DESIGN AND ANALYSIS OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION

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DESIGN AND ANALYSIS OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION

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Submitted by **Hassan ALDULAIMI**

Approval of the Graduate School of Natural and Applied Sciences, Çankaya University.

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

DESIGN AND ANALYSIS OF AN OVERCONSTRAINED MANIPULATOR FOR REHABILITATION

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A parallel manipulator is a closed loop kinematic chain mechanism that is attached to the base by at least two limbs in parallel. Compared with the serial, the parallel manipulator has higher load-carrying capacities, higher accuracy, higher stiffness and lower inertia. Due to these advantages the parallel manipulator become viable alternative for wide applications, therefore it have been the subject of study of many research during the recent three decades.

However, these kinematic closed loops also have some disadvantages, such as limited workspace, complicated forward kinematics, high cost and complicated structures. To overcome these disadvantages parallel manipulators with less than 6-DoF can be used. Also, designing a parallel manipulator with less than six degree of freedom for a subspace can relatively reduce the complexity.

This thesis deals with one of the applications of parallel manipulator as a rehabilitation robot. This device is an over –constrained parallel manipulator 5 degree of freedom with 3 legs. This manipulator consists of a moving platform which is connected to a fixed base via three legs. Each leg is made of RRR(RR) (revolute) joints where the first three joint in all legs are parallel and the recent two joint are intersecting .

Inverse kinematics of this device including position velocity and acceleration are studied. In addition, the workspace of the parallel manipulator is analyzed. A design optimization is conducted for the prescribed workspace of the device. Finally, this thesis also deals with the dynamic modeling the manipulator using an approach of the principle of virtual work. The equations of motion of the manipulator are derived by considering its motion characteristics. To reduce computations, inverse Jacobian matrices for manipulators are derived to give relations between input and output variables.

Keywords: Rehabilitation Robotics, Overconstrained Manipulators, Workspace analysis, Dynamics Analysis.

ÖZ

REHABİLİTASYON İÇİN KULLANILACAK BİR KISITLI MANİPÜLATÖRÜN TASARIMI VE ANALİZİ

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Paralel manipülatörler yere birkaç bacak ile bağlanan sistemlerdir. Seri manipülatörler ile karşılaştırıldığında daha yüksek yük taşıma kapasitelerine, daha yüksek hassasiyet değerlerine, yüksek dayanım ve düşük eylemsizliğe sahiptirler. Bu özellikleri sayesinde paralel manipülatörler birçok alanda kullanılmaktadır ve ayrıca son 30 yıldır birçok araştırmalara konu olmaktadır.

Ancak paralel manipülatörlerin kısıtlı çalışma alanı karmaşık düz kinematik hesapları, yüksek maliyet ve karmaşık yapıya sahip olma gibi bazı dezavantajları da mevcuttur. Bu dezavantajların üstesinden gelebilmek için alt serbestlikten daha düşük sayıda serbestlik içeren manipülatörlerin çalışılmasına hız verilmiştir. Özel bir alt uzay için tasarlanmış bir manipülatör sistemin karmaşıklığını düşürecektir.

Bu tezde paralel bir manipülatörün rehabilitasyon amacı ile tasarımı ve analizi gerçekleştirilmiştir. Önerilen sistem beş serbestlikte ve yere 3 bacaktan bağlanmaktadır. Her bir bacak RRR(RR) yapılandırmasına sahiptir. Bütün

bacaklardaki ilk üç mafsal birbirine paralel iken son iki mafsallar eksenleri çakışacak şekilde yerleştirilmiştir.

Verilen pozisyon hız ve ivme değerlerine göre motor pozisyon hız ve ivme değerlerinin hesaplanması için ters kinematik yöntemler kullanılmıştır. Ek olarak sistemin çalışma alanı analizi yapılmış ve optimum bir sonuç elde edilmiştir. Son olarak edimsiz iş prensibi kullanılarak sistemin dinamik modellemesi yapılmıştır. Hesaplamaları kolaylaştırmak adına sistemin Jacobi matrisleri ortaya çıkarılmıştır.

Anahtar Kelimeler: Rehabilitasyon Robotları, Kısıtlı Manipülatörler, Çalışma Alanı Analizi, Dinamik Analiz.

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LIST OF ABBREVIATIONS

DoF	Degree of freedom
SPM	Spherical parallel manipulator
RRR	Three revolute joints with axes are parallel
(RR)	Two revolute joints with axes are intersecting
P	Prismatic joint
U	Universal joint
S	Spherical joint
R	Revolute joint

CHAPTER 1

INTRODUCTION

1.1 Background

At the outset, most manufacturing produced by craftsmen and the quality of product depended on dexterity of the craftsmanship which is known as manual production. At the beginning of the last century the industrial witness improve in manufacturing processes, the concept of mass production was introduced by the Ford Motor Company where most manufacturing processes accomplish by special purpose machines .This method reduces the cost of manufacturing .However, the machine is designed to perform a special task, and every model introduced can be very expensive.

Recently, manufacturing industries introduced robot manipulators to perform certain production tasks, such as spot welding, assembling and material handling and improve robot manipulator for different tasks and products became affordable. In recent years, the robot manipulator are controlled by computer or microprocessors (flexible automation) and its reprogrammed to perform different tasks and no need to change these machines in a new model. Flexible automation has high accuracy and it is more economical.

A robot can be defined as a mechanical system under automatic control that is designed to perform operations such as moving materials, tools, parts or devices through variable programmed motions for variety tasks. Another definition includes walking machines, mechanical manipulators, numerical control machines and device that operate automatically or by remote control.

A robotic system usually consists of a mechanical manipulator (links and joints), external sensing devices, a controller, a computer and an end effector, which is a mediator between the environment and its manipulator. A manipulator can be classified as a machine which is the transformation external energy by electrical, hydraulic or pneumatic components in to useful work. These works can be exemplified as welding, carrying loads, pick and place operations, assembly and other industrial operations. Although, many definitions of robotic, the best one describes it as a mechanical being built to do routine manual work for human beings.
A parallel manipulators is a device which has a moving platform that is attached to the base via several independent kinematic chains .Compared with the serial, the parallel manipulator has higher accuracy, higher load-carrying capacity, higher stiffness, lower inertia and static balance. Due to these qualities the parallel manipulator become viable alternatives for wide applications, such as simulators, medical applications, industrial application, rehabilitation and health care applications.

Rehabilitation robotics is one of the technology-based solutions that use to assist persons who becomes unable to do necessary activities or that provide therapy for persons who are in need of improving their physical or cognitive functions. Also it is used for other aspects such as prosthetics development, technology for the diagnosis and monitoring and functional neural stimulation. The robotic therapy for the upper extremity to improve the hand and wrist motion will be studied in this thesis.
In general, during the design of rehabilitation robotics or any robot many criteria should be considered such as workspace, dexterity, singularity avoidance, dynamics, and stiffness. These estimation criteria can be classified into two set: one relates to the kinematic performance and the other relates to the dynamic performance of the manipulator.

The kinematics model can be used to find analytical relations between the values of the actuated joints (input variables) and the location of the moving platform (output variables) .Where, the equations that represent relations between the input and the output variables of a robotics are called the kinematic equations. The equations that represent relations between input and output velocities in a robotics are called the instantaneous kinematic variables of the mechanism. Forward kinematics and inverse kinematics are the two types of kinematics problem. The forward kinematics problem is used for finding the required positions and orientations of the moving platform for a given actuator displacement or rotation. In contrast, the inverse kinematics problem, deal with finding the set of input variables that correspond to desired configuration of the moving platform.

The workspace of robot related with kinematics and design of robotics. Where, the workspace of the parallel manipulator can be defined as configuration set of the end-effector which can be reached by describing architectural parameter values.

The dynamic analysis is essential part in describing the behavior of mechanical systems and getting their best performance. Dynamical problems are listed in two categories. First, the forward dynamics problem, the applied actuator forces and/or moments of a robot are given and the aim to find the response arm. Second, in the inverse dynamics problem the desired force of the manipulator on a trajectory is given and the aim is to find the required actuator forces and/or moments.

There are three main approaches to solve the dynamical equations: The principle of virtual work, Newton–Euler laws and the Lagrangian multiplier. The Newton–Euler formulates the motion equations of each body of a mechanism should be written, thus that leads to a large number of equations. The forces of the actuators and the reactions in the joints are calculated from simple equilibrium equations of the platform and the limbs. The principle of virtual work is the active approach for dynamic equations of parallel manipulators where, the constraint forces and moment at the joints are eliminated from the motion equations. The Lagrangian multipliers method is based on the differentiation of kinetic energy and potential energy with respect to variables of the system and time. The benefits of this approach all forces, reaction and moments are eliminated at the beginning.

1.2 Classification of Robot Manipulators

We can classify robots according to kinematic structure to three basic architectures. These architectures are characterized by the model of kinematic chains connecting the base link of the manipulator to the output link. The three basic parallel manipulators are:

- Serial manipulator.

- Parallel manipulator.
- Hybrid manipulator.

1.2.1 Serial Manipulators

Serial manipulators can be defined as open loop kinematic chains which consists of several links this links that are connected by joints. One end of chain is attached to the base and the other end is free moving and attached with a gripper or a mechanical hand.

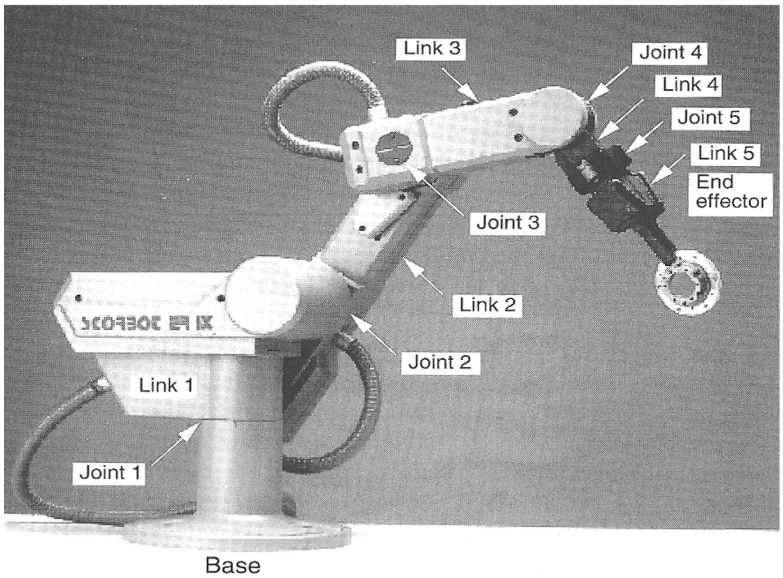


Figure 1 Serial manipulator

Scorbot robot (Courtesy of Eshed Robotec Inc., Princeton, New Jersey)

(Source: Tsai-Robot Analysis)

Serial manipulators exhibit several advantages such as large workspace , high dexterity and simple with single-valued solution of direct kinematics problem. The direct kinematics problem is defined as calculating the location of the end effector for a given set of the joint variables. However, serial manipulators suffer from several disadvantages which include low accuracy, low payload-to-weight ratio and high inertia. The low accuracy is due to the joint errors are cumulative and deflections in the links. The low payload-to-weight ratio due to the weight of the successor links are affected on every actuator. The high inertia stems from the large number of moving parts that are connected in series, thus forming long beams with high inertia, there for serial manipulators cannot be used with applications which are requiring high acceleration and agility. Complicated with multiple solutions of inverse kinematics problem is another disadvantage of serial manipulators. The inverse kinematics problem is defined as finding the required values of joint variables that correspond to a desired location of the end effector.

1.2.2. Parallel Manipulator

Parallel manipulators can be defined as a closed kinematic loop mechanism with a moving platform (end-effector) attached to the base with several limbs in parallel. The motion of the platform is achieved by simultaneous actuation of the kinematic chains extremities, and these manipulators are working in cooperation. The parallel manipulators support the end effector with at least two chains and these chains contains at least one simple actuator. Therefore parallel manipulators can bear heavy loads because the load is distributing on the chains and the several legs shares the external loads, this is the most important advantage for parallel manipulators.
The number of actuators in parallel manipulators is equal the degree of freedom of platform. Therefore the number of actuators is minimal, thus the number of sensors for parallel manipulators control of mechanism is minimal. Additionally, high stiffness is one of the advantages of parallel manipulators and the stiffness has direct effect for position accuracy. Another advantage of parallel manipulator high precision, high agility, low inertia and the inverse kinematics problem is simple solution. The high precision of these robots stems from non- cumulative of joint errors and sharing the load by several kinematic chains. Also parallel manipulators exhibit several disadvantages which are limited workspace, low dexterity, singularities occur in side and envelope of workspace and the direct kinematics solution are complicated.

This advantage and disadvantage of parallel manipulator can be exploited for suitable implementation include limited workspace, high precision ,high agility and high payload-to weight ratio for used parallel manipulators in industry and scientific facilities such as flight simulation ,earthquake simulation ,medical applications and industrial application.

Figure 2 Parallel manipulator

(Source: J.P.Merlett - Parallel Robots_2nd_Edition)

1.2.3 Hybrid Manipulator

The combination of both serial and parallel manipulator of kinematic chains in a mechanism leads to a third type of architecture, which is called the hybrid architecture. The hybrid architecture combines both advantages and disadvantages of the parallel and serial mechanisms. Fig .3 presents the hybrid manipulator which consist from two parallel manipulators connected as serial.

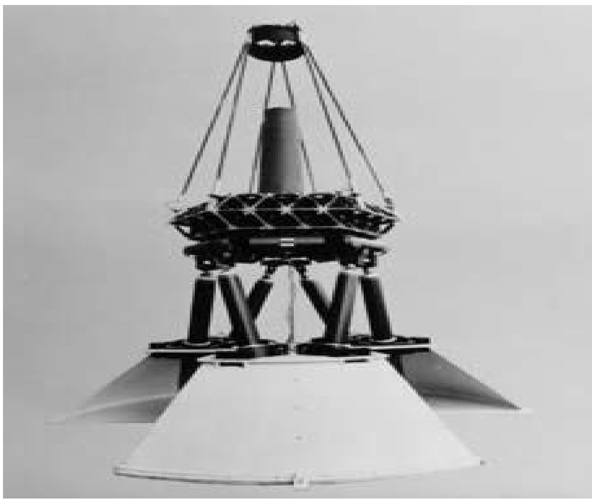


Figure 3 Vertex Antennentechnik a telescope pointing system(Hybrid manipulator)

(Source: J.P.Merlett - Parallel Robots_2nd_Edition)

1.3. Application of Parallel Manipulator

Parallel manipulators have found numerous applications in industries ranging

1.3.1. Application As Simulators

In 1947 , the first octahedral hexapod was invented by Dr. Eric Gough and became the most popular. The moving platform is attached to the ground by six links with varying lengths and the one that has been replicated over a thousand times.

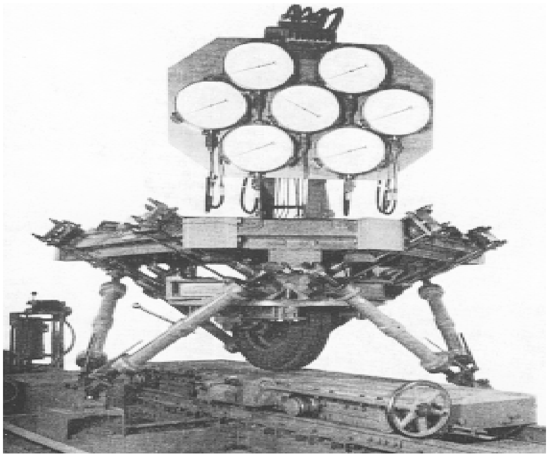


Figure 4 Gough platform simulator
(Source:Merlet- parallel manipulator)

Another flight simulator was fabricated by Klaus Cappel in 1964 shown in Fig.5 which have three rotations and three translations to simulate a flying object in space. At the same time the Stewart's paper describe motions of platform for flight simulation by six legs constructed by pneumatic cylinders.

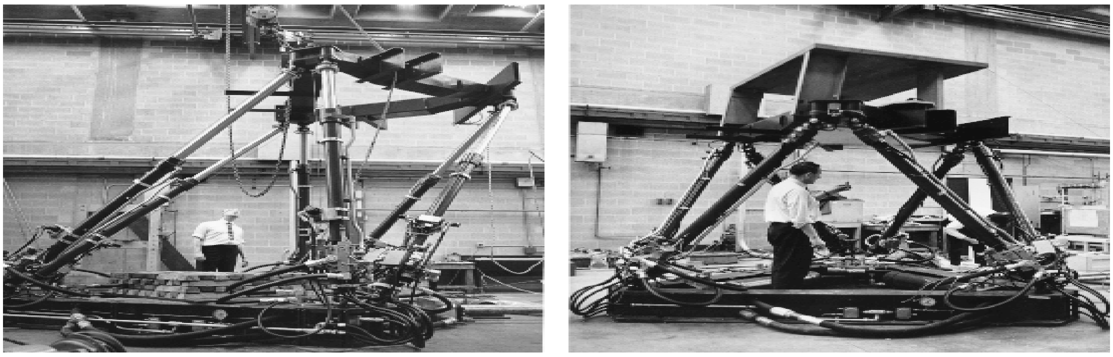


Figure.5 Flight simulator based on an octahedral hexapod

Furthermore, the parallel architecture is utilized for earthquake simulators by (casino parallel manipulator) Fig .6.a the vibration of the ground is described by a single platform which is connected to base by three identical legs. Another studied of earthquake simulator in laboratory in university of Nevada Fig. 6.b this type of parallel manipulators is utilized to describe the shake.

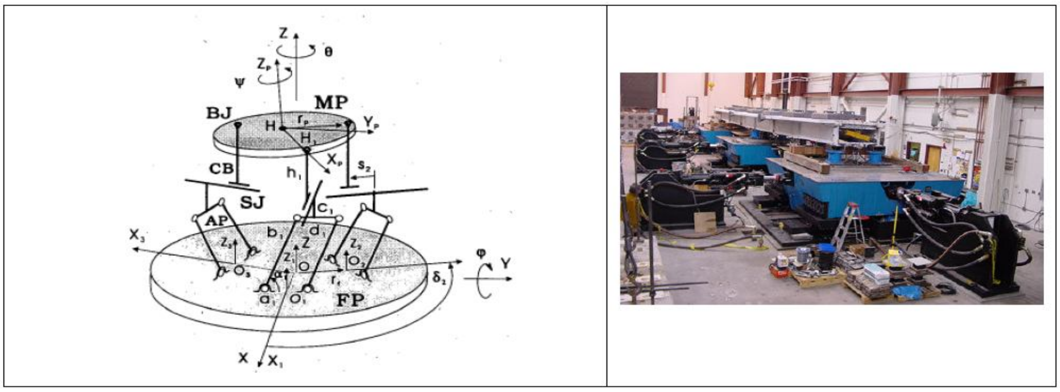


Figure 6 (a) earthquake simulations (b) shake simulations

(Source: ceccarelli et.al)

1.3.2 Medical Applications

Parallel manipulators feature high precision and high stiffness there for becoming popular in medical area due to these advantages. As a result, the robotic devices are produced by several companies in medical market. The microdex alpha prototype is one product of medical device that have high precision shown in Fig.7. Therefore, this mechanism is utilized for special operations such as, certain aneurysms, brain tumors and cervical spine problems and this type of manipulators are controlled by a doctor in surgery process.

