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Research Article

Local Fractional Discrete Wavelet Transform for Solving Signals on Cantor Sets

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The discrete wavelet transform via local fractional operators is structured and applied to process the signals on Cantor sets. An illustrative example of the local fractional discrete wavelet transform is given.

1. Introduction

In recent years, the classical wavelet theory [1–7] has played an important role in many scientific fields such as signal processing [8], electrical systems [9], image processing [10], and differential equations [11]. The continuous wavelet transform is applied to handle the analyzing nonstationary signals, which have some characteristics of instantaneous peaks or discontinuities, where the mother wavelet met scaling and translation operations [3]. Two major categories of wavelet transforms are continuous and discrete [5]. When the mother wavelet functions are orthonormal, the discrete wavelet transform [12] gives multiresolution algorithm decomposing signals into scales with different time and frequency resolution, which leads to finite number of wavelet comparisons of signals, and improves the computational speeds because of the functions that are stretched or compressed and placed at many positions along the signals [13].

Based on the fractional Fourier transform [14–17], the fractional wavelet transform, which was a good tool for processing transient signals and compressing images, was structured in [18, 19]. The fractional wavelet transform has some applications in various branches of science and engineering [20–23]. For example, the simultaneous spectral analysis of a binary mixture system was presented in [20] by using the fractional wavelet transform. Application of the fractional wavelet transform to the simultaneous determination of ampicillin sodium and sulbactam sodium in a binary mixture was considered in [21]. The fractional wavelet transform for the quantitative spectral resolution of the composite signals of the active compounds in a two-component mixture was suggested in [22]. The optical image encryption based on fractional wavelet transform was given in [23]. By discretizing continuous fractional wavelet transform, the discrete fractional wavelet transform was reported and its application to multiple encryptions was considered in [24].

The wavelet method and its fractional counterpart have many applications in various branches of science and engineering. However, they are invalid for solving the signals defined on Cantor sets. The local fractional calculus theory [25–34] was applied to handle the functions defined on Cantor sets, which are local fractional continuous. A natural question is to generalize signals concepts on the Cantor set, which are the nondifferentiable functions defined on Cantor sets [24, 26] and the Cantor function [35]. The mathematical theory of the local fractional wavelet transform of the local fractional continuous signal was structured in [25, 36] based on the basic idea.

One of the open problems in this area is how to improve the computational speeds of the local fractional wavelet theory as in the classical one. The aim of this paper is to structure the discrete version of the local fractional wavelet transform based on the generalized inner production space. The paper has been organized as follows. In Section 2, we introduce some basic notations and theorems of the generalized inner product space. In Section 3, we propose the local fractional discrete wavelet transform. In Section 4, one example is presented. Finally, Section 5 is conclusions.

2. Preliminaries

In this section, we give some basic notations and theorems of the generalized inner product space.

Let [25]

$$L_{2,\alpha}[R]=\left\{f(x)\in C_\alpha[R]:\left(\frac{1}{\Gamma(1+\alpha)}\int_{-\infty}^{\infty}|f(x)|^p(dx)^\alpha\right)^{1/p}<\infty,1\le p<\infty\right\}. \quad (1)$$

Here, the local fractional integral operator $f(x)$ in the interval $[a,b]$ was defined in [25–30] as

$$\begin{aligned}{}_aI_b^{(\alpha)}f(x)&=\frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(dt)^\alpha\\&=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to 0}\sum_{j=0}^{j=N-1}f(t_j)(\Delta t_j)^\alpha,\end{aligned} \quad (2)$$

where a partition of the interval $[a,b]$ is denoted as $\Delta t_j=t_{j+1}-t_j$, $\Delta t=\max\{\Delta t_0,\Delta t_1,\Delta t_j,\dots\}$ and $j=0,\dots,N-1$, $t_0=a$, $t_N=b$. Local fractional operators were applied to model some nondifferentiable problems [25–32].

From (1) the generalized inner product space of $L_{2,\alpha}[R]$ is defined as follows [25]:

$$\langle f,g\rangle_\alpha=\frac{1}{\Gamma(1+\alpha)}\int_{-\infty}^{\infty}f(x)\overline{g(x)}(dx)^\alpha. \quad (3)$$

The two useful theorems are presented as follows.

