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Applications of some fixed point theorems for fractional differential equations with Mittag-Leffler kernel

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Abstract

Using some fixed point theorems for contractive mappings, including α-γ-Geraghty type contraction, α-type F-contraction, and some other contractions in $\mathcal{F}$-metric space, this research intends to investigate the existence of solutions for some Atangana–Baleanu fractional differential equations in the Caputo sense.

Keywords: Atangana–Baleanu derivative in the Caputo sense; $\mathcal{F}$-metric space; α-type F-contractive mapping; Atangana–Baleanu fractional operator in the Caputo sense

1 Introduction

It is well known that several physical phenomena are described by nonlinear differential equations (both ODEs and PDEs). Therefore, the study of the many analytical and numerical methods used for solving the nonlinear differential equations is a very important topic for the analysis of engineering practical problems [1–19].

In 2016, the interesting and new derivatives without singular kernel were introduced by Atangana and Baleanu, which generalized the Caputo–Fabrizio definition [8]. Atangana–Baleanu derivative contains Mittag-Leffler function as a nonlocal and nonsingular kernel. Many authors showed their interest in this definition as it holds the profits of Riemann–Liouville and Caputo derivatives [20–30]. Last year, Atangana et al. provided the numerical approximation to the fractional advection-diffusion equation whose fractional derivatives are Atangana–Baleanu derivative of Riemann–Liouville type [14].

In the last decades, two topics have been densely studied: "fixed point theory" and "fractional differential/integral equations". Recently, several significant results have been recorded [7, 31, 32].

In 2012, Samet et al. [33] studied the concept of α-admissible mappings that was expanded by Karapınar and Samet in [34]. Also, Wardowski [35] proposed a new inequality to guarantee the existence and uniqueness of a given mapping in the framework of standard metric space. This inequality has been known as F-contraction.

In 2016, Gopal et al. considered new concepts of α-type F-contractive mappings (see [12]). Very recently, Jleli and Samet [36] mentioned the concept of $\mathcal{F}$-metric space and obtained the generalization of Banach contraction principle.

In [1–4], the authors studied generalized Geraghty contractive mappings and their applications in b-metric spaces.

In this paper by applying some fixed point theorems for contractive mappings, like α-γ-Geraghty type, α-type F-contraction, and some other contractions in $\mathcal{F}$-complete $\mathcal{F}$-metric space, we study the existence of solutions for some Atangana–Baleanu fractional differential equations in the Caputo sense. Throughout the article J denotes $[0,1]$.

Suppose that (M,d) is a complete b-metric space (with constant s_1), also let Ω be a set of all increasing and continuous functions $\gamma:[0,\infty)\to[0,\infty)$ satisfying: $\gamma(cx)\le c\gamma(x)\le cx$ for all $c>1$, and Λ is the family of all nondecreasing functions $\lambda:[0,\infty)\to[0,\frac{1}{{s_1}^2})$, $s_1\ge 1$.

Definition 1.1 ([2]) The mapping $g:M\to M$ is a generalized α-γ-Geraghty contraction mapping whenever there exists $\alpha: M\times M\to[0,\infty)$ with

$$\alpha(w,z)\gamma\big({s_1}^3 d(gw,gz)\big)\le\lambda\big(\gamma\big(d(w,z)\big)\big)\gamma\big(d(w,z)\big) \tag{1}$$

for $w,z\in M$, $\lambda\in\Lambda$, and $\gamma\in\Omega$.

Definition 1.2 ([33]) Let $\varphi:M\to M$, where M is nonempty, and $\alpha:M\times M\to[0,\infty)$ be given, g is α-admissible if

$$\alpha(w,z)\ge 1 \quad\Longrightarrow\quad \alpha(\varphi w,\varphi z)\ge 1,\quad \forall w,z\in M. \tag{2}$$

Theorem 1.3 ([2]) *Let (M,d) be a complete b-metric space and $\varphi:M\to M$ be a generalized α-γ-Geraghty contraction such that*

(i) *φ is α-admissible;*
(ii) *$\exists\, w_0\in M$ with $\alpha(w_0,\varphi w_0)\ge 1$;*
(iii) *$\{w_n\}\subseteq M$, $w_n\to u$ in M and $\alpha(w_n,w_{n+1})\ge 1$, then $\alpha(w_n,w)\ge 1$.*

Then φ has a fixed point.