Figure 7 Microdex alpha prototype

(Source: advanced robotics for medicine and industry 2008)

Another product, Mazor that includes both close and open kinematic chain as shown in Fig.8(a),which is fabricated to make the surgical environment more accurate and more safety . Furthermore, the prototype robot, shown in Fig. 8(b), has been developed for testing in medical tasks such as Arthroscopic knee surgery and manipulating a laparoscope.

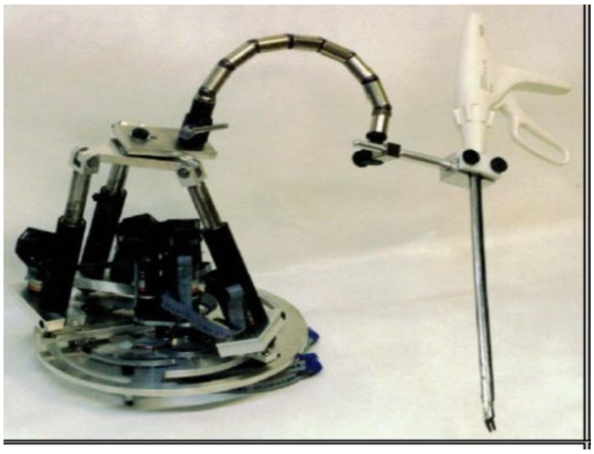


Figure 8(a) Cutting-edge technology (Mazor) . (b) the device for arthroscopic knee surgery (Source: SmartAssist 2008).

Also parallel manipulators are used in scanning operations. One product is Headfix shown in Fig. 9. This system is specifically designed to overcome the drawbacks of conventional invasive fixation and non-invasive thermoplastic masks.

Figure 9. HeadFIX prototype

(Source: Medical Intelligence 2008)

Another device is manufactured for scanning brain or neurology in medical area Surgiscope shown in Fig.10. The mechanism is constructed on three identical legs to make necessary motions. A camera is placed on the moving platform and manipulator is mounted downward.

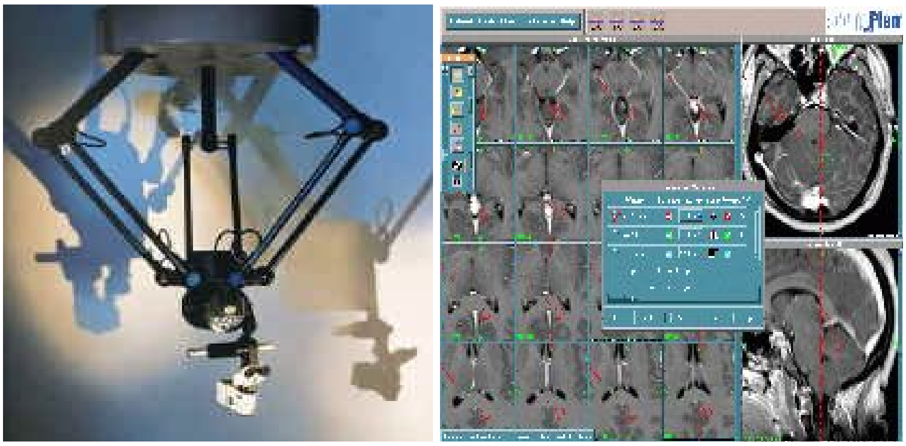


Figure 10. The SurgiScope prototype
(Source: iSiS 2008)

1.3.3 Industrial Applications

The applications of parallel robots in machine tools have the largest economic impact in industry .The first milling machine was given by the Giddings & Lewis company called Variax. Its possesses six degrees of freedom, it was based on the principle of the Gough platform which was five times stiffer than a classical machine and high speed.

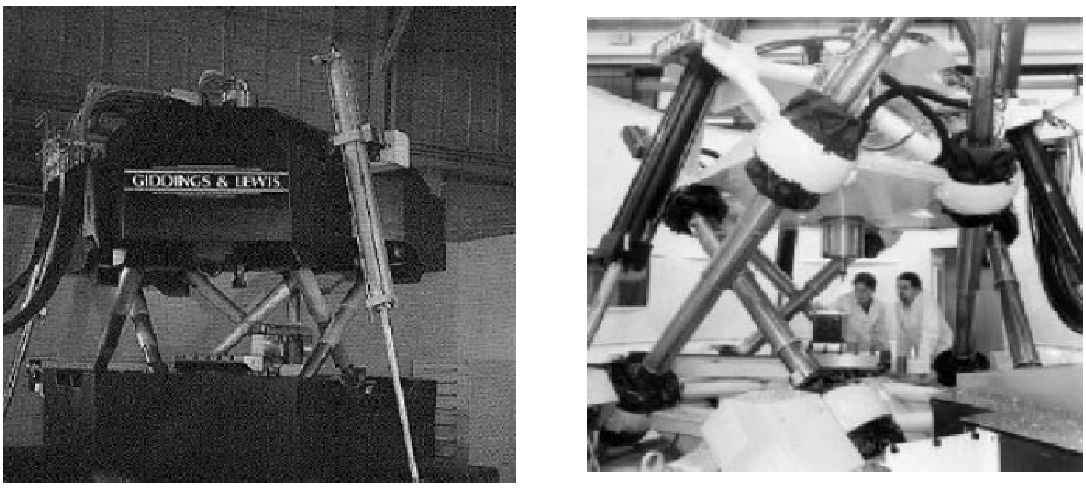


Figure 11 The Variax of Gidding & Lewis, presented in 1994
(source: J.P.Merlett - Parallel Robots_2nd_Edition)

Anthor applecation of parallel structures in CNC machines.

Figure 12 (a) the Genius 500 (b) the Trijoint 900H
(source: J.P.Merlett - Parallel Robots_2nd_Edition)

Parallel robots are favorite in applications of positioning devices and several companies produce fine positioning devices and the down picture show that device.

Figure 13. (a) the Paros robot. (b) the HR1 robot
(source: J.P.Merlett - Parallel Robots_2nd_Edition)

1.3.4 Application in Rehabilitation and Health Care

The field of rehabilitation robotics is one of technology-based solutions that use to assist persons who become unable to do necessary activities and disability to interact physically with environment and do major life activities. In general the field of rehabilitation robotics represent by therapy and assistance robots. However, its includes functional neural stimulation, artificial limb development, and technology for the diagnosis and monitoring of people during ADLs .In addition, rehabilitation robotics used in another filed such as, therapy for walking, movement therapy after neurologic injury and therapy for the upper extremity.

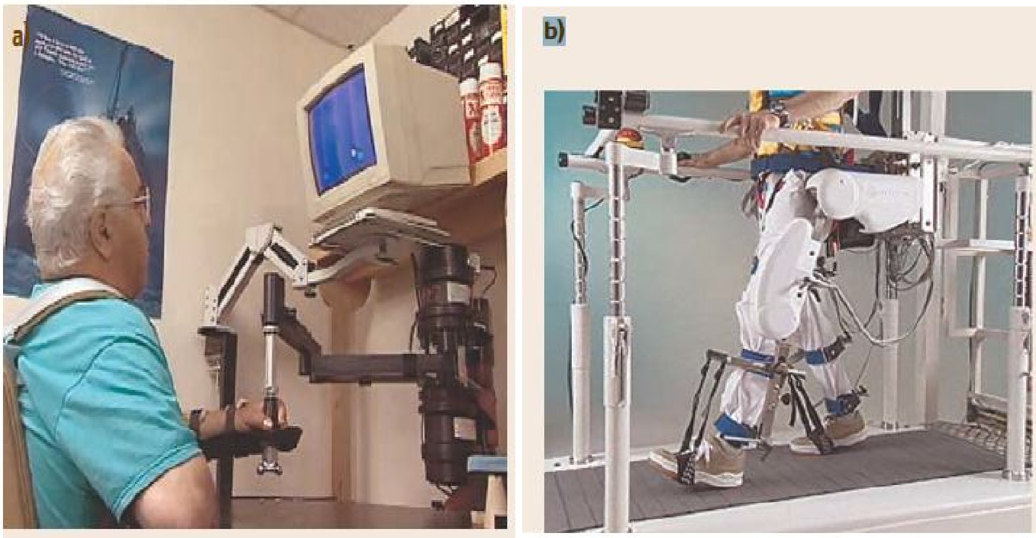


Figure 14
(a)Arm-therapy robotic systems (b) Gait-training robotic systems
(source: rehabilitation and health care, H.F. Machiel Van der Loos, David. J)

1.4 Literature Survey

Properties and usage areas of parallel manipulators led scientists to work in this area. Recently, researchers have tried to use these features to develop parallel manipulator. Merlet [1] presents famous treatise on parallel robots in his book {Parallel robots}. He introduces the studies on the kinematic analysis, static, dynamic analysis, singular configurations, workspace and finally design. A rather detailed analysis of Parallel Manipulators has been studied by Merlet [2] general mathematical framework inverse Jacobian matrices was estimated. It focuses on motions of the end-effector that cannot be detected by measurement of the actuated joint space parameters. Tsai [3] provided in his book (Robot analysis) both fundamental and advanced topics of position analysis, Jacobian analysis, statics and stiffness and dynamics analysis for both serial and parallel manipulator. In addition, he studied with wrist mechanisms and tendon-driven manipulators. Another study of parallel manipulator by Niku[4] is done where the kinematics analysis is developed which including forward and inverse kinematics that explores position , transformations and orientation analysis as well as the Denavit- Hartenberg method is represented and continues with velocity analysis. Also dynamics analysis base on Lagrangian equations is developed. In addition, its covers motion controls system, trajectory planning and sensors. The Stewart-Gough parallel platform manipulator is the most common robot. The moving platform is attached to the fixed base with six extensible legs via spherical joint. Each leg consist two links that are connected via prismatic joint. This structure is called (SPS) with 6-DOF. Therefore, it is based on much research topic [5-7].

The rehabilitation robotics one of the applications of parallel manipulator that use to assist patients who becomes unable to do necessary activities. Machiel [8] provide descriptions of the main achievements of rehabilitation robotics field and domains of this robotics with short history. Also, describes both of therapy robots for physical therapy and training and assistance robots for people with disabilities. For the hand rehabilitation device, it is important to provide some criterion to guide the design or evaluate the performance .Li-Chieh [9]proposed the functional workspace of palmar opposition of with respect to maximal workspace of the other fingers with two-dimensional to characterize the motor capability. Zheng [10] proposed an exoskeleton of hand rehabilitation assistive device. The workspace and kinematics of this device for thumb and index finger rehabilitation are considered.

The frequency of movement repetitions in robot-aided therapy is more practical. To provide passive movements, should be selected pre- determined trajectory (trajectory tracking) [11-13]. Also the safety requirement plays important role in design of rehabilitation robotics. Kang [14] studied an adaptive control strategy for exoskeleton robotic of 5 DoF upper limbs. To improve the safety of robotic should be considered the architecture of robot, unknown variances and actuator faults.

In the past most of the attention was given to parallel mechanisms which has six-degree of freedom (DoF). Alternative six-DoF platforms have been proposed and analyzed by different authors. But, it is seen that six DoF is not needed in many applications. Parallel manipulators with low mobility have simpler mechanical structures, fewer actuators and simple control mechanisms.

Yi Lu [15] proposed a 5-DoF 3UPS parallel manipulator with two composite rotational/linear active legs. A prototype of this mechanism and its displacement are constructed. The formulas of velocity and acceleration of composite active leg are derived, in addition a reachable workspace is analyzed by using geometric approach. Finally, kinematics and statics are solved. Si J. Zhu [16] presented six practicable of 5-DoF 3R2T parallel manipulator fully-symmetrical and studied the singularity for each type with screw theory and Grassman geometry, and a prototype 5-RRR(RR) as a mechanism example is analyzed with symbol expression and numerical figure. Dynamics model analysis was developed of five degrees of freedom (5-DoF) parallel manipulator for aero engine spindle dual-rotor by Qinghua [17].

Structural Synthesis is one of main studies on parallel manipulators that studied mechanical architectures. The number of joints and type of joints is calculated by knowing the motion of the platform, degree of freedom, number of legs, number of hinges and branch loops. We can obtain different structures by using exchangeability of kinematic pairs.

Selvi [18] described a generalized approach for structural synthesis and creation of new overconstrained manipulators and to describe a potentially generalizable approach for function and motion generation synthesis of overconstrained mechanism. Feng [19] based on Plucker coordinates for design of structures of parallel manipulator of several new types of composite pairs and obtain several new types of parallel robotic mechanisms with 2, 3, 4 and 5-DOF. Huafeng [20] presented The structural synthesis of 5-DOF parallel manipulator by a computer-aided method is proposed for both symmetrical and asymmetrical, where databases are established corresponding kinematic structure.

The kinematics model is essential part for insure the positioning of the end –effector and find analytical relations between the input variables with respect to output variables (location of the end –effector) to obtain optimal design of mechanism. Medical robot is proposed by Nicolae[21] ,present kinematics modeling of 5- DoF parallel manipulator, which could be utilized for brachytherapy. That use to deliver high doses of radiation inside the tumors for cancer treatment. Both the forward and inverse kinematics are analyzed, workspace and singularities are studied. The analytic formulae for inverse and direct of displacement, velocity and acceleration for 4-DoF over-constrained parallel manipulators with three legs (RRPU + 2UPU) are derived also reachable workspace are analyzed by Lu [22].

Inverse kinematics can be used to obtain optimal design of mechanism .many researchers used and present new methods for inverse kinematics. Zhao [23] studied the inverse kinematics of three rotational DoF ,where position, velocity and acceleration are considered. Also dynamic model is analysis by using the principle of virtual work and presented. The total actuating torques related to velocity, gravity, and the acceleration. In addition, calculated torque with respect to external force. Williams [24] proposed 3-DoF parallel manipulator with three limbs. Inverse kinematics of position and velocity are calculated for abroad class (RRR, RRP, RPR, RPP, PRR, PRP, and PPR) of planar manipulator. Where, the actuated joint rates are fined by derived inverse Jacobian matrices.

The design of the workspace for parallel robots includes finding optimal set of configurations of the end-effector which can be reached by some choice architectural parameter values. Therefore, the workspace of manipulators is an influential issue for the design of robots. The problem of determining the workspace received considerable attention by many researchers. Feng Gao [25] was investigated the relationships between the link lengths and the shapes of the workspaces of (3-DOF PPMs) and presented various of shapes of the workspaces for each architecture to obtain optimal design of manipulator.

Different types of workspaces have been proposed, such as maximal workspace, constant orientation workspace, inclusive maximal workspace, and dexterous workspace. The reachable and dexterous workspaces for parallel manipulators are the most useful. Several researchers have deal with the reachable and dexterous workspace. Gallant [26] presented the geometric method to determine of the dexterous workspace of two architectures the n-RRRR and the n-RRPR. The dexterous workspace of three degree of freedom parallel manipulator is studied and the changes of the workspace volume are presented with change of the architectural parameters and find the size, shape and workspace relative to the particular set architectural parameters of mechanism [27]. Novel idea was presented by Z. Wang and Shiming Ji [28] for determining the dexterous and reachable workspace of a general parallel manipulators and calculating their boundary by used a new numerical method, called ‘‘stratified boundary search technique’’.

A numerical method was developed to prescribe the workspace and obtain optimal design of spherical parallel manipulators (SPM) for maximum dexterity [29]. A general method was developed to prescribe a compatible orientation workspace for a six degree of freedom parallel manipulator [30]. Where, the boundary of workspace for any manipulator can be determined by solving the equations of inverse kinematics and use the bisection method, if the equations are fourth or higher degree polynomials. The effect of the passive joint limits, the size of the platform, the link interactions and singularity are also determined. The trajectory planning inside the workspace of parallel manipulators is very difficult because of limited workspace, voids inside it and singularities. Dash [31] was presented a numerical technique to present an algorithm that can be planning singularity-free path. Jing [32] presented new algorithm to develop the limb lengths of a spatial parallel manipulator to obtain a desired dexterous workspace rather than the whole reachable workspace. With the analysis of mobility, choose the least number of variables to describe the kinematic constraints of each leg. The optimum parameters can be calculated from the objective functions with the given dexterous workspace.

The dynamic analysis is significant part in expectation the behavior of mechanism and realization their best performance and there are three main approaches to solve the dynamical equations: the principle of virtual work, Newton–Euler laws and the Lagrangian multiplier. Tsai [3] mentions, the Newton–Euler method leads to a large number of equations because of the motion equations of each body of a mechanism should be written. The forces of the actuators and the reactions in the joints are calculated simply. Several researchers applied this approach. Xi and Sinatra [33] presented Newton-Euler formulations for calculation the inverse dynamics of hexapods. The hexapod consists from platform connecting to the base by six legs with (Prismatic-Universal-Spherical) for each leg. Also inverse kinematic is analyzed including determination the Jacobian matrix by using loop-closure equations. The principle of virtual work is the active approach for dynamic equations of parallel manipulators where, the constraint forces and moment at the joints are eliminated from the motion equations. Alexei and Paul [34] use the approach of the principle of virtual work to solve the dynamic analysis for a 3-DOF parallel manipulator and obtain the motion equations with R–P–S joint structure. The inverse dynamics of three rotational 3-DOF is investigated by use the principle of virtual work method and studied the total actuating torque related to the acceleration, gravity , velocity and external force [35]. Lagrangian multipliers are simple way because all forces, reaction and moments are eliminated at the beginning. Guanglei [36] solved the inverse dynamics and geometric synthesis of 3-DOF spherical parallel manipulator by using method of Lagrange multipliers and motion equations which associated with the expressions for the kinetic energy were derived

1.5. An Overconstrained Manipulator For Rehabilitation Purposes

A large clinical testing in several studies was developed by MIT-MANUS[8] for study advantage of the therapy robotics for the patients. Investigations show that patients who received arm movement by robot therapy make more improvement of movement ability than who received arm movement by conventional therapy, in additional the robotic therapy do without any adverse effects. Also, the robot group patients received more therapy by a robotic device for an hour each day, five days per week, after several weeks, the robot group have recovery of a brain and arm movement ability. To improve the movement ability and movement practice stimulating, the robotic make equivalent to computer mouse. Due to these features of the robotic device, it becomes necessary. The aim of this thesis work, the focus is placed on the study of robot therapy to improve physical or cognitive function for persons who becomes unable to move our hands. However this device can provide the hand by necessary movement.

