GROUP DELAY REDUCTION IN FINITE IMPULSE RESPONSE DIGITAL FILTERS

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GROUP DELAY REDUCTION IN FINITE IMPULSE RESPONSE DIGITAL FILTERS

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ABSTRACT

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Linear-phase finite impulse response (FIR) filters are excessively utilized in digital signal processing programs due various advantage. These advantages include that there is no distorted phase, unrestricted stability, and lower filter-coefficient insecurity. Most important shortcoming of linear-phase FIR filter is that the total group delay is (N-1)/2 where N is scope of filter. The amount becomes large to filter orders that are higher in telecommunication programs. Many algorithms have been proposed to reduce this delay and its distortion. Typically, block involution mechanism like overlap-add method (OAM) and overlap-save method (OSM) are used for a long input sequence. Yet, with respect to input, by using these techniques, the output series includes a finite group delay. In this thesis, the performance of enhanced modified overlap and save method is investigated. First, the impulse response is made causal and then it is shifted left (circular) by an amount of $(N-1)/2$ for N odd and $N/2$ for N even. Finally, the samples to be excluded from the final convolution are defined. It is expected that this results in a reduction in the causal delay and also in the group delay. Simulations are carried out by MATLAB. The performance of the method is compared with the results obtained from the OSM based filter.

Keywords: Impulse Response, FIR Filter, Linear Phase FIR Filter, Group Delay.

ÖZ

SONLU DÜRTÜ CEVAPLI DİJİTAL FILTRELERINDE GRUP GECİKME AZALTMA YÖNTEMLERİ

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Doğrusal fazlı sonlu dürtü cevaplı (FIR) süzgeçler çok çeşitli avantajlarından dolayı, sayısal sinyal işleme uygulamalarında yaygın olarak kullanılmaktadırlar. Bu avantajlar, faz bozunumu içermeyen, sınırsız kararlılık ve süzgeç katsayılarına olan az duyarlılık olarak sıralanabilir. Ancak, doğrusal fazlı FIR süzgeçlerinin en önemli dezavantajı ise toplam grup gecikmesinin (N-1)/2 olarak ortaya çıkmasıdır. Buradaki N, süzgeçteki katsayı sayısını temsil etmektedir. Haberleşme uygulamalarında, toplam gecikmenin miktarı süzgeç katsayılarının sayıları ile doğru orantılı olarak artmaktadır.

Bu gecikmeyi ve ondan dolayı oluşan bozunumu azaltmak için birçok algoritma önerilmiştir. Tipik olarak, uzun giriş dizileri için blok konvolüsyon olarak bilinen üstüste binik toplama metodu (OAM) ve üstüste binik saklama metodu (OSM) kullanılır. Bu metodlar kullanıldığında, çıkış dizisinin girişe göre sonlu bir grup gecikmesi vardır.

Bu tezde, iyileştirilip modifiye edilmiş üstüste binik saklama metodunun performansı incelemiştir. İlk olarak, süzgeçin dürtü cevabı nedensel yapılmıştır ve bu dürtü cevabı daha sonra, N tek olduğu zaman (N-1)/2 kadar sola, çift olduğu zaman ise N/2 kadar yine sola kaydırılmıştır.

Son olarak, son konvolüsyon sonucundan dışlanacak olan örnekler tanımlanmıştır. Bunun, nedensel gecikme ve grup gecikmesinin azalmasına neden olacağı beklenmektedir. Benzetim çalışmaları MATLAB ortamında yapılarak, bu metodun performansı, OSM metodundan elde edilen sonuçlarla karşılaştırılmıştır.

Anahtar kelimeler: Dürtü Cevabı, FIR Süzgeç, Doğrusal Fazlı FIR Süzgeç, Grup Gecikmesi.

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LIST OF ABBREVIATIONS

ZDMOSM	Zero Delay Modified Overlap-Save Method
RDMOSM	Reduced Delay Modified Overlap-Save Method
DHT	Discrete Hilbert Transform
ERDMOSM	Enhanced Reduced Delay Modified Overlap-Save Method
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
OSM	Overlap-Save Method
OAM	Overlap-Add Method
IR	Impulse Response
IFFT	Inverse Fast Fourier Transform
WLS	Weighted Least Squares
DFT	Discrete Fourier Transform
FFT	Fast Fourier Transform

CHAPTER 1

INTRODUCTION

Finite impulse response (FIR) filter is defined as the digital filters execute mathematical procedures at piece of detached-time signal due changes some of its characteristics in a desired manner. Linear-phase FIR filters are preferred in digital signal treating programs due to their various advantages. These advantages include stabilization, and that there is no any distorted phase and lower filter-coefficient insecurity.

The linear-phase FIR filter also has a main disadvantage which is the overall group delay $(N - 1)/2$ in specific applications, where N represents the filter length. It is obvious that this amount becomes larger when high order filters are considered. Also, such a large amount of group delay leads to untolerable echoes of the transmitted signals in communication applications. There are a lot of important applications of linear phase filters, in which the most delayed groups caused by linear phase is important (for instance in electrocardiography in which modification could be applied for the delay location of the QRS complex [1], in two- route speech telecommunication systems calling for a low round-trip delay).

On the other hand, large delay in discrete-time control applications is also unacceptable. The group delay slows down the speed of processor. A multirate digital signal processing approach has been suggested to minimize this delay in active noise control systems [2].

A recent category of maximally nonsymmetrical the improvement has been effective for the flat FIR lowpass filter as in [3] to improve performance of the designing of the filter. In comparative manner, this improvement gives a constant of group delay, unlike the symmetric filter with no collapse of frequency response volume. A robust non-iterative algorithm is presented in [4] in order to plan optimum minimum-phase digital FIR filters with arbitrary magnitude responses depending on discrete Hilbert transform (DHT). When the DHT is extended to the complicated case where the minimum-phase filters require less memory and lower arithmetic computation than linear-phase filters in order to satisfy the same restrictions on the response of delay and volume. So through this algorithm, the magnitude scale of the amputated minimum-phase series would differ from the actual magnitude scale. As a result, it is possible to compute the minimum delayed outcome response without making any change in the impulse response (IR) of the filter. It looks like unattainable in accordance with the most common filtering algorithms, for instance overlap add method (OAM), and overlap save method (OSM) mentioned in [5] and this technique is utilized by the following convolution equation:

$$y(n) = \sum_{k=0}^{N} h(k)x(n-k) \tag{1.1}$$

Here x(n) represents the input signal which will be filtered, h(n) represents the filter impulse response, y(n) represents filtered signal, and N is the filter length. The design of low delay FIR bandpass filters with maximally flat passband and equiripple stopband by doing consecutive projections approach which is shown in [6]. It is well known that linear-phase FIR filter delay increases by increasing the filter length. A weighted least squares (WLS) technique has been proposed to design a near-equiripple FIR filter having variable fractional delay [7]. On the other hand, the design of arbitrary variable fractional-delay FIR filters has been achieved which is based on the complex version of WLS [8].

Apaydin showed a new method for decreasing the delay in FIR digital filters with equiripple passband, and peak restrained minimum squares stop-bands for programs in the actual time [9]. In this technique, the reduction of the group delay that can be reached is between 12-22 % in passband compared with the present techniques. Kene [10] proposed doing some adjustments to overlap-save method (OSM) and overlap-add method (OAM) algorithms to be used for filtering real signals by an N tap FIR filter. This technique requires two complex 2N-point discrete Fourier transform (DFT) in order to get two blocks of N samples, unlike the actual method which only needs four real 2N-point DFT to generate the exact data quantities. In this method, when it is compared with the classical algorithms, the time needed for processing the delay increases by N samples. Although the polyphase decomposition method reduces the delay considerably, it increases the computational complexity. A recent filtering algorithm has been designed using the DFT based orbicular involution and OSM ways of block involution when the series of the data are long and need to be filtered [11]. Zero group delay has been obtained by zero delay modified overlap-save method (ZDMOSM). Although this method achieves a zero group delay, it needs some changes in the acquisition duration in the process period, which is not likely preferred. Furthermore, J., S. Fouda [11] proposed a reduced delay modified overlap-save method (RDMOSM), in which the group delay reduction was achieved by a factor of 0.5 compared to OSM.

Group delay has some importance in the audio field and especially in the sound reproduction field. Many components of an audio reproduction chain, notably loudspeakers and multiway loudspeaker crossover networks, introduce group delay in the audio signal. It is therefore important to know the threshold of audibility of group delay with respect to frequency, especially if the audio chain is supposed to provide high fidelity reproduction.

In this thesis, the RDMOSM technique has been enhanced by defining the samples again and exclude them from the result of the final circular involution. According to the simulation results, the proposed method is very close to that of OSM with small alterations in the delay. Subsequently, it can be compared to linear involution. This thesis is organized as follows: Chapter 2 deals with the FIR filter structures and their types. In Chapter 3, the reduction of group delay will be explained with delay reduction methods such as RDMOSM and ZDMOSM and the proposed enhanced

RDMOSM (ERDMOSM) technique will be discussed. In Chapter 4, results and discussions will be discussed. Finally, Chapter 5 gives conclusions of the work.

CHAPTER 2

FIR FILTERS

2.1 Structures of FIR Systems

A polynomial system function in z^{-1} for a basic FIR filter is:

$$H(z)=\sum_{n=0}^{N} h(n)z^{-n} \tag{2.1}$$

the function of the transfer FIR filter is $H(z)$, impulse response is h(n), a delay of one sample time denoted by z^{-1}, N represents the filter length (number of coefficients) and n represents discrete time. For an input $x\ (n)$, the output is as follows:

$$y(n)=\sum_{k=0}^{N} h(k)x(n-k) \tag{2.2}$$

Equation (2.2) is identified as the convolution sum equation. Computation of this sum requires N additions and $(N\ +\ 1)$ multiplications for every n value.

2.1.1 Direct Form

The realization of equation (2.2) by using a tapped delay line method is shown in Fig. 1.

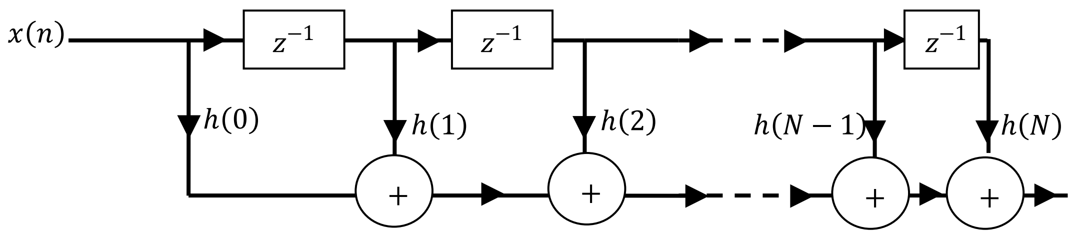

Figure 1 Direct Form Structure

Each output sample y(n) needs N additions, $N + 1$ multiplications, and N delays. Otherwise, if any similarity appears in the response of the factory sample .we have the ability to make the multiplication number less than it where.

2.1.2 Cascade Form

While for basic FIR filter, transfer orders we could put them into first-instruction factors,

$$H(z)=\sum_{n=0}^{N} h(n)z^{-n} = A\prod_{k=1}^{N}(1 - a_{\kappa}\, z^{-1}) \qquad (2.3)$$

While a_k for $k = 1\ldots\ldots N$ are the zeros of $H\,(z)$. The complex roots of $H(z)$ happen in complex conjugated pairs if $h(n)$ is real and the conjugated couples can be mixed to make a second- instruction factors with real coefficients,

$$H(z)= A\prod_{k=1}^{N_s}[1 + b_k(1)\, z^{-1} + b_k(2)z^{-2}] \qquad (2.4)$$

the structure of equation (2.4) is illustrated in Figure 2.

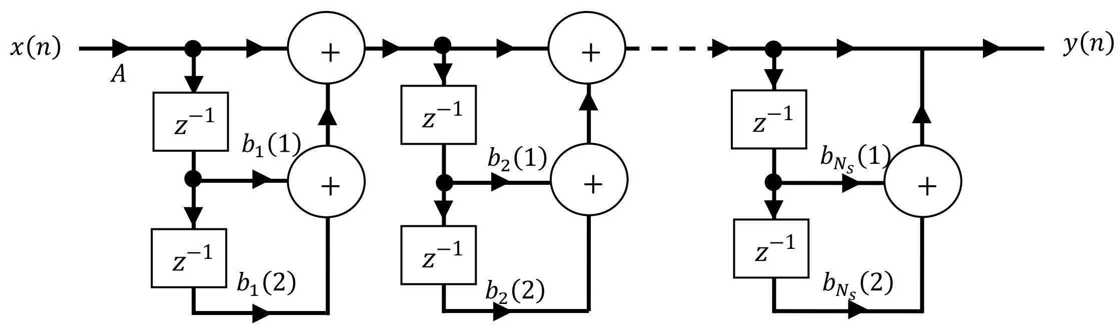

Figure 2 An FIR Filter Implemented as a Cascade of Second-order Systems

2.1.3 Linear Phase Filters

Impulse responses of linear phase filters are either symmetric or anti-symmetric

$$h(n) = h(N-n) \tag{2.5}$$

$$h(n) = -h(N-n) \tag{2.6}$$

respectively. The harmony could be exploit to make a simple network building. For instance, if $h(n)$ is symmetric, and N is even (type I filter),

$$y(n) = \sum_{k=0}^{N} h(k)x(n-k) = \sum_{k=0}^{\frac{N}{2}-1} h(k)[x(n-k)+x(n-N+k)] + h\left(\frac{N}{2}\right)x\left(n-\frac{N}{2}\right) \tag{2.7}$$

Consequently, making the combination $[x(n-k)+x(n-N+k)]$ before multiplying by $h(k)$ reduces the multiplications number. The out coming building is in Fig. 3 (a). While, if N is odd and $h(n)$ symmetric (type II filter), resulting building is presented in Fig. 3(b). We have many identical anti-symmetric buildings (type III and IV) linear phase filters.