Definition 1.4 ([8]) Let $\delta\in H^1(a,b)$, $a<b$, and $0\le\kappa\le 1$. The Atangana–Baleanu fractional derivative in the Caputo sense of δ of order κ is defined by

$$\big({}^{ABC}_{a}D^{\kappa}\delta\big)(s)=\frac{B(\kappa)}{1-\kappa}\int_a^s\delta'(\nu)E_\kappa\left[-\kappa\frac{(s-\nu)^\kappa}{1-\kappa}\right]d\nu, \tag{3}$$

where E_κ is the Mittag-Leffler function defined by $E_\kappa(z)=\Sigma_{n=0}^{\infty}\frac{z^n}{\Gamma(n\kappa+1)}$ and $B(\kappa)$ is a normalizing positive function satisfying $B(0)=B(1)=1$ (see [15, 19]). The associated fractional integral is defined by

$$\big({}^{AB}_{a}I^{\kappa}\delta\big)(s)=\frac{1-\kappa}{\kappa}\delta(s)+\frac{\kappa}{B(\kappa)}\big({}_aI^\kappa\delta\big)(s), \tag{4}$$

where ${}_aI^\kappa$ is the left Riemann–Liouville fractional integral given as

$$\big({}_aI^\kappa\delta\big)(s)=\frac{1}{\Gamma(\kappa)}\int_a^s(s-\nu)^{\kappa-1}\delta(\nu)\,d\nu. \tag{5}$$

Consider $d : M \times M \to [0,\infty)$ given by

$$d(\delta,\sigma) = \left\|(\delta-\sigma)^2\right\|_\infty = \sup_{s\in J}\bigl(\delta(s)-\sigma(s)\bigr)^2,$$

where $M = C(J,\mathbb{R})$ denotes the set of continuous functions, (M,d) is a complete b-metric space with $s_1 = 2$.

We discuss the problem

$$\bigl({}_0^{ABC}D^\kappa\delta\bigr)(s) = h\bigl(s,\delta(s)\bigr), \quad s\in J, 1\le\kappa\le 1, \tag{6}$$

$$\delta(0) = \delta_0, \tag{7}$$

where D^κ is the Atangana–Baleanu derivative in the Caputo sense of order κ and $h : J\times M \to M$ is continuous with $h(0,\delta(0)) = 0$.

Proposition 1.5 ([10]) *For* $0<\kappa<1$, *we have*

$$\bigl({}^{AB}I_b^{\kappa\,ABC}D^\kappa\delta\bigr)(s) = \delta(s)-\delta(b). \tag{8}$$

2 Main result

Theorem 2.1 *Suppose*

(i) $\exists\omega : \mathbb{R}^2\to\mathbb{R}$ *such that*

$$\begin{aligned}&\bigl|h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr|\\&\quad\le\frac{1}{2\sqrt{2}}\frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}\sqrt{\lambda\bigl(\gamma\bigl(\left|\delta(s)-\sigma(s)\right|^2\bigr)\bigr)\gamma\bigl(\left|\delta(s)-\sigma(s)\right|^2\bigr)}\end{aligned}$$

for $s\in J$, $\gamma\in\Omega$, *and* $\delta,\sigma\in\mathbb{R}$ *with* $\omega(\delta,\sigma)\ge 0$;

(ii) $\exists\delta_1\in C(J)$ *with* $\omega(\delta_1(s),T\delta_1(s))\ge 0$ *for* $s\in J$, *where* $T : C(J)\to C(J)$ *is defined by*

$$(T\delta)(s) = \delta_0 + {}_0^{AB}I^\kappa h\bigl(s,\delta(s)\bigr);$$

(iii) *for* $s\in J$ *and* $\delta,\sigma\in C(J)$, $\omega(\delta(s),\sigma(s))\ge 0$ *implies* $\omega(T\delta(s),T\sigma(s))\ge 0$;

(iv) $\{\delta_n\}\subseteq C(J)$, $\delta_n\to\delta$ *in* $C(J)$ *and* $\omega(\delta_n,\delta_{n+1})\ge 0$, *then* $\omega(\delta_n,\delta)\ge 0$, $n\in\mathbb{N}$.