In this study a manipulator for the rehabilitation for human arm is proposed as a 5-DoF 3RRR-(RR) mechanism.

The selection of this structure has some advantages to the other similar parallel manipulators such as;

- Its subspace includes both planar and spherical movements which are needed for the rehabilitation of both hand and wrist.
- Parallel structure can carry more loads and don’t need to carry all motors.

- 3 legs are optimum for this kind of subspace so it is simpler in structure and less in potential interference than same 5-DOF PM with 5 or 4 active legs.
- Its workspace is larger than that of 5-DOF with 5 active legs PM.

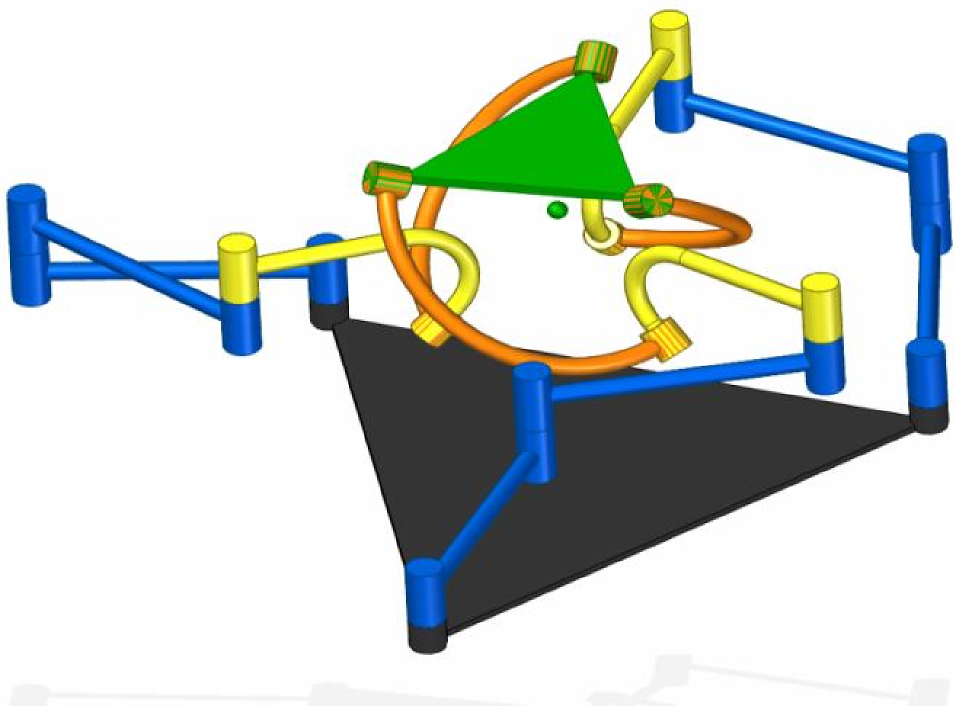


Figure 15 5 DoF over –constrained parallel manipulator

In chapter 2 the kinematics of the Over-constrained parallel manipulators are investigated, including the inverse kinematics analysis to obtain the values of the actuated joints with respect to the end-effector pose. Where, inverse kinematics of the manipulator is described by separating the system to its sub-subspaces and relating with an intersecting joint.

In chapter 3 the workspace is analyzed and optimum values for the pre-described position and orientation range of the arm are found. Discretization method is applied for workspace synthesis along with and algorithm.

In chapter 4 dynamic Analysis of the overconstrained manipulator is analyzed by using the principle of virtual work method. Jacobian matrices are derived from the constraint equations of two subspaces and merged in to one.

CHAPTER 2

KINEMATIC MODELING

The kinematic model of the manipulator is used to describe the motion of the end – effector and perform a controlled task of corresponding to the base frame. Kinematics is also used to intend to identify the real parameters which describe the actual position and orientation of the end effector with respect to the base frame. The kinematics model investigate the analytical relations between the values of the actuated joints (input variables) and the location of the moving platform (output variables). The inverse kinematics problem, deal with finding the set of joint variables that correspond to the desired configuration of the moving platform. This chapter deals with finding the inverse kinematics of an overconstrained manipulator for rehabilitation purposes.

2.1 Geometry of the Manipulator

The selected parallel manipulator is shown in Figure(16). This manipulator is composed of a moving platform attached to the base by three kinematic independent chains. The point O represented the center of the reference coordinate system O _xyz.

Each limb contains five joints where first three from the ground have parallel axes and later two joints axes are intersecting in a point P. The mechanism needs five actuators thus one motor is attached to the base revolute joint of the first leg ($\varphi 1$) and two motors are attached to first two revolute joints from the ground on the second leg ($\varphi 2$, $\varphi 3$). For the third leg same procedure is carried with two motors ($\varphi 4$, $\varphi 5$).

Manipulator is designed symmetrical. To supply symmetry the distances between base joints are selected equal and also angle between platforms joints are selected equal. All three legs have identical architectures, first two link lengths are defined as a_i and b_i where i is the index for the limb. Connection between 3^{rd} and 4^{th} joint is described by distance A_iP as r_i and angle between the axis of the 4th joint and A_iP is $\alpha_{1,i}$. Angle between 4th and 5th joint is described as $\alpha_{2,i}$. orientation of the platform joint is described by δ_i.

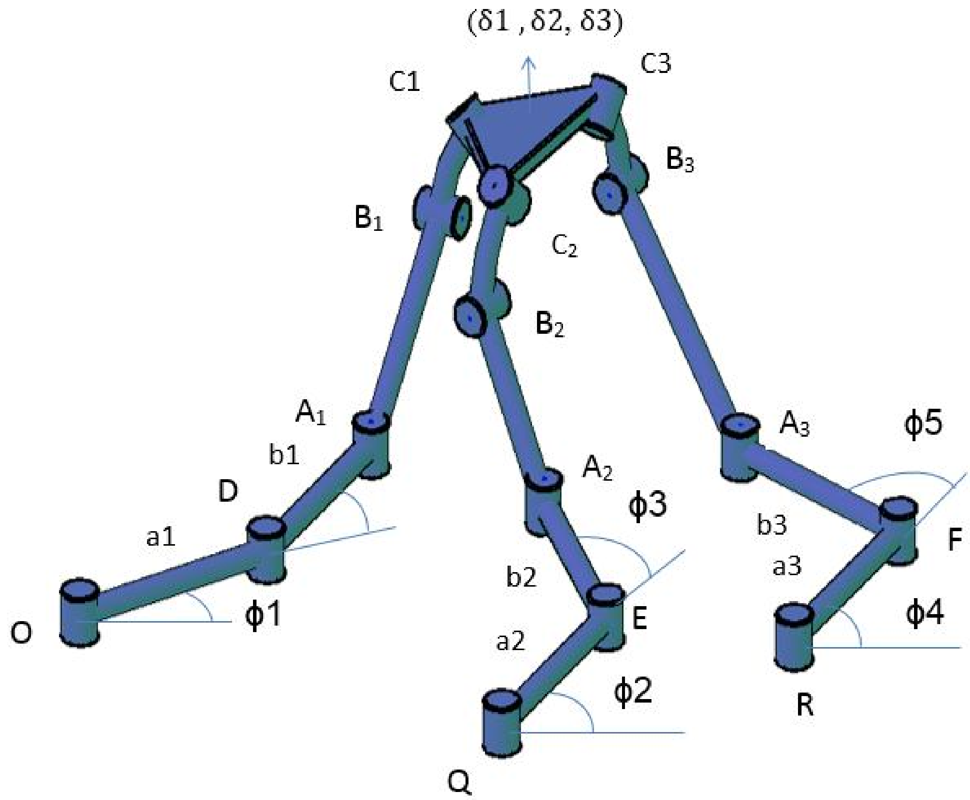


Figure 16 The Geometry of the Manipulator

2.2 Methodology (Inverse kinematics)

The kinematics model investigate the analytical relations between the values of the actuated joints (input variables) and the location of the moving platform (output variables). This section dealt with finding the inverse kinematics to solve the values of the actuated joints with respect to the end-effector pose.

The inverse kinematics model consists in finding the value of the joints displacements with respect to configuration of the platform. The inverse kinematics is essential for control of location of the platform of parallel robots and an urge for the analysis or synthesis of the workspace of the manipulator.

For designed manipulator 5 degree of freedom with 3 legs the desired position and orientation of the output link (x , y , $\delta 1$, $\delta 2$, $\delta 3$) is given and the problem is finding the required values of the input actuated joints ($\varphi 1$, $\varphi 2$, $\varphi 3$, $\varphi 4$, $\varphi 5$).

To solve inverse kinematics, the manipulator will be divided in to two sub-manipulators in subspace $\lambda=3$ with the help of three imaginary joints placed at the intersection of platform joints with a direction parallel to base joints as shown in figure 3a. The upper part will be a 3 Dof -3(RRR) spherical manipulator where input axes are coaxial as shown in figure 3b. The lower part wil be a redundant 5 Dof - 3 RRRR manipulator as shown in figure 3c.

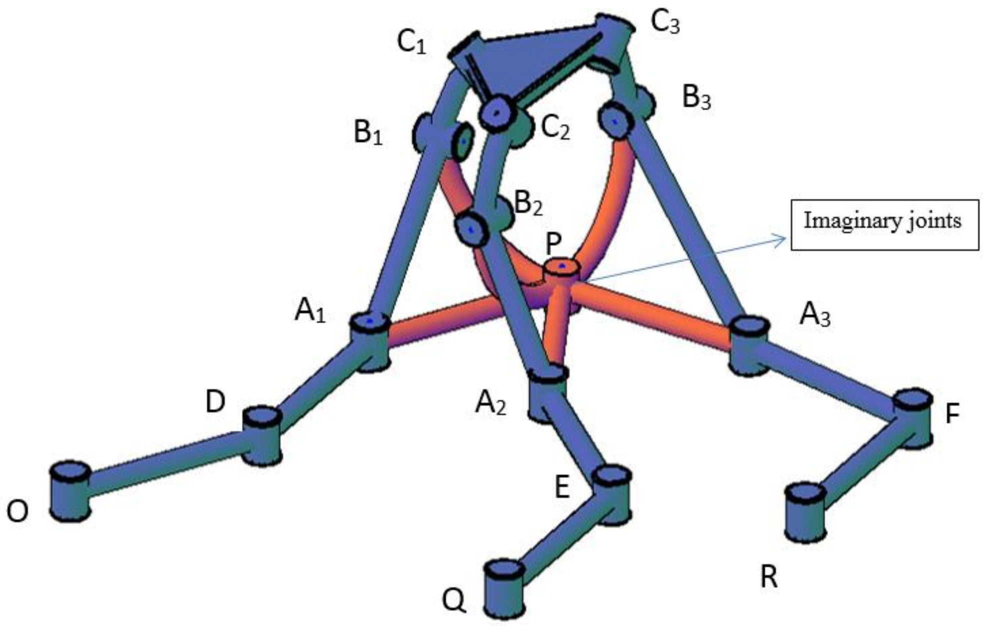


Figure 17 (a)The geometric of manipulator with imaginary joints

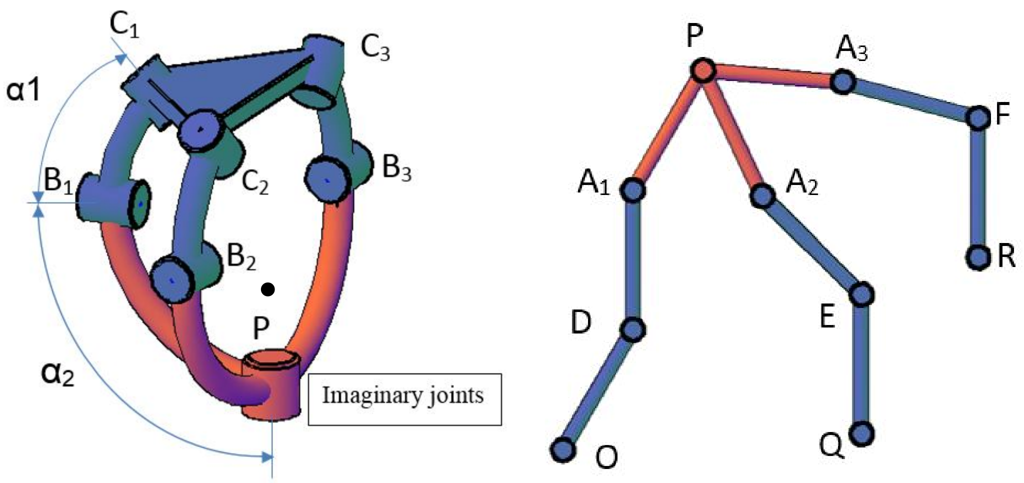


Figure 17 (b) The upper spherical mechanism (c)The lower planar mechanism

The orientation of the platform is either given as a 3x3 matrix or calculated using angles δz, δy and δx as the rotations around z, y, x in Euler angles.

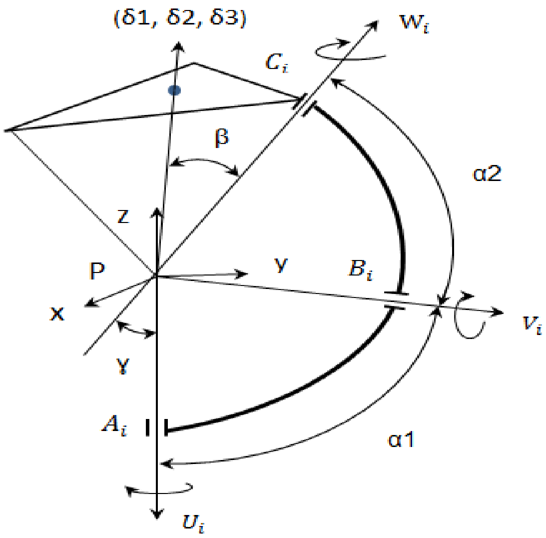


Figure 18 The vectors of spherical part

$$\delta = \begin{pmatrix} \mathrm{Cos}[\delta 1] & -\mathrm{Sin}[\delta 1] & 0 \\ \mathrm{Sin}[\delta 1] & \mathrm{Cos}[\delta 1] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \mathrm{Cos}[\delta 2] & 0 & \mathrm{Sin}[\delta 2] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[\delta 2] & 0 & \mathrm{Cos}[\delta 2] \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\delta 3] & -\mathrm{Sin}[\delta 3] \\ 0 & \mathrm{Sin}[\delta 3] & \mathrm{Cos}[\delta 3] \end{pmatrix}$$

As the orientation of the platform is known orientation of the axes of each joint on the platform as shown in Fig. 18 which are defined by unit vectors $w_i = \{w_{x,i}, w_{y,i}, w_{z,i}\}$ can be calculated by using Eq. (2.1).

$$w_i = \begin{pmatrix} wx_i \\ wy_i \\ wz_i \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[_i] & -\mathrm{Sin}[_i] & 0 \\ \mathrm{Sin}[_i] & \mathrm{Cos}[_i] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\beta_i] & -\mathrm{Sin}[\beta_i] \\ 0 & \mathrm{Sin}[\beta_i] & \mathrm{Cos}[\beta_i] \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad (2.1)$$

Where β_i is the orientation with respect to z axis for each joint around x axis and γ_i is the orientation around z axis for each joint for i=1,2,3.

Now for each leg, orientation of each joint on the platform $W_i = \{ Wxi , Wyi , Wzi \}$ with respect to ground can be calculated with respect to rotations (δ1 , δ2, δ3) as shown in Eq. (2. 2) for i=1,2,3

$$W_i = \delta \cdot w_i \qquad (2.2)$$

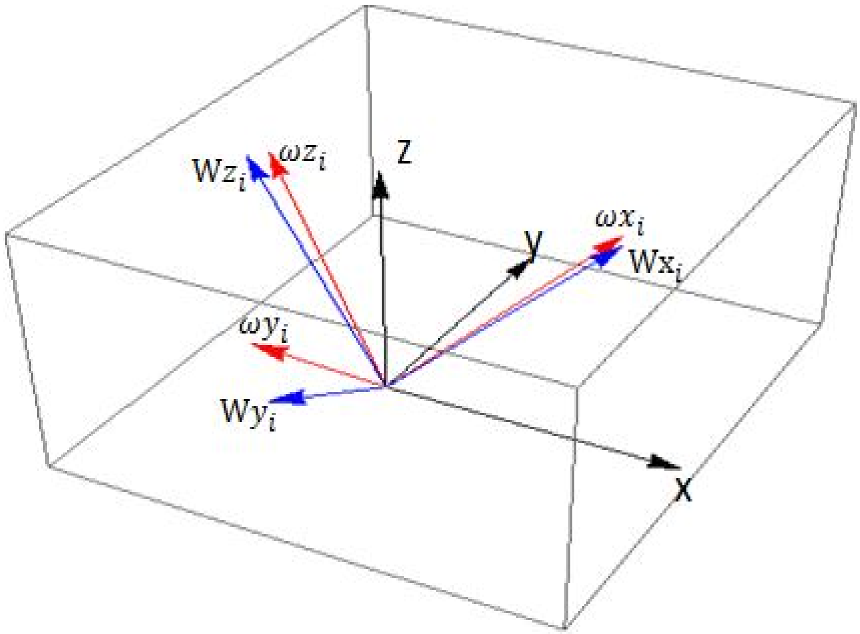


Figure 19 The orientations of the platform and platform joints in space

According to the structure of mechanism in spherical part the orientation of joints on platform W_i is equal to the orientation of the coordinate system at point P to the W_i using joint angles θ1,i and θ2,i and link angles α1,i and α2,i.

$$W_i = R_{\theta_{1,i},z} \cdot R_{\alpha_{1,i},z} \cdot R_{\theta_{2,i},z} \cdot R_{\alpha_{2,i},z} \cdot [0,0,1]^T \quad (2.3)$$

That leads:

$$\cos[\alpha 2]\sin[\alpha 1]\sin[\theta 1_i] + \sin[\alpha 2](\cos[\alpha 1]\cos[\theta 2_i]\sin[\theta 1_i] + \cos[\theta 1_i]\sin[\theta 2_i]) = Wxi \quad (2.4)$$

$$-\cos[\alpha 2]\cos[\theta 1_i] + \sin[\alpha 1]\sin[\alpha 2](-\cos[\alpha 1]\cos[\theta 1_i]\cos[\theta 2_i] + \sin[\theta 1_i]\sin[\theta 2_i]) = Wyi \quad (2.5)$$

$$\cos[\alpha 1]\cos[\alpha 2] - \cos[\theta 2]\sin[\alpha 1]\sin[\alpha 2] = Wzi \quad (2.6)$$

Solving equations (2.4) and (2.5) for Cos$[\theta 2_i]$ and Sin$[\theta 2_i]$ yields

$$\cos[\theta 2_i] = \csc[\alpha 2]\sec[\alpha 1](Wyi\text{Cos}[\theta 1_i] + Wxi\text{Sin}[\theta 1_i]) - \cot[\alpha 2]\tan[\alpha 1] \quad (2.7)$$

$$\sin[\theta 2_i] = \csc[\alpha 2](Wxi\ \text{Cos}[\theta 1_i] + Wyi\ \text{Sin}[\theta 1_i]) \quad (2.8)$$

we can eliminate $\theta 2_i$ by substitute equations (2.7), (2.8) in equation (2.6) we obtain

$$\cos[\alpha 1]\cos[\alpha 2] - \sin[\alpha 1]\sin[\alpha 2](\csc[\alpha 2]\sec[\alpha 1](-Wyi\cos[\theta 1_i] + Wxi\sin[\theta 1_i]) - \cot[\alpha 2]\tan[\alpha 1]) = Wzi \quad (2.9)$$

substituting trigonometric identities:

$\cos[\theta 1_i] = \frac{1-t^2}{1+t^2}$, $\sin[\theta 1_i] = \frac{2t}{1+t^2}$ where t =tan[$\theta 1_i/2$] in equation (2.9)

We obtain:

$$\frac{\mathrm{Sec}[\alpha 1]((1+t^2)\mathrm{Cos}[\alpha 2]+(-2t\,Wxi+Wyi-t^2\,Wyi)\mathrm{Sin}[\alpha 1])}{1+t^2} = Wz \qquad (2.10)$$

We can solve equation (2.10) and obtain the value of t

$$t = \frac{\mathrm{Sin}[\alpha 1]Wx_i \pm \sqrt{\mathrm{Sin}[\alpha 1]^2 Wx_i^2 + \mathrm{Sin}[\alpha 1]^2 Wy_i^2 - (\mathrm{Cos}[\alpha 2] - 2\,\mathrm{Cos}[\alpha 1]Wz_i)^2}}{\mathrm{Cos}[\alpha 2] - \mathrm{Sin}[\alpha 1]Wy_i - 2\,\mathrm{Cos}[\alpha 1]Wz_i}$$

$\theta 1_i$ =2*Arctan [t]

We get two solution for $\theta 1_i$, for i=1,2,3

In the second step for the inverse kinematics of the planar manipulator the point (P={X,Y}) which is the position of the end-effector is given and $\theta 1_i$ which are found in the previous part represent the orientation for r_i around z axis. Thus the position of A_i can be found.