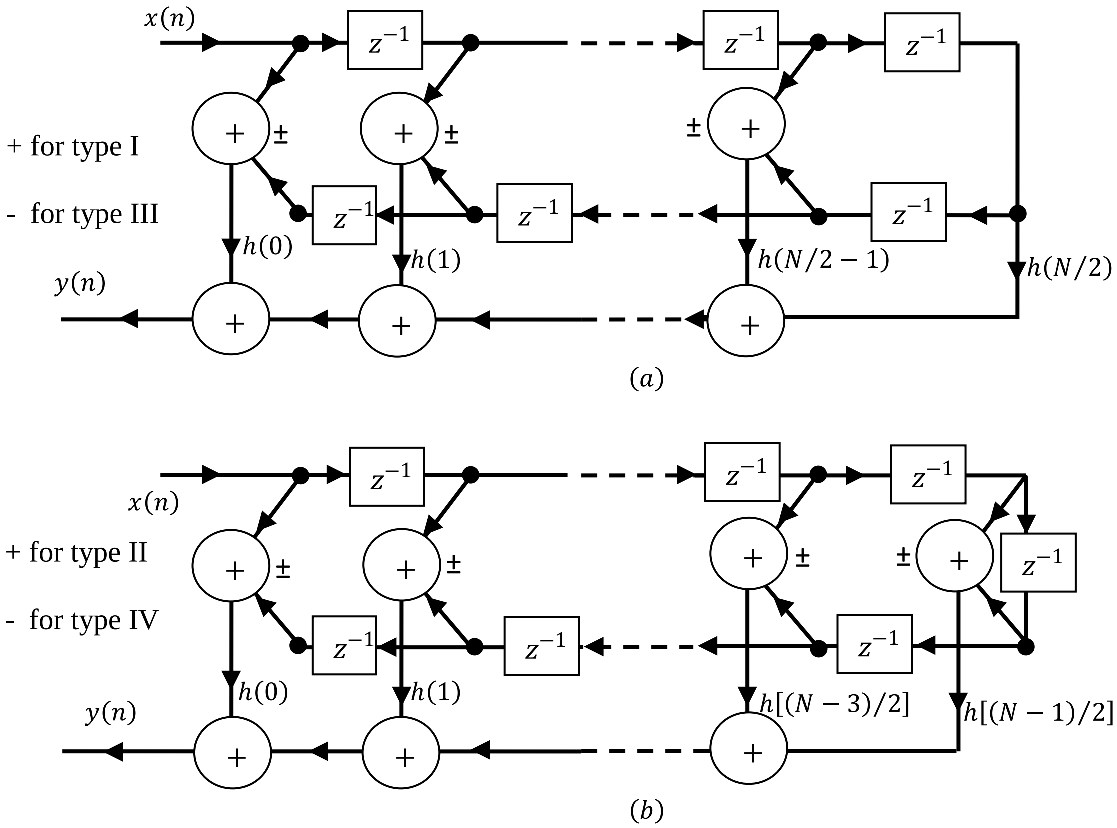

Figure 3 Direct Form Implementations for Linear Phase Filters. (a) Type I, III (b) Type II, IV

2.1.4 Frequency Sampling

A filter is parameterized after the implementation of frequency sampling structure in terms of its discrete Fourier transform (DFT) coefficients. Particularly, if $H(k)$ is the $\mathrm{N}-$point DFT of an FIR filter with $h(n) = 0$ for $0 < n > N$, then the unit sample response of the filter written as:

$$h(n) = \frac{1}{N}\sum_{k=0}^{N} H(k)e^{j2\pi nk/N} \tag{2.8}$$

The transfer function can be written as

$$\left.\begin{aligned} H(z) &= \sum_{n=0}^{N-1} h(n)z^{-n} = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} H(k)e^{j2\pi nk/N}\right]z^{-n} \\ &= \frac{1}{N}\sum_{k=0}^{N-1} H(k)\sum_{n=0}^{N-1} e^{j2\pi nk/N}z^{-n} \end{aligned}\right\} \tag{2.9}$$

Computing the sum over n gives

$$H(z) = \frac{1}{N}(1 - z^{-N})\sum_{k=0}^{N-1}\frac{H(k)}{1 - e^{j2\pi k/N}\,z^{-1}} \tag{2.10}$$

which corresponds to an FIR filter cascade $\frac{1}{\mathrm{N}}(1-\mathrm{z}^{-\mathrm{N}})$ with one-pole parallel network filters:

$$H(z) = \frac{H(k)}{1 - e^{j2\pi k/N}\,z^{-1}} \tag{2.11}$$

For a filter in narrowband that has the majority of its DFT coefficients that match to zero, structure of frequency sampling will be effective way of applying. The structure of frequency sampling is given in Figure 4. If $h(n)$ is real, $H(k) = H^*(N - k)$, the structure could be simplified . For instance of $H(k)$, if N is even [12]

$$H(z) = \frac{1}{N}(1 - z^{-N})\left[\frac{H(0)}{1 - z^{-1}} + \frac{H(N/2)}{1 + z^{-1}} + \sum_{k=1}^{N/2-1} \frac{A(k) - B(k)z^{-1}}{1 - 2\cos(2\pi k/N)z^{-1} + z^{-2}}\right] \quad (2.12)$$

where

$$A(k) = H(k) + H(N - k) \quad (2.13)$$

$$B(k) = H(k)e^{-j2\pi k/N} + H(N - k)e^{j2\pi k/N} \quad (2.14)$$

On the other hand, when N is odd, same definition results can be obtained.

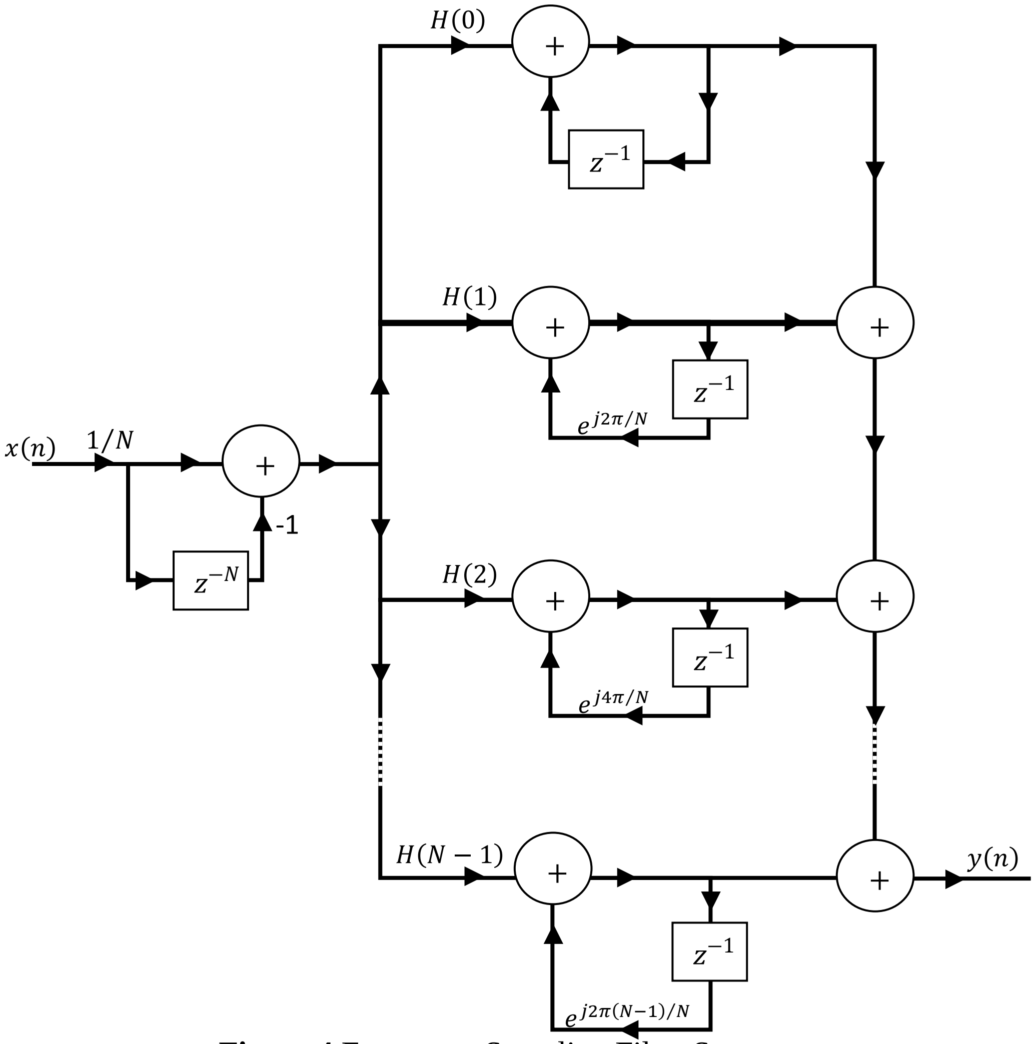

Figure 4 Frequency Sampling Filter Structure

2.2 Linear Phase Response

Linear phase response is one of the most important properties of FIR filters. When an input signal is applied to filter, it appears at the output of the filter with modifications done in amplitude and/or phase. The extent of this modification depends on the amplitude and phase characteristics of the filter. The phase delay or group delay of the filter gives important information about how the filter makes this modification in the phase of the signal. The phase delay of the filter is defined as the part of delaying time for each frequency elements of the input signal bears in going into the filter. On other hand, group delay is the middle time delay the complex signal bears in each frequency. Mathematically, the phase delay can be defined as the negative of the phase angle divided by frequency and the delays in the combination is recognized as the opposite of the derivative of phase with a considering to frequency.

If a filter has nonlinear phase characteristics, it causes a phase distortion in the signal passing through it. Such a distortion is undesired in many applications such as music, data transmission, video, and biomedicine. Therefore, the filters having linear phase characteristics are widely used in these applications. A linear shift-invariant system has a linear phase response if it is written in the following form

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{-j\alpha\omega} \tag{2.15}$$

where α can be a real number which defines the group delay,

$$\tau_h(\omega) = \alpha \tag{2.16}$$

A system have overall linear phase when the frequency replays as conceder in this equation

$$H(e^{j\omega}) = A(e^{j\omega}) e^{-j(\alpha\omega-\beta)} \tag{2.17}$$

While α and β are constants. Now, consider the FIR system with an impulse response

$$h(n) = \begin{cases} 1 & n = 0,1,\ldots N \\ 0 & else \end{cases} \tag{2.18}$$

The frequency response is

$$H(e^{j\omega}) = e^{-jN\omega/2} \frac{\sin\left(\frac{N+1}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)} \tag{2.19}$$

Thus, this system have an overall linear phase, with α = N / 2 and β = 0.

While for a normal system with a partial transfer function to own linear phase, impulse response have to be finite in length. FIR filter that has a real-valued impulse replays of length N + 1, has generalized linear phase if its impulse response is symmetric,

$$h(n) = h(N - n) \tag{2.20}$$

At this situation, α = N / 2 and β = 0 or π. In other appropriate circumstances that is h(n) be antisymmetric [12]

$$h(n) = -h(N - n) \tag{2.21}$$

which corresponds to case through which α = N/2 and β = π/2 or 3π/2.

2.3 Types of Linear Phase FIR Filters

Let us taken consideration the unique kinds of FIR filters when the coefficients $h(n)$ of the transmit function is as follows:

$$H(z^{-1}) \ = \sum_{n=0}^{N} h(n)z^{-n} \tag{2.22}$$

are supposed to be symmetric or anti-symmetric. As long as the organization of the polynomial in each of these two kinds we could either have it as odd or even, there are four kinds of filters with diverse characteristic, they will be explained next [13].

Type I. Coefficients are symmetric [$h(n) \ = \ h(N \ - \ n)$], and the order N is even.

In general, coefficients can be expressed in some other forms. Let us assume that the order is even. The transfer function in equation (2.22) can be expanded as:

$$H(z^{-1}) = \sum_{n=0}^{N} h(n)z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(N-1)z^{-N+1} + h(N)z^{-N} \tag{2.23}$$

For type I filter with N order, as shown in Fig. 5, it is noted that $h(0) = h(N)$, $h(1) = h(N-1), \ldots, h(n) = h(N-n)$. Applying these relationships in the equation above, we get

$$H(z^{-1}) = h(0)[1 + z^{-N}] + h(1)[z^{-1} + z^{-N+1}] + \cdots + h(\frac{N}{2})z^{\frac{-N}{2}} \tag{2.24}$$

This can also be shown as in the following form

$$H(z^{-1}) = z^{\frac{-N}{2}}\left\{h(0)\left[z^{\frac{N}{2}} + z^{\frac{-N}{2}}\right] + h(1)\left[z^{-1+\frac{N}{2}} + z^{-N+1+\frac{N}{2}}\right] + \cdots + h\left(\frac{N}{2}\right)\right\} \qquad (2.25)$$

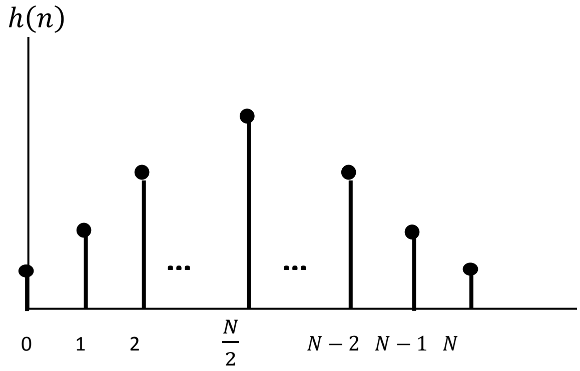

Figure 5 Unit Impulse Responses of the Type I FIR Linear Phase Filters

The frequency response of equation (2.25) is given by

$$H\left(e^{-j\omega}\right) = e^{j\theta\omega}\{H_R(\omega)\} \qquad (2.26)$$

In this formula, the term $H_R(\omega)$ is an actual valued role; while it could be negative or positive in any frequency, so that when we are transmitting from a positive value to a negative value, the phase angle changes by amount of π radians $(180°)$. The phase angle $\theta(\omega) = -3\omega$ is a linear role of ω, and the group delay τ is the same as three patterns. Remember that the group delay is three patterns on the regular frequency standard, but real group delay is equal to $3T$ seconds, where T represents the period of the sampling.

