power and size of SSE and PAD methods for Example 2.

Hypothesis		Method	N			
H_0	H_1		100	200	500	1000
$(\omega_1, \omega_2) = (0.25, 0.5)$	$(\omega_1, \omega_2) = (0.25, 0.5)$	Proposed Method	0.051	0.050	0.050	0.049
		PAD	0.052	0.051	0.051	0.049
	$(\omega_1, \omega_2) = (0.75, 1)$	Proposed Method	0.895	0.889	1.000	1.000
		PAD	0.802	0.834	0.954	0.973
	$(\omega_1, \omega_2) = (1.5, 2)$	Proposed Method	0.925	0.966	1.000	1.000
		PAD	0.831	0.889	0.961	0.982
$(\omega_1, \omega_2) = (0.75, 1)$	$(\omega_1, \omega_2) = (0.25, 0.5)$	Proposed Method	0.893	0.911	0.993	1.000
		PAD	0.801	0.853	0.912	0.973
	$(\omega_1, \omega_2) = (0.75, 1)$	Proposed Method	0.051	0.050	0.049	0.049
		PAD	0.051	0.051	0.050	0.049
	$(\omega_1, \omega_2) = (1.5, 2)$	Proposed Method	0.880	0.952	0.976	0.999
		PAD	0.828	0.875	0.926	0.992
$(\omega_1, \omega_2) = (1.5, 2)$	$(\omega_1, \omega_2) = (0.25, 0.5)$	Proposed Method	0.882	0.952	1.000	1.000
		PAD	0.813	0.881	0.952	0.987
	$(\omega_1, \omega_2) = (0.75, 1)$	Proposed Method	0.882	0.930	1.000	1.000
		PAD	0.804	0.869	0.951	0.991
	$(\omega_1, \omega_2) = (1.5, 2)$	Proposed Method	0.051	0.050	0.049	0.049
		PAD	0.052	0.051	0.049	0.049

$$T_1(x) = x, T_2(x) = x + \omega, T_3(x) = x - \omega, T_4(x) = x - 2\omega, T_5(x) = x + 2\omega.$$

Fig. 1 shows the spectral square$[0, 2\pi)^2$, for $\omega = \{0.5, 1, 2\}$.

The results of the estimation procedure are shown in Table 1. The columns show the mean square error (MSE) of $\widehat{T}_j$ in different frequencies $\frac{\pi}{4}, \frac{\pi}{2}$, and $\frac{3\pi}{4}$, for different values of ω and N. It can be concluded the values of MSE are approaching zero. In other words, the proposed method can estimate accurately, especially as the value of N is increasing. The 1th, 9th and 17th rows of Table 2 report the estimated probabilities of the type 1 error (Reject H_0 when H_0 is true) of the SSE method for $\omega = 0.5, 1$ and 2, respectively. The estimated probabilities of the type 1 error for the PAD method are also summarized in the 2th, 10th and 18th rows of Table 2. The powers (Reject H_0 when H_1 is true) of two methods are also reported on other rows. The probability of the type 1 error and the power are estimated by

$$\frac{\textit{The number of Monte Carlo replicates for which the H0 is rejected}}{1000}.$$

Table 4 Multiple testing technique to detect ACS structure for the first difference of centered moving average filter 2 × 12 MA applied for logarithm of IPI in Poland (2005 = 100%) since January 1995 until December 2009.

	SSE Estimation Results			Multiple Testing Method			
	Coefficient	Standard Error	t Value	P-Value	BPV	q-value	LFDR
α_1	0.0612	0.0017	−0.4584	0.647	1.000	0.916	1.000
α_2	0.1509	0.0017	−1.2588	0.208	0.624	0.779	1.000
α_3	0.2599	0.0034	0.5503	0.582	1.000	0.908	1.000