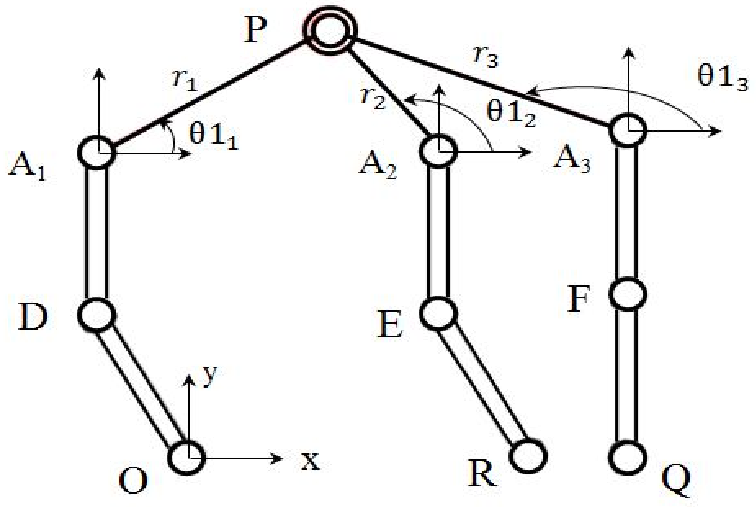


Figure 20 The planar part

Where $A_i = \{XA_i, YA_i\}$ represent the position of third joint in each legs as show in figure for i=1,2,3.

$XA_i = r_i*\mathrm{Cos}[\theta 1_i] + X$ (2.11)

$YA_i = r_i*\mathrm{Sin}[\theta 1_i] + Y$ (2.12)

Where:

r_i: represent the distance of link from point P to A_i

$\theta 1_i$: represent the rotation angle for this link about z axis.

From equation (2.11) and (2.12), we obtain the position of (XA_i, YA_i) for each leg. finding position (XA_i, YA_i) leads to find the input angles (φ1,φ2,φ3,φ4,φ5).

For the first leg as shown in Fig. 21.

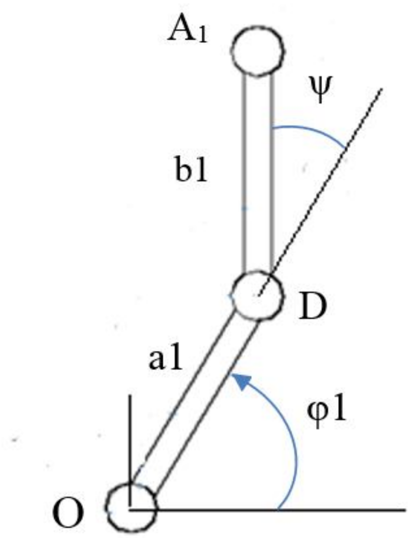


Figure 21 The first leg of manipulator

From the geometry of the leg we can write a vector- loop equation

$$OA_1 = OD + DA_1 \tag{2.13}$$

Expressing the vector loop equation in fixed coordinate frame gives

$$XA_1 = a1\cos[\varphi 1] + b1\cos[\varphi 1 + \psi 1] \tag{2.14}$$

$$YA_1 = a1\sin[\varphi 1] + b1\sin[\varphi 1 + \psi 1]$$

Since ψ is passive joint angle, It should be eliminated from the equation above by summing the squares of two equation in(2.2) yields

$$(XA_1 - a1\cos[\varphi 1])^2 - (b1\cos[\varphi 1 + \psi 1])^2 + (YA_1 - a1\sin[\varphi 1])^2 - (b1\sin[\varphi 1 + \psi 1])^2 = 0$$

By expanding and simplifying the equation yields

$$XA_1{}^2 + YA_1{}^2 - 2XA_1\, a_1\cos[\varphi 1] - 2YA_1\, b_1\sin[\varphi 1] + a_1{}^2 - b_1{}^2 = 0 \tag{2.15}$$

Equation (2.3) can be written as

$$e1\sin[\varphi 1] + e2\cos[\varphi 1] + e3 = 0 \tag{2.16}$$

where:

$$e1 = -2\, YA_1\, a1$$

$e2 = -2\, XA_1\, a1$

$e3 = XA^2 + YA^2 + a1^2 - b1^2$

Substituting the trigonometric identities:

$\cos[\varphi 1] = \frac{1-t^2}{1+t^2}$ and $\sin[\varphi 1] = \frac{2t}{1+t^2}$ where $t = \tan[\varphi 1/2]$

In equation (2.4), we obtain

$$e2 + e3 + 2e1t + (-e2 + e3)t^2 = 0 \tag{2.17}$$

By solving equation (2.17)

$$t = \frac{-e1 \pm \sqrt{e1^2 + e2^2 - e3^2}}{e3 - e2}$$

$$\varphi 1 = 2\mathrm{Arctan}\frac{-e1 \pm \sqrt{e1^2 + e2^2 - e3^2}}{e3 - e2} \tag{2.18}$$

from the equation (2.18), there are two solution of $\varphi 1$ and there for two configurations of leg 1.

[when equation (2.16)yields a double root ,the two links OB and BA are in a folded – back or fully stretched out configuration called the singular configuration. when equation (2.16) yields no real root , the specified location of moving platform in not reachable].

For the leg 2 which is shown in Fig. 22. The location of point A_2 is calculated from the second step and the problem is to find the joint angles (φ2,φ3).

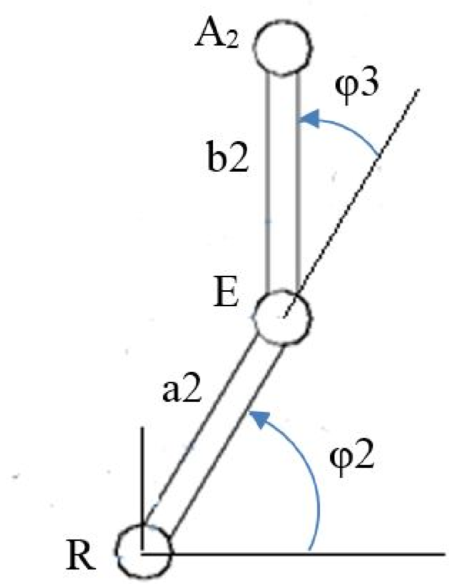


Figure 22 The second leg of manipulator

From the geometry of Figure (22) we can write a vector- loop equation

OA_2=OR+ RE +EA_2 (2.19)

OA_2 –OR=RE +EA_2

Expressing the vector loop equation in fixed coordinate frame gives

$$XA_2 - XR = a2 * \cos[\varphi 2] + b2 * \cos[\varphi 2 + \varphi 3] \quad (2.20)$$

$$YA_2 - YR = a2 * \sin[\varphi 2] + b2 * \sin[\varphi 2 + \varphi 3] \quad (2.21)$$

Since φ3 is active joint angle and shouldn't be eliminated.

From the figure22 we observe that the distance from point R to A_2 is independent of φ2, hence we can eliminate φ2 by summing the squares of equations (2.20) and (2.21) yields:

$$(XA_2 - XR)^2 + (YA_2 - YR)^2 = a_2{}^2 + b_2{}^2 + 2a2b2\mathrm{Cos}[\varphi 3] \quad (2.22)$$

Solving equation(2.22), we obtain

$$\varphi 3 = \mathrm{Cos}^{-1} k \quad (2.23)$$

where $k = \frac{(XA_2 - XR)^2 + (YA_2 - YR)^2 - b2^2 - a2^2}{2b2a2}$

[equation (2.23) yields (1) one double root if |k|=1, (2) two real roots if $|k| < 1$ and (3) no real roots if $|k| > 1$. In general , if $|k| < 1$, $\varphi 3 = \varphi 3$ is a solution, $\varphi 3 = -\varphi 3$ is also solution where $\pi \geq \varphi 3 \geq 0$. We call $\varphi 3 = \varphi 3$ the elbow-down solution and $\varphi 3 = -\varphi 3$ elbow-up solution. If |k|=1, the arm is in a fully stretched or folded configuration . If $|k| > 1$, the position is not reachable].

Corresponding to each φ3 ,we can find φ2 by expanding equations (2.20) and (2.21) as follow:

$$XA_2 - XR = a2 + b2 \cos[\varphi 3]\cos[\varphi 2] - (b2 \sin[\varphi 3]) \sin[\varphi 2] \quad (2.24)$$

$$YA_2 - YR = b2 \sin[\varphi 3] \cos[\varphi 2] + (a2 + b2 \cos[\varphi 3]) \sin[\varphi 2] \quad (2.25)$$

Solving equations (2.24) and (2.25) for Cos[φ2] and Sin[φ2], yields:

$$\sin[\varphi 2] = (-(XA2 - XR)b2 \sin[\varphi 3] + (YA2 - YR)(a2 + b2 \cos[\varphi 3]))/\rho$$

$$\cos[\varphi 2] = ((XA2 - XR)(a2 + b2 \cos[\varphi 3]) + (YA2 - YR) b2 \sin[\varphi 3])/\rho$$

Where $\rho = b2^2 + a2^2 + 2b\,2a2 \cos[\varphi 3]$

Hence, corresponding to each φ3, we obtain a unique solution for φ2:

$$\varphi 2 = \mathrm{Atan2}(\mathrm{Sin}[\varphi 2], \mathrm{Cos}[\varphi 2]) \quad (2.26)$$

From equation (2.26)we can obtain unique solution for φ2 by using the function Atan2(x,y) In a computer program. However, these equation yields real or complex solution. A complex solution mean the end _effector position that is not reachable.

The third leg has same form with the second leg and same procedure can be applied to find φ4 and φ5 for known A_3 position and lengths a3,b3,r3.

CHAPTER 3

WORKSPACE ANALYSIS

Determining the workspace of a general manipulator has received considerable attention and studied by many researchers in recent years, because of its significance in determining functionality and performance of the manipulators and obtaining the optimal design.

The parallel manipulators have some advantages, such as high load-carrying capacities, high accuracy, high stiffness and low inertia .However, there also have some disadvantages associated with parallel manipulators. One of the main drawbacks of parallel manipulator is a limited workspace and poor dexterity because of the additional constraints imposed by the closed kinematic chains of the parallel manipulator. Additionally, the performance of parallel manipulators depends on the design parameters and configuration of the end-effector that leads to difficult design of the manipulator. Solving these problems of parallel manipulators in the design process gives good results for manipulation of objects.

The workspace of parallel manipulator is limited by three different factors such as links interference, mechanical limited on the passive joints and limitation of actuators. The range of the link length and the effect of these constraints limiting on the workspace was studied by Merlet [37] and an algorithm enabling to determine the boundary of the end effector which its rotation around a fixed point for 6 DoF parallel manipulator is presented and found that the workspace of the Parallel manipulator can be defined as the set of configurations of the end-effector which can be reached by some choice architectural parameter values.

The researchers working on the parallel manipulators always wish to design them with the maximal workspace that can be used. Thus, the determination boundary of the reachable and dexterity workspaces are very important in the selection of the appropriate mechanism and calculate optimal design of the manipulators correspond to the sizes and shapes of reachable workspaces and prescribed the dexterity.

As reported in Merlet [1], there are three classes for determining the boundary workspace of parallel manipulators such as geometrical methods, discretization methods, and numerical methods.

A geometrical method was proposed by weng [38], where the boundary of the workspace is defined by a singularity zone and depend on structural limited of the architecture . Gosselin [39] suppose a method represented for finding the boundaries of the workspace of each kinematic chain and finding the intersection between the boundaries and employing this technique for determining the reachable workspace of a 6-DoF parallel manipulator. Another geometrical method by Gallant [26] was proposed to determine the geometry of the dexterous workspace of two architectures n-RRRR and n-RRPR. Also geometrical approach is applied by Merlet [40], Liu [41], and Zhang [42], etc. The disadvantage of this method is an ignoring the mechanical interference by links and other physical constraints.

In the discretization method approach, the workspace described with a regular grid of nodes. Each node tested to see for the effects on the workspace. The boundary of the workspace represented by the discretization of the pose parameters and the precision of the boundary depends on the sampling steps. This method also has some disadvantage such as limit on the accuracy due to the workspace possesses voids and time consuming. This approach was used by Stamper [43] and Arai [44] in determining the workspace boundary of the 3-DoF and 6-DoF parallel manipulators respectively.

The numerical methods approach was suggested by Jo [45] where this method transforms the inequalities that are imposed by the constraints on the articular coordinates into equalities by introducing extra variables by derived of Jacobian matrix. The generalized coordinates are considered by a vector X, articular coordinates by a vector Θ and the variables considered by a vector W and all of these vectors are represented by a vector q. The workspace boundary is obtained as a set of vector q.

This method was improved by Shaoping [29] to prescribe the workspace and obtain optimal design of spherical parallel manipulators (SPM) which including architecture of parameters and link dimensions.
Even if the numerical methods were proposed by many researchers, this method has some drawbacks such us, control of other constraints that lead to a Jacobian become a large and quite difficult to solve and efficient boundary search technique. Where, the boundary search technique plays main role in this method. Novel idea was presented by Z. Wang and Shiming Ji [28] for determining the dexterous and reachable workspace of a general parallel manipulators and calculating their boundary by using a new numerical method, called ‘‘stratified boundary search technique’’.

There are different types of workspaces. Constant orientation workspace or translation workspace is a set of locations of the end effector can be reached with the constant orientation. Orientation workspace is a set of location of the end effector by rotations around a fixed point. Reachable workspace or maximal workspace is a set of locations of the end effector that can be reached with at least one orientation. Inclusive orientation workspace is a set of locations of the end effector that can be reached with at least one orientation among ranges of orientation parameters. Dexterous workspace is a set of location of the end effector which all orientations are possible. Total orientation workspace is a set of locations of the end effector that can be reached with all orientations among ranges of orientation parameters. In this study reachable workspace of the 5RRR-RR manipulator is examined. Because of we need maximum workspace with all orientations are possible for optimal design.

3.1. Method for Describing Reachable Workspace

In order to evaluate the reachable workspace of the mechanism, we apply the discretization procedure implemented in a program. In this approach, the workspace of the mechanism is generated by sampling joint angles with the length links and the boundary is evaluated by the resulting set of the end effector motions. To study the changes of the workspace with changes of the architectural parameters, several plots of the workspace volume are presented. The shape and the size of the workspace is dependent on the particular set of architectural parameters of the mechanism which are chosen.

In the present work the effect of the architectural parameters on the workspace of variations is studied. In this work the mechanism analyzed is a 5-DOF mechanism, the workspace is defined with three rotations δ1, δ2, δ3 and two translation with x, y. The procedure starts with selected the desired orientation (δ1, δ2, δ3) and translation (x , y).

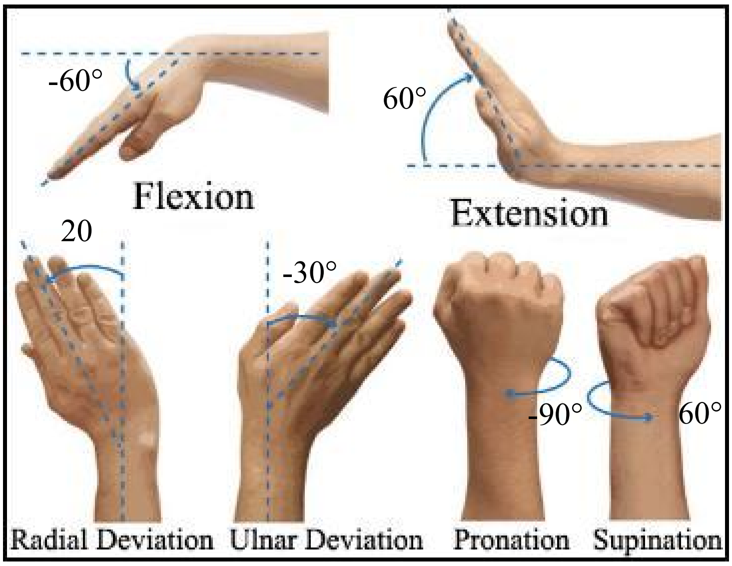


Figure 23 Wrist movement at the joint is called extension or flexion

As mentioned in chapter 1 the mechanism is proposed as a rehabilitation manipulator. This manipulator will assist persons who have a disability to move their hands. To improve physical or cognitive function the patient sits across from the device, with the weaker hand attached to the end-effecter as show in Fig. 24 and repeat the motions multiple times by the help of the manipulator.