Theorem 1 (see [25]). *Let X be an inner product space. If $\{e_n^\alpha\}$ is an orthonormal system in X, then one has that*

$$\|f\|_\alpha^2=\sum_{i=1}^{\infty}\left|\langle f,e_i^\alpha\rangle_\alpha\right|^2, \quad (4)$$

$$f=\sum_{i=1}^{\infty}\langle f,e_i^\alpha\rangle_\alpha e_i^\alpha \quad (5)$$

are equivalent, where $\|f\|_\alpha^2$ is a norm of the function f and $\{e_n^\alpha\}$ has the following properties:

$$\begin{aligned}\|e_n^\alpha\|_\alpha&=1,\\ \langle e_i^\alpha,e_j^\alpha\rangle&=\begin{cases}0, & i\ne j,\\ 1, & i=j.\end{cases}\end{aligned} \quad (6)$$

Proof. See [25]. □

Theorem 2 (see [25]). *Let X be an inner product space and $\{e_n^\alpha\}$ be an orthonormal system in X. If $x^\alpha\in\operatorname{span}\{e_1^\alpha,\dots,e_n^\alpha\}$, then for all $x^\alpha\in X$ one has*

$$x^\alpha=\sum_{i=1}^{n}\langle x^\alpha,e_i^\alpha\rangle_\alpha e_i^\alpha, \quad (7)$$

where $\operatorname{span}\{x_1^\alpha,\dots,x_n^\alpha\}$ is the linear subspace of X of the linear span of the local fractional vectors [25], namely,

$$\operatorname{span}\{x_1^\alpha,\dots,x_n^\alpha\}=\left\{x^\alpha=\sum_{i=1}^{n}a_ix_i^\alpha:a_i\in E\right\}. \quad (8)$$

Proof. See [25]. □

3. Local Fractional Discrete Wavelet Transform for Signals on Cantor Sets

3.1. Local Fractional Continuous Wavelet Transformation for Signals on Cantor Sets. The local fractional continuous wavelet transform of the local fractional continuous signal $f(t)$ was presented in [25, 26, 36] as

$$W_{\varphi,\alpha}f(a,b)=\frac{a^{-(\alpha/2)}}{\Gamma(1+\alpha)}\int_{-\infty}^{\infty}f(t)\overline{\varphi_{a,b,\alpha}(t)}(dt)^\alpha,\quad 0<\alpha\le 1, \quad (9)$$

where the local fractional daughter's wavelets were suggested in [25, 26, 36] by

$$\varphi_{a,b,\alpha}(t)=\frac{1}{a^{\alpha/2}}\varphi\left(\frac{t-b}{a}\right), \quad (10)$$

where a is the dyadic dilation, b is the dyadic position, and $a^{-(\alpha/2)}$ is the normalization Cantor factor. The inverse

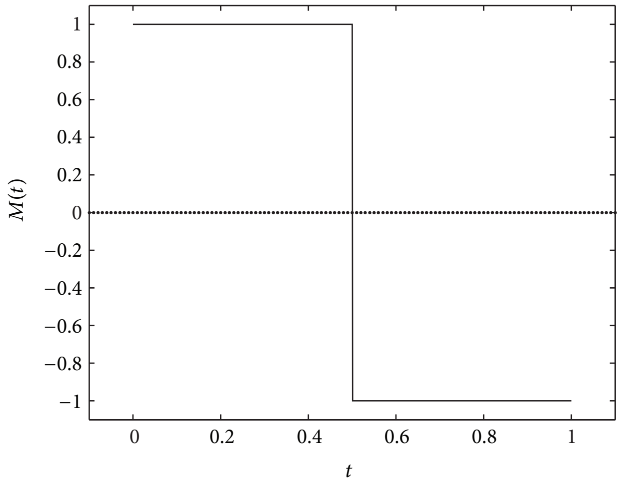

Figure 1: The graph of the local fractional mother wavelet.

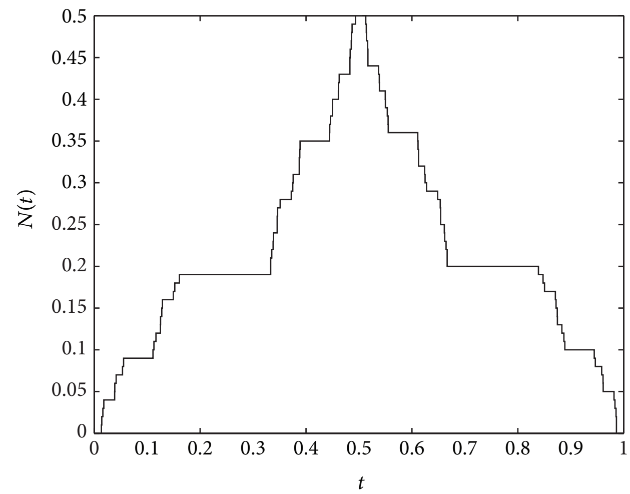

Figure 2: The graph of the local fractional integral of local fractional mother wavelet.