Generally, $H(e^{j\omega})$ can be expressed in some other forms

$$\left.\begin{aligned} H\left(e^{j\omega}\right) &= \sum_{n=0}^{N} h(n)e^{-jn\omega} \\ &= h(0) + h(1)e^{-j\omega} + h(2)e^{-2j\omega} + \cdots + h(N-1)e^{-j(N\omega)} \\ &= e^{-j[(N/2)\omega]}\left\{2h(0)\cos\left(\frac{N\omega}{2}\right)\right. \\ &\quad + 2h(1)\cos\left(\left(\frac{N}{2}-1\right)\omega\right) + 2h(2)\cos\left(\left(\frac{N}{2}-2\right)\omega\right) + \cdots \\ &\quad \left. + h\left(\frac{N}{2}\right)\right\} \end{aligned}\right\} \quad (2.27)$$

and now in a more compact form:

$$H\left(e^{j\omega}\right) = e^{-j[(N/2)\omega]}\left\{h\left(\frac{N}{2}\right) + 2\sum_{n=1}^{N/2} h\left[\frac{N}{2}-n\right]\cos(n\omega)\right\} = e^{j\omega}\{H_R(\omega)\} \quad (2.28)$$

The total group delay is fixed $\tau = N/2$ in the every case, for I FIR filter.

Type II. Coefficients are symmetric $[h(n) = h(N - n)]$, and the order N is odd.

$$H(z^{-1}) = \sum_{n=0}^{N} h(n)z^{-n} = h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(N)z^{-N} \quad (2.29)$$

and due to symmetry

$$h(0) = h(N), h(1) = h(N-1), h(2) = h(N-2), \ldots, h(n) = h(N-n) \quad (2.30)$$

Now, if we consider symmetric coefficients with N odd, the impulse response is shown in Figure 6.

The frequency response is in the type II filter for general case can be written as

$$
\left.
\begin{aligned}
H\left(e^{-j\omega}\right) &= \sum_{n=0}^{N} h(n)e^{-jn\omega} = e^{j\theta(\omega)}\{H_R(\omega)\} \\
&= e^{-j\left(\frac{N}{2}\omega\right)}\left\{\sum_{n=1}^{(N+1)/2} 2h\left[\frac{N+1}{2}-n\right]\cos\left(\left(n-\frac{1}{2}\right)\omega\right)\right\}
\end{aligned}
\right\} \quad (2.31)
$$

which demonstrates a linear phase $\theta(\omega) = -[(N/2)\omega]$ and a constant group delay $\tau = N/2$ samples.

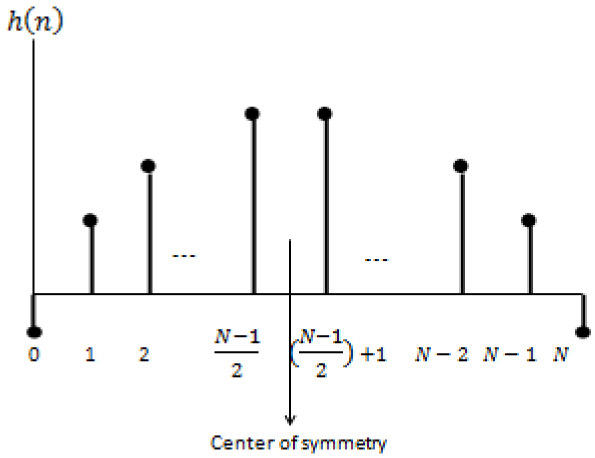

Figure 6 Unit Impulse Responses of the Type II FIR of Linear Phase FIR Filters

Type III. The coefficients are anti-symmetric $[h(n) = -h(N-n)]$, and the order N is even. Figure 7 shows that $h(0) = -h(N), h(1) = -h(N-1), h(2) = -h(N-2)$ and $h(N/2) = 0$ to preserve anti-symmetry for these samples:

$$H(z^{-1}) = h(0)[1 - z^{-N}] + h(1)[z^{-1} - z^{-N+1}] + \cdots + h(\frac{N}{2})z^{\frac{-N}{2}} \quad (2.32)$$

This can also be shown as in the following form

$$H(z^{-1}) = z^{\frac{-N}{2}} \left\{ h(0) \left[z^{\frac{N}{2}} - z^{\frac{-N}{2}} \right] + h(1) \left[z^{-1+\frac{N}{2}} - z^{-N+1+\frac{N}{2}} \right] + \cdots + h\left(\frac{N}{2}\right) \right\} \quad (2.33)$$

Here if we place $z = e^{j\omega}$, and $e^{j\omega} - e^{-j\omega} = 2j\, sin(\omega) = 2e^{j(\pi/2)}\, sin(\omega)$, we get the frequency response in the general case as

$$H\left(e^{-j\omega}\right) = e^{-j[(N\omega - \pi)/2]} \left\{ 2 \sum_{n=1}^{N/2} h\left[\frac{N}{2} - n\right] \sin(n\omega) \right\} \quad (2.34)$$

and it has a linear phase $\theta(\omega) = -[(N\omega - \pi)/2]$ and the group delay $\tau = N/2$ samples.

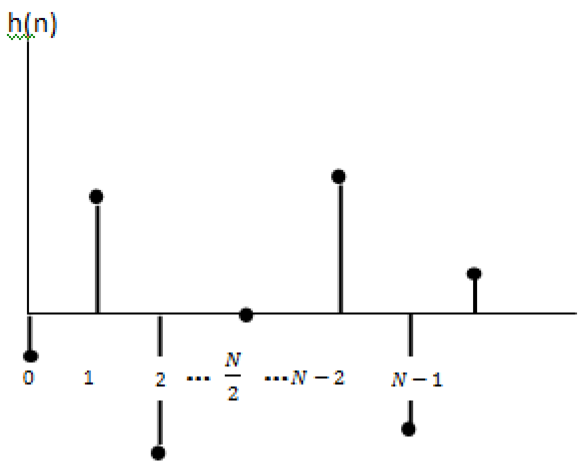

Figure 7 Unit Impulse Responses of the Type III FIR of Linear Phase FIR Filters

Type IV. Coefficients are anti-symmetric $[h(n) = -h(N - n)]$, and the order N is odd. As in Figure 8, in which $h(0) = -h(N),\ h(1) = -h(N-1), h(2) = -h(N-2), \dots,\ h(n) = h(N-n)$. Its transfer function can be written as

$$H(z^{-1}) = h(0)[1 - z^{-N}] + h(1)[z^{-1} - z^{-N+1}] + \cdots + h(\frac{N-1}{2})z^{\frac{-N-1}{2}} \tag{2.35}$$

The frequency response of the transfer function of the IV linear phase filter is often taken by

$$H\left(e^{-j\omega}\right) = e^{-j[(N\omega-\pi)/2]}\left\{2\sum_{n=1}^{(N+1)/2} h\left[\frac{N+1}{2} - n\right]\sin\left(\left(n - \frac{1}{2}\right)\omega\right)\right\} \tag{2.36}$$

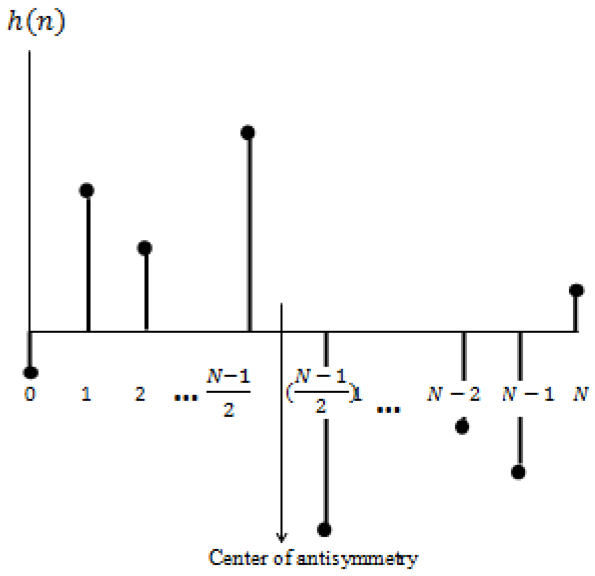

Figure 8 Unit Impulse Responses of the Type IV FIR of Linear Phase FIR Filters

2.4 Properties of Linear Phase FIR Filters

Discussion of kinds of FIR filters showed that FIR filters with symmetric or anti-symmetric coefficients offer equivalently constant group delay or (linear phase); these coefficients represent the impulse response samples. It was noticed before that an FIR filter with symmetrical or non-symmetrical coefficients has a linear phase and as a result a fixed group delay. Theoretically, [14] confirmed that an FIR filter with a fixed group delay is required to own symmetrical or non-symmetrical coefficients. These properties are practical in designing a FIR filters and their applications. The volume response of standard FIR filters with linear phase have been calculated to observe some extra properties of these four filter types [13] presented in Figure 9.

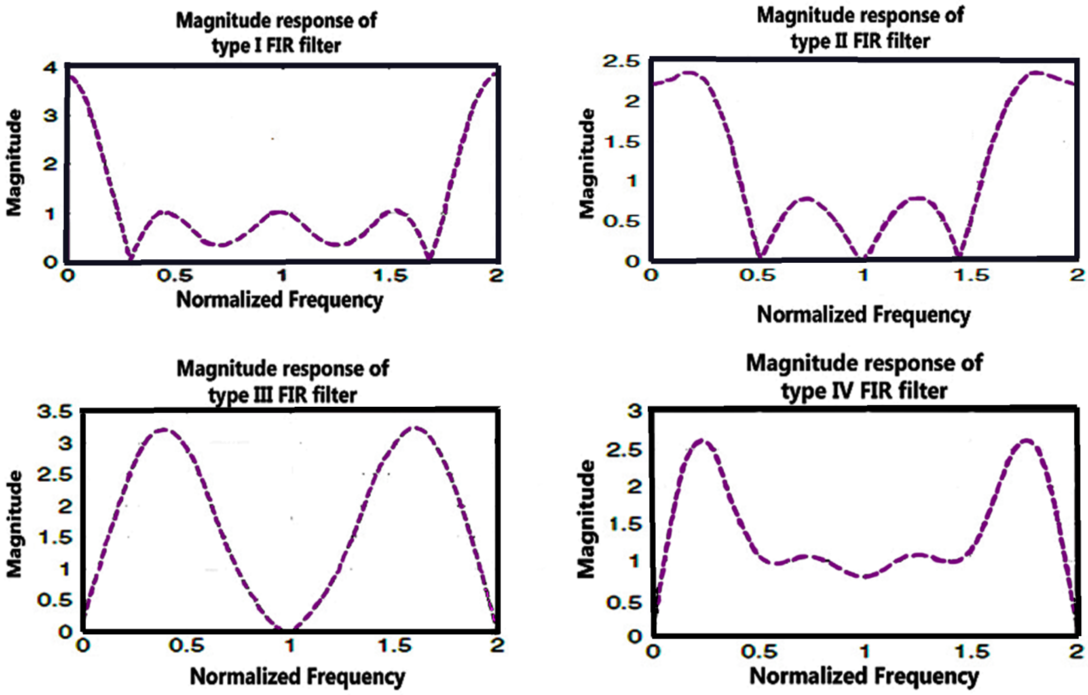

Figure 9 Magnitude Responses of the Four Types of Linear Phases FIR Filter

The following explanations about these standard magnitude responses will be useful in creating suitable alternatives at the beginning of their design. For example, type I filters have a non-zero magnitude at $\omega = 0$ as well as a non-zero value at the normalized frequency $\omega/\pi = 1$ (corresponding to Nyquist frequency), while type II filters have non-zero volume at $\omega = 0$ however a zero value at Nyquist frequency. Therefore, these filters are clearly not appropriate for the design of bandpass and highpass filters, while the two types are appropriate for lowpass filters. Type III filters is with zero volume at $\omega = 0$ and at $\omega/\pi = 1$, thus they are appropriate for the design of bandpass filters nevertheless it is not appropriate for lowpass and bandstop filters. Whereas, type IV filters have zero volume at $\omega = 0$ and a non-zero at $\omega/\pi = 1$. They are not appropriate for the design of lowpass and bandstop filters however they can be used for bandpass and highpass filters.

In Figure 10 (a), the linear relationship is shown by plotting filter phase response (type I). The phase responding presents a big gap of π radians at an identical frequency when the transfer role is with a zero unit circle in the z plane, and the plot applies a big gap of 2π at any time the phase responding moves beyond $\pm\pi$ therefore the whole amount phase responding still in primary range of $\pm\pi$. If on the unit circle there are no zeros, that is, if there are no big gap by π radians, the phase response, when it is unwrapped, becomes a constant function of ω. The result of unwrapping the phase (as shown in Figure 10 (a)) is to eliminate the big gap in phase response in such way that phase response stays in the rang of $\pm\pi$ (as shown in Fig. 10 (b)). Group delay is an integer multiple of samples match to $N/2$ samples if the order N of the FIR filter is even. Whereas, when the order N is odd, the group delay is (an integer plus half) a sample.