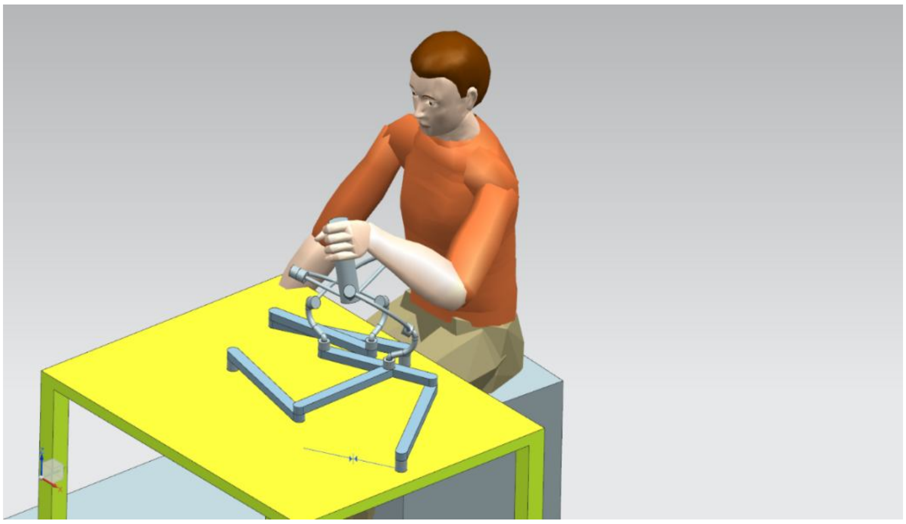


Figure 24 The human hand with mechanism

We can select the optimal values of orientations of end effector (δ1, δ2, δ3) for Wrist motion

δ1= from -30 Degree to 20 Degree

δ2= from -90 Degree to 60 Degree

δ3= from -60 Degree to 60 Degree

and the dimensions of (x ,y) should be described by at least 35 cm. Also to analyze the boundary of workspace, we should separate the mechanism in two parts:

The upper part as spherical parallel manipulator with three legs which ith leg consists of two curved links (α1,α2) connected by three revolute joints which extension of joints are intersected in point (p) and equilateral triangles of moving platform. The orientation of the platform is described by using angles (δ1, δ2, δ3). Inverse kinematics analysis in previous section and the equations are used for the calculation of the workspace . If the solutions of inverse kinematics are imaginary, the workspace cannot be reachable.

By solving this equations as mentioned in Chapter 2 we get two solution of $(\theta 1_i)$ for each legs , where i=1,2,3

$$\theta 1_i = 2\text{Arctan}\left[\frac{\sin[\alpha 1]Wx_i \pm \sqrt{\sin[\alpha 1]^2 Wx_i^2 + \sin[\alpha 1]^2 Wy_i^2 - (\cos[\alpha 2] - 2\cos[\alpha 1]Wz_i)^2}}{\cos[\alpha 2] - \sin[\alpha 1]Wy_i - 2\cos[\alpha 1]Wz_i}\right]$$

And one solution of $(\theta 2_i)$

$$\theta 2_i = \frac{-Wz_1 + \cos[\alpha 1]\cos[\alpha 2]}{\sin[\alpha 1]\sin[\alpha 2]}$$

To get real solution of$(\theta1_i, \theta2_i)$ that achieving real workspace ,we should achieve these conditions

$$\sin[\alpha1]^2 Wx_i^2 + \sin[\alpha1]^2 Wy_i^2 - (\cos[\alpha2] - 2\cos[\alpha1]Wz_i)^2 \geq 0 \tag{3.1}$$

$$-1 \geq \frac{-Wz_i+\cos[\alpha1]\cos[\alpha2]}{\sin[\alpha1]\sin[\alpha2]} \geq 1 \tag{3.2}$$

$(\theta1_i)$ and $(\theta2_i)$ are depended on the unit vectors of joints with orientation(δ1, δ2, δ3) and values of the curved links (α1,α2) as in equations in chapter
Therefore, the parameter can be changed to achieve the above conditions and the desired orientation of the end effector (δ1, δ2, δ3). We apply The discretization procedure implemented in a program to show the effect of changes of the architectural on the boundary of workspace for each smaller part of orientation(δ1, δ2, δ3) by use repetition factor (J),where we should select a large number of repetition to avoid a void .
After that values for (α1,α2) are selected that will give real solutions for the selected range of orientations δ1, δ2, δ3 and a useful range $(\theta1_i)$ is obtained that w used in the planar part of the mechanisms. The workspace is obtained as show in Fig. 25

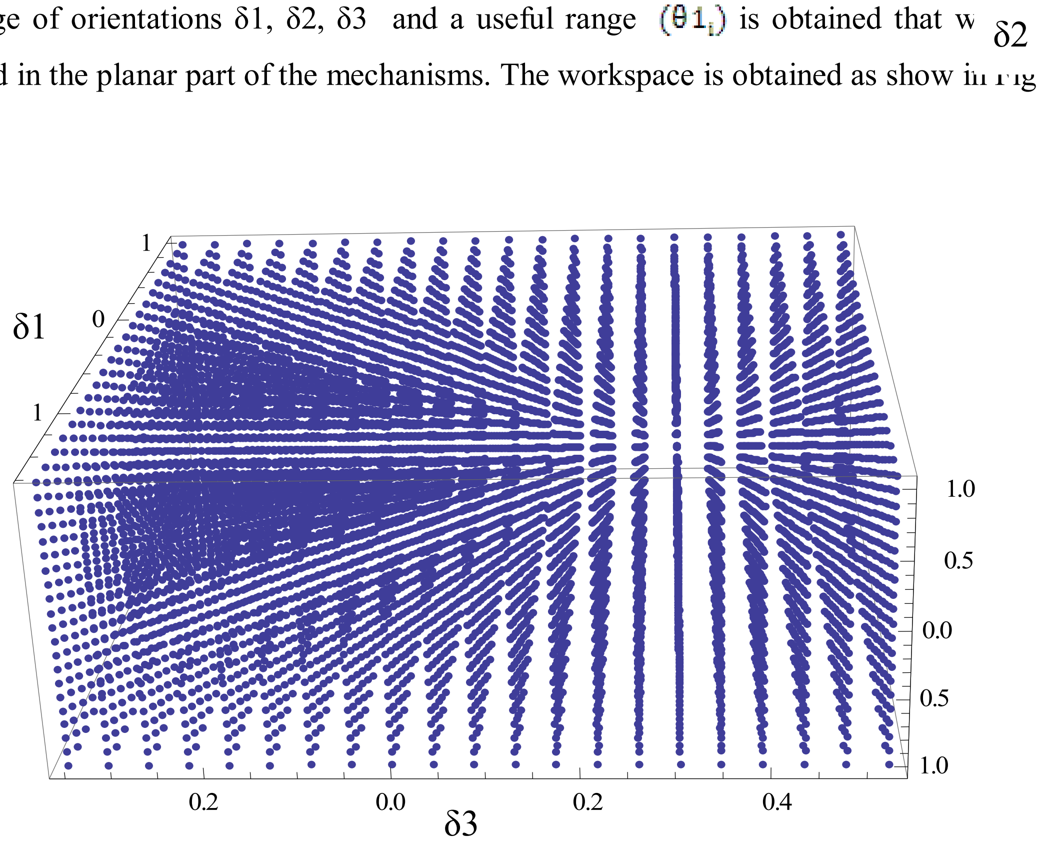


Figure 25 The boundary workspace of orientation (δ1, δ2, δ3)

After solving the spherical part of the manipulator the range of $\theta1_i$ is used as the orientation workspace of the planar part and described together with work space of x and y. Thus the optimal workspace of the planar part will be in the range of($x, y, \theta 1_i$). As mentioned earlier the needed range of x , y is 35 cm in each direction to achieve rehabilitation for the arm.

Solve inverse kinematics which are described in chapter 2 for the legs we get actuator angles as;

$$\varphi 1 = 2\mathrm{Arctan}\left[\frac{2\,YA_1\ a1 \pm \sqrt{4a1^2 XA_1{}^2 + 4a1^2 YA_1{}^2 - (a1^2 - b1^2 + XA_1{}^2 + YA_1{}^2)^2}}{a1^2 - b1^2 + XA_1{}^2 + YA_1{}^2 + 2\,XA_1\ a1}\right]$$

$$\varphi 2 = \mathrm{Atan2}(\mathrm{Sin}[\varphi 2], \mathrm{Cos}[\varphi 2])$$

Where:

$$\sin[\varphi 2] = (-(XA2 - XR)b2\ \mathrm{Sin}[\varphi 3] + (YA2 - YR)\ (a2 + b2\ \mathrm{Cos}[\varphi 3]))/\rho$$

$$\cos[\varphi 2] = ((XA2 - XR)\ (a2 + b2\ \mathrm{Cos}[\varphi 3]) + (YA2 - YR)\ b2\ \mathrm{Sin}[\varphi 3])/\rho$$

Where $\rho = b2^2 + a2^2 + 2b2\ a2\ \mathrm{Cos}[\varphi 3]$

$$\varphi 3 = \mathrm{Cos}^{-1}\left[\frac{(XA_2 - XR)^2 + (YA_2 - YR)^2 - b2^2 - a2^2}{2\,b2\,a2}\right]$$

$$\varphi 4 = \mathrm{Atan2}(\mathrm{Sin}[\varphi 5], \mathrm{Cos}[\varphi 5])$$

$$\sin[\varphi 4] = (-(XA3 - XQ)b3\ \sin[\varphi 5]) + (YA3 - YQ)\ (a3 + b3\ \cos[\varphi 5]))/\rho$$

$$\cos[\varphi 2] = (XA3 - XQ)\ (a3 + b3\ \cos[\varphi 5]) + (YA3 - YQ)\ b3\ \sin[\varphi 5])/\rho$$

Where $\rho = b3^2 + a3^2 + 2b3\ a3\ \cos[\varphi 5]$

$$\varphi 5 = \mathrm{Cos}^{-1}\left[\frac{(XA_3 - XQ)^2 + (YA_3 - YQ)^2 - b3^2 - a3^2}{2\,b3\,a3}\right]$$

To get real solution of $(\varphi 1, \varphi 2, \varphi 3, \varphi 4, \varphi 5)$ must be get to achieve the workspace thus conditions will be:

$$4a1^2 xa^2 + 4a1^2 ya^2 - (a1^2 - b1^2 + xa^2 + ya^2)^2 \geq 0$$

$$1 \geq \frac{(XA_2 - XR)^2 + (YA_2 - YR)^2 - b2^2 - a2^2}{2\,b2\,a2} \geq -1$$

$$1 \geq \frac{(XA_3 - XQ)^2 + (YA_3 - YQ)^2 - b3^2 - a3^2}{2\,b3\,a3} \geq -1$$

$$-a2^2(YA_2 - YR)^2\big(a2^4 + (-b2^2 + (XA_2 - XR)^2 + (YA_2 - YR)^2)^2 - 2a2^2(b2^2 + (XA_2 - XR)^2 + (YA_2 - YR)^2)\big) \geq 0$$

$$-a3^2(YA_3 - YQ)^2\big(a3^4 + (-b3^2 + (XA_3 - XQ)^2 + (YA_3 - YQ)^2)^2 - 2a3^2(b3^2 + (XA_3 - XQ)^2 + (YA_3 - YQ)^2)\big) \geq 0$$

The translations (x , y) depended on the position of A_i , the length r_i and the angels value $\theta 1_i$

For each leg

$XA_i = r_i * \cos[\theta 1_i] + X$

$YA_i = r_i * \sin[\theta 1_i] + Y$

The ranges of $\theta 1_i$ is presented in spherical part and the position of A_i depended on the set of architectural for each legs.

Where:

A_1 depend on length links (a1 , b1 , r_1) and input angle ($\varphi 1$)

A_2 depend on length links (a2, b2, r_2) and input angles ($\varphi 2$, $\varphi 3$)

A_3 depend on length links (a3 , b3 , r_3) and input angles ($\varphi 4$, $\varphi 5$)

Therefore, the boundary of workspace of each leg is dependent on the changes of its parameters and achieving above constraint equations.

The intersection of these boundaries represent the actual workspace coupled with translation(x , y) and orientation (δ1, δ2, δ3).

Changing the link dimensions or the joint limitations has effect on the boundary workspace. With several trials of these values finally one solution is found for achieving the described workspace.

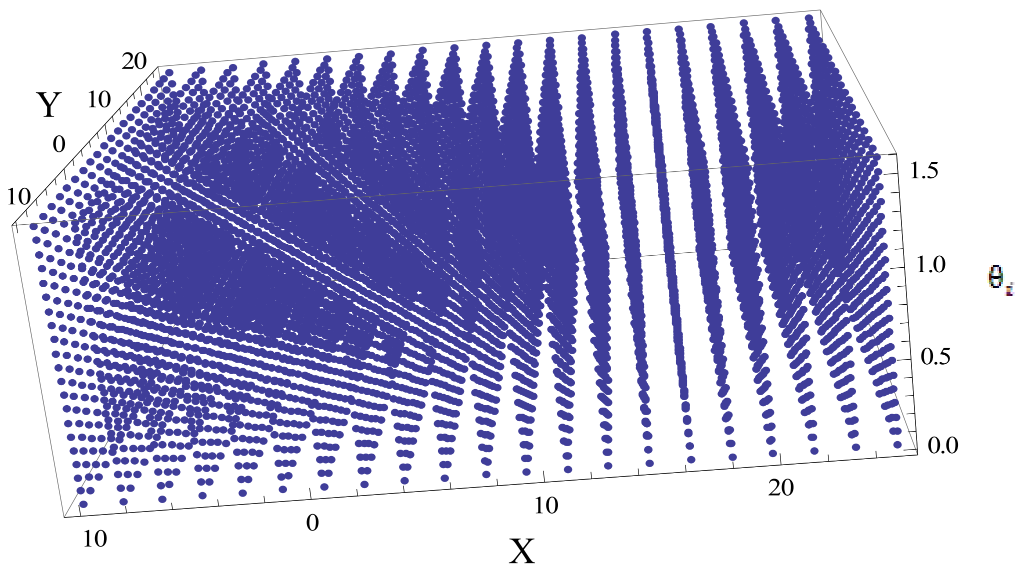
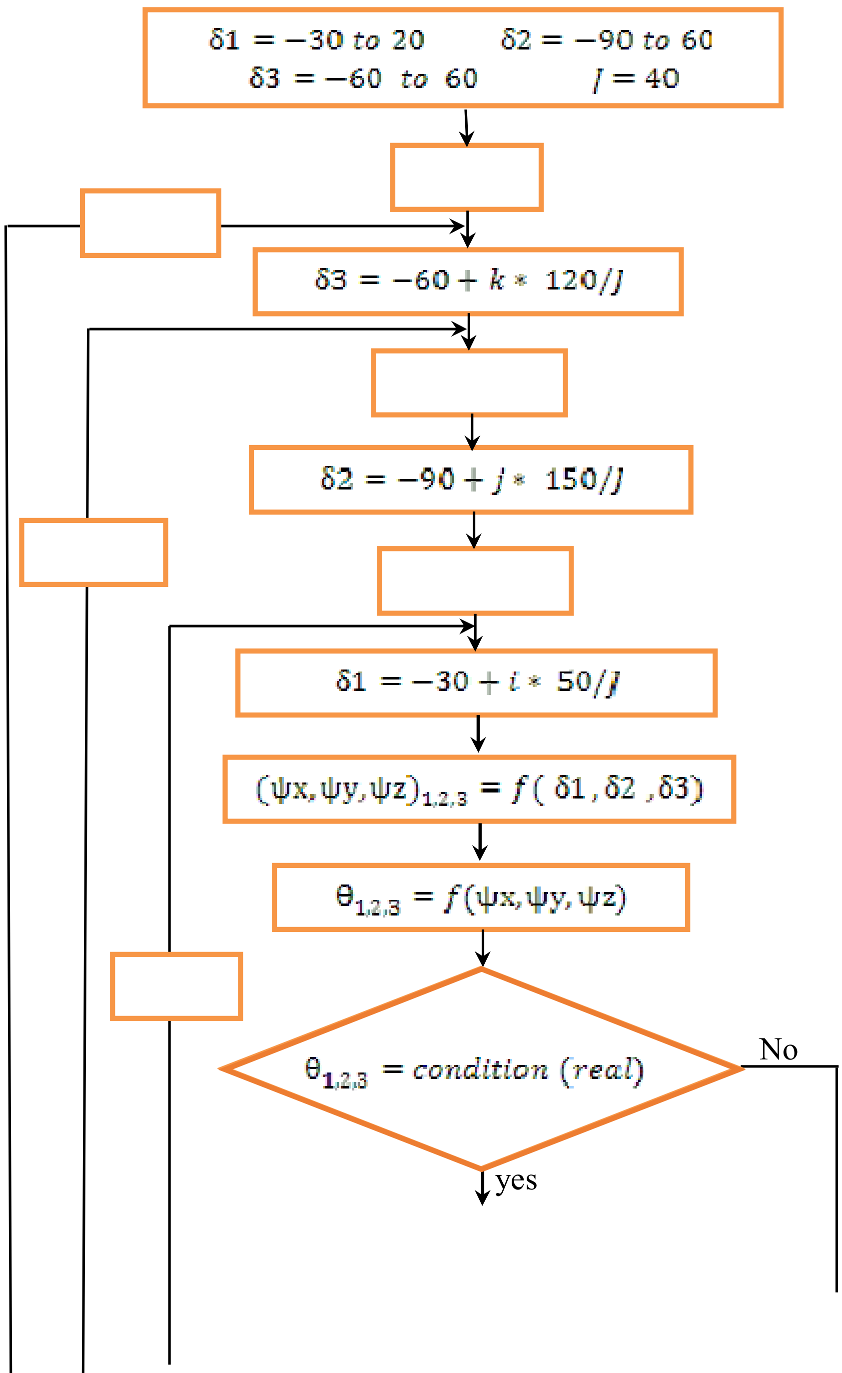


Figure 26 The translation of platform with orientation θi

As a result, we can select optimal values for parameters coupled with the desired workspace with taking into consideration the shape and the size of the mechanism which is stable and appropriate with dynamics.