formula of local fractional wavelet transform was given in [25, 36] by

$$f(x) = \frac{C_{\varphi,\alpha}}{\Gamma^2(1+\alpha)} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a^{-2\alpha} W_{\varphi,\alpha} f(a,b)\, \varphi_{a,b,\alpha}(t)(da)^{\alpha}(db)^{\alpha}, \quad 0<\alpha\leq 1, \tag{11}$$

where the parameter is [25, 36]

$$C_{\varphi,\alpha} = \frac{1}{\Gamma(1+\alpha)}\int_{-\infty}^{\infty} \frac{|f(x)|^2}{|x|^{\alpha}}(dx)^{\alpha}, \quad 0<\alpha\leq 1. \tag{12}$$

We notice that the classical continuous wavelet transform is the local fractional one in case of fractal dimension $\alpha = 1$.

3.2. Local Fractional Discrete Wavelet Transform for Signals on Cantor Sets. Let us structure the local fractional daughter wavelet in the form

$$\varphi_{a,b,\alpha}(t) = \frac{1}{a^{\alpha/2}}\varphi\left(\frac{t-b}{a}\right), \tag{13}$$

where $\varphi \in L_{2,\alpha}[R]$.

When $a = 2^{-j}$ and $b = k2^{-j}$, we get

$$\varphi_{a,b,\alpha}(t) = \varphi_{j,k,\alpha}(t) = \varphi_{2^{-j},k2^{-j},\alpha}(t) = 2^{j\alpha/2}\varphi\left(2^{j}t-k\right) \tag{14}$$

for integers $j, k \in Z$.

Let $\varphi_{j,k,\alpha}(t) = 2^{j\alpha/2}\varphi(2^{j}t-k)$ be orthogonal set of local fractional wavelets. Then we can obtain

$$\left\langle \varphi_{j,k,\alpha}, \varphi_{m,n,\alpha}\right\rangle_{\alpha} = \delta_{j,m}^{\alpha}\delta_{k,n}^{\alpha}, \quad j,k,m,n\in Z, \tag{15}$$

where $\delta_{j,m}^{\alpha}$ and $\delta_{k,n}^{\alpha}$ are local fractional Kronecker delta [27].

Making use of (7), for $j, k, m \in Z$ we have

$$f(x) = \sum_{j=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} a_{j,k,\alpha} e_{j,k}^{\alpha}, \tag{16}$$

where its coefficients are

$$a_{j,k} = \left\langle f(x), e_{j,k}^{\alpha}\right\rangle_{\alpha} = W_{\varphi,\alpha} f\left(2^{-j}, k2^{-j}\right). \tag{17}$$

Here, $a_{j,k}$ is called as the local fractional discrete wavelet transform of the signal $f(x)$.

4. An Illustrative Example

Local fractional mother wavelet is defined in [26] as

$$\varphi_{H(\alpha)}(t) = M(t) = \begin{cases} 1, & 0 \leq t < \frac{1}{2} \\ -1, & \frac{1}{2} \leq t < 1 \\ 0, & \text{else} \end{cases} \tag{18}$$

and local fractional integral of local fractional mother wavelet reads as

$$\phi_{H(\alpha)}(t) = N(t) = \begin{cases} \frac{t^{\alpha}}{\Gamma(1+\alpha)}, & 0 \leq t < \frac{1}{2} \\ \frac{(1-t)^{\alpha}}{\Gamma(1+\alpha)}, & \frac{1}{2} \leq t < 1 \\ 0, & \text{else.} \end{cases} \tag{19}$$

Figure 1 shows the graph of the local fractional mother wavelet and Figure 2 shows the graph of the local fractional integral of local fractional mother wavelet.