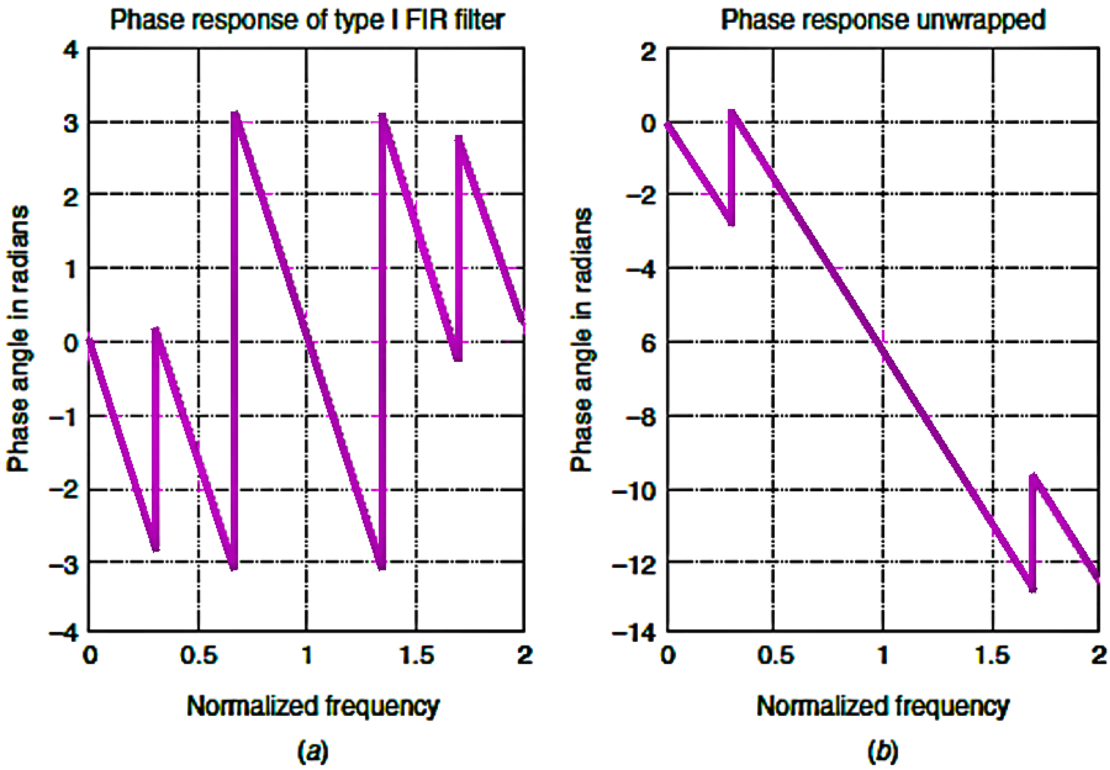

Figure 10 Linear Phase Responses of Type I FIR Filter

CHAPTER 3

GROUP DELAY REDUCTION METHODS

3.1 General Overview

Fast Fourier Transform (FFT) based circular convolution is used by Overlap-Save Method (OSM) to construct equivalent results as in the linear convolution. After that, the aliasing that occurs because of the circular involution cannot be eliminated unless using zero padding after the last nonzero impulse response (IR) sample. The convolution length for two signals having size N and M is $L = N + M - 1$. Therefore, the minimum number of added zeros for an IR with size N, is $M - 1$. For a considered piece period of signal with duration M, $N - 1$ unwanted samples are to be deleted. While the OSM is in use, these patterns are first on the circular involution outcome provided by $X(k)\ H(k)$. OSM outcome will provide a group delay equal to $(\mathrm{N} - 1)/2$ for N odd and $N/2$ for N even if $h(n)$ is a linear phase IR. Nevertheless, it is seen that Discrete Fourier Transform (DFT) of a samples series relies on its layout and the phase can be reduced for a certain layout [11]. To prove that, consider a linear phase IR of length N, denoted $h(n)$ (Figure 11). Its DFT $H(k)$ is given by:

$$H(k) = \sum_{n=0}^{N-1} h(n) W_L^{-kn} \tag{3.1}$$

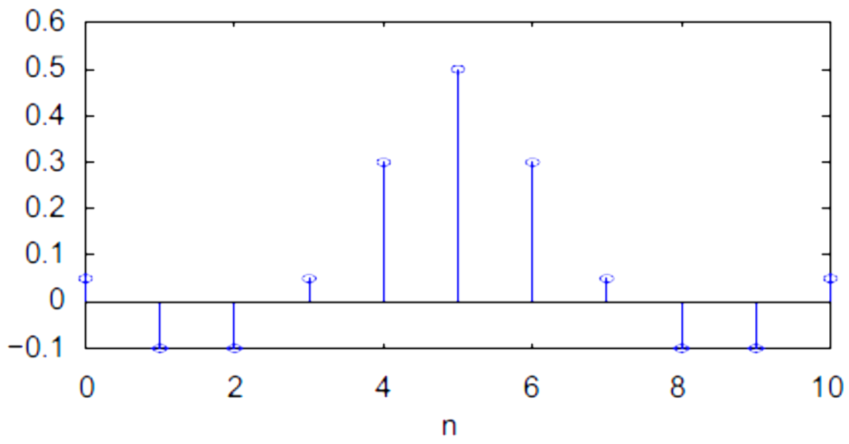

Figure 11 Linear Phase IR h(n)

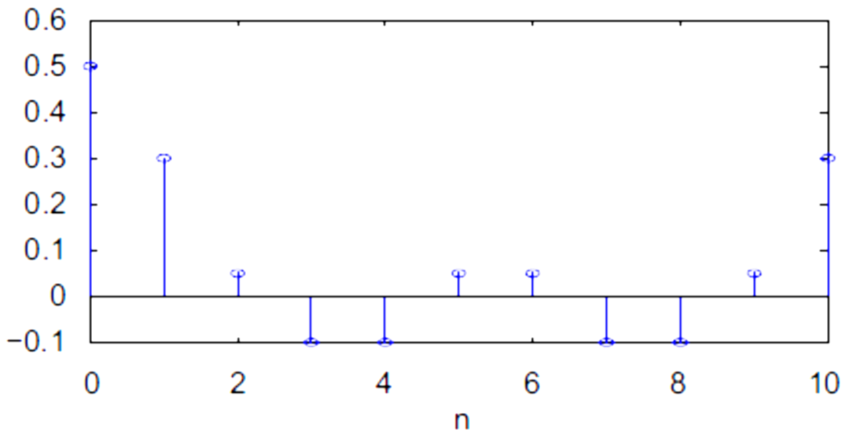

Figure 12 Circularly Shifted Linear Phase IR $h_1(n)$

with $W_L^{kn} = e^{j2\pi kn/L}$, $L = N + M - 1$. During the calculation of the DFT, it is assumed that the samples series are periodic, and are situated on a circle. The circle has origin which synchronizes with the first pattern of $h(n)$. The kind of $h(n)$ being $N - 1$, its phase is presented by

$$\varphi = -2\pi \frac{N-1}{2} f \tag{3.2}$$

Therefore the group delay is $(N-1)/2$. So, by shifting IR of $(N-1)/2$ samples circularly toward left the origin is changed as in Fig. 12. The DFT of the new IR is represented by $h_1(n)$ will be:

$$H_1(k) = \sum_{n=0}^{N-1} h_1(n) W_L^{-kn} \tag{3.3}$$

Equation (3.3) may be written also as below

$$H_1(k) = \sum_{n=0}^{N-1} h\left(n + \frac{N-1}{2}\right) W_L^{-kn} \tag{3.4}$$

The relation in equation (3.4), according to the DFT characteristics, lastly can be written as follows:

$$H_1(k) = W_L^{k(N-1)/2} \sum_{n=0}^{N-1} h(n) W_L^{-kn} \tag{3.5}$$

Equation (3.5) demonstrates that the phase of $h_1(n)$ compared to that of $h(n)$, is given by

$$\varphi_1 = 2\pi \frac{N-1}{2} f \tag{3.6}$$

Which shows the cancellation of the dephasing φ because $h(n)$. Consequently, $h_1(n)$ shows a zero phase if *N* is odd and its phase is $\varphi_1 = \pi f$ if N is even. This type which is named as zero phase IR could be utilized in the FFT based circular convolution to achieve zero group delay in filtering. However, this kind of operation is impossible since OSM and overlap-add method (OAM) techniques which depends on utilizing DFT, advise zero padding at IR end. Thus, DFT phase of the IR $h_2(n)$ acquired after zero padding to $h_1(n)$ would become, as in Fig. 13, unspecified. Fig. 13 shows the frequency response of $h_1(n)$ after zero padding [11]. As a result, it appears essential in order to evolve a new algorithm according to employment of DFT, which keeps the zero phase IR properties [15].

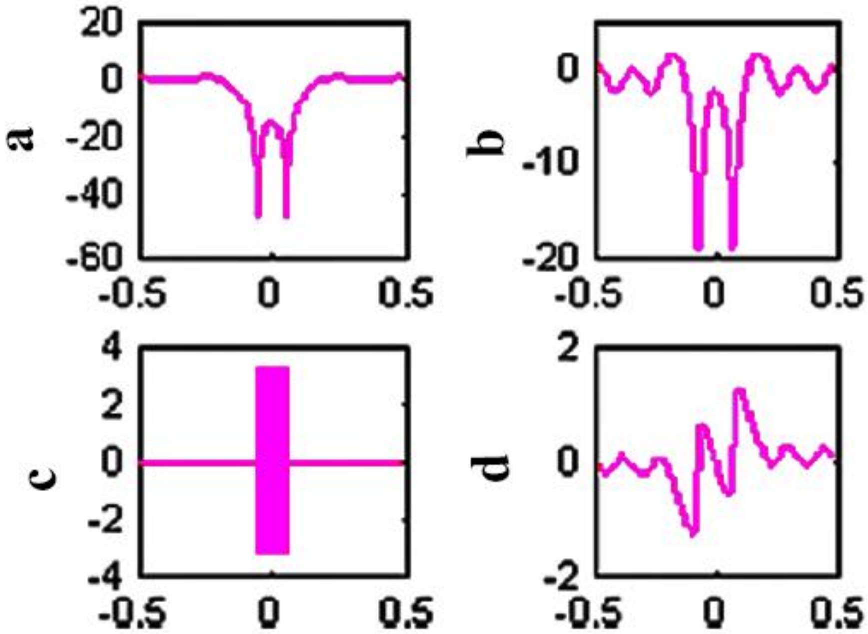

Figure 13 Frequency Response of $h_1(n)$ after zero padding: (a) magnitude response of $h_1(n)$, (b) magnitude response of $h_2(n)$, (c) phase response of $h_1(n)$, (d) phase response of $h_2(n)$.

3.2 Modified Overlap and Save Method (MOSM)

Here, a modern algorithm is presented for filtering with the use of the FFT based circular convolution and the OSM. The idea here is to make it easier to realize the zero group delay in the filtering. Here, no zero padding occurs after zero phase IR Means that we have the ability to contain it is ghostly futures [16]. Let $y(n)$ be the circular convolution result of $h(n)$ and $x(n)$, its DFT will be written as

$$Y(k) = X(k)H(k) \tag{3.7}$$

According to the OSM, if the size of $y(n)$ is $\mathrm{L} = \mathrm{N} + \mathrm{M} - 1$, N the size of $h(n)$, and then $N - 1$ first samples of $y(n)$ must be deleted. Through determining the circular involution $y_1(n)$ of $h_1(n)$ and $x(n)$, one gets

$$Y_1(k) = W_L^{k(N-1)/2} X(k)H(k) \tag{3.8}$$

This formula demonstrates that the result which is gotten by the zero phase IR is circularly shifted of $(N - 1)/2$ patterns to the left like $h_1(n)$. As a result, $(N -$

$1)/2$ patterns of the $N-1$ to be omitted must be repressed by each of the two margins of the circular involution result. Next, it seems that after the $N - 1$ first samples of $y(n)$ are deleted by the OSM they will be not suitable to the employ of filters of the zero phase. Thus, we call the MOSM whose idea is to omit $(N - 1)/2$ samples at the two opposite edge of the circular involution result margins. It can be noticed that this involution result rotating will not make any changes of the group delay which is produced by the filter, thus the MOSM produces similar conclusions compared with other involution filtering methods. In order to decrease the group delay, redefining of the patterns to be kept is needed.

3.3 Zero Delay MOSM (ZDMOSM)

It is possible to perform zero delay filtering by the use of the MOSM. Therefore, rather than adding $N - 1$ zero patterns to the starting of file which will be filtered as proposed by the OSM, as explained in Fig. 14, only $(N-1)/2$ patterns will be considered [11]. Next, by deleting $(N - 1)/2$ patterns on the two sides of the filtered signal, the group delay is repressed. At this method it is called as the zero delay MOSM (ZDMOSM). A filtered signal having no group delay is presented in Fig. 15 [17].