Then problem (6) *has at least one solution.*

Proof Applying the Atangana–Baleanu integral to both sides of (6) and using Proposition 1.5, we get

$$\delta(s) = \delta_0 + {}_0^{AB}I^\kappa h\bigl(s,\delta(s)\bigr).$$

We show that T has a fixed point:

$$\begin{aligned}&\left|T\delta(s)-T\sigma(s)\right|^2\\&\quad=\left|{}_0^{AB}I^\kappa\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]\right|^2\\&\quad\le\left|\left[\frac{1-\kappa}{B(\kappa)}\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]+\frac{\kappa}{B(\kappa)}{}_0I^\kappa\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]\right]\right|^2\end{aligned}$$

$$
\begin{aligned}
&\leq \left\{ \frac{1-\kappa}{B(\kappa)} \left| h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right| + \frac{\kappa}{B(\kappa)} {}_0I^{\kappa} \left| h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right| \right\}^2 \\
&\leq \left\{ \frac{1}{2\sqrt{2}} \frac{1-\kappa}{B(\kappa)} \times \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} \sqrt{\lambda\big(\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)\big)\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)} \right. \\
&\quad \left. + \frac{1}{2\sqrt{2}} \frac{\kappa}{B(\kappa)} \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} {}_0I^{\kappa}(1) \sqrt{\lambda\big(\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)\big)\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)} \right\}^2 \\
&= \left\{ \frac{1}{2\sqrt{2}} \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} \sqrt{\lambda\big(\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)\big)\gamma\big(\left|\delta(s)-\sigma(s)\right|^2\big)} \right\}^2 \\
&\quad \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{\kappa}{B(\kappa)} \frac{1}{\kappa\Gamma(\kappa)} \right\}^2 \\
&\leq \left\{ \frac{1}{2\sqrt{2}} \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} \sqrt{\lambda\Big(\gamma\Big(\sup_{s\in J}\left|\delta(s)-\sigma(s)\right|^2\Big)\Big)\gamma\Big(\sup_{s\in J}\left|\delta(s)-\sigma(s)\right|^2\Big)} \right\}^2 \\
&\quad \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{\kappa}{B(\kappa)} \frac{1}{\kappa\Gamma(\kappa)} \right\}^2 \\
&= \left\{ \frac{1}{2\sqrt{2}} \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} \sqrt{\lambda\big(\gamma\big(d(\delta,\sigma)\big)\big)\gamma\big(d(\delta,\sigma)\big)} \right\}^2 \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{1}{B(\kappa)\Gamma(\kappa)} \right\}^2 \\
&= \frac{1}{8}\lambda\big(\gamma\big(d(\delta,\sigma)\big)\big)\gamma\big(d(\delta,\sigma)\big).
\end{aligned}
$$

Hence, for $\delta,\sigma \in C(J)$, $s \in J$ with $\omega(\delta(s),\sigma(s)) \geq 0$, we have

$$
8\left\|(T\delta - T\sigma)^2\right\|_{\infty} \leq \lambda\big(\gamma\big(d(\delta,\sigma)\big)\big)\gamma\big(d(\delta,\sigma)\big).
$$

Put $\alpha : C(J) \times C(J) \to [0,\infty)$ by

$$
\alpha(\delta,\sigma) = \begin{cases} 1 & \omega(\delta(s),\sigma(s)) \geq 0 \text{ for all } s \in J, \\ 0 & \text{else,} \end{cases}
$$

and

$$
\begin{aligned}
\alpha(\delta,\sigma)\gamma\big(8d(T\delta,T\sigma)\big) &\leq 8d(T\delta,T\sigma) \\
&\leq \lambda\big(\gamma\big(d(\delta,\sigma)\big)\big)\gamma\big(d(\delta,\sigma)\big).
\end{aligned}
$$

Then T is an α-γ-contractive mapping. From (iii),

$$
\begin{aligned}
\alpha(\delta,\sigma) \geq 1 \quad &\Rightarrow \quad \omega\big(\delta(s),\sigma(s)\big) \geq 0 \\
&\Rightarrow \quad \omega\big(T(\delta),T(\sigma)\big) \geq 0 \\
&\Rightarrow \quad \alpha\big(T(\delta),T(\sigma)\big) \geq 1,
\end{aligned}
$$

for $\delta,\sigma \in C(J)$. Therefore, T is α-admissible. From (ii), there exists $\delta_0 \in C(J)$ with $\alpha(\delta_0, T\delta_0) \geq 1$. By (iv) and Theorem 1.3, we conclude there exists $\delta^* \in C(J)$ with $\delta^* = T\delta^*$. Hence, δ^* is a solution of the problem. □