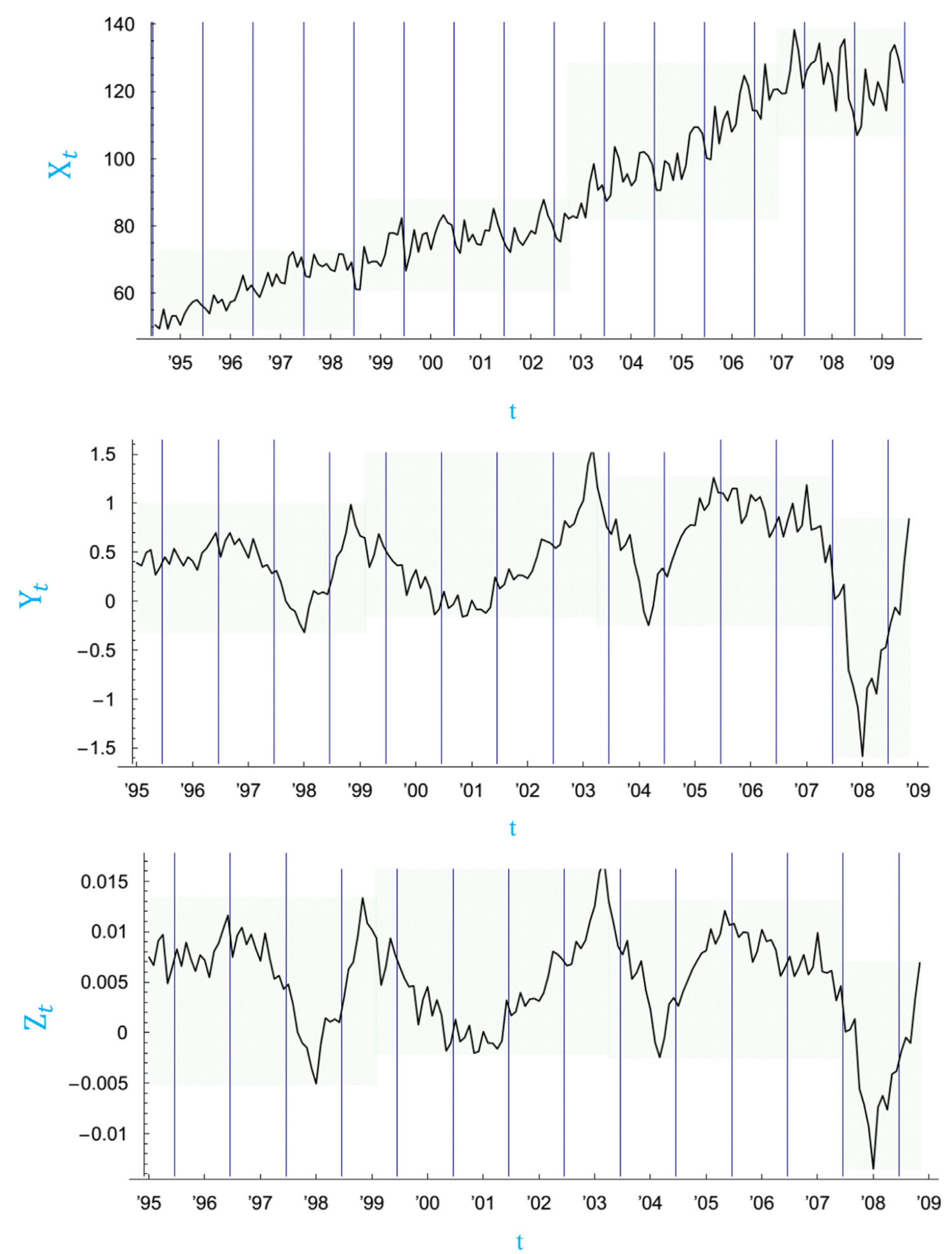

Fig. 2 The IPI (X_t), the first difference of the centered 2 × 12 MA filter applied on the IPI (Y_t) and the first difference of the centered 2 × 12 MA filter applied on the logarithm of the IPI (Z_t) in Poland since January 1995 until December 2009.