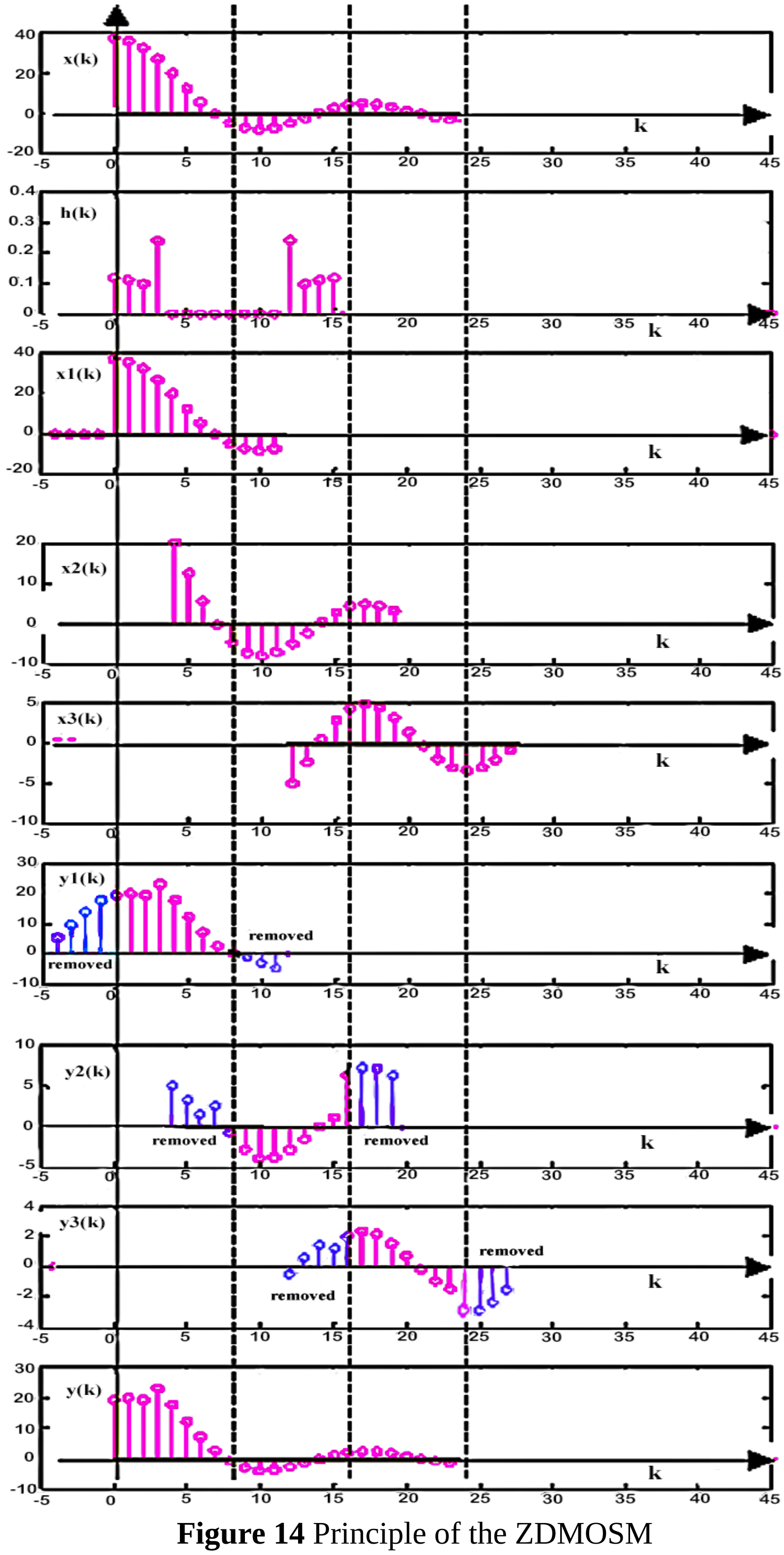

Figure 14 Principle of the ZDMOSM

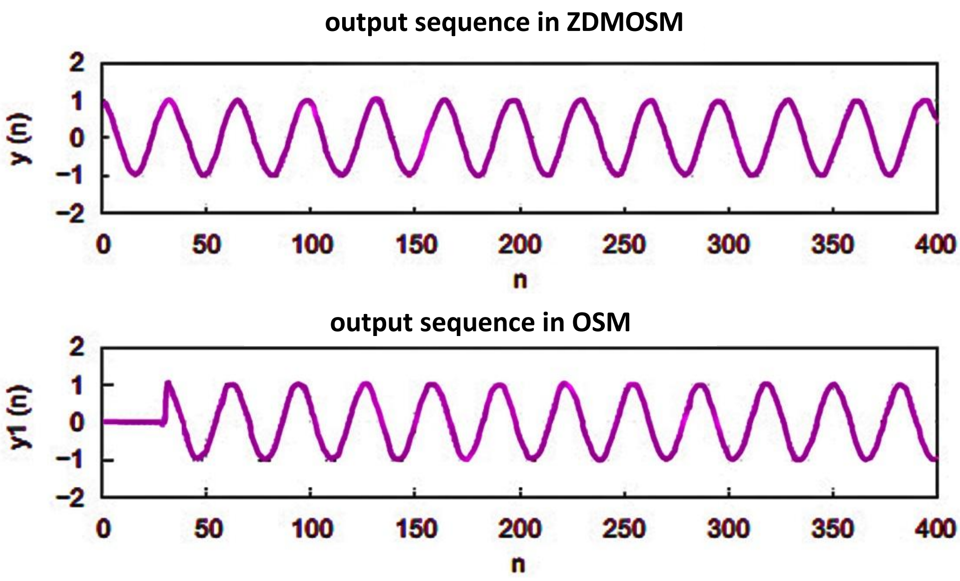

Figure 15 Zero Group Delay Filtering using the ZDMOSM

3.4 Reduced Delay MOSM (RDMOSM)

Here ZDMOSM method cannot be applied since it would need altering the acquisition time during the processing, which is not easy. Next, on beginning the filtering as in the OSM, $\mathrm{N}-1$ zero patterns are taken into consideration. In general, the $N-1$ deleted patterns show some vibrations. These vibrations, when emerging happened on filtered signal can create an inappropriate influence (as in filtering audio). But, their amplitude dropped slowly from left side to right side as it shown in in Fig. 16. In actual time processing, decrease of the group delay can be regarded by deleting, following orbicular involution with zero phase filter, rather than $(N-1)/2$ samples each sides as explained in the MOSM, however $3(N-1)/4$ at left end side and just $(N-1)/4$ at right side of the result. This method maintains at right end side $(N-1)/4$ of patterns which normally must be omitted, in accordance with the OSM. Whereas, $(N-1)/4$ of samples carrying the group delay is deleted, this reduces the group delay into a half. This method is named as the reduced delay MOSM (RDMOSM). Fig. 17 shows a filtered signal example (in the actual time) where the groups delay was decreased [11].

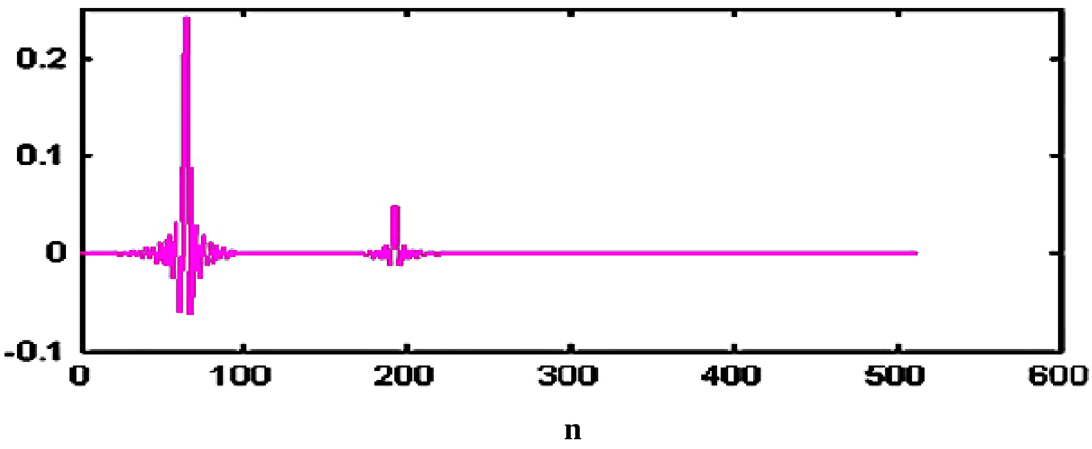

Figure 16 Evolution of the Ripples Amplitude

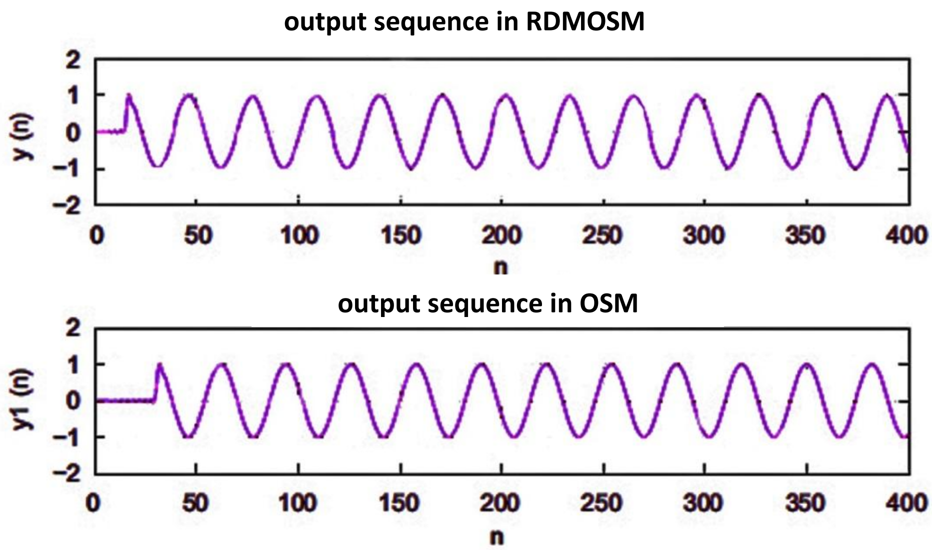

Figure 17 The use of RDMOSM to Reduce Group Delay Filtering

3.5 Enhanced RDMOSM (ERDMOSM)

In this section, the new proposed filtering algorithm via circular involution which is based on DFT and the OSM will be examined.

The primary purpose of this technique is to realize zero group delay filtering. In this method, there exists no zero padding to zero phase IR. Let us consider $y(n)$ as the result of circular convolution of $h(n)$ and $x(n)$. Now, the DFT of $y(n)$ can be written as follows

$$Y(k) = X(k)H(k) \tag{3.9}$$

According to the OSM, if the size of $y(n)\ is\ L = N + M - 1$ where N is the size of $h(n)$, after that leftmost $N - 1$ samples of $y(n)$ have to be deleted. By evaluating the circular convolution $y_1(n)$ of $h_1(n)$ and $x(n)$, we are able to show that:

$$\left.\begin{aligned} Y_1(k) &= H_1(k)X(k) \\ &= e^{j(2\pi/L)k((N-1)/2)}\, H(k)X(k) \\ &= W_L^{k((N-1)/2)} H(k)X(k) \end{aligned}\right\} \tag{3.10}$$

Taking inverse DFT gives

$$y_1(n) = \left(y\left(n + \frac{N-1}{2}\right)\right) \tag{3.11}$$

This equation demonstrates that result obtained by zero phase IR is a orbicular wag version of $y(n)$ wag by $(N-1)/2$ pattern to the left. Therefore, $(N-1)/2$ pattern out of $N-1$ pattern to be deleted have to be repressed on the two directions of the orbicular involution conclusion margins. After that, it is discovered that the OSM which omits the $N-1$ first samples of $y(n)$ is inappropriate to be used in zero phase filters. As a result, the MOSM method was defined and its working principle is to delete $(N-1)/2$ samples on both ends of the orbicular involution result. The convolution result rotation $y\ (n)$ would not alter the group delay which is generated by the filter. Thus, the MOSM offers similar result to those gotten from the OSM or other usual filtering method. Afterward, it is noted that a reduction in the group delay could be gotten through re-defining the patterns to be kept from last rotated result of the circular involution.

In situation of RDMOSM, a reducing in the group delay can reach half. This technique has been extended to one where reduction in the group delay could be gotten using a factor which is higher than half. In performing DFT based orbicular involution the $N-1$ zero patterns should be considered which will to be filtered as in case of the OSM. Following circular involution of $x(n)$ with $h_1(n)$, $4(N-1)/5$

and $9(N-1)/10$ of the patterns are deleted from the left end side. Similarly, $(N-1)/5$ and $(N-1)/10$ of the patterns are deleted from the right end side of the result. These are named as enhanced RDMOSM, ERDMOSM1 and ERDMOSM2, respectively. This process preserves at the left end side $(N-1)-4(N-1)/5 = (N-1)/5$ samples in ERDMOSM1 case and $(N-1)-9(N-1)/10 = (N-1)/10$ samples in the ERDMOSM2 case, which would usually be deleted according to the OSM. Whereas, $4(N-1)/5$ or $9(N-1)/10$ samples carrying the group delay are deleted, correspondingly, for the ERDMOSM1 or ERDMOSM2 cases. These lead to the reduction of the group delay by a factor given by

$$\left(1-\frac{\frac{N-1}{5}}{\frac{N-1}{2}}\right)=\frac{3}{5} \tag{3.12}$$

and

$$\left(1-\frac{\frac{N-1}{10}}{\frac{N-1}{2}}\right)=\frac{4}{5} \tag{3.13}$$

Figures 18 and 19 show a sample of a filtered signal (in the actual time) in which reduction in the group delay happened by a factor 3/5 and 4/5, correspondingly, in ERDMOSM1 and ERDMOSM2 cases.

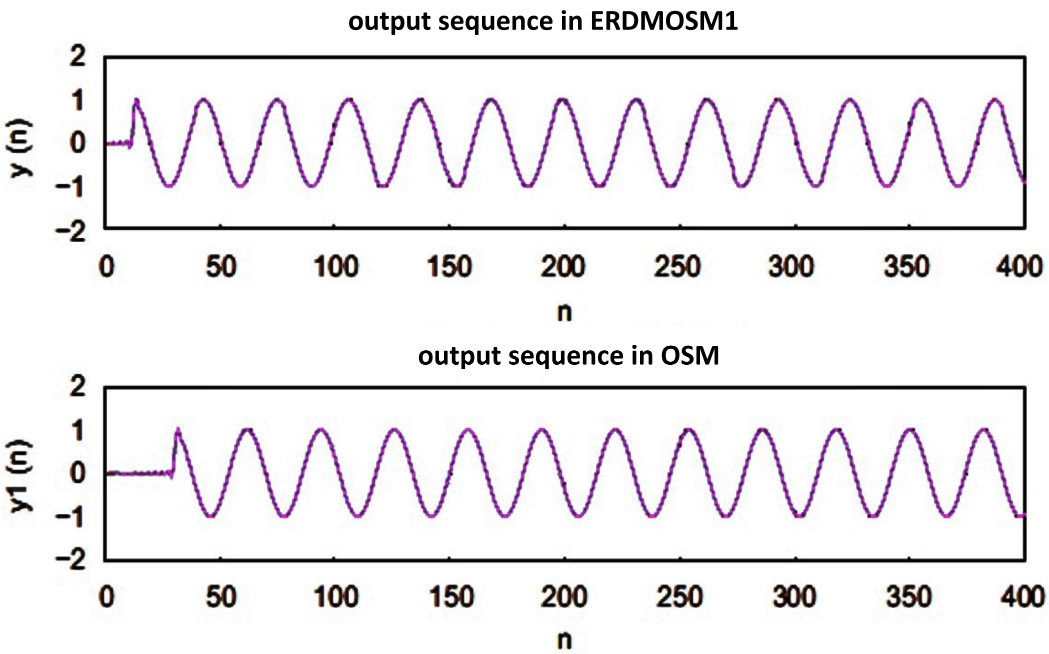

Figure 18 Reduced Group Delay Filtering using ERDMOSM1

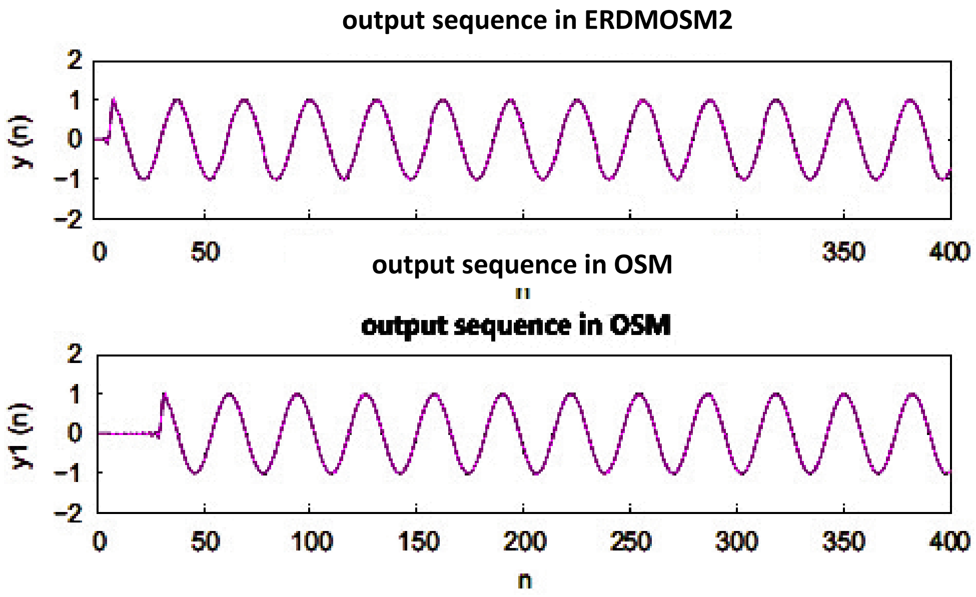

Figure 19 Reduced Group Delay Filtering using ERDMOSM2

It is noticed that for a filter with the order of 60, the result of the outgoing signal beginning from 31^{st} pattern in the OSM case (see Fig. 17) or linear involution (the group delay is $(N-1)/2 = 30$ patterns), where in the RDMOSM case (see Fig. 17), the output sequence begins from the 16^{th} pattern (i.e., the group delay is $\frac{1}{2} * (N - 1)/2 = 15$ patterns) and in ZDMOSM, the output series starts from the 1^{st} patterns which leads to that there is no group delay. On the opposing side, for the same input signal, the output sequence of ZDMOSM varies a lot from the OSM filtered output which shows a deviation from linear involution outcome. In the current ERDMOSM1 case (see Fig. 18), the filtered output series beginning from the 13^{th} pattern (i.e., the group delay is $\frac{2}{5} * \frac{N-1}{2} = 12$ samples) and for ERDMOSM2 case (see Fig 19), the filtered output sequence begins after 6 samples, from 7^{th} sample (i.e, the group delay is $\frac{1}{5} * (N - 1)/2 = 6$ samples).