We denote by $\mathcal{F}$ the family of all functions that satisfy the following conditions:

(i) $F:\mathbb{R}_+\to\mathbb{R}$ is a strictly increasing mapping;

(ii) $\lim_{n\to\infty}F(\alpha_n)=-\infty$ if and only if, for each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers, $\lim_{n\to\infty}\alpha_n=0$;

(iii) there exists $k\in(0,1)$ such that $\lim_{\alpha\to 0^+}\alpha^k F(\alpha)=0$.

Definition 2.2 Let (M,d) be a metric space, $g:M\to M$ is said to be an α-type F-contraction on M if there exist $\nu>0$ and two functions $F\in\mathcal{F}$ and $\alpha:M\times M\to\{-\infty\}\cup(0,\infty)$ such that, for all $\delta,\sigma\in M$ satisfying $d(g\delta,g\sigma)>0$, we have

$$\nu+\alpha(\delta,\sigma)F\bigl(d(g\delta,g\sigma)\bigr)\le F\bigl(d(\delta,\sigma)\bigr).$$

Theorem 2.3 ([12]) *Let (M,d) be a metric space and $g:M\to M$ be an α-type F-contraction such that*:

(i) $\exists\delta_0\in M$ *with* $\alpha(\delta_0,g\delta_0)\ge 1$,

(ii) g *is α-admissible*,

(iii) *if* $\{\delta_n\}\subseteq M$ *with* $\alpha(\delta_n,\delta_{n+1})\ge 1$ *and* $\delta_n\to\delta$, *then* $\alpha(\delta_n,\delta)\ge 1$, $n\in N$,

(iv) F *is continuous*.

Then g has a fixed point $\delta^*\in M$ and, for every $\delta_0\in M$, the sequence $\{g^n\delta_0\}_{n\in N}$ is convergent to δ^*.

Theorem 2.4 *Suppose*

(i) $\exists\omega:\mathbb{R}^2\to\mathbb{R}$ *such that*

$$\bigl|h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr|\le\frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}e^{\frac{-\nu}{2}}\bigl|\delta(s)-\sigma(s)\bigr|$$

for $s\in J$ *and* $\delta,\sigma\in\mathbb{R}$ *with* $\omega(\delta,\sigma)\ge 0$;

(ii) $\exists\delta_1\in C(J)$ *such that* $\omega(\delta_1(s),T\delta_1(s))\ge 0$ *for* $s\in J$, *where* $T:C(J)\to C(J)$ *is defined by*

$$(T\delta)(s)=\delta_0+{}_0^{AB}I^{\kappa}h\bigl(s,\delta(s)\bigr);\tag{9}$$

(iii) *for* $s\in J$ *and* $\delta,\sigma\in C(J)$, $\omega(\delta(s),\sigma(s))\ge 0$ *implies* $\omega(T\delta(s),T\sigma(s))\ge 0$:

(iv) $\{\delta_n\}\subseteq C(J)$, $\delta_n\to\delta$ *in* $C(J)$ *and* $\omega(\delta_n,\delta_{n+1})\ge 0$, *then* $\omega(\delta_n,\delta)\ge 0$, $n\in N$.

Then problem (6) has at least one solution.

Proof Similar to the previous theorem, we demonstrate that T has a fixed point:

$$\begin{aligned}
&\bigl|T\delta(s)-T\sigma(s)\bigr|^2\\
&\quad=\bigl|{}_0^{AB}I^{\kappa}\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]\bigr|^2\\
&\quad\le\left|\left[\frac{1-\kappa}{B(\kappa)}\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]+\frac{\kappa}{B(\kappa)}{}_0I^{\kappa}\bigl[h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr]\right]\right|^2\\
&\quad\le\left\{\frac{1-\kappa}{B(\kappa)}\bigl|h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr|+\frac{\kappa}{B(\kappa)}{}_0I^{\kappa}\bigl|h\bigl(s,\delta(s)\bigr)-h\bigl(s,\sigma(s)\bigr)\bigr|\right\}^2\\
&\quad\le\left\{\frac{1-\kappa}{B(\kappa)}\times\frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}e^{\frac{-\nu}{2}}\sqrt{\bigl|\delta(s)-\sigma(s)\bigr|^2}\right.
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{\kappa}{B(\kappa)} \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} {}_0I^{\kappa}(1) e^{\frac{-\nu}{2}} \sqrt{\left|\delta(s)-\sigma(s)\right|^2} \Bigg\}^2 \\
&= \left\{ \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} e^{\frac{-\nu}{2}} \sqrt{\left|\delta(s)-\sigma(s)\right|^2} \right\}^2 \\
&\quad \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{\kappa}{B(\kappa)} \frac{1}{\kappa\Gamma(\kappa)} \right\}^2 \\
&\leq \left\{ \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} e^{\frac{-\nu}{2}} \sqrt{\sup_{s\in J}\left|\delta(s)-\sigma(s)\right|^2} \right\}^2 \\
&\quad \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{\kappa}{B(\kappa)} \frac{1}{\kappa\Gamma(\kappa)} \right\}^2 \\
&= \left\{ \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1} e^{\frac{-\nu}{2}} \sqrt{d(\delta,\sigma)} \right\}^2 \times \left\{ \frac{1-\kappa}{B(\kappa)} + \frac{1}{B(\kappa)\Gamma(\kappa)} \right\}^2 \\
&= e^{-\nu} d(\delta,\sigma).
\end{aligned}
$$

Hence, for $\delta,\sigma \in C(J)$, $s \in J$ with $\omega(\delta(s),\sigma(s)) \geq 0$, we have

$$d(T\delta,T\sigma) \leq e^{-\nu} d(\delta,\sigma).$$

So

$$\ln\bigl(d(T\delta,T\sigma)\bigr) \leq \ln\bigl(e^{-\nu} d(\delta,\sigma)\bigr),$$

therefore

$$\nu + \ln\bigl(d(T\delta,T\sigma)\bigr) \leq \ln\bigl(d(\delta,\sigma)\bigr).$$

Now, let $F : [0,\infty) \to R$ given by $F(u) = \ln u$, $u > 0$, then $F \in \mathcal{F}$.

Put $\alpha : C(J) \times C(J) \to \{-\infty\} \cup [0,\infty)$ by

$$\alpha(\delta,\sigma) = \begin{cases} 1 & \omega(\delta(s),\sigma(s)) \geq 0 \text{ for all } s \in J, \\ -\infty & \text{else.} \end{cases}$$

Therefore $\nu + \alpha(\delta,\sigma)F(d(T\delta,T\sigma)) \leq F(d(\delta,\sigma))$ for $\delta,\sigma \in M$ with $d(T\delta,T\sigma) > 0$. For this reason, T is an α-type F-contraction. From (iii),

$$
\begin{aligned}
\alpha(\delta,\sigma) \geq 1 \quad &\Rightarrow \quad \omega\bigl(\delta(s),\sigma(s)\bigr) \geq 0 \\
&\Rightarrow \quad \omega\bigl(T(\delta),T(\sigma)\bigr) \geq 0 \\
&\Rightarrow \quad \alpha\bigl(T(\delta),T(\sigma)\bigr) \geq 1,
\end{aligned}
$$

for all $\delta,\sigma \in C(J)$. Thus, T is α-admissible. From (ii), there exists $\delta_0 \in C(J)$ with $\alpha(\delta_0, T\delta_0) \geq 1$. By (iv) and Theorem 2.3, we conclude $\delta^* \in C(J)$ with $\delta^* = T\delta^*$. Hence, δ^* is a solution of the problem. □

Now let $\mathcal{F}$ be the set of functions $g : (0,\infty) \to \mathbb{R}$ with the conditions:

($\mathcal{F}_1$) If $0 < s < t$, then $g(s) \leq g(t)$;
($\mathcal{F}_2$) If $\{s_n\} \subset (0, +\infty)$, then

$$\lim_{n\to+\infty} s_n = 0 \quad \text{if and only if} \quad \lim_{n\to+\infty} g(s_n) = -\infty.$$

The space of an $\mathcal{F}$-metric is defined as follows.

