

Research Article

Maxwell's Equations on Cantor Sets: A Local Fractional Approach

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Maxwell's equations on Cantor sets are derived from the local fractional vector calculus. It is shown that Maxwell's equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. Local fractional differential forms of Maxwell's equations on Cantor sets in the Cantorian and Cantor-type cylindrical coordinates are obtained. Maxwell's equations on Cantor set with local fractional operators are the first step towards a unified theory of Maxwell's equations for the dynamics of cold dark matter.

1. Introduction

Nondifferentiability, complexity, and similarity represent the basic properties of the nature. Fractals [1] are the basic characteristics of nature, which are that fractal geometry of substances generalizes to noninteger dimensions. Microphysics reveals the fractal behaviors of matter distribution in the universe [2] and soft materials [3].

Fractal time was used to describe the transport of charges and defects in the condensed matter [4]. In fractal space-time, the geometric analogue of relativistic quantum mechanics was presented in [5–8]. In fractal-Cantorian space-time $\Omega^{4\alpha} \subset \Omega^4$ [9, 10], the unified field theory, quantum physics, cosmology, and chaotic systems were discussed in [11–13].

Based on the fractal distribution of charged particles, the electric and magnetic fields in time-space $\Omega^{3\alpha+1} \subset \Omega^4$ were developed in [14] and fractional Maxwell's equations were proposed in [15]. In [16, 17], the concept of static fractional

electric potential was developed. Recently, based on the Hausdorff derivative, fractal continuum electrodynamics in time-space $\Omega^{4\alpha} \subset \Omega^4$ was proposed [18]. The fractional differential form of Maxwell's equations on fractal sets was suggested in [19]. In [20], the Maxwell equations of fractional electrodynamics in time-space $\Omega^{3+\alpha} \subset \Omega^4$ were considered. The Maxwell equations on anisotropic fractal media in time-space $\Omega^{3\alpha+1} \subset \Omega^4$ were developed in [21].

The local fractional calculus theory [22, 23] was applied to model some dynamics systems with nondifferentiable characteristics. In [22–26], the heat-conduction equation on Cantor sets was considered. In [27], the Navier-Stokes equations on Cantor sets based on local fractional vector calculus were proposed. Helmholtz and diffusion equations via local fractional vector calculus were reported in [28]. The Fokker-Planck equation with local fractional space derivative was suggested in [29]. In [30], Lagrangian and Hamiltonian

mechanics with local fractional space derivative was presented. The measuring structures of time in fractal, fractional, classical, and discrete electrodynamics are shown in Figure 1.

The aim of this paper is to structure Maxwell's equations on Cantor sets from the local fractional calculus theory [23, 27, 28] point of view. This paper is structured as follows. In Section 2, we introduce the basic definitions and theorems for local fractional vector calculus. In Section 3, Maxwell's equations on Cantor sets in the local fractional vector integral form are presented. Maxwell's equations on Cantor sets in the Cantorian and Cantor-type cylindrical coordinates are given in Section 4. Finally, Section 5 is devoted to conclusions.

2. Fundamentals

In this section, we recall the basic definitions and theorems for local fractional vector calculus, which are used throughout the paper.

Local fractional gradient of the scale function ϕ is defined as [23, 27]

$$\nabla^\alpha \phi = \lim_{dV^{(\gamma)} \rightarrow 0} \left(\frac{1}{dV^{(\gamma)}} \oint_{S^{(\beta)}} \phi dS^{(\beta)} \right), \quad (1)$$

where $S^{(\beta)}$ is its bounding fractal surface, V is a small fractal volume enclosing P , and the local fractional surface integral is given by [23, 27, 28]

$$\iint u(r_P) dS^{(\beta)} = \lim_{N \rightarrow \infty} \sum_{P=1}^N u(r_P) \mathbf{n}_P \Delta S_P^{(\beta)}, \quad (2)$$

with N elements of area with a unit normal local fractional vector \mathbf{n}_P , $\Delta S_P^{(\beta)} \rightarrow 0$ as $N \rightarrow \infty$ for $\gamma = (3/2)\beta = 3\alpha$, and ∇^α is denoted as the local fractional Laplace operator [22, 23].