The full algorithms are shown in Figures 20 and 21. The results obtained from each of these algorithms are better than the usual RDMOSM.

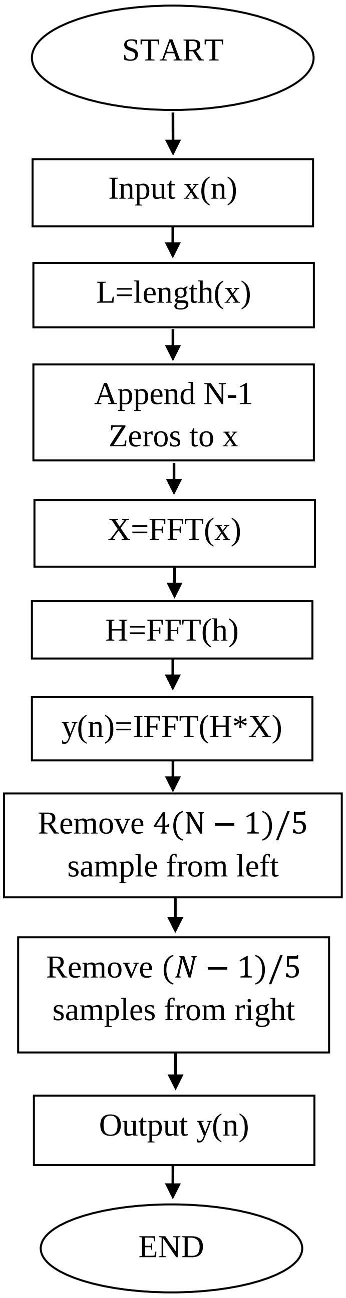

Figure 20 Algorithm for ERDMOSM1

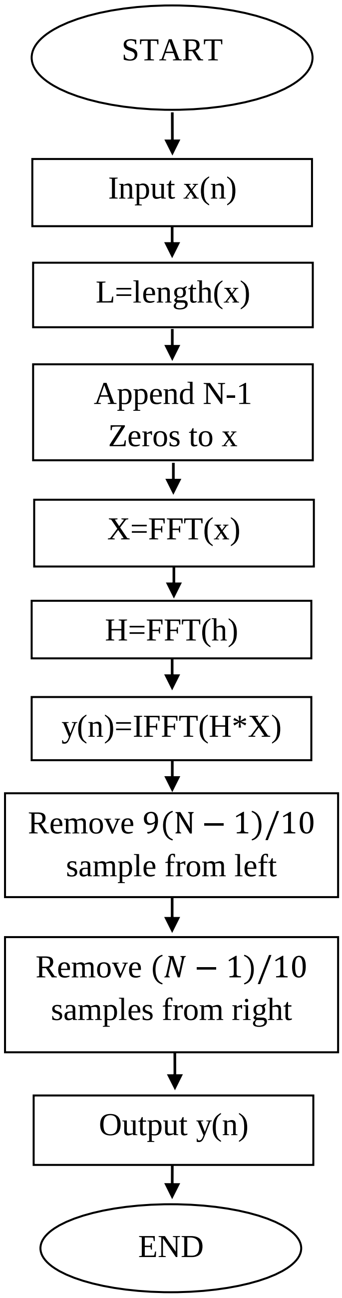

Figure 21 Algorithm for ERDMOSM2

In our present work, it has been observed that if the movement in the amount of patterns is grown from the left side of the orbicular involution outcome, there will be a reduction in the group delay that increases with the increase of the movement of the number of samples at the cost of increased ripple amplitude. If $N-1$ samples are deleted from the left of the circular involution outcome side, group delay will be completely repressed so there will be no pattern that carries the group delay will stay, which shall raise the deviation of the resulting filtered series from the OSM outcome. Thus, the result shall be invalid. Therefore, patterns must be omitted from each sides of the circular involution result to get better efficiency.

The reason of removing $(N-1)/5, (N-1)/10$ numbers of samples from the right and $4(N-1)/5, 9(N-1)/10$ numbers of samples from the left can be explained as follows. Three important conditions should be followed for re-defining the patterns to be kept after circular involution:

- The overall number of patterns that will be deleted from the left side and the right side of final orbicular involution outcome of length $L=(M+N-1)$ have to be $(N-1)$ as the original input length is M.
- More patterns will be omitted from the left side instead of the right end side of the orbicular involution outcome since the reduction factor of the group delay depends on how many samples are deleted from the left.
- Samples should be deleted from each sides of the circular convolution result to obtain better performance [17].

CHAPTER 4

COMPUTER SIMULATIONS

In this chapter, two cases will be presented to show the effectiveness of group delay reduction to the recently suggested algorithms defined in chapter 3. In order to be able to compare these filtering methods with the traditional OSM, the results that have decreased delay are every time in comparison to those obtained with the OSM. The wrong signal represented by $e(n)$ could be defined as follows

$$e(n) = y_1(n) - y(n) \quad (4.1)$$

Where $y_1(n)$ and y(n) represent the filtered output OSM and the filtered output regarding each filtering technique, respectively. The input signal that will be filtered is referred by $x(n)$ and the zero phase impulse responding is represented by $h(n)$. In the simulation studies, sine and random waves are considered as the input.

4.1 Simulation Study 1: Sine Wave

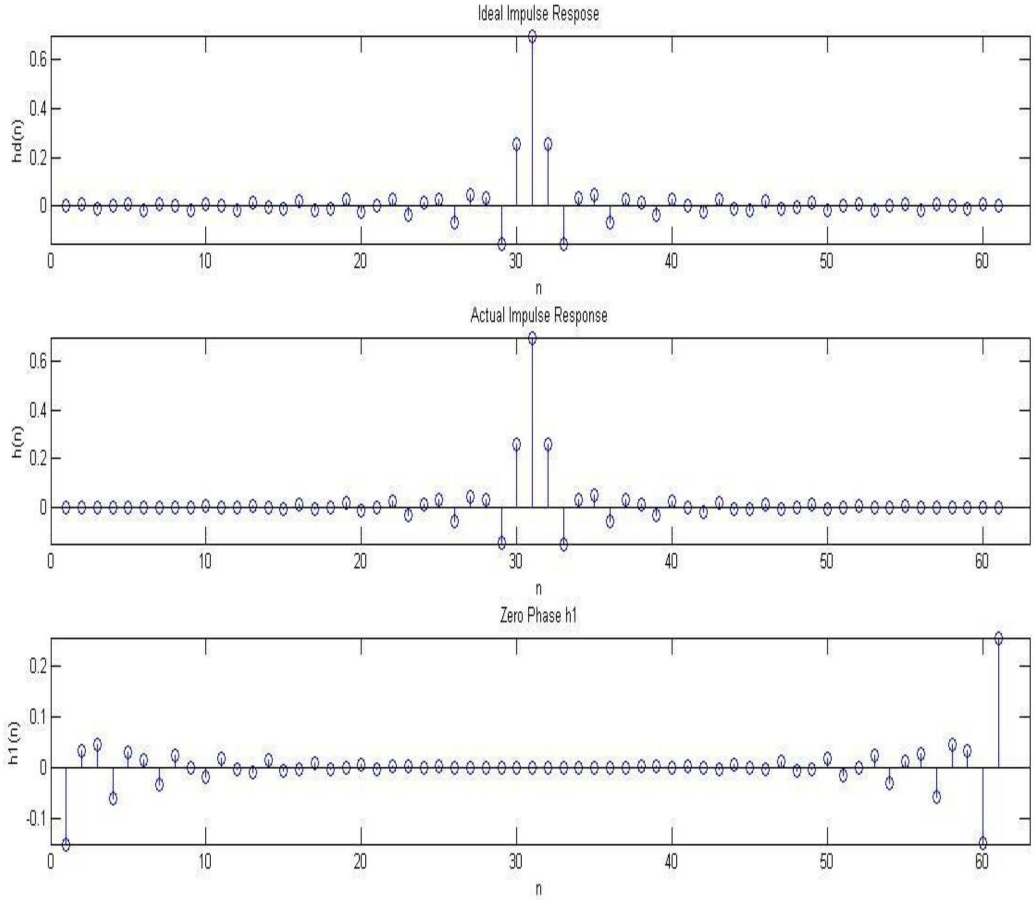

Figure 22 Impulse Response of FIR Filter Lowpass Equiripple Filter & Actual Impulse Response and Zero Phase $h_1(n)$

Fig. 22 shows the desired impulse response, actual impulse response and zero phase impulse response of FIR filter. It is clear that zero phase impulse response can be obtained by circularly shifting the impulse response of the filter by (N-1)/2 samples toward left.

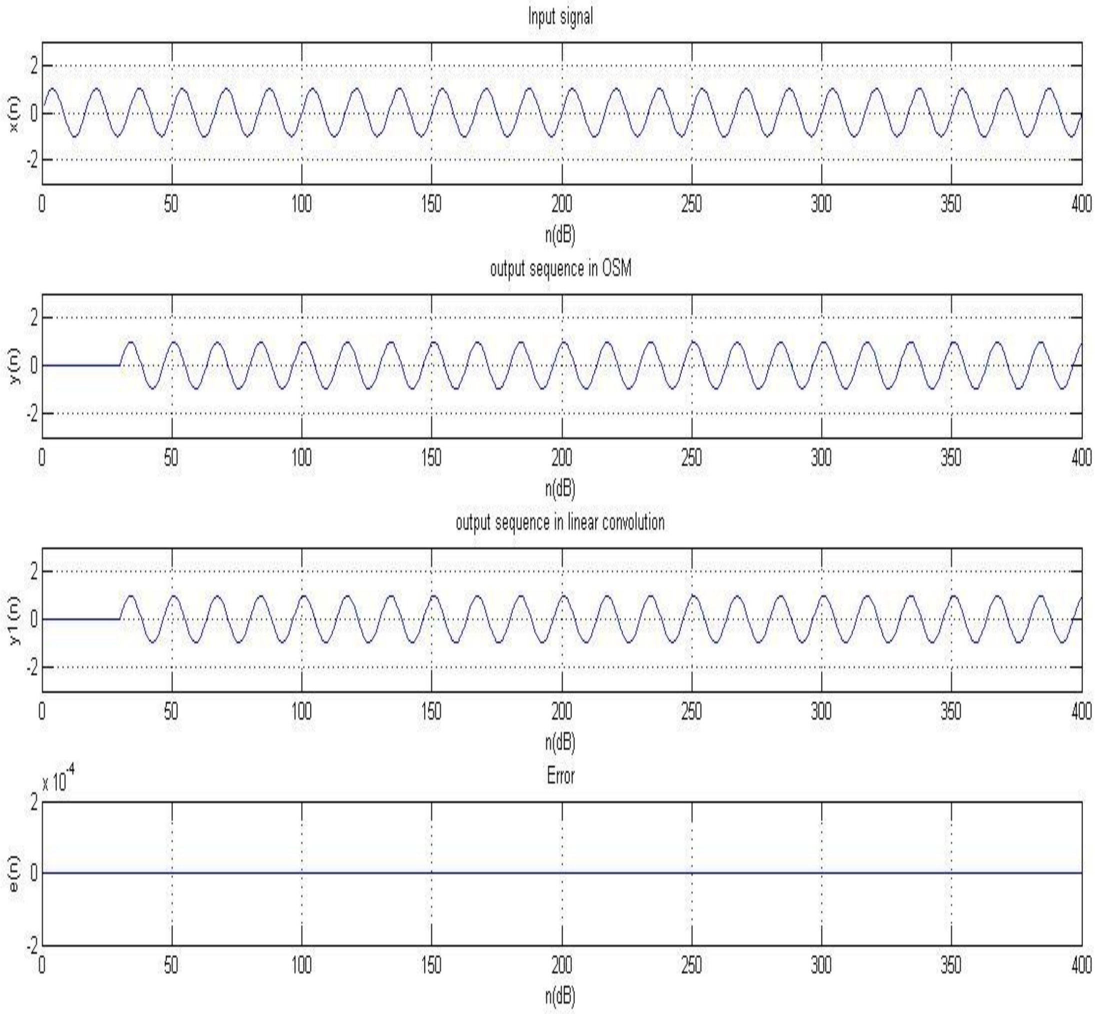

Figure 23 Comparison Between the Linear Convolution and OSM method

Fig. 23 shows the comparison between linear convolution and OSM method. Since two methods are equivalent, the OSM method can be considered as the reference in the other parts of this discussion. It has been observed that for a filter length of 60 coefficients, the resultant output signal starts from 31^{st} sample in the case of OSM (or linear convolution) which shows that the group delay is (N-1)/2=30 samples.