Definition 2.5 ([36]) Let M be nonempty, $d : M \times M \to [0, +\infty)$ and $(g, a) \in \mathcal{F} \times [0, +\infty)$ such that
(d_1) $(\delta, \sigma) \in M \times M$, $d(\delta, \sigma) = 0 \Leftrightarrow \delta = \sigma$;
(d_2) $d(\delta, \sigma) = d(\sigma, \delta)$, for $(\delta, \sigma) \in M \times M$;
(d_3) For $(\delta, \sigma) \in M \times M$, $N \in \mathbb{N}$, $N \geq 2$, and for $(u_i)_{i=1}^N \subset M$ with $(u_1, u_N) = (\delta, \sigma)$, we have

$$d(\delta, \sigma) > 0 \quad \text{implies} \quad g\bigl(d(\delta, \sigma)\bigr) \leq g\left(\sum_{i=1}^{N-1} d(u_i, u_{i+1})\right) + a.$$

Then d is said to be an $\mathcal{F}$-metric on M, and the pair (M, d) is said to be an $\mathcal{F}$-metric space.

A sequence $\{\delta_n\}$ in (M, d) is convergent to δ with respect to the $\mathcal{F}$-metric d if

$$\lim_{n\to\infty} d(\delta_n, \delta) = 0.$$

A sequence $\{\delta_n\}$ in (M, d) is called $\mathcal{F}$-Cauchy if

$$\lim_{n,m\to+\infty} d(\delta_n, \delta_m) = 0.$$

(M, d) is $\mathcal{F}$-complete if every $\mathcal{F}$-Cauchy sequence in M is $\mathcal{F}$-convergent to a specified element in M. Let Γ be the set of functions $\gamma : [0, \infty) \to [0, \infty)$ such that
(γ_1) γ is nondecreasing;
(γ_2) $\sum_{n=1}^{\infty} \gamma^n(s) < \infty$ for $s \in \mathbb{R}^+$, where γ^n is the nth iterate of γ.

Definition 2.6 ([37]) Let $\alpha : M \times M \to [0, \infty)$, then $g : M \to M$ is said to be an α-orbital admissible if, for $s \in M$, we have

$$\alpha(s, gs) \geq 1 \quad \Rightarrow \quad \alpha\bigl(gs, g^2 s\bigr) \geq 1. \tag{10}$$

Theorem 2.7 ([9]) *Assume (M, d) to be an $\mathcal{F}$-complete metric space and $g : M \to M$ such that*

$$\alpha(\delta, \sigma) d(g\delta, g\sigma) \leq \gamma\bigl(d(\delta, \sigma)\bigr)$$

for $\delta, \sigma \in M$, where $\gamma \in \Gamma$. Suppose
(i) *g is orbital α-admissible;*
(ii) *there exists $\delta_0 \in M$ with $\alpha(\delta_0, g\delta_0) \geq 1$;*
(iii) *$g \in \mathcal{F}$ verifying (d_3) is assumed to be continuous; also, γ is chosen to be continuous and to satisfy that $g(u) > g(\gamma(u)) + a$, $u \in (0, \infty)$, where a is also given in (d_3);*
then f has a fixed point.

Consider the $\mathcal{F}$-metric $d : M \times M \to [0,\infty)$ with $M = C(J,\mathbb{N})$, given as

$$d(\delta,\sigma) = \begin{cases} e^{|\delta-\sigma|} & \text{if } \delta \neq \sigma, \\ 0 & \text{if } \delta = \sigma, \end{cases}$$

where $M = \{0,1,2,\ldots\}$, $g(s) = -\frac{1}{s}$ for $s > 0$, $a = 1$ and g is continuous on $(0,\infty)$. The condition $g(u) > g(\gamma(u)) + a$, $u > 0$, becomes $-\frac{1}{u} > \frac{1}{\gamma(u)} > 1$, that is, γ is chosen to be continuous such that

$$\gamma(u) < \frac{u}{u+1}.$$

Also consider that γ satisfies the following additional condition:

$$e^{\gamma(s)} \leq \gamma\left(e^{s}\right), \quad s \in \{0,1,2,3,\ldots\}.$$

Theorem 2.8 *Assume*

(i) $\exists \omega : \mathbb{R}^2 \to \mathbb{R}$ *with*

$$\left|h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right| \leq \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}\gamma\left(\left|\delta(s)-\sigma(s)\right|\right)$$

for $s \in J$ *and* $\delta, \sigma \in \mathbb{R}$ *with* $\omega(\delta,\sigma) \geq 0$;

(ii) $\exists\, \delta_1 \in C(J)$ *with* $\omega(\delta_1(s), T\delta_1(s)) \geq 0$ *for* $s \in J$, *where* $T : C(J) \to C(J)$ *is defined by*

$$(T\delta)(s) = \delta_0 + {}_0^{AB}I^{\kappa} h\big(s,\delta(s)\big); \tag{11}$$

(iii) *for* $s \in J$ *and* $\delta \in C(J)$, $\omega(\delta(s), T\delta(s)) \geq 0$ *implies* $\omega(T\delta(s), T^2\delta(s)) \geq 0$.