The local fractional divergence of the vector function \mathbf{u} is defined through [23]

$$\nabla^\alpha \cdot \mathbf{u} = \lim_{dV^{(\gamma)} \rightarrow 0} \left(\frac{1}{dV^{(\gamma)}} \oint_{S^{(2\alpha)}} \mathbf{u} \cdot dS^{(\beta)} \right), \quad (3)$$

where the local fractional surface integral is suggested by [23, 27, 28]

$$\iint \mathbf{u}(r_P) \cdot dS^{(\beta)} = \lim_{N \rightarrow \infty} \sum_{P=1}^N \mathbf{u}(r_P) \cdot \mathbf{n}_P \Delta S_P^{(\beta)}, \quad (4)$$

with N elements of area with a unit normal local fractional vector \mathbf{n}_P , $\Delta S_P^{(\beta)} \rightarrow 0$ as $N \rightarrow \infty$ for $\gamma = (3/2)\beta = 3\alpha$.

The local fractional curl of the vector function \mathbf{u} [23] is defined as follows:

$$\nabla^\alpha \times \mathbf{u} = \lim_{dS^{(\beta)} \rightarrow 0} \left(\frac{1}{dS^{(\beta)}} \oint_{l^{(\alpha)}} \mathbf{u} \cdot d\mathbf{l}^{(\alpha)} \right) \mathbf{n}_P, \quad (5)$$

where the local fractional line integral of the function \mathbf{u} along a fractal line $l^{(\alpha)}$ is given by [23]

$$\begin{aligned} & \int_{l^{(\alpha)}} \mathbf{u}(x_P, y_P, z_P) \cdot d\mathbf{l}^{(\alpha)} \\ &= \lim_{N \rightarrow \infty} \sum_{P=1}^N \mathbf{u}(x_P, y_P, z_P) \cdot \Delta \mathbf{l}_P^{(\alpha)}, \end{aligned} \quad (6)$$

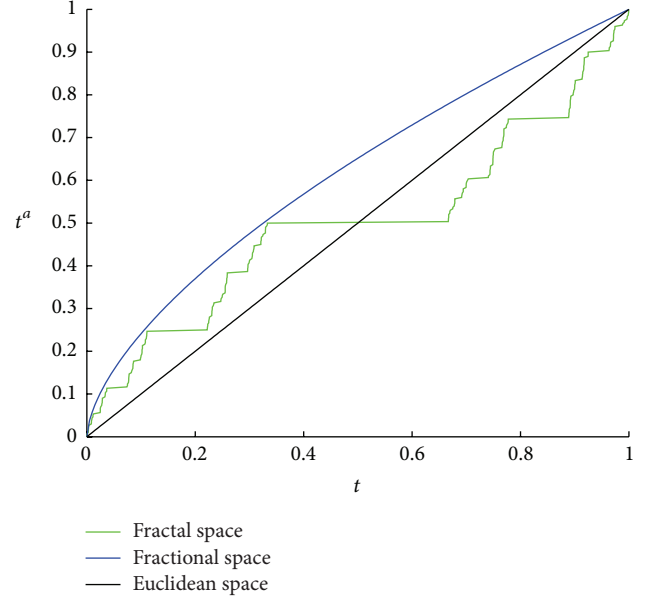


FIGURE 1: Graph for comparison of the measuring structures of time in fractal, fractional, and classical electrodynamics.

with the elements of line $\Delta \mathbf{l}_P^{(\alpha)}$ requiring that all $|\Delta \mathbf{l}_P^{(\alpha)}| \rightarrow 0$ as $N \rightarrow \infty$ and $\beta = 2\alpha$.

The local fractional Gauss theorem of the fractal vector field states that [23, 27]

$$\iiint_{V^{(\gamma)}} \nabla^\alpha \cdot \mathbf{u} dV^{(\gamma)} = \oint_{S^{(\beta)}} \mathbf{u} \cdot dS^{(\beta)}, \quad (7)$$

where the local fractional volume integral of the function \mathbf{u} is written as [23]

$$\iiint \mathbf{u}(r_P) dV^{(\gamma)} = \lim_{N \rightarrow \infty} \sum_{P=1}^N \mathbf{u}(r_P) \Delta V_P^{(\gamma)}, \quad (8)$$

with the elements of volume $\Delta V_P^{(\gamma)} \rightarrow 0$ as $N \rightarrow \infty$ and $\gamma = (3/2)\beta = 3\alpha$.