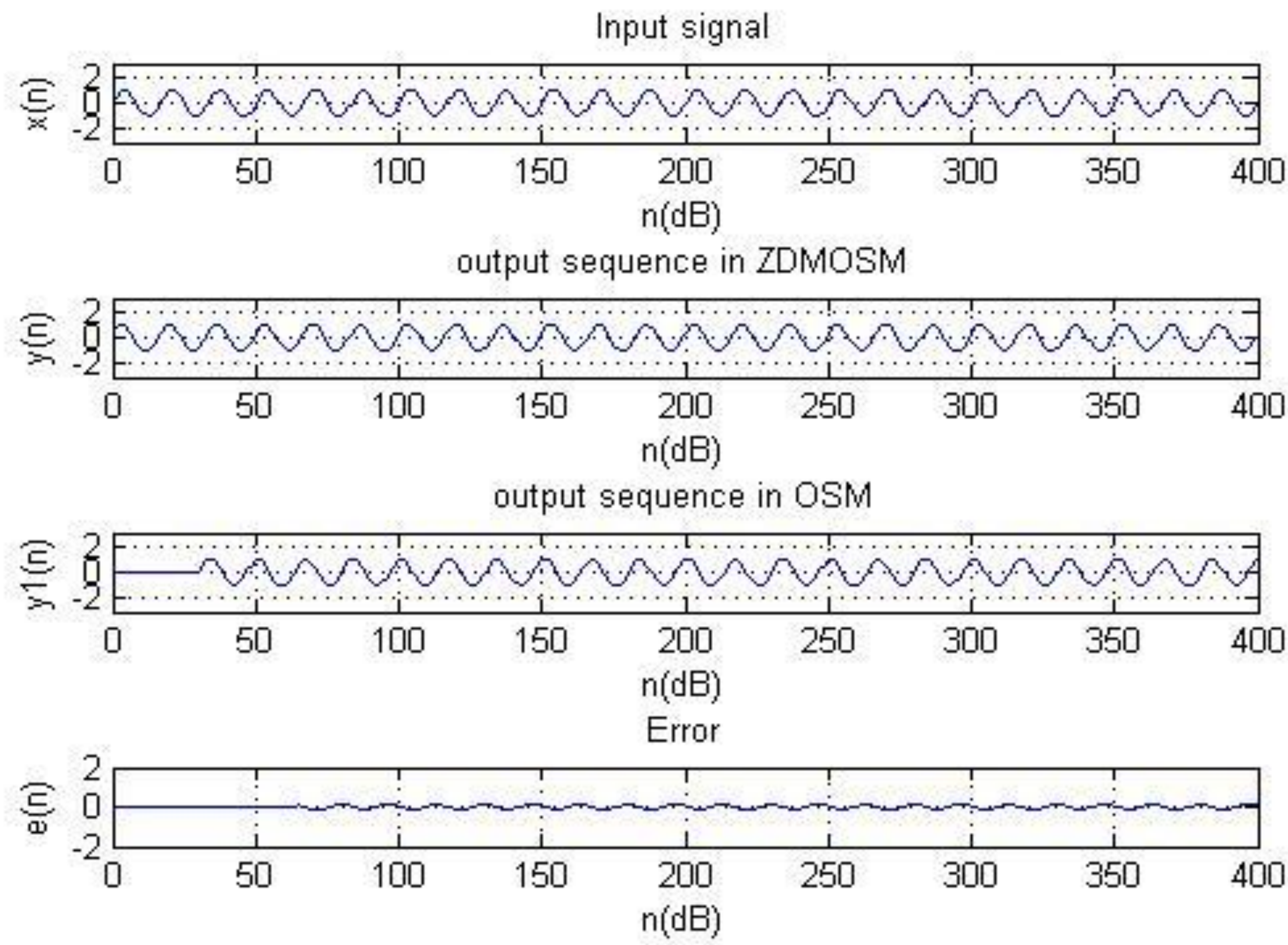

Figure 24 Comparison between ZDMOSM and OSM methods

Fig. 24 shows the output of the filter obtained by OSM and MOSM methods for the off-line processing case. The input signal, $x(n)$, the output of the filter obtained by the ZDMOSM, the output of the filter obtained by the OSM are illustrated, from top to bottom, in Fig. 24. It is clear the output obtained by ZDMOSM is different from that of obtained by the OSM.

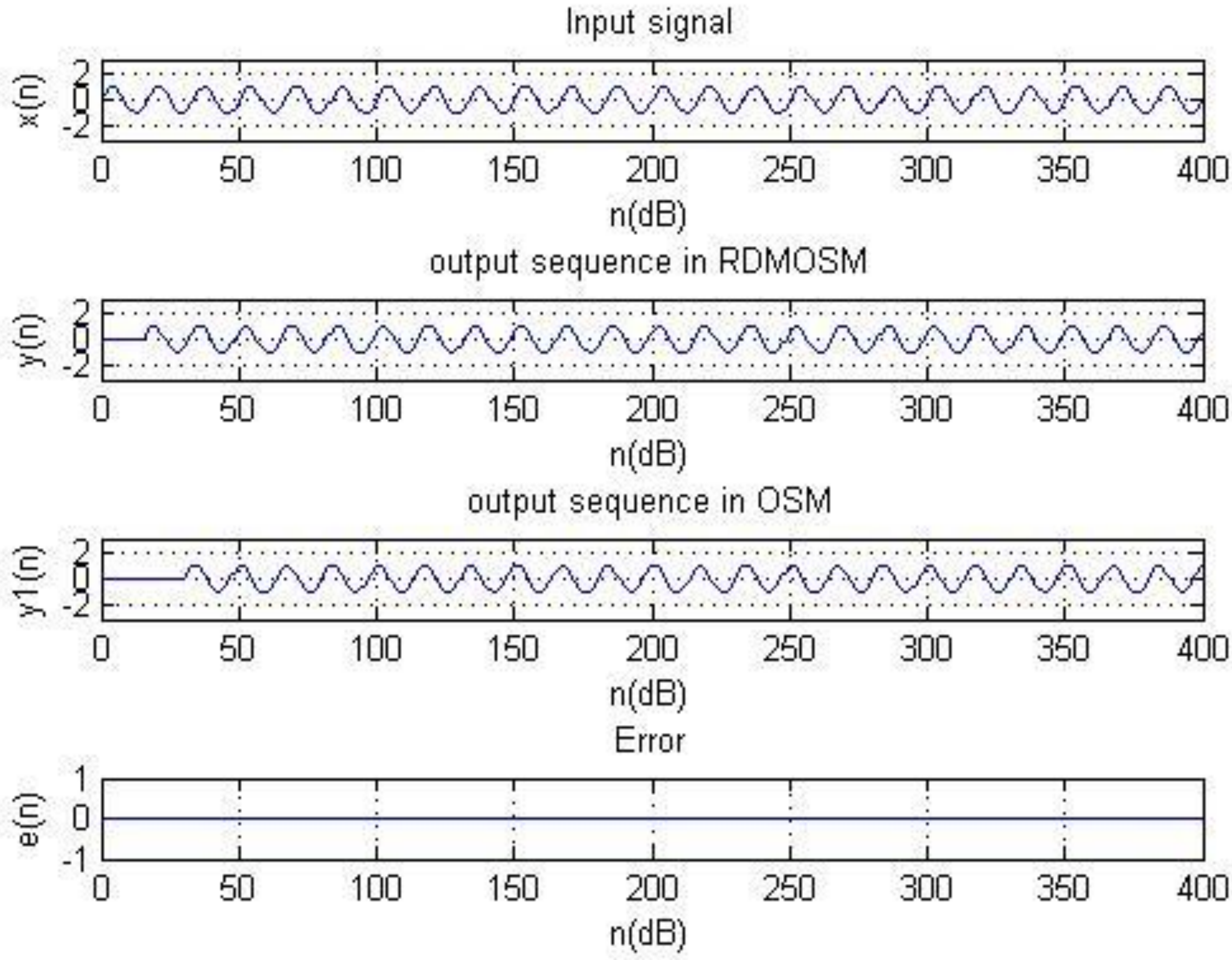

Figure 25 Comparison between RDMOSM and OSM methods.

Fig. 25 shows the output of the filter obtained by OSM and RDMOSM methods for the same input shown in Fig. 24. If the output signals obtained by ZDMOSM in Fig. 24 and by RDMOSM in Fig. 25 are compared, one can easily see that the group delay has been reduced by a factor of 1/2. Also, since the outputs obtained by OSM and RDMOSM are same, the resulting error signal becomes zero. The linear convolution can be approximated by using the MOSM method with the reduced group delay. However, this needs a zero phase filter which should be obtained by circularly shifting a linear phase impulse response.

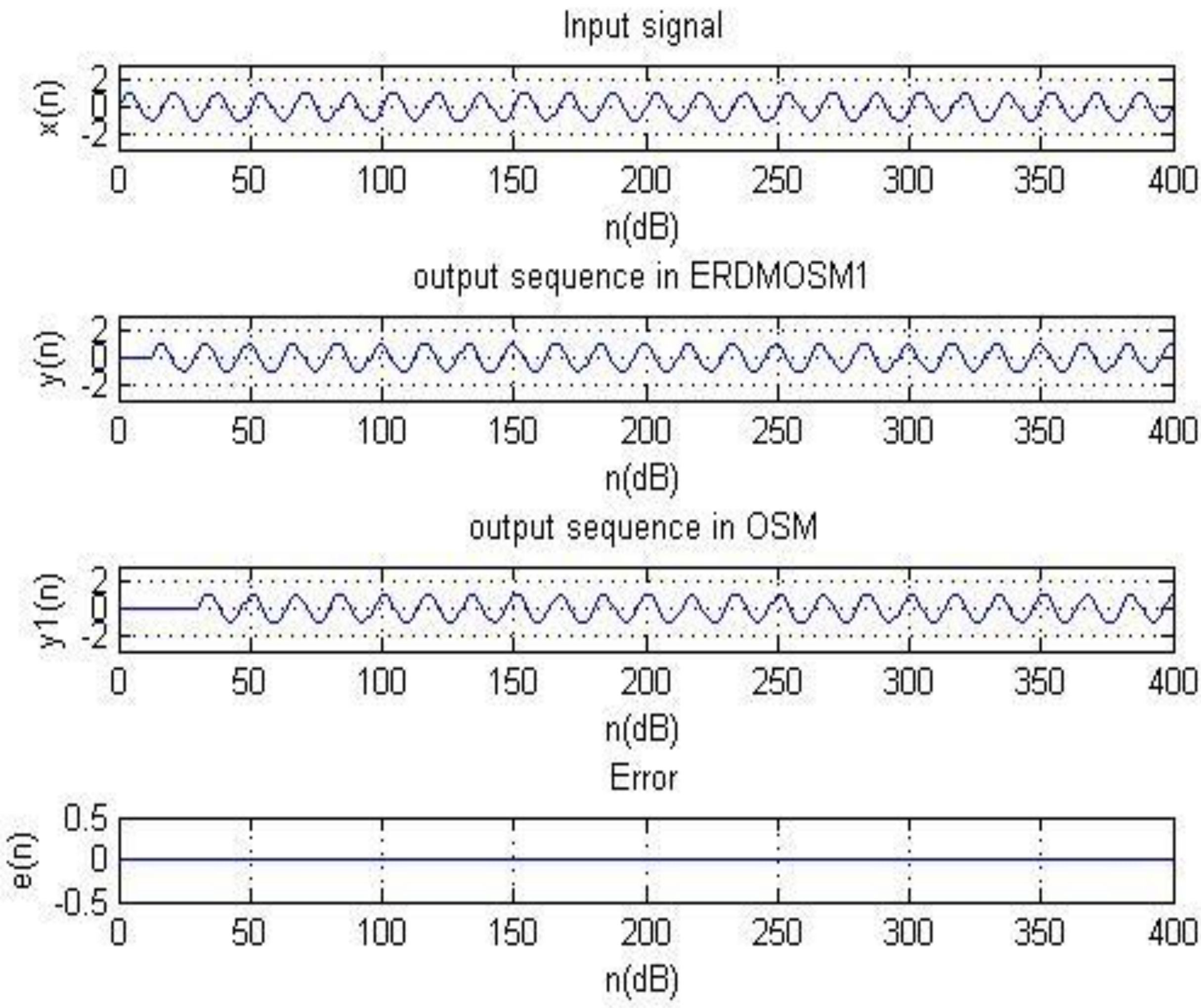

Figure 26 Comparison between ERDMOSM1 and OSM methods.

Fig. 26 and 27 show the results obtained from the OSM and enhanced-RDMOSM (ERDMOSM1 and ERDMOSM2). In Fig. 26, the results obtained by OSM and ERDMOSM1 are compared. Similarly, in Fig. 27, the results obtained from the OSM and ERDMOSM2 are compared. It can be clearly seen from these figures that ERDMOSM1 and ERDMOSM2 methods result in better delay reduction. However, the ripple amplitude in the error signal may increase slightly. The delay reductions obtained by the ERDMOSM 1 and ERDMOSM2 are approximately 3/5 and 4/5, respectively.

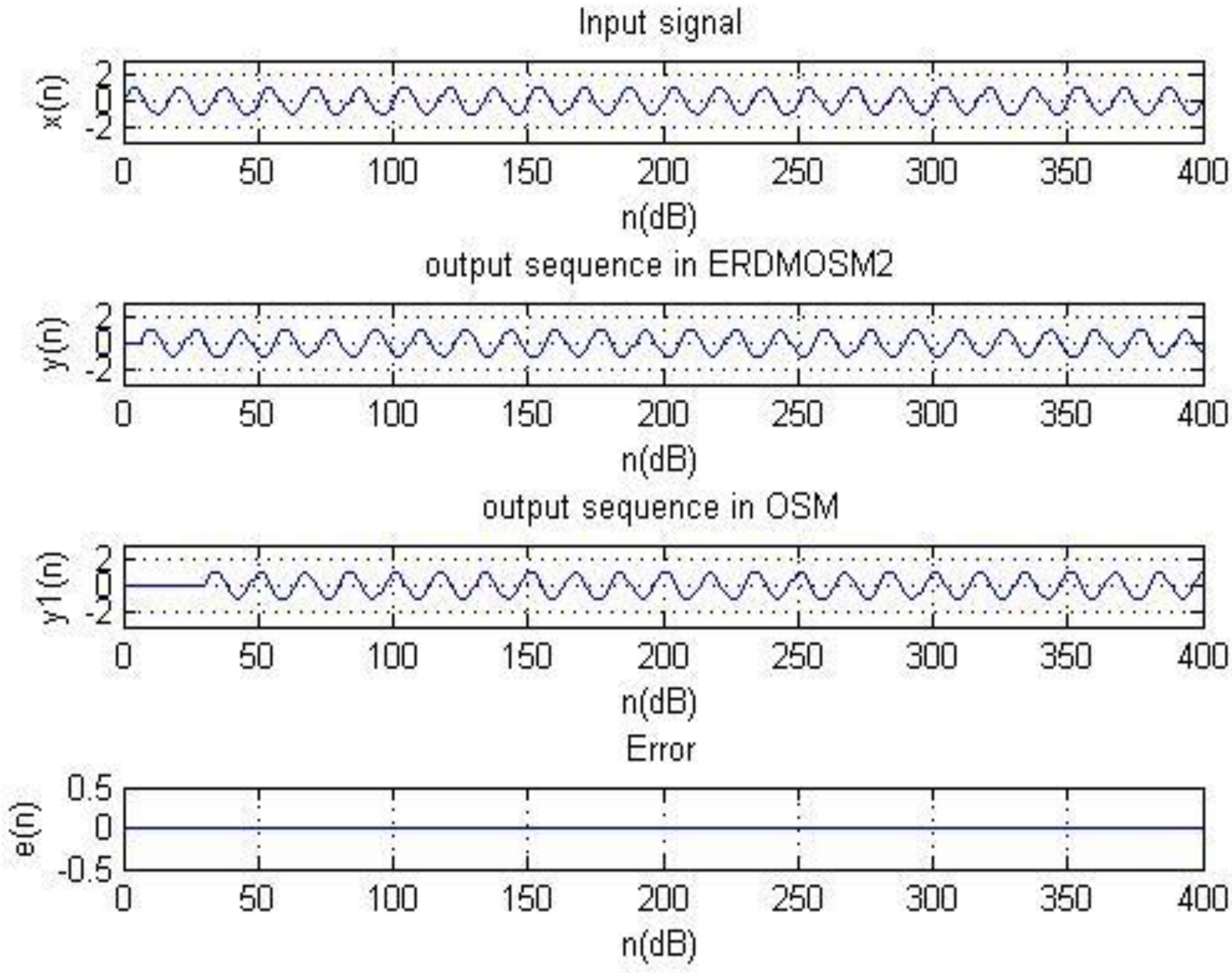

Figure 27: Comparison between ERDMOSM2 and OSM methods.