Then (6) *has at least one solution.*

Similar to the previous theorem, we demonstrate that T has a fixed point:

$$\begin{aligned}
&\left|T\delta(s) - T\sigma(s)\right| \\
&\quad= \left|{}_0^{AB}I^{\kappa}\left[h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right]\right| \\
&\quad\leq \left|\left[\frac{1-\kappa}{B(\kappa)}\left[h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right] + \frac{\kappa}{B(\kappa)}{}_0I^{\kappa}\left[h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right]\right]\right| \\
&\quad\leq \left\{\frac{1-\kappa}{B(\kappa)}\left|h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right| + \frac{\kappa}{B(\kappa)}{}_0I^{\kappa}\left|h\big(s,\delta(s)\big) - h\big(s,\sigma(s)\big)\right|\right\} \\
&\quad\leq \left\{\frac{1-\kappa}{B(\kappa)} \times \frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}\gamma\left(\left|\delta(s)-\sigma(s)\right|\right)\right\} \\
&\qquad + \left\{\frac{\kappa}{B(\kappa)}\frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}{}_0I^{\kappa}(1)\gamma\left(\left|\delta(s)-\sigma(s)\right|\right)\right\} \\
&\quad= \left\{\frac{B(\kappa)\Gamma(\kappa)}{(1-\kappa)\Gamma(\kappa)+1}\gamma\left(\left|\delta(s)-\sigma(s)\right|\right)\right\}\left\{\frac{1-\kappa}{B(\kappa)} + \frac{\kappa}{B(\kappa)}\frac{1}{\kappa\Gamma(\kappa)}\right\} \\
&\quad= \gamma\left(\left|\delta(s)-\sigma(s)\right|\right).
\end{aligned}$$

Hence, for $\delta, \sigma \in C(J)$, $s \in J$ with $\omega(\delta(s), \sigma(s)) \geq 0$, we have

$$d(T\delta, T\sigma) = e^{|T\delta(s)-T\sigma(s)|} \leq e^{\gamma(|\delta(s)-\sigma(s)|)} \leq \gamma\left(e^{|\delta(s)-\sigma(s)|}\right) = \gamma\left(d(\delta, \sigma)\right).$$

Put $\alpha : C(J) \times C(J) \to [0, \infty)$ by

$$\alpha(\delta, \sigma) = \begin{cases} 1 & \omega(\delta(s), \sigma(s)) \geq 0 \text{ for all } s \in J, \\ 0 & \text{else.} \end{cases}$$

Therefore $\alpha(\delta, \sigma)d(T\delta, T\sigma) \leq d(T\delta, T\sigma) \leq \gamma(d(\delta, \sigma))$ for all $\delta, \sigma \in M$ with $d(T\delta, T\sigma) > 0$. From (iii),

$$\begin{aligned} \alpha(\delta, T\delta) \geq 1 \quad & \Rightarrow \quad \omega\big(\delta(s), T\delta(s)\big) \geq 0 \\ & \Rightarrow \quad \omega\big(T(\delta), T^2(\delta)\big) \geq 0 \\ & \Rightarrow \quad \alpha\big(T(\delta), T^2(\delta)\big) \geq 1, \end{aligned}$$

for $\delta \in C(J)$. Thus, T is orbital α-admissible. From (ii), there exists $\delta_1 \in C(J)$ with $\alpha(\delta_1, T\delta_1) \geq 1$. By (iii) and Theorem 2.7, we get $\sigma^* \in C(J)$ with $\delta^* = T\delta^*$. Hence, δ^* is a solution of the problem.

3 Conclusion

In this manuscript, we extend some of the fractional differential equations of Riemann–Liouville and Caputo type to the fractional differential equations of Atangana–Baleanu in the Caputo sense.

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