The local fractional Stokes theorem of the fractal field states that [23, 27]

$$\oint_{l^{(\alpha)}} \mathbf{u} \cdot d\mathbf{l}^{(\alpha)} = \iint_{S^{(\beta)}} (\nabla^\alpha \times \mathbf{u}) \cdot dS^{(\beta)}. \quad (9)$$

The Reynolds transport theorem of the local fractional vector field states that [27]

$$\begin{aligned} & \frac{D^\alpha}{Dt^\alpha} \iiint_{V^{(\gamma)}} F(x, t) dV^{(\gamma)} \\ &= \iiint_{V^{(\gamma)}} \frac{\partial^\alpha}{\partial t^\alpha} F(x, t) dV^{(\gamma)} + \oint_{S^{(\beta)}} F(x, t) \mathbf{v} \cdot dS^{(\beta)}, \end{aligned} \quad (10)$$

where \mathbf{v} is the fractal fluid velocity.

3. Local Fractional Integral Forms of Maxwell's Equations on Cantor Sets

According to fractional complex transform method [31], the fact that the classical differential equations always transform

into the local fractional differential equations leads to the idea of yielding Maxwell's equations on Cantor sets using the local fractional vector calculus.

3.1. Charge Conservations in Local Fractional Field. Let us consider the total charge, which is described as follows:

$$Q = \iiint_{V^{(\gamma)}} \rho(r, t) dV^{(\gamma)}, \quad (11)$$

and the total electrical current is as follows

$$I = \iint_{S^{(\beta)}} J(r, t) \cdot d\mathbf{S}^{(\beta)}, \quad (12)$$

where $\rho(r, t)$ is the fractal electric charge density and $J(r, t)$ is the fractal electric current density.

The Reynolds transport theorem in the fractal field gives

$$\begin{aligned} \frac{D^\alpha Q}{Dt^\alpha} &= \frac{D^\alpha}{Dt^\alpha} \iiint_{V^{(\gamma)}} \rho(r, t) dV^{(\gamma)} \\ &= \iiint_{V^{(\gamma)}} \frac{\partial^\alpha \rho(r, t)}{\partial t^\alpha} dV^{(\gamma)} + \iint_{S^{(\beta)}} \rho(r, t) \mathbf{v} \cdot d\mathbf{S}^{(\beta)} \quad (13) \\ &= 0, \end{aligned}$$

which leads to

$$\iiint_{V^{(\gamma)}} \frac{\partial^\alpha \rho(r, t)}{\partial t^\alpha} dV^{(\gamma)} + \iint_{S^{(\beta)}} \rho(r, t) \mathbf{v} \cdot d\mathbf{S}^{(\beta)} = 0. \quad (14)$$

From (14), we have

$$\begin{aligned} \iiint_{V^{(\gamma)}} \frac{\partial^\alpha \rho(r, t)}{\partial t^\alpha} dV^{(\gamma)} + \iint_{S^{(\beta)}} J(r, t) \cdot d\mathbf{S}^{(\beta)} \\ = \iiint_{V^{(\gamma)}} \left(\frac{\partial^\alpha \rho(r, t)}{\partial t^\alpha} + \nabla^\alpha \cdot J(r, t) \right) dV^{(\gamma)} \quad (15) \\ = 0, \end{aligned}$$

where $J = \rho \mathbf{v}$ represents the current density in the fractal field.

Hence, from (15), we get

$$\frac{\partial^\alpha \rho(r, t)}{\partial t^\alpha} + \nabla^\alpha \cdot J(r, t) = 0. \quad (16)$$

By analogy with electric charge density in the fractal field, we obtain the conservation of fractal magnetic charge, namely,

$$\frac{\partial^\alpha \rho_m(r, t)}{\partial t^\alpha} + \nabla^\alpha \cdot J_m(r, t) = 0, \quad (17)$$

where $\rho_m(r, t)$ is the fractal magnetic charge density and $J_m(r, t)$ is the fractal magnetic current density in the fractal field.

3.2. Formulation of Maxwell's Equations on Cantor Sets. We now derive Maxwell's equations on Cantor set based on the local fractional vector calculus.

3.2.1. Gauss's Law for the Fractal Electric Field. From (3), the electric charge density can be written as

$$\nabla^\alpha \cdot D = \lim_{dV^{(\gamma)} \rightarrow 0} \left(\frac{1}{dV^{(\gamma)}} \iint_{S^{(2\alpha)}} D \cdot d\mathbf{S}^{(\beta)} \right) = \rho, \quad (18)$$

where D is electric displacement in the fractal electric field.