3.2. Implemented Algorithm

In this thesis, we are interested the changed of the workspace with the different design parameters. This interest need to represent an algorithm that can obtain the boundary workspace for a given set of architectural. The procedure start by the desired orientation (δ1, δ2, δ3) and describing the procedure to find orientation of imaginary links Fig.27 then in the second part using these found values and given x, y movements all actuator rotations are found Fig.28.



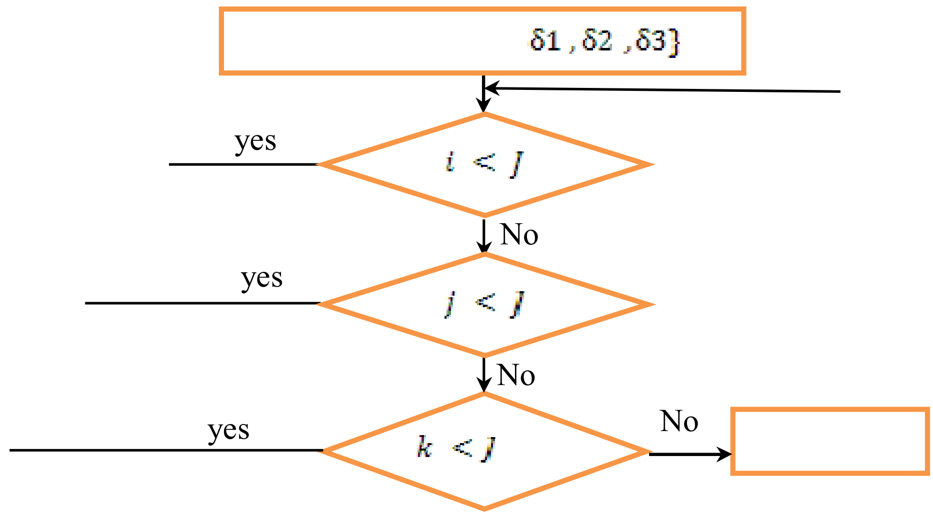
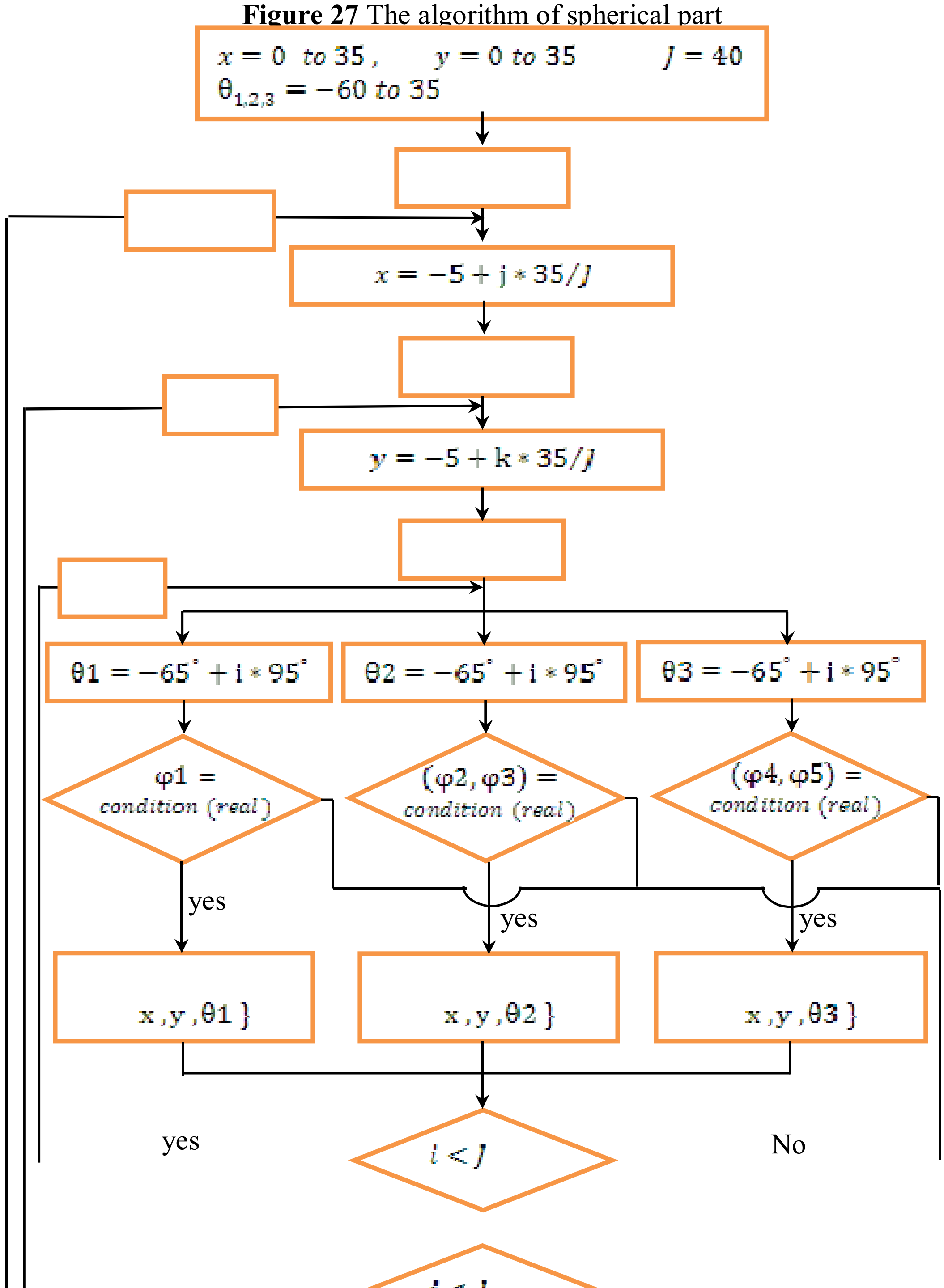


Figure 27 The algorithm of spherical part



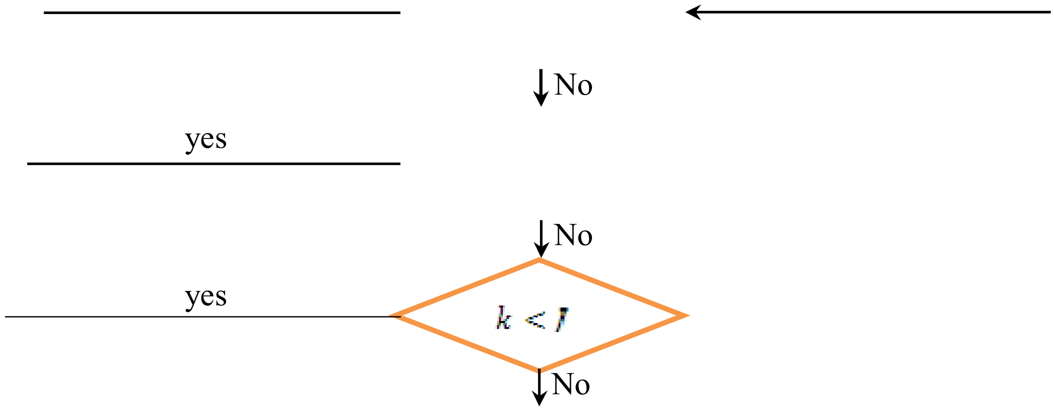


Figure 28 The algorithm of planar part

The results are shown in table 1 and table 2.

Table 1. The design variables of spherical part

α1(deg)	α2(deg)	βi(deg)	□i(deg)	θ1(deg)	θ 2 (deg)	θ 3(deg)
91	91	5	120	-150 to150	-180 to 0	-60 to 60

Table 2. The design variables of planar part

ai	bi	ri	xb	yb	xc	yc
22	22	5	20	0	10	20

CHAPTER 4

DYNAMICS MODELING

The dynamics of robot deal with loads, masses, acceleration and inertia. It is an essential part in expectation of the behavior of mechanical systems and realization their best performance.

In the dynamics to accelerate the mass we need to spend a force on it.

$$F= m .a \tag{4.1}$$

Where F is applied force an a is acceleration of the mass.

Similarly in a rotating body we need to exert a torque to cause an angular acceleration.

$$T=I.\alpha \tag{4.2}$$

To move and accelerate a robot's links to desired velocity and acceleration, it is necessary to have actuators enough force and torques. The links must be moved with respect to desired positional accuracy. Therefore, it is necessary to calculate how strong each actuator must be. According to Equations (4.1) and (4.2), we can calculate the required torque of actuators corresponding to the external loads.

However, the dynamical analysis of parallel manipulator is quite complicated due to the presence of multiple closed-loop chains and kinematic constraints. There are two types of dynamical problems. First, the forward dynamics problem, the applied actuator forces and/or moments of a robot are given and the aim to find the response arm. Second, the inverse dynamics problem, the desired generate a trajectory of the manipulator is given and the aims to find the required actuator forces and/or moments.

To solve the dynamic equations, there are three main approaches of the dynamical equations have been proposed: Newton–Euler formalism, the principle of virtual work and Lagrangian multiplier method. All of these three methods have advantages and disadvantages.

Newton–Euler formalism is applied by several researchers [46-48] where the disadvantage is that body of a mechanism should be written which gives large number of equations. Therefore, this approach has poor computational efficiency. The advantage this approach, the forces of the actuators and the reactions in the joints are calculated from simple equilibrium equations of the platform and the limbs. Several researchers applied this method [46-48].

The benefits of the Lagrangian multipliers method is that all forces, reaction and moments are not included in the equations. Nevertheless because of the numerous constraints imposed by the closed loops deriving the equations of motion in terms of a set of independent generalized coordinates is a difficult task. To simplify the task coordinates including input and output variables with a set of Lagrangian multipliers must be described. This approach was considered by different researchers [49-50]

The principle of virtual work is the active approach for dynamic equations of parallel manipulators where, the constraint forces and moment at the joints are eliminated from the motion equations. The coordinates of the end-effector could be chosen as the generalized ones. Several researchers present this approach [51-53].

In this chapter we investigate the dynamics of the 3(RR)RRR parallel manipulator using the principle of virtual work.

4.1 Principle of Virtual Work

Let $\mathbf{F}_i$ denote the sum of all applied and inertia forces about the center mass of links

$$\mathbf{F}_i = \begin{pmatrix} f_i \\ M_i \end{pmatrix} \tag{4.3}$$

f_i =resulting force (excluding the actuator force) exerted at the center of mass of link i + inertia force exerted at the center of mass of link i $(-m_i \dot{v}_i)$.

M_i =resulting moment (excluding the actuator force) exerted at the center of mass of link i + inertia moment exerted at the center of mass of link i.

Let F_p denote sum of all applied and inertia forces about the center of mass of the moving platform.

$$F_p = \begin{pmatrix} f_p \\ M_p \end{pmatrix} \tag{4.4}$$

f_p =Resulting force (excluding the actuator force) exerted at the center of mass of platform + inertia force exerted at the center of mass of platform $(-m_p \dot{v}_p)$

M_p =resulting moment (excluding the actuator force) exerted at the center of mass of platform + inertia moment exerted at the center of mass of platform m_p.

The virtual work done by the manipulator should be equal to zero($\delta u = 0$).

Thus virtual work of actuators forces on the platform and forces in the links can be written as:

$$\delta q^T \tau + \delta X_p^T F_p + \sum_i \delta X_i^T F_i = 0 \tag{4.5}$$

Where the summation denote all links of legs. The Eq.(4.5) is isolated the actuator torque from other applied forces for convenience of deivation.

The virtual displacement of actuated joints δq is related to virtual displacement of moving platform δX_p by Jacobian matrix (J_p).

$$\delta q = J_p \ \delta X_p \tag{4.6}$$

Also, the virtual displacement of actuated joints δX_i is related to virtual displacement of moving platform δX_p by Jacobian matrix (J_i).

$$\delta X_i = J_i \ \delta X_p \tag{4.7}$$

Substituting equations (4.6) and (4.7) in Eq. (4.5),we obtain:

$$\delta X_p^T \left(J_p^T \tau + F_p + \sum_i j_i^T F_i \right) = 0 \tag{4.8}$$

For any virtual displacement of the platform δX_p

$$J_p^T \tau + F_p + \sum_i j_i^T F_i = 0 \tag{4.9}$$

Thus:

$$\tau = -J_p^{-T}\left(F_p + \sum_i j_i^T F_i \right) \tag{4.10}$$

Eq (4.10) described the general form to find the actuated torque of parallel manipulator.

$$F_p = \begin{pmatrix} f_p \\ M_p \end{pmatrix} = \begin{pmatrix} f_{ex} + m_p g - m_p \dot{v}_p \\ M_{ex} + I_p \dot{\omega}_p - \omega_p \times (I_p \omega_p) \end{pmatrix}$$

Where f_{ex} denote the external force, M_{ex} external moment, $m_p$mass of platform I_p moment of inertia of platform $\omega_p$the angular velocity of platform.

$$F_i = \begin{pmatrix} f_i \\ M_i \end{pmatrix} = \begin{pmatrix} m_i g - m_i \dot{v}_p \\ I_i \dot{\omega}_i - \omega_i \times (I_i \omega_i) \end{pmatrix}$$

Where f_{ex} denote the external force $m_i$mass of platform I_i moment of inertia of platform $\omega_i$the angular velocity of platform.

In the name of simplicity and by looking the selected sizes of the manipulator and effecting forces in the rehabilitation , the masses of the platform and links are neglected. That leads:

$$F_p = \begin{pmatrix} f_{ex} \\ M_{ex} \end{pmatrix}$$

$$F_i = 0$$

Therefore, the equation of actuator torque in Eq.(4.10) can be written as

$$\tau = -J_p^{-T} F_p \tag{4.11}$$

To find Jacobian matrix of the mechanism, again the mechanism is separated as a spherical manipulator and a redundant planar manipulator part.

4.2 Jacobian (Velocity) Analysis of the Manipulator

The spherical part of this manipulator consist three identical legs ,which each leg consist of two curve links (α1 ,α2) and 3 (RRR), whose axes are coaxial to the unit vectors ui ,vi and wi. The orientation of the platform manipulator is described by (δ1, δ2, δ3) as show in fig. 29.

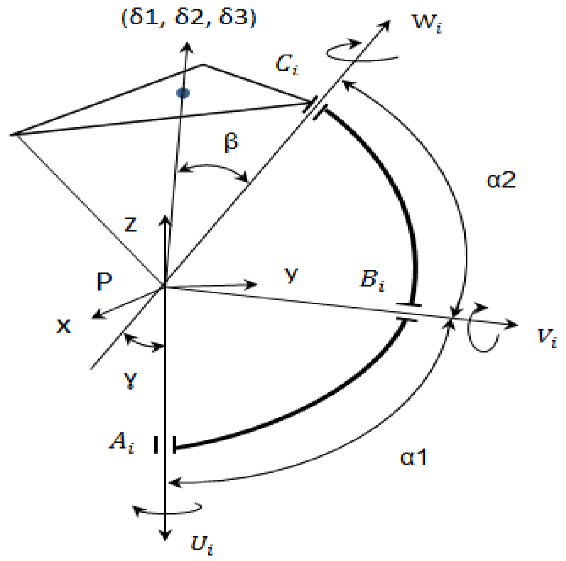


Figure 29 The vectors of spherical part

The unit vector Ui of revolute joint A in the ith leg can be expressed as:

$U_i=[0\ 0\ 1]^T$

Unit vector vi of revolute joint B in the ith leg can be expressed as:

$V_i = R_z(\theta_i).R_x(\alpha_1).U_i$

$V_i = \{Sin[\alpha 1]Sin[\theta_i], -Cos[\theta_i]Sin[\alpha 1], Cos[\alpha 1]\}$

Unit vector Wi of the top revolute joint C in the ith leg ,which are connected with platform can be expressed as:

$W_i = W_i^*.\delta$

Where:

W_i^*: the axes of each joint on the platform with respect z axes

$W_i^* = R_z(\gamma_i).R_x(\beta_i).[0\ 0\ 1]^T$

$W_i^* = \{Sin[\beta_i]Sin[\gamma_i], -Cos[\gamma_i]Sin[\beta_i], Cos[\beta_i]\}$

δ : The orientation of the platform in reference frame by (δ1, δ2, δ3)

$\delta = R_x(\delta_1).R_y(\delta_2).R_z(\delta_3)$

In Jacobian of the spherical manipulator , we should find the relationship between the angular velocity of the end-effector $\omega = [\omega x\ \omega y\ \omega z]$ and the angular velocity of imaginary joints as an input velocity $\dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$

The angular velocity of platform ω can be described with respect to rates of described Euler angles $(\dot{\delta}_1, \dot{\delta}_2, \dot{\delta}_3)$ where

$$\omega = \frac{d\delta}{d\delta 1}.\dot{\delta}^T + \frac{d\delta}{d\delta 2}.\dot{\delta}^T + \frac{d\delta}{d\delta 3}.\dot{\delta}^T \quad (4.12)$$

That leads to:

$$\omega x = \dot{\delta}3\ \mathrm{Cos}[\delta 1]\mathrm{Cos}[\delta 2] - \dot{\delta}2\ \mathrm{Sin}[\delta 1]$$

$$\omega y = \dot{\delta}2\ \mathrm{Cos}[\delta 1] + \dot{\delta}3\ \mathrm{Cos}[\delta 2]\ \mathrm{Sin}[\delta 1]$$

$$\omega z = \dot{\delta}1 - \dot{\delta}3\ \mathrm{Sin}[\delta 2]$$

writing in the matrix vector form will give us

$$\omega = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0 & \mathrm{Sin}[\delta 1] & \mathrm{Cos}[\delta 1]\mathrm{Cos}[\delta 2] \\ 0 & \mathrm{Cos}[\delta 1] & \mathrm{Cos}[\delta 2]\mathrm{Sin}[\delta 1] \\ 1 & 0 & \mathrm{Sin}[\delta 2] \end{pmatrix} \begin{pmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \end{pmatrix} \quad (4.13)$$

For the closed chain of the spherical parallel manipulator, the following equation holds: for each leg

$$V_i.W_i = \mathrm{Cos}[\alpha_2] \qquad \text{for i=1,2,3} \quad (4.14)$$

The Jacobian matrix of SPMs can be obtained through differentiating Eq. (4.14), which gives

$$\dot{V}_i.W_i + V_i.\dot{W}_i = 0 \quad (4.15)$$

Note that

$$\dot{V}_i = \dot{\theta}_i U_i \times V_i$$

$$\dot{W}_i = \omega \times W_i$$

Where ω is the angular velocity of the platform.