When fractal dimension $\alpha = 1$, we have

$$\varphi_{H(1)}(t) = M(t) \tag{20}$$

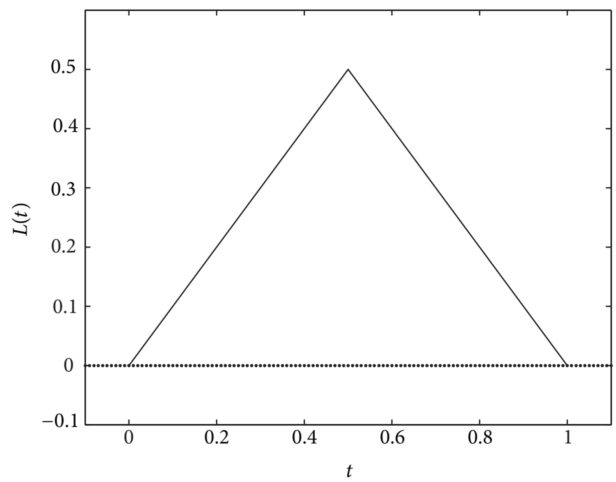

Figure 3: The graph of the integral of the mother wavelet.

so that

$$\phi_{H(1)}(t) = L(t) = \begin{cases} t, & 0 \le t < \dfrac{1}{2} \\ 1-t, & \dfrac{1}{2} \le t < 1 \\ 0, & \text{else.} \end{cases} \quad (21)$$

Figure 3 shows the graph of the integral of mother wavelet $\varphi_{H(1)}(t)$.

For integers $j, k \in \mathrm{Z}$, we have [26]

$$\varphi_{H(\alpha)}^{j,k}(t) = 2^{j\alpha/2}\varphi_{H(\alpha)}\left(2^j t - k\right), \quad (22)$$

where

$$\varphi_{H(\alpha)}(t) = \begin{cases} 1, & 0 \le t < \dfrac{1}{2}, \\ -1, & \dfrac{1}{2} \le t < 1, \\ 0, & \text{else.} \end{cases} \quad (23)$$

Hence, we have

$$\begin{aligned} &\left\langle \varphi_{H(\alpha)}^{j,k}, \varphi_{H(\alpha)}^{m,n} \right\rangle_\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} \varphi_{H(\alpha)}^{j,k}(t)\, \varphi_{H(\alpha)}^{m,n}(t)\,(dt)^\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} 2^{j\alpha/2}\varphi_{H(\alpha)}\left(2^j t - k\right) 2^{m\alpha/2}\varphi_{H(\alpha)} \\ &\qquad\qquad \times \left(2^m t - n\right)(dt)^\alpha \\ &\quad = 2^{(j+m)\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}\left(2^j t - k\right)\varphi_{H(\alpha)}\left(2^m t - n\right)(dt)^\alpha \\ &\quad = 2^{(m-j)\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\,\varphi_{H(\alpha)}\left(2^{m-j}(s+k) - n\right)(ds)^\alpha, \end{aligned} \quad (24)$$

where $s = 2^j t - k$.

In view of (24), we obtain [15]

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} \left[\varphi_{H(\alpha)}^{j,k}(t)\right]^2 (dt)^\alpha = 1, \\ &\frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} \varphi_{H(\alpha)}^{j,k}(t)\,(dt)^\alpha = 0, \end{aligned} \quad (25)$$

where $j = m$ and $k = n$, $j, k \in \mathrm{Z}$.

When $j = m$, $j, k, m \in \mathrm{Z}$, from (24) we obtain

$$\begin{aligned} &\left\langle \varphi_{H(\alpha)}^{j,k}, \varphi_{H(\alpha)}^{j,n} \right\rangle_\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} \varphi_{H(\alpha)}^{j,k}(t)\, \varphi_{H(\alpha)}^{m,n}(t)\,(dt)^\alpha \\ &\quad = 2^{(j+m)\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}\left(2^j t - k\right)\varphi_{H(\alpha)}\left(2^m t - n\right)(dt)^\alpha \\ &\quad = 2^{(m-j)\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\,\varphi_{H(\alpha)}\left(2^{m-j}(s+k) - n\right)(ds)^\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\,\varphi_{H(\alpha)}(s+k-n)\,(ds)^\alpha \\ &\quad = \delta_{0,k-n}^\alpha \\ &\quad = \delta_{k,n}^\alpha, \end{aligned} \quad (26)$$

where $s = 2^j t - k$.