From (7), (18) becomes

$$\begin{aligned} \iiint_{V^{(\gamma)}} \nabla^\alpha \cdot D dV^{(\gamma)} &= \iint_{S^{(\beta)}} D \cdot d\mathbf{S}^{(\beta)}, \quad (19) \\ \iiint_{V^{(\gamma)}} \nabla^\alpha \cdot D dV^{(\gamma)} &= \iiint_{V^{(\gamma)}} \rho dV^{(\gamma)}. \end{aligned}$$

Hence, we obtain Gauss's law for the fractal electric field in the form

$$\iint_{S^{(\beta)}} D \cdot d\mathbf{S}^{(\beta)} = \iiint_{V^{(\gamma)}} \rho dV^{(\gamma)}. \quad (20)$$

3.2.2. Amperé's Law in the Fractal Magnetic Field. Mathematically, Amperé's law in the fractal magnetic field can be suggested as [23]

$$\oint_{I^{(\alpha)}} H \cdot d\mathbf{l}^{(\alpha)} = I, \quad (21)$$

where H is the magnetic field strength in the fractal field.

The current density in the fractal field can be written as

$$\nabla^\alpha \times H = \lim_{dS^{(\beta)} \rightarrow 0} \left(\frac{1}{dS^{(\beta)}} \oint_{I^{(\alpha)}} H \cdot d\mathbf{l}^{(\alpha)} \right) \mathbf{n}_p = J, \quad (22)$$

which leads to

$$\begin{aligned} \oint_{I^{(\alpha)}} H \cdot d\mathbf{l}^{(\alpha)} &= \iint_{S^{(\beta)}} (\nabla^\alpha \times H) \cdot d\mathbf{S}^{(\beta)}, \quad (23) \\ \iint_{S^{(\beta)}} J \cdot d\mathbf{S}^{(\beta)} &= \iint_{S^{(\beta)}} (\nabla^\alpha \times H) \cdot d\mathbf{S}^{(\beta)}. \end{aligned}$$

The total current in the fractal field reads

$$I = \iint_{S^{(\beta)}} (J_a + J_b) \cdot d\mathbf{S}^{(\beta)}, \quad (24)$$

where J_a is the conductive current and

$$J_b = \frac{\partial^\alpha D}{\partial t^\alpha}, \quad (25)$$

which satisfies the following condition:

$$\begin{aligned} I_b &= \iint_{S^{(\beta)}} J_b \cdot d\mathbf{S}^{(\beta)} \\ &= \iint_{S^{(\beta)}} \frac{\partial^\alpha D}{\partial t^\alpha} \cdot d\mathbf{S}^{(\beta)} \quad (26) \\ &= \frac{\partial^\alpha}{\partial t^\alpha} \iint_{S^{(\beta)}} D \cdot d\mathbf{S}^{(\beta)}. \end{aligned}$$

Hence, Amperé's law in the fractal field is expressed as follows:

$$\oint_{I^{(\alpha)}} H \cdot d\mathbf{l}^{(\alpha)} = \iint_{S^{(\beta)}} \left(J_a + \frac{\partial^\alpha D}{\partial t^\alpha} \right) \cdot d\mathbf{S}^{(\beta)}. \quad (27)$$

3.2.3. *Faraday's Law in the Fractal Electric Field.* Mathematically, Faraday's law in the local fractional field is expressed as

$$E = -\frac{\partial^\alpha \varphi}{\partial t^\alpha}, \quad (28)$$

where φ is the magnetic potential in the fractal field and E is the electrical field strength in the fractal field.

From (28), we have

$$\begin{aligned} \nabla^\alpha \times E &= \lim_{dS^{(\beta)} \rightarrow 0} \left(\frac{1}{dS^{(\beta)}} \oint_{l^{(\alpha)}} E \cdot d\mathbf{l}^{(\alpha)} \right) \mathbf{n}_P \\ &= -\frac{\partial^\alpha}{\partial t^\alpha} (\nabla^\alpha \times \varphi) = -\frac{\partial^\alpha B}{\partial t^\alpha}, \end{aligned} \quad (29)$$

where B is the magnetic induction in the fractal field.