Substituting ω in equation (4.15) results in

$$\dot{\theta}_i (U_i \times V_i).W_i + \omega(W_i \times V_i) = 0 \text{ for i=1,2,3} \quad (4.16)$$

From three equations of (4.16) we can write in the form of variable jacobian and input jacobian

$$J_{sx}\omega = J_{sq}\dot{\theta}_i \quad (4.17)$$

Multipliying both sides with the inverse of Jsx we get

$$J_s.\omega = \dot{\theta}_i \quad (4.18)$$

Where $J_s = J_{sq}\ J_{sx}^{-1} = \frac{(W_i \times V_i)}{(U_i \times V_i).W_i}$

From equation (4.18), where ω is calculated, we can find $\dot{\theta}_i$ where,

$\dot{\theta}_i = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$

The planar part is a 4 RRRR. For this manipulator the input vector is q= [φ1, φ2, φ3, φ4, φ5] and the output vector is x=[Px, Py, θ1, θ2, θ3]

The loop –closure equation for leg 1

$$OP + PA_1 = OD + DA_1 \tag{4.19}$$

Due to planar manipulator properties the angular velocity vectors of all links are parallel to Z-direction. By taking the derivative of equation (4.1) with respect to time, we have obtained the velocity vector- loop equation as

$$V_p + \dot{\theta}_1(U \times r_1) = \dot{\phi}_1(U \times a_1) + (\dot{\varphi}_1 + \dot{\psi}_1)(U \times b_1) \tag{4.20}$$

Where V_p is the velocity of point P and U is a unit vector with respect to Z-direction.The passive variable ($\dot{\psi}_1$) should be eliminated from equation (4.20). Both sides of equation (4.20) is dot- multiplied by b_1 which leads to:

$$b_1 V_p + \dot{\theta}_1 U.(r_1 \times b_1) = \dot{\varphi}_1 U.(a_1 \times b_1) \tag{4.21}$$

$$b_{1x}V_{Px} + b_{1y}V_{Py} + (r_{1x}b_{1y} - r_{1y}b_{1x})\dot{\theta}_1 = (a_{1x}b_{1y} - a_{1y}b_{1x})\dot{\varphi}_1 \tag{4.22}$$

Which is the equation of motion for the first leg.

The loop –closure equation for leg 2 is

$$PQ + PA_2 = QE + EA_2 \tag{4.23}$$

The velocity vector- loop equation will be

$$V_p + \dot{\theta}_2(U \times r_2) = \dot{\varphi}_2(U \times a_2) + (\dot{\phi}_2 + \dot{\phi}_3)(U \times b_2) \tag{4.24}$$

Separating vector parts of the equation leads to equation of motion for leg 2

$$V_{Px} + r_{2y}\,\dot{\theta}_2 = a_{2y}\ \dot{\varphi}_2\ + b_{2y}\,(\dot{\varphi}_2 + \dot{\varphi}_3) \tag{4.25}$$

$$V_{Py} + r_{2x}\,\dot{\theta}_2 = a_{2x}\ \dot{\varphi}_2\ + b_{2x}\,(\dot{\varphi}_2 + \dot{\varphi}_3) \tag{4.26}$$

The loop –closure equation for leg 3 is

$$PR + PA3 = RF + FA3 \tag{4.27}$$

The velocity vector- loop equation will be

$$V_p + \dot{\theta}_3(U \times r_3) = \dot{\varphi}_4(U \times a_3) + (\dot{\varphi}_4 + \dot{\varphi}_5)(U \times b_3) \tag{4.28}$$

That leads to the equation of motion for the third leg

$$V_{Px} + r_{3y}\,\dot{\theta}_3 = a_{3y}\ \dot{\varphi}_4 + b_{3y}\,(\dot{\varphi}_4 + \dot{\varphi}_5) \tag{4.29}$$

$$V_{Py} + r_{3x}\,\dot{\theta}_3 = a_{3x}\ \dot{\varphi}_4\ + b_{3x}\,(\dot{\varphi}_4 + \dot{\varphi}_5) \tag{4.30}$$

We can be writing equations (4.22), (4.25), (4.26), (4.29), (4.30) to find equation of motion for the planar manipulator

$$J_{px}\dot{x}_p = J_{pq}\dot{q}_p$$

Where:

$$J_{px} = \begin{pmatrix} b_{1x} & b_{1y} & r_{1x}b_{1y} - r_{1y}b_{1x} & 0 & 0 \\ 1 & 0 & 0 & r_{2y} & 0 \\ 0 & 1 & 0 & r_{2x} & \\ 1 & 0 & 0 & 0 & r_{3y} \\ 0 & 1 & 0 & 0 & r_{3x} \end{pmatrix}$$

$$J_{pq} = \begin{pmatrix} a_{1x}b_{1y} - a_{1y}b_{1x} & 0 & 0 & 0 & 0 \\ 0 & a_{2y} + b_{2y} & b_{2x} & 0 & 0 \\ 0 & a_{2x} + b_{2y} & b_{2x} & 0 & 0 \\ 0 & 0 & 0 & a_{3y} + b_{3y} & b_{3y} \\ 0 & 0 & 0 & a_{3x} + b_{3x} & b_{3x} \end{pmatrix}$$

$$\dot{x}_p = \mathbf{J}_p \ \dot{q}_p \qquad (4.31)$$

Where $J_p = J_{px}{}^{-1} \cdot J_{pq}$

$\dot{x}_p = [V_{px}, V_{py}, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$

$\dot{q}_p = [\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4, \dot{\varphi}_5]^T$

V_{px}, V_{py} is given and $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ is calculated from the spherical part.

From equation (4.31), we can find the angular velocity of actuated joint $[\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3, \dot{\varphi}_4, \dot{\varphi}_5]^T$

4.3 Acceleration Analysis Using Jacobian

After finding the Jacobian for the manipulator acceleration for the inverse kinematics can be easily found by derivation of velocity equations by time.

Derivation of the spherical part

$$\frac{d}{dt}\left(J_s \cdot \omega = \dot{\theta}_i \right)$$

$$\ddot{\theta}_i = \mathbf{J}_s \cdot \dot{\omega} + \dot{J}_s \cdot \omega$$

Derivation of the planar part

$$\frac{d}{dt}(J_p \dot{x}_p = \dot{q}_p)$$

$$\ddot{q}_p = \dot{J}_p \dot{x}_p + J_p \ddot{x}_p$$

Results with the formulation of accelerations of motors with respect to a given output.

4.4 Force Analysis Using Jacobian

By using the Jacobian matrix, we can be write the relation between external moments on platform and torque on point (P) which the center of intersection of spherical joint: .

$$J_s . M = \tau_{\theta_i} \quad (4.32)$$

Where $M = [Mx, My, Mz]^T$ is given external moment effecting on platform.

From equation (4.32), the torque of imaginary joints can be found

$$\tau_{\theta_i} = [\tau_{\theta_1}, \tau_{\theta_2}, \tau_{\theta_3}]^T \quad (4.33)$$

Also by using the Jacobian matrix of planar part, we can write the relation between external forces on point (O) which the center of intersection of spherical joint and the input torque by actuated joints τ_{φ_i}.

$$J_p . F = \tau_{\varphi_i} \quad (4.34)$$

Where $F = [f_{px}, f_{py}, \tau_{\theta_1}, \tau_{\theta_2}, \tau_{\theta_3}]^T$ external force for the planar part

Where f_{px}, f_{py} is given external forces effecting on the platform and $\tau_{\theta_1}, \tau_{\theta_2}, \tau_{\theta_3}$ are calculated moments in the spherical part.

From equation (4.34) ,we can find the required input torque of actuated joints for external forces and moments.

$$\tau_{\varphi_i} = [\tau_{\varphi_1}, \tau_{\varphi_2}, \tau_{\varphi_3}, \tau_{\varphi_4}, \tau_{\varphi_5}]^T$$

Now we will describe the position and orientation of platform and Simulation results of position, velocity and torque of input actuators with figures

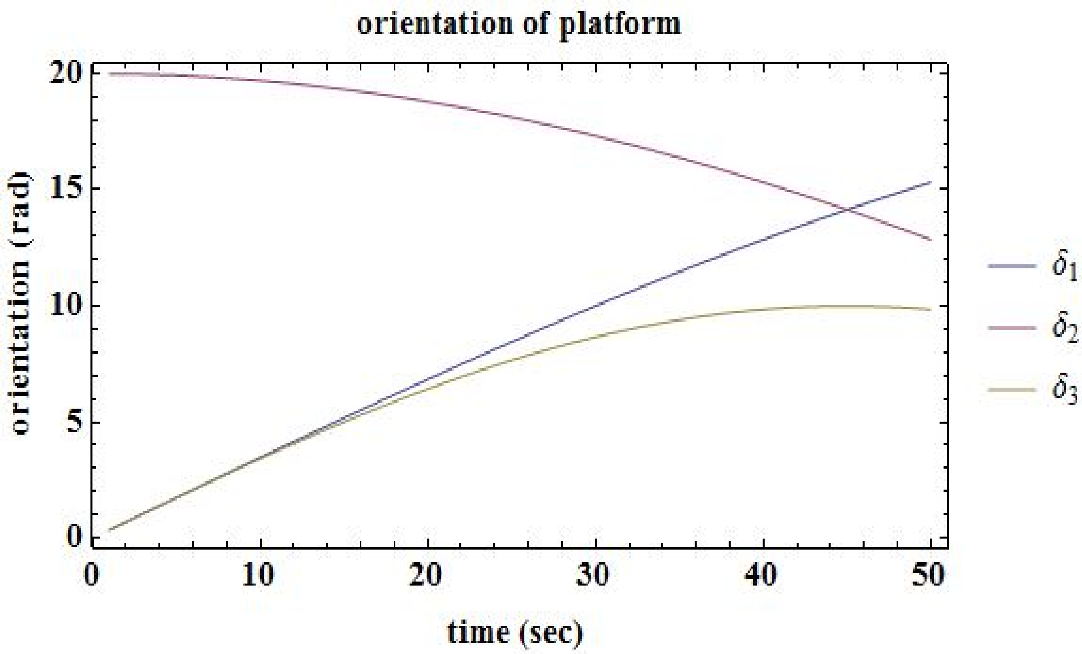


Figure 30 The orientation of platform with respect to time

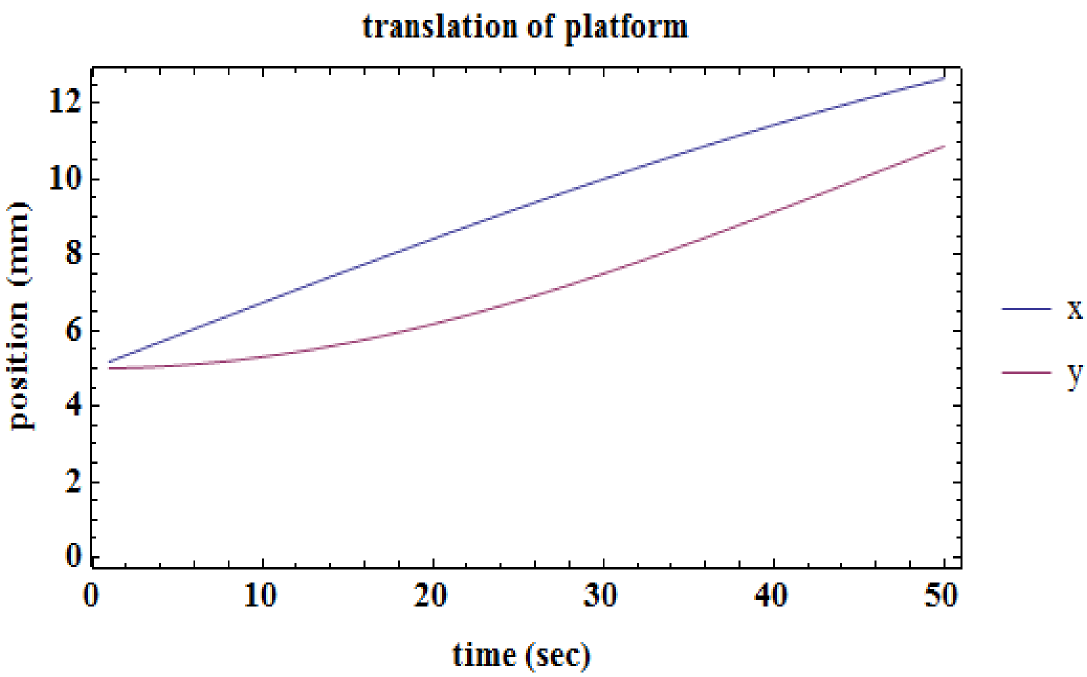


Figure 31 The translation of platform with respect to time

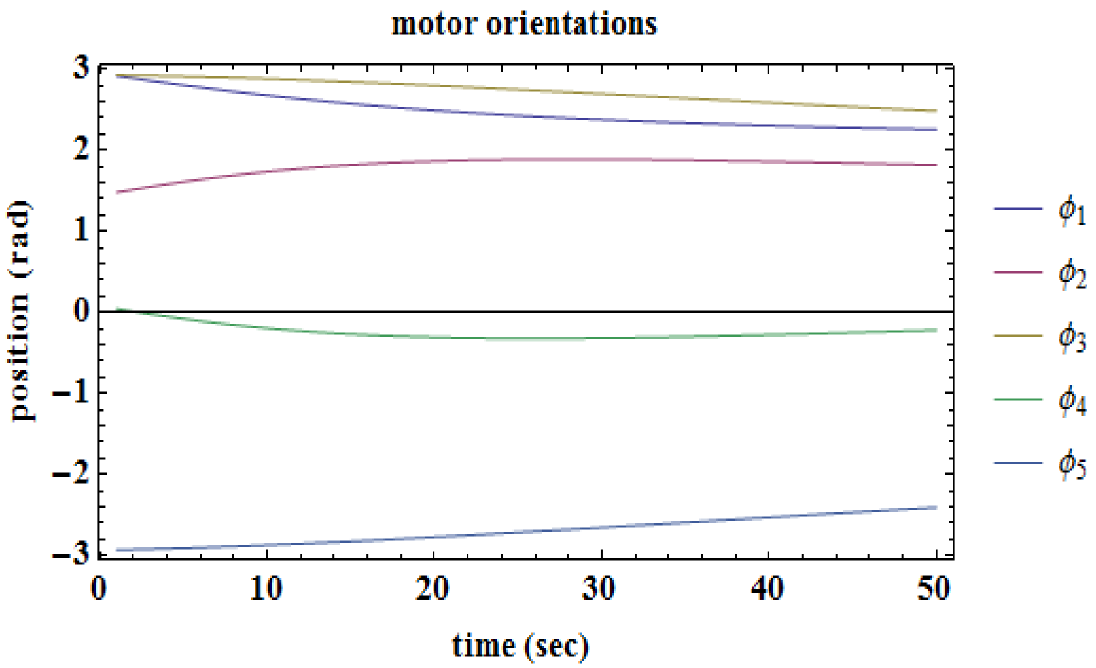


Figure 32 The position of motors orientation with respect to time

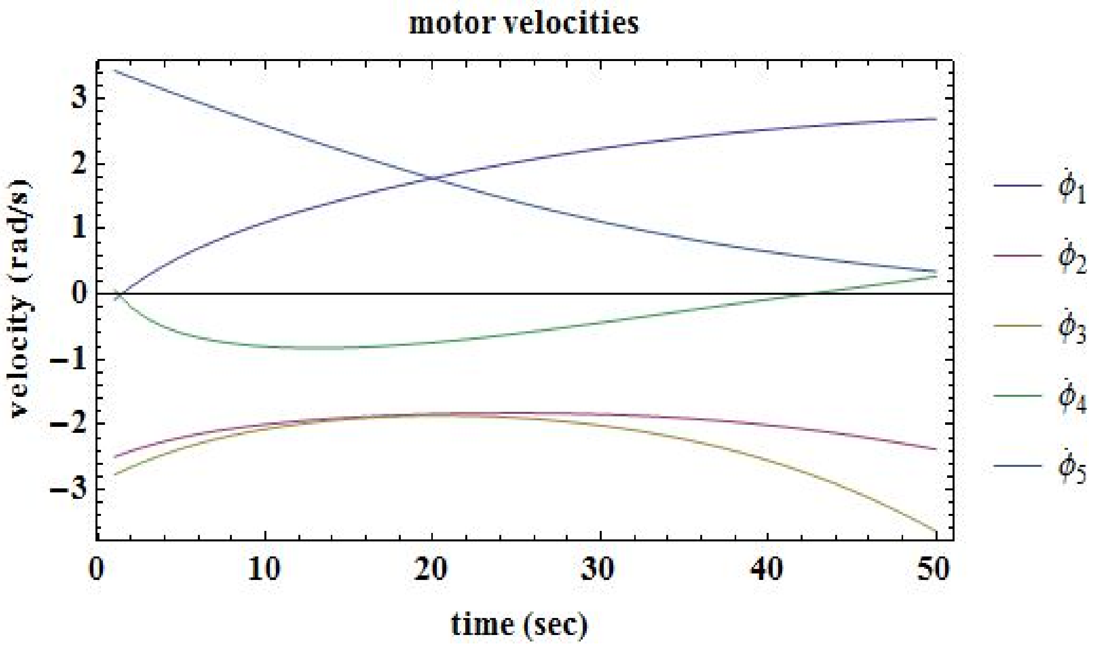


Figure 33 The velocity of motors with respect to time

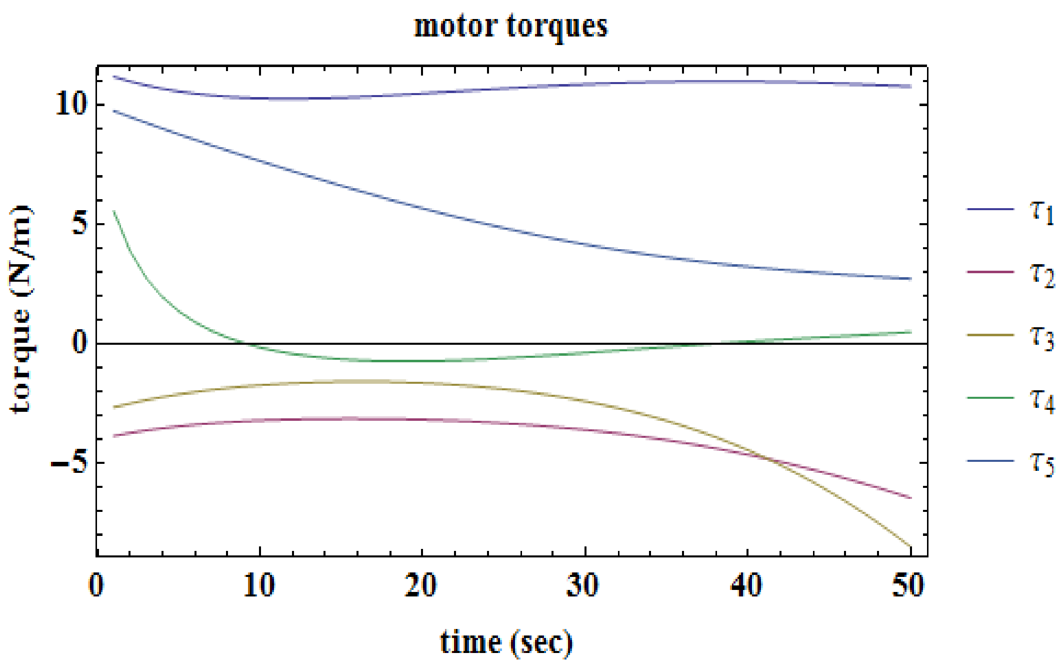


Figure 34 The torque of motors with respect to time

CHAPTER 5

CONCLUSION

This thesis deals with one of the applications of parallel manipulator in rehabilitation robotics . Where, the aim of this thesis work, the focus is placed on the study of robot therapy to improve physical or cognitive function for persons who becomes unable to move their hands. However, this device can provide the hand by necessary movement. It is seen that selected manipulator is able to do necessary movement for the rehabilitation of the human arm.