When $g = m - j > 0$, $j, k, m, n \in \mathrm{Z}$, from (24) we have

$$\begin{aligned} &\left\langle \varphi_{H(\alpha)}^{j,k}, \varphi_{H(\alpha)}^{m,n} \right\rangle_\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} \varphi_{H(\alpha)}^{j,k}(t)\, \varphi_{H(\alpha)}^{m,n}(t)\,(dt)^\alpha \\ &\quad = 2^{g\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\,\varphi_{H(\alpha)}\left(2^g(s+k) - n\right)(ds)^\alpha \\ &\quad = 2^{g\alpha/2}\frac{1}{\Gamma(1+\alpha)} \\ &\qquad \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\,\varphi_{H(\alpha)}\left(2^g s + \eta\right)(ds)^\alpha, \end{aligned} \quad (27)$$

where $s = 2^j t - k$ and $\eta = 2^g k - n$. Consider

$$\begin{aligned}
&\left\langle \varphi_{H(\alpha)}^{j,k}, \varphi_{H(\alpha)}^{m,n} \right\rangle_\alpha \\
&= 2^{g\alpha/2} \frac{1}{\Gamma(1+\alpha)} \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\, \varphi_{H(\alpha)}(2^g s + \eta)\,(ds)^\alpha \\
&= 2^{g\alpha/2} \frac{1}{\Gamma(1+\alpha)} \times \int_{-\infty}^{\infty} \varphi_{H(\alpha)}(s)\, \varphi_{H(\alpha)}(2^g s + \eta)\,(ds)^\alpha \\
&= 2^{g\alpha/2} \left[\frac{1}{\Gamma(1+\alpha)} \int_0^{1/2} \varphi_{H(\alpha)}(2^g s + \eta)\,(ds)^\alpha - \frac{1}{\Gamma(1+\alpha)} \int_{1/2}^{1} \varphi_{H(\alpha)}(2^g s + \eta)\,(ds)^\alpha \right] \\
&= 2^{-g\alpha/2} \left[\frac{1}{\Gamma(1+\alpha)} \int_{\eta}^{2^{g-1}+\eta} \varphi_{H(\alpha)}(q)\,(dq)^\alpha - \frac{1}{\Gamma(1+\alpha)} \int_{2^{g-1}+\eta}^{2^g+\eta} \varphi_{H(\alpha)}(q)\,(dq)^\alpha \right],
\end{aligned} \tag{28}$$

where

$$\begin{gathered}
q = 2^g s + \eta, \\
\frac{1}{\Gamma(1+\alpha)} \int_{\eta}^{2^{g-1}+\eta} \varphi_{H(\alpha)}(q)\,(dq)^\alpha = 0, \\
\frac{1}{\Gamma(1+\alpha)} \int_{2^{g-1}+\eta}^{2^g+\eta} \varphi_{H(\alpha)}(q)\,(dq)^\alpha = 0,
\end{gathered} \tag{29}$$

with $\eta > 1$, $2^{g-1} + \eta > 1$, and $2^g + \eta > 1$.

Hence, taking $e_{j,k}^{\alpha} = \varphi_{H(\alpha)}^{j,k}$ gives

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_{j,k,\alpha} \varphi_{H(\alpha)}^{j,k}(x), \tag{30}$$

where

$$\begin{aligned}
a_{j,k} &= \left\langle f(x), \varphi_{H(\alpha)}^{j,k}(x) \right\rangle_\alpha \\
&= W_{\varphi,\alpha} f\left(2^{-j}, k2^{-j}\right) \\
&= 2^{j\alpha/2} \frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} f(x)\, \overline{\varphi}_{H(\alpha)}^{j,k}(x)\,(dx)^\alpha.
\end{aligned} \tag{31}$$

Appling (4), we have

$$f^2(x) = \sum_{i=1}^{\infty} \left| a_{j,k} \right|^2 \tag{32}$$

with

$$a_{j,k} = 2^{j\alpha/2} \frac{1}{\Gamma(1+\alpha)} \int_{-\infty}^{\infty} f(x)\, \overline{\varphi}_{H(\alpha)}^{j,k}(x)\,(dx)^\alpha. \tag{33}$$

Hence, from (32) we find that the energy is conserved.

5. Conclusions

In this work the local fractional discrete wavelet transform based on the local fractional calculus theory was proposed. By using the basic theorems of generalized inner product space, the local fractional discrete wavelet transform and its reconstruction formula were discussed. We find that the energy of the signal on Cantor sets is conserved. An illustrative example for the local fractional wavelet transform of the signal on Cantor sets was given. It is shown that the classical discrete wavelet transform is the local fractional one in case of fractal dimension $\alpha = 1$.

Conflict of Interests

The authors declare that they have no conflict of interests regarding this paper.

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