In view of (9), we rewrite (29) as

$$\begin{aligned} \oint_{l^{(\alpha)}} E \cdot d\mathbf{l}^{(\alpha)} &= \iint_{S^{(\beta)}} (\nabla^\alpha \times E) \cdot d\mathbf{S}^{(\beta)}, \\ \iint_{S^{(\beta)}} (\nabla^\alpha \times E) \cdot d\mathbf{S}^{(\beta)} &= \iint_{S^{(\beta)}} \left(-\frac{\partial^\alpha B}{\partial t^\alpha} \right) \cdot d\mathbf{S}^{(\beta)} \\ &= -\frac{\partial^\alpha}{\partial t^\alpha} \iint_{S^{(\beta)}} B \cdot d\mathbf{S}^{(\beta)}. \end{aligned} \quad (30)$$

So, from (30), Faraday's law in the fractal field reads as

$$\oint_{l^{(\alpha)}} E \cdot d\mathbf{l}^{(\alpha)} + \frac{\partial^\alpha}{\partial t^\alpha} \iint_{S^{(\beta)}} B \cdot d\mathbf{S}^{(\beta)} = 0. \quad (31)$$

3.2.4. *Magnetic Gauss's Law for the Fractal Magnetic Field.* From (3), we derive the local fractional divergence of the magnetic induction in the fractal field, namely,

$$\nabla^\alpha \cdot B = \lim_{dV^{(\gamma)} \rightarrow 0} \left(\frac{1}{dV^{(\gamma)}} \iiint_{S^{(2\alpha)}} B \cdot d\mathbf{S}^{(\beta)} \right) = 0. \quad (32)$$

Furthermore, the magnetic Gauss' law for the fractal magnetic field reads as

$$\iiint_{S^{(\beta)}} B \cdot d\mathbf{S}^{(\beta)} = 0. \quad (33)$$

3.2.5. *The Constitutive Equations in the Fractal Field.* Similar to the constitutive relations in fractal continuous medium mechanics [23], the constitutive relationships in fractal electromagnetic can be written as

$$\begin{aligned} D &= \varepsilon_f E, \\ H &= \mu_f B, \end{aligned} \quad (34)$$

where ε_f is the fractal dielectric permittivity and μ_f is the fractal magnetic permeability.

4. Local Fractional Differential Forms of Maxwell's Equations on Cantor Sets

In this section, we investigate the local fractional differential forms of Maxwell's equations on Cantor sets.

The Cantor-type cylindrical coordinates can be written as follows [26]:

$$\begin{aligned} x^\alpha &= R^\alpha \cos_\alpha \theta^\alpha, \\ y^\alpha &= R^\alpha \sin_\alpha \theta^\alpha, \\ z^\alpha &= z^\alpha, \end{aligned} \quad (35)$$

with $R \in (0, +\infty)$, $z \in (-\infty, +\infty)$, $\theta \in (0, \pi]$, and $x^{2\alpha} + y^{2\alpha} = R^{2\alpha}$.

Making use of (35), we have

$$\begin{aligned} \nabla^\alpha \cdot \mathbf{r} &= \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha} \frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha}, \\ \nabla^\alpha \times \mathbf{r} &= \left(\frac{1}{R^\alpha} \frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha r_\theta}{\partial z^\alpha} \right) \mathbf{e}_R^\alpha \\ &\quad + \left(\frac{\partial^\alpha r_R}{\partial z^\alpha} - \frac{\partial^\alpha r_z}{\partial R^\alpha} \right) \mathbf{e}_\theta^\alpha \\ &\quad + \left(\frac{\partial^\alpha r_\theta}{\partial R^\alpha} + \frac{r_R}{R^\alpha} - \frac{1}{R^\alpha} \frac{\partial^\alpha r_R}{\partial \theta^\alpha} \right) \mathbf{e}_z^\alpha, \end{aligned} \quad (36)$$

where

$$\begin{aligned} \mathbf{r} &= R^\alpha \cos_\alpha \theta^\alpha \mathbf{e}_1^\alpha + R^\alpha \sin_\alpha \theta^\alpha \mathbf{e}_2^\alpha + z^\alpha \mathbf{e}_3^\alpha \\ &= r_R \mathbf{e}_R^\alpha + r_\theta \mathbf{e}_\theta^\alpha + r_z \mathbf{e}_z^\alpha. \end{aligned} \quad (37)$$

From (7) and (20), the local fractional differential form of Gauss's law for the fractal electric field is expressed by

$$\nabla^\alpha \cdot D = \rho, \quad (38)$$

which leads to

$$\frac{\partial^\alpha D_R}{\partial R^\alpha} + \frac{1}{R^\alpha} \frac{\partial^\alpha D_\theta}{\partial \theta^\alpha} + \frac{D_R}{R^\alpha} + \frac{\partial^\alpha D_z}{\partial z^\alpha} = \rho(R, \theta, z, t), \quad (39)$$

where $D = D_R \mathbf{e}_R^\alpha + D_\theta \mathbf{e}_\theta^\alpha + D_z \mathbf{e}_z^\alpha$.