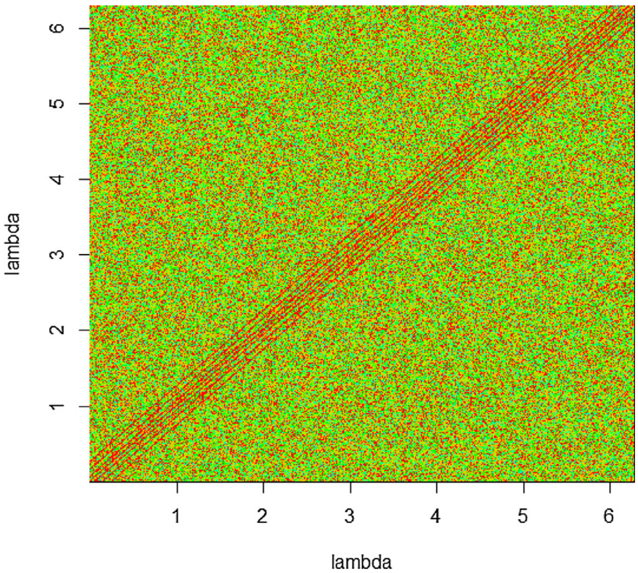

Fig. 3 CSS graph for the first difference of centered moving average filter 2 × 12 MA applied for logarithm of IPI in Poland (2005 = 100%) since January 1995 until December 2009.

The results indicated that the estimated probabilities of the type 1 error for two methods are very close to the desired level (0.05), especially as N is increasing. As can be seen, the values of the proposed method are less than PAD method. In other words, the proposed method is more accurate to control the desired level. On the other hand, the power values of two methods verify that the SSE method is efficiently more able to discriminate H_0 from H_1. Therefore, the introduced approach is more efficient than comparative method.

Example 2. Consider the process

$$X_t = \cos(\omega_1 t) Y_{1(t)} + \cos(\omega_2 t) Y_{2(t)}, \omega_1, \omega_2 \in (0, \infty),$$

Such that Y_{1t} and Y_{2t} are two uncorrelated MA(1) time series

$$Y_{1(t)} = 0.5 Y_{1(t-1)} + W_t,$$

$$Y_{2t} = 0.3 Y_{2(t-1)} + Z_t,$$

And the processes W_t and Z_t are two uncorrelated standard normal noises.

The 1th, 9th and 17th rows of Table 3 report the estimated probabilities of the type 1 error (Reject H_0 when H_0 is true) of the SSE method for $\omega = 0.5, 1$ and 2, respectively. The estimated probabilities of the type 1 error for the PAD method are also summarized in the 2th, 10th and 18th rows of Table 3. The powers (Reject H_0 when H_1 is true) of two methods are also reported on other rows. The results indicated that the estimated probabilities of the type 1 error for two methods are very close to the desired level (0.05), especially as N is increasing. As can be seen, the values of the proposed method are less than PAD method. In other words, the proposed method is more accurate to control the desired level. On the other hand, the power values of two methods verify that the SSE method is efficiently more able to discriminate H_0 from H_1. Therefore, the introduced approach is more efficient than comparative method. The SSE method is powerful than the PAD method, because in estimating the spectral functions $T_j, j = 1, \cdots, m$, PAD method uses the asymptotic distribution of DFT and then regression analysis; But SSE method acts straightly by using the estimation of the joint spectral coherency $C\left(\lambda, \lambda'\right)$(see Table 4).

3.2. Real data

Now, we illustrate a real example to show the ability of the SSE method in the real world applications. The dataset includes the first difference of centered moving average filter 2 × 12 moving average (MA) applied for logarithm of industrial production index (IPI) in Poland (2005 = 100%) since January 1995 until December 2009, Lenart and Pipien [18]. Fig. 2 shows the IPI, the first difference of centered moving average filter 2 × 12 MA applied for IPI and the first difference of centered moving average filter 2 × 12 MA applied for logarithm of IPI, respectively. Lenart and Pipien [18] detected an ACS time series with spectra on the lines $T_j(x) = x \pm \alpha, \alpha \in \{0.062, 0.153, 0.258\}$. Fig. 3 also shows the CSS graph. This graph also reveals that the considered ACS time series by Lenart and Pipien [18] can be a good choice to fit dataset. Then, the desired null hypothesis is $H_0 : X_t$ is ACS with spectra on the lines $T_j(x) = x \pm \alpha, \alpha \in \{0.062, 0.153, 0.258\}$.We illustrate P-Values, Bonferroni correction P-Values (BPV), qvalues and LFDRs in Table 3. Note that all of these values are more than 0.05. Hence through these methods, the null hypothesis cannot be rejected. This result is consistent with the model building Lenart and Pipien [18].

Declaration of Competing Interest

The authors have no conflict of interest.

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