The selection of this structure leads to advantages to the other similar parallel manipulators such as; the subspace including both planar and spherical movements which are needed for the rehabilitation of both hand and wrist, by using parallel structure to carry more load and don't need to carry all motors.3 legs are found to be optimum for this kind of subspace so it is simpler in structure and less in potential interference than same 5-DOF PM with 5 or 4 active legs.

For solving the inverse kinematics and dynamics and reduce complexity of the manipulator, it was divided in to two sub-manipulators in subspace $\lambda=3$ with the help of three imaginary joints placed at the intersection of platform joints with a direction parallel to base joints. The upper part being a 3 Dof -3(RRR) spherical manipulator where input axes are coaxial and the lower part being a redundant 5 Dof - 3 RRRR manipulator. This division is resulted in the easiness of calculations and then the inverse kinematic analysis is made for the manipulator.

The workspace is analyzed by using the discretization method approach the workspace described with a regular grid of nodes. Each node tested to see for the effects on the workspace. The boundary of the workspace represented by the discretization of the pose parameters and chooses the optimal design for the pre-described position and orientation range of the end-effector. Optimum parameters of the manipulator are found by using two algorithms.

The inverse dynamic analysis of the over-constrained Manipulator is analyzed by using the principle of virtual work method. Inverse Jacobian matrices is derived to get the relation between input torque by actuated joints and output forces on platform. For spherical and planar parts two Jacobian matrices are derived to find the relation of the external force with the input torques of the actuators.

During this study the selected manipulator is investigated both in kinematics and dynamics along with the workspace analysis. The results are a manipulator which can be controlled using these calculations and achieve the necessary motion for the rehabilitation of the human arm.

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APPENDICES A

Mathematica code for workspace analysis:

```
Clear ["Global`*"]
Remove ["Global`*"]
kk1 = {5Degree, 5Degree, 5Degree};
kk2 = {0Degree, 120Degree, 240Degree};
```

$\text{Do}[k1_i = kk1[[i]]; k2_i = kk2[[i]], \{i, 1, 3\}]$

$$\text{Do}\left[\omega_i = \begin{pmatrix} Ux_i \\ Uy_i \\ Uz_i \end{pmatrix} = \begin{pmatrix} \text{Cos}[k2_i] & -\text{Sin}[k2_i] & 0 \\ \text{Sin}[k2_i] & \text{Cos}[k2_i] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[k1_i] & -\text{Sin}[k1_i] \\ 0 & \text{Sin}[k1_i] & \text{Cos}[k1_i] \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \{i, 1, 3\}\right] //N;$$

$\text{For}[kk = 0; \text{Results} = \{\}; \delta 1 = 0; \delta 2 = 0; \delta 3 = 0, kk \leq J, kk++, J := 20$

$\text{For}[jj = 0, jj \leq J, jj++,$

$\text{For}[ii = 0, ii \leq J, ii++, \delta 1 = -30\text{Degree} + ii * 50\ \text{Degree}/J;$

$\delta 2 = -90\text{Degree} + jj * 140\ \text{Degree}/J;\ \delta 3 = -60\text{Degree} + kk * 120\ \text{Degree}/J;$

$$\text{Do}\left[\begin{pmatrix} \psi x_i \\ \psi y_i \\ \psi z_i \end{pmatrix} = \begin{pmatrix} \text{Cos}[\delta 1] & -\text{Sin}[\delta 1] & 0 \\ \text{Sin}[\delta 1] & \text{Cos}[\delta 1] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \text{Cos}[\delta 2] & 0 & \text{Sin}[\delta 2] \\ 0 & 1 & 0 \\ -\text{Sin}[\delta 2] & 0 & \text{Cos}[\delta 2] \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\delta 3] & -\text{Sin}[\delta 3] \\ 0 & \text{Sin}[\delta 3] & \text{Cos}[\delta 3] \end{pmatrix} . \begin{pmatrix} Ux_i \\ Uy_i \\ Uz_i \end{pmatrix}\right]$$

$$\text{Do}[\theta 1_i = 2\text{ArcTan}\left[\frac{\text{Sin}[\alpha 1]\psi x_i - \sqrt{\text{Sin}[\alpha 1]^2 \psi x_i^2 + \text{Sin}[\alpha 1]^2 \psi y_i^2 - (\text{Cos}[\alpha 2] - 2\text{Cos}[\alpha 1]\psi z_i)^2}}{\text{Cos}[\alpha 2] - \text{Sin}[\alpha 1]\psi y_i - 2\text{Cos}[\alpha 1]\psi z_i}\right];$$

$$\theta 2_i = 2\text{ArcTan}\left[\frac{\text{Sin}[\alpha 1]\psi x_i + \sqrt{\text{Sin}[\alpha 1]^2 \psi x_i^2 + \text{Sin}[\alpha 1]^2 \psi y_i^2 - (\text{Cos}[\alpha 2] - 2\text{Cos}[\alpha 1]\psi z_i)^2}}{\text{Cos}[\alpha 2] - \text{Sin}[\alpha 1]\psi y_i - 2\text{Cos}[\alpha 1]\psi z_i}\right];$$

$, \{i, 1, 3\}]$

$\text{If}[150\text{Degree} \geq \tau 1_1 \geq -150\text{Degree} || 150\text{Degree} \geq \tau 2_1 \geq -150\text{Degree} \&\&$

$0\text{Degree} \geq \tau 1_2 \geq -180\text{Degree} || 0\text{Degree} \geq \tau 2_2 \geq -180\text{Degree} \&\&$

```
225Degree ≥ τ1_3 ≥ 135Degree||225Degree ≥ τ2_3 ≥ 135Degree&&
Sin[α1]^2ψx_1^2 + Sin[α1]^2ψy_1^2 - 1.(Cos[α2] - 2.Cos[α1]ψz_1)^2 ≥ 0&&
Sin[α1]^2ψx_2^2 + Sin[α1]^2ψy_2^2 - 1.(Cos[α2] - 2.Cos[α1]ψz_2)^2 ≥ 0&&
Sin[α1]^2ψx_3^2 + Sin[α1]^2ψy_3^2 - 1.(Cos[α2] - 2.Cos[α1]ψz_3)^2 ≥ 0&&
1 ≥ (-ψz_1 + Cos[α1]Cos[α2])/(Sin[α1]Sin[α2]) ≥ -1&&
1 ≥ (-ψz_2 + Cos[α1]Cos[α2])/(Sin[α1]Sin[α2]) ≥ -1&&
1 ≥ (-ψz_3 + Cos[α1]Cos[α2])/(Sin[α1]Sin[α2]) ≥ -1, Results = Append[Results, {δ1, δ2, δ3}]]];]];
ListPointPlot3D[Results]
(*For planar part*)
(*the optimization value *)
J:=20   a1:=22   b1:=22   a2:=22   b2:=22   a3:=22   b3:=22
r1:=5   r2:=5    r3:=5    xb0:=20  yb0:=0   xc0:=10  yc0:=20
(*loop*)
For[kk = 0; Results01 = {}; Results02 = {}; Results03 = {}, kk ≤ J, kk + +, For[jj
        = 0, jj ≤ J, jj + +, For[ii = 0, ii <= J, ii + +,
θ1 = -150Degree + ii * 300 Degree/J ; θ2 = 0Degree + ii * -180 Degree/J; θ3
        = 135Degree + ii * 90 Degree/J ;
x=-10+jj*35/J;          y=-10+kk*35/J;          xa=r1 Cos[θ1]+x;
ya=r1 Sin[θ1]+y;        xb=r2 Cos[θ 2]+x;       yb=r2 Sin[θ 2]+y;
xc=r3 Cos[θ 3]+x;       yc=r3 Sin[θ 3]+y;
(*conditions*)
If[4a1^2xa^2 + 4a1^2ya^2 - (a1^2 - b1^2 + xa^2 + ya^2)^2 ≥ 0, Results01
        = Append[Results01, {x, y, θ1 + 150Degree}]];
If[1 ≥ ((xb - xb0)^2 + (yb - yb0)^2 - b2^2 - a2^2)/(2b2a2) ≥ -1&&
-a2^2(yb - yb0)^2(a2^4 + (-b2^2 + (xb - xb0)^2 + (yb - yb0)^2)^2
        - 2a2^2(b2^2 + (xb - xb0)^2 + (yb - yb0)^2)) ≥ 0,
Results02 = Append[Results02, {x, y, θ2 + 180Degree}]];
```

```
If[1 ≥ ((xc - xc0)^2 + (yc - yc0)^2 - b3^2 - a3^2)/(2b3a3) ≥ -1&&
-a3^2(yc - yc0)^2(a3^4 + (-b3^2 + (xc - xc0)^2 + (yc - yc0)^2)^2
        - 2a3^2(b3^2 + (xc - xc0)^2 + (yc - yc0)^2)) ≥ 0,
Results03 = Append[Results03, {x, y, θ3 - 135Degree}]]];
ListPointPlot3D[Results01]
ListPointPlot3D[Results02]
ListPointPlot3D[Results03]
ListPointPlot3D[Intersection[Results01, Results02, Results03]]
```

APPENDICES B

Mathematica code for workspace analysis:

```
Clear["Global`*"]
Remove["Global`*"]
STEPS=50
Do[δ01_J = 2Sin[JDegree]; δ02_J = 2Cos[JDegree]; δ03_J
        = 2Sin[JDegree] * Cos[JDegree]
x0J=5+10Sin[J Degree];y0J=5+10Sin[J Degree]2;,{J,1,STEPS}]
```

$$Fs = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} //Flatten;$$

```
x0t=10;   y0t=5;   Fx0t=50;   Fy0t=40;   r1=5;   a1=22;   b1=21.9;
a2=a1;   b2=b1;   a3=a1;   b3=b1;   α1=91Degree;   α2=91Degree;
βvector= {5Degree,5Degree,5Degree};
□vector= {0Degree,120Degree,240Degree};
For[J = 20; realtorques = {}; realvelocity = {}; velocity = {}; motors
        = {}; torques = {}; realmotors = {};, J < STEPS + 1, J + +,
```

$$\begin{pmatrix} \delta 1 \\ \delta 2 \\ \delta 3 \end{pmatrix} = \begin{pmatrix} \delta 01_J \text{Degree} \\ \delta 02_J \text{Degree} \\ \delta 03_J \text{Degree} \end{pmatrix};$$

$$xst = \begin{pmatrix} 0 & \text{Sin}[\delta 1] & \text{Cos}[\delta 1]\text{Cos}[\delta 2] \\ 0 & \text{Cos}[\delta 1] & \text{Sin}[\delta 1]\text{Cos}[\delta 2] \\ 1 & 0 & \text{Sin}[\delta 2] \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ 0.5 \end{pmatrix} //Flatten$$

```
Do[β_i = βvector[[i]]; γ_i = γvector[[i]], {i, 1, 3}]
```

$$\mathrm{Do}\left[\omega_i = \begin{pmatrix} wx_i \\ wy_i \\ wz_i \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[\gamma_i] & -\mathrm{Sin}[\gamma_i] & 0 \\ \mathrm{Sin}[\gamma_i] & \mathrm{Cos}[\gamma_i] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\beta_i] & -\mathrm{Sin}[\beta_i] \\ 0 & \mathrm{Sin}[\beta_i] & \mathrm{Cos}[\beta_i] \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \{i, 1, 3\}\right]$$

$$\mathrm{Do}\left[W_i = \begin{pmatrix} Wx_i \\ Wy_i \\ Wz_i \end{pmatrix} = \begin{pmatrix} \mathrm{Cos}[\delta 1] & -\mathrm{Sin}[\delta 1] & 0 \\ \mathrm{Sin}[\delta 1] & \mathrm{Cos}[\delta 1] & 0 \\ 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} \mathrm{Cos}[\delta 2] & 0 & \mathrm{Sin}[\delta 2] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[\delta 2] & 0 & \mathrm{Cos}[\delta 2] \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\delta 3] & -\mathrm{Sin}[\delta 3] \\ 0 & \mathrm{Sin}[\delta 3] & \mathrm{Cos}[\delta 3] \end{pmatrix} . \begin{pmatrix} Ux_i \\ Uy_i \\ Uz_i \end{pmatrix}\right]$$

$$\mathrm{Do}[\theta_{i,j} = 2\mathrm{ArcTan}\left[\frac{\mathrm{Sin}[\alpha 1]\psi x_i + (-1)^j \sqrt{\mathrm{Sin}[\alpha 1]^2 \psi x_i^2 + (\mathrm{Cos}[\alpha 1] + \mathrm{Sin}[\alpha 1]\psi y_i)(-\mathrm{Cos}[\alpha 2] + \mathrm{Sin}[\alpha 1]\psi y_i + 2\mathrm{Cos}[\alpha 1]\psi z}}{\mathrm{Cos}[\alpha 2] - \mathrm{Sin}[\alpha 1]\psi y_i - 2\mathrm{Cos}[\alpha 1]\psi z_i}\right.$$

;

$, \{j, 1, 2\}], \{i, 1, 3\}]; \quad \mathrm{Do}[\theta_i = \theta_{i,j}, \{i, 1, 3\}, \{j, 2, 2\}];$

xa=r1 Cos[θ1]+x0J; ya=r1 Sin[θ1]+y0J; xb=r1 Cos[θ2]+x0J;

yb=r1 Sin[θ2]+y0J; xc=r1 Cos[θ3]+x0J; yc=r1 Sin[θ3]+y0J;

e1=-2ya a1; e2=-2 xa a1; e3=xa2+ya2+a12-b12;

kb=(xb2+yb2-b22-a22)/(2b2 a2);kc=(xc2+yc2-b32-a32)/(2b3 a3);

$$\mathbf{Do}[\phi_{1,j} = 2\mathbf{ArcTan}[\frac{-\mathbf{e1} + (-1)^j\sqrt{\mathbf{e1}^2 + \mathbf{e2}^2 + \mathbf{e3}^2}}{\mathbf{e3} - \mathbf{e2}}]//\mathbf{N}, \{\mathbf{j}, 1, 2\}];$$

$$\mathbf{Do}[\psi 1, \mathbf{j} = \mathbf{ArcTan}[(\mathbf{xa} - \mathbf{a1}\ \mathbf{Cos}[\phi_{1,j}])/\mathbf{b1}, (\mathbf{ya} - \mathbf{a1}\ \mathbf{Sin}[\phi_{1,j}])/\mathbf{b1}] - \phi_{1,j}//\mathbf{N}, \{\mathbf{j}, 1, 2\}];$$

$$\mathbf{Do}[\phi_{2,j} = \mathbf{ArcTan}[\left(-\mathbf{xbb2Sin}[\phi_{3,j}] + \mathbf{yb}(\mathbf{a2} + \mathbf{b2Cos}[\phi_{3,j}])\right)/(\mathbf{b2}^2 + \mathbf{a2}^2 + \mathbf{2b2a2Cos}[\phi_{3,j}]),$$

$$\frac{\mathbf{xb}(\mathbf{a2} + \mathbf{b2Cos}[\phi_{3,j}]) + \mathbf{ybb2Sin}[\phi_{3,j}]}{\mathbf{b2}^2 + \mathbf{a2}^2 + \mathbf{2b2a2Cos}[\phi_{3,j}]}], \{\mathbf{j}, 1, 2\}]//\mathbf{N};$$

$$\mathbf{Do}[\phi_{3,j} = (-1)^j\mathbf{ArcCos}[\mathbf{kb}]//\mathbf{N}, \{\mathbf{j}, 1, 2\}];$$

$$\mathbf{Do}[\phi_{4,j} = \mathbf{ArcTan}[\frac{\mathbf{xb}(\mathbf{b2} + \mathbf{a2Cos}[\phi_{5,j}]) + \mathbf{yba2Sin}[\phi_{5,j}]}{\mathbf{b2}^2 + \mathbf{a2}^2 + \mathbf{2b2a2Cos}[\phi_{5,j}]};$$

$$(-\mathbf{xba2Sin}[\phi_{5,j}] + \mathbf{yb}(\mathbf{b2} + \mathbf{a2Cos}[\phi_{5,j}]))/(\mathbf{b2}^2 + \mathbf{a2}^2 + \mathbf{2b2a2Cos}[\phi_{5,j}])], \{\mathbf{j}, 1, 2\}] //\mathbf{N};$$

```
Do[ϕ5,j = (−1)^j ArcCos[kc]//N,{j,1,2}];
Do[ϕi= ϕi,j,{i,1,5},{j,2,2}];
Do[ψi=ψi,j,{i,1,1},{j,2,2}];
Jxs = ( ) Jqs = ( ) Jxp = ( ) Jqp = ( )
qt=Prepend[Prepend[qst,y0t],x0t];
realqt =-Inverse[Jqp].Jxp.qt;
torq=-Inverse[Jqs].Jxs.Fs;
forceplanar=Prepend[Prepend[torq,Fy0t],Fx0t];
realtorq=-Inverse[Jqp].Jxp.forceplanar;
motors=Append[motors,{ θ1, θ2, θ3}];
realmotors=Append[realmotors,{ ϕ1, ϕ2, ϕ3, ϕ4, ϕ5}];
velocity=Append[velocity,qst];
torques=Append[torques,torq];
realvelocity=Append[realvelocity,realqt];
realtorques=Append[realtorques,realtorq]
```

CURRICULUM VITAE

PERSONAL INFORMATION

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EDUCATION

Degree	Institution	Year of Graduation
M.Sc.	Çankaya Univ., Mechanical Engineering	2015
B.Sc.	Tikrit Univ., Mechanical Engineering	1998
High School	Al Mashroa High School	1993

WORK EXPERIENCE

Year	Place	Enrollment
2007- Present	Directorate of Education in Banylon	Teacher

FOREIN LANGUAGES

English.

HOBBIES

Science, Books, Swimming. Football.