In view of (9) and (27), we can present the local fractional differential forms of Ampere's law in the fractal magnetic field

$$\begin{aligned} \nabla^\alpha \times H &= J_a + \frac{\partial^\alpha D}{\partial t^\alpha}, \\ \left(\frac{1}{R^\alpha} \frac{\partial^\alpha H_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha H_\theta}{\partial z^\alpha} \right) \mathbf{e}_R^\alpha &+ \left(\frac{\partial^\alpha H_R}{\partial z^\alpha} - \frac{\partial^\alpha H_z}{\partial R^\alpha} \right) \mathbf{e}_\theta^\alpha \\ &+ \left(\frac{\partial^\alpha H_\theta}{\partial R^\alpha} + \frac{H_R}{R^\alpha} - \frac{1}{R^\alpha} \frac{\partial^\alpha H_R}{\partial \theta^\alpha} \right) \mathbf{e}_z^\alpha \\ &= J_a(R, \theta, z, t) + \frac{\partial^\alpha D(R, \theta, z, t)}{\partial t^\alpha}, \end{aligned} \quad (40)$$

where $H = H_R \mathbf{e}_R^\alpha + H_\theta \mathbf{e}_\theta^\alpha + H_z \mathbf{e}_z^\alpha$.

Using (9) and (31), the local fractional differential forms of Faraday's law in the fractal electric field can be written as

$$\begin{aligned} \nabla^\alpha \times E &= -\frac{\partial^\alpha B}{\partial t^\alpha}, \\ \left(\frac{1}{R^\alpha} \frac{\partial^\alpha E_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha E_\theta}{\partial z^\alpha} \right) \mathbf{e}_R^\alpha + \left(\frac{\partial^\alpha E_R}{\partial z^\alpha} - \frac{\partial^\alpha E_z}{\partial R^\alpha} \right) \mathbf{e}_\theta^\alpha \\ + \left(\frac{\partial^\alpha E_\theta}{\partial R^\alpha} + \frac{E_R}{R^\alpha} - \frac{1}{R^\alpha} \frac{\partial^\alpha E_R}{\partial \theta^\alpha} \right) \mathbf{e}_z^\alpha \\ &= -\frac{\partial^\alpha B(R, \theta, z, t)}{\partial t^\alpha}, \end{aligned} \quad (41)$$

where $E = E_R \mathbf{e}_R^\alpha + E_\theta \mathbf{e}_\theta^\alpha + E_z \mathbf{e}_z^\alpha$.

From (7) and (33), the local fractional differential forms of the magnetic Gauss law for the fractal magnetic field read as follows:

$$\begin{aligned} \nabla^\alpha \cdot B &= 0, \\ \frac{\partial^\alpha B_R}{\partial R^\alpha} + \frac{1}{R^\alpha} \frac{\partial^\alpha B_\theta}{\partial \theta^\alpha} + \frac{B_R}{R^\alpha} + \frac{\partial^\alpha B_z}{\partial z^\alpha} &= 0, \end{aligned} \quad (42)$$

where $B = B_R \mathbf{e}_R^\alpha + B_\theta \mathbf{e}_\theta^\alpha + B_z \mathbf{e}_z^\alpha$.

5. Conclusions

In this work, we proposed the local fractional approach for Maxwell's equations on Cantor sets based on the local fractional vector calculus. Employing the local fractional divergence and curl of the vector function, we deduced Maxwell's equations on Cantor sets. The local fractional differential forms of Maxwell's equations on Cantor sets in the Cantorian and Cantor-type cylindrical coordinates were discussed. Finding a formulation of Maxwell's equations on Cantor set within local fractional operators is the first step towards generalizing a simple field equation which allows the unification of Maxwell's equations to the standard model with the dynamics of cold dark matter. We noticed that the classical case was debated in [32].

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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