The performances of the group delay reduction methods are compared and the results are reported in Table 1. Except ZDMOSM, it is obvious that ERDMOSM1 and ERDMOSM2 performs better than the other methods.

Methods	**Group Delay (samples)**
Linear Convolution	30
OSM	30
ZDMOSM	0
RDMOSM	15
ERDMOSM1	12
ERDMOSM2	6

Table 1 Comparison of Group Delay Reduction Methods

4.2 Simulation Study 2: Random Wave

Another FIR, equiripple filter having passband frequency of 5 kHz, stopband frequency of 6 kHz, patterning frequency of 45 kHz, maximum passband ripple of 0.1 dB and lower stopband attenuation of 60.1 dB has been considered.

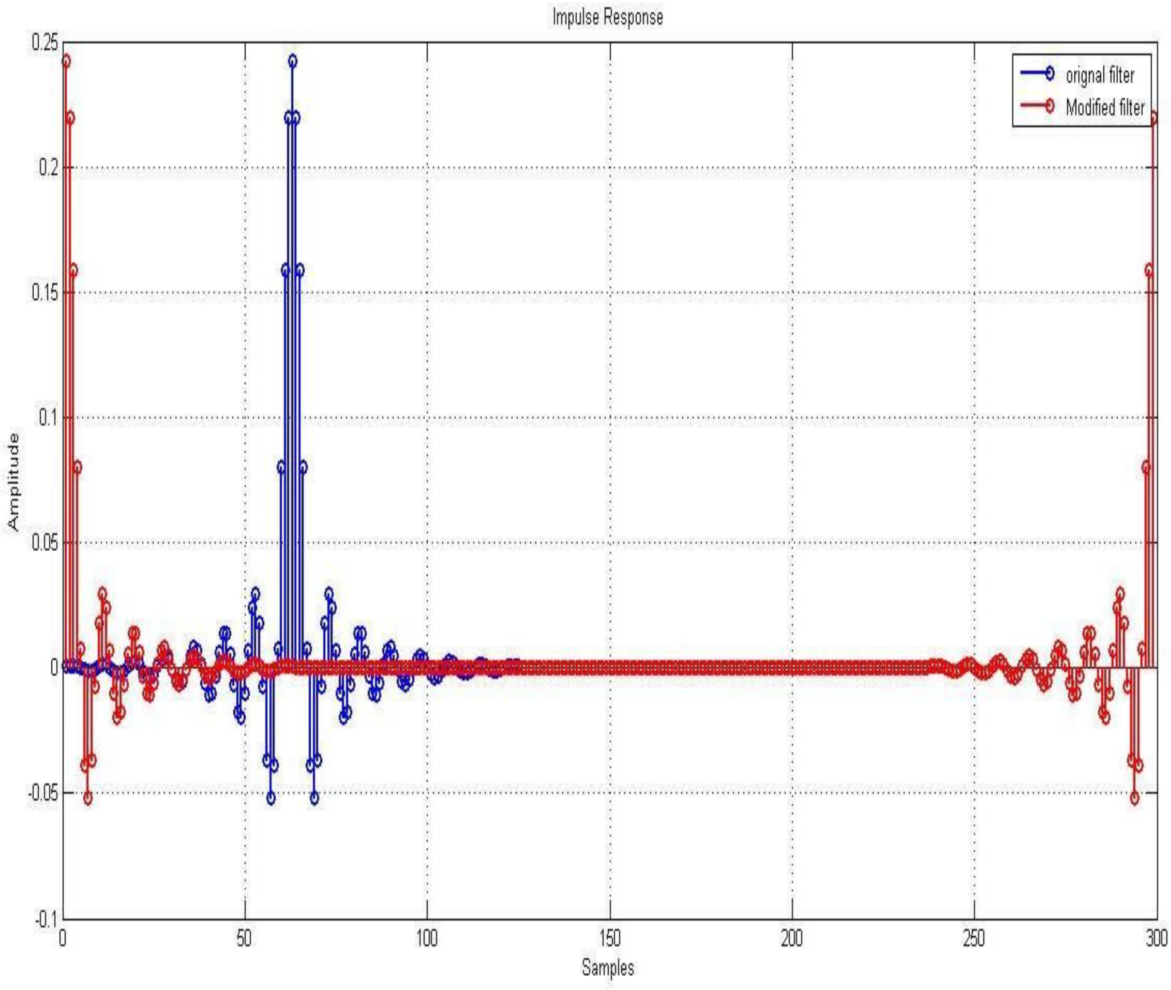

Figure 28 Impulse Response (blue and solid), the Equiripple Linear-phase Filters (red and dash) same Filter after Zero Padding and Circular Left Shifting

In this study, in order to make the filter length equal to the block length L=N+M-1, M-1 zeros have been added following the original filter coefficients. Also, the filter coefficients are circularly shifted to the left by an amount (N-1)/2. Fig. 28 shows the impulse response of the original filter and the modified filter. The parameters in this figure are: length of original filter (N)=121, data points in the segmented input sequence (M)=180, block length (L)=N+M-1=300. Thus, M-1=179 number of zeros have been added following the original filter coefficients. Also, the amount of the circular left shift is for (N-1)/2 = 60 number of samples.

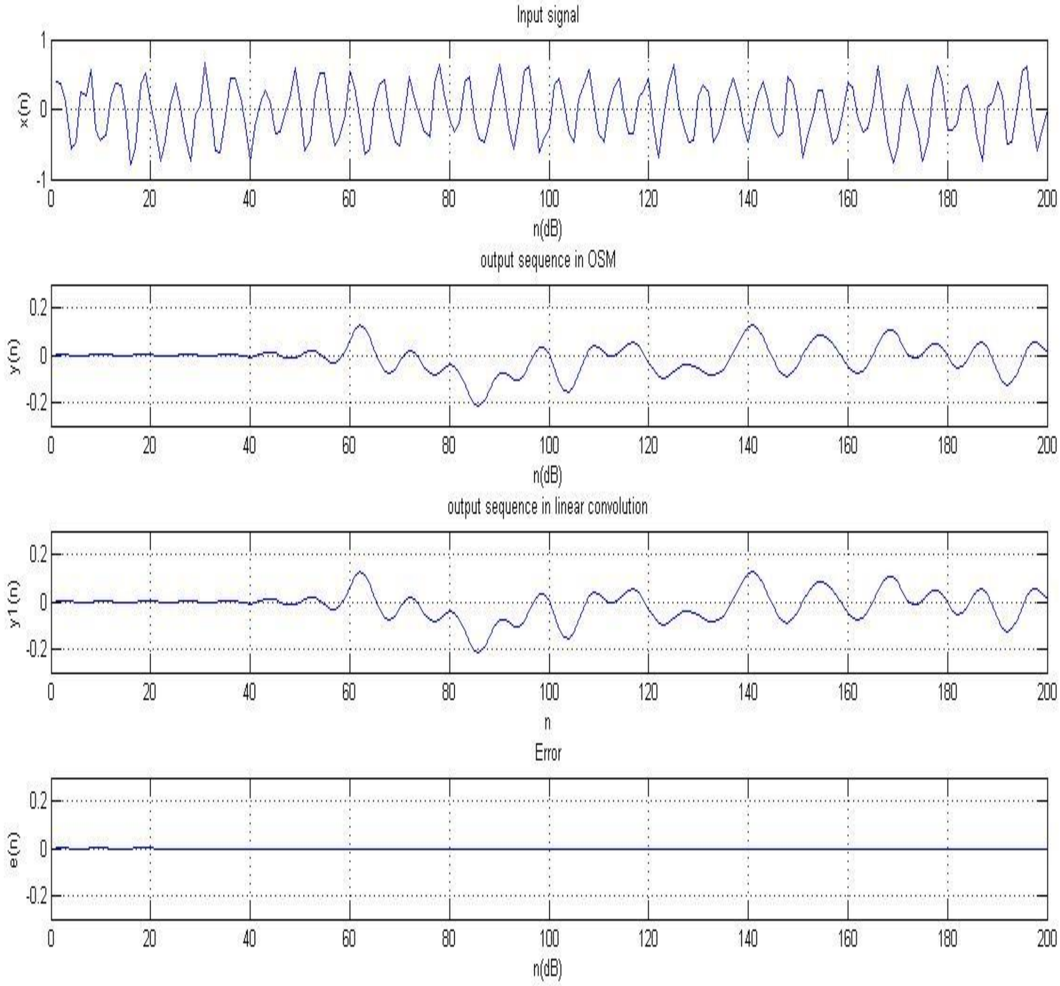

Figure 29 Comparison between OSM method and the linear convolution

Fig. 29 shows the comparison between linear convolution and OSM method. Since two methods are equivalent, the OSM method can be considered as the reference in the other parts of this discussion.

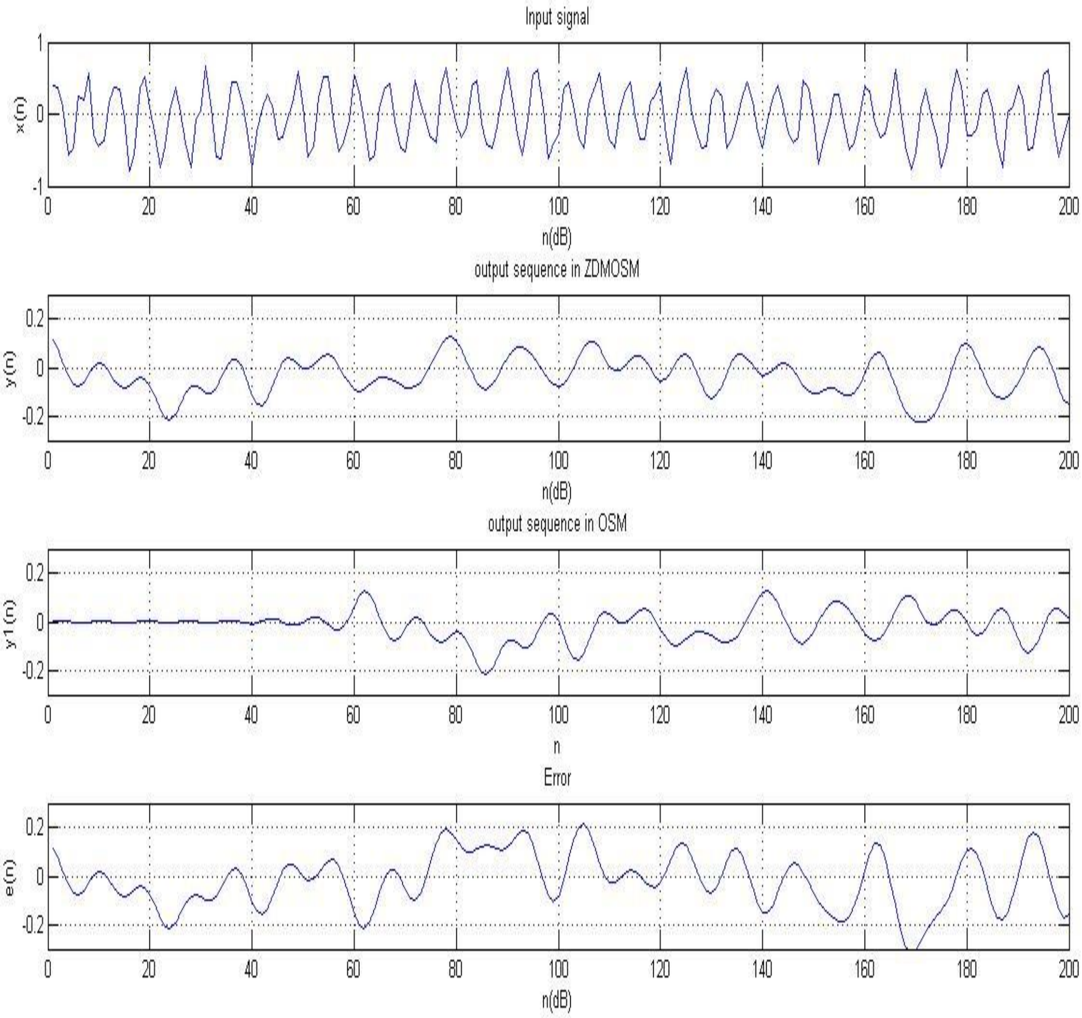

Figure 30 Comparison between ZDMOSM and OSM methods

Fig. 30 shows the output of the filter obtained by OSM and MOSM methods for the off-line processing case. The input signal, $x(n)$, the output of the filter obtained by the ZDMOSM, the output of the filter obtained by the OSM are illustrated, from top to bottom, in Fig. 30. It is clear the output obtained by ZDMOSM is different from that of obtained by the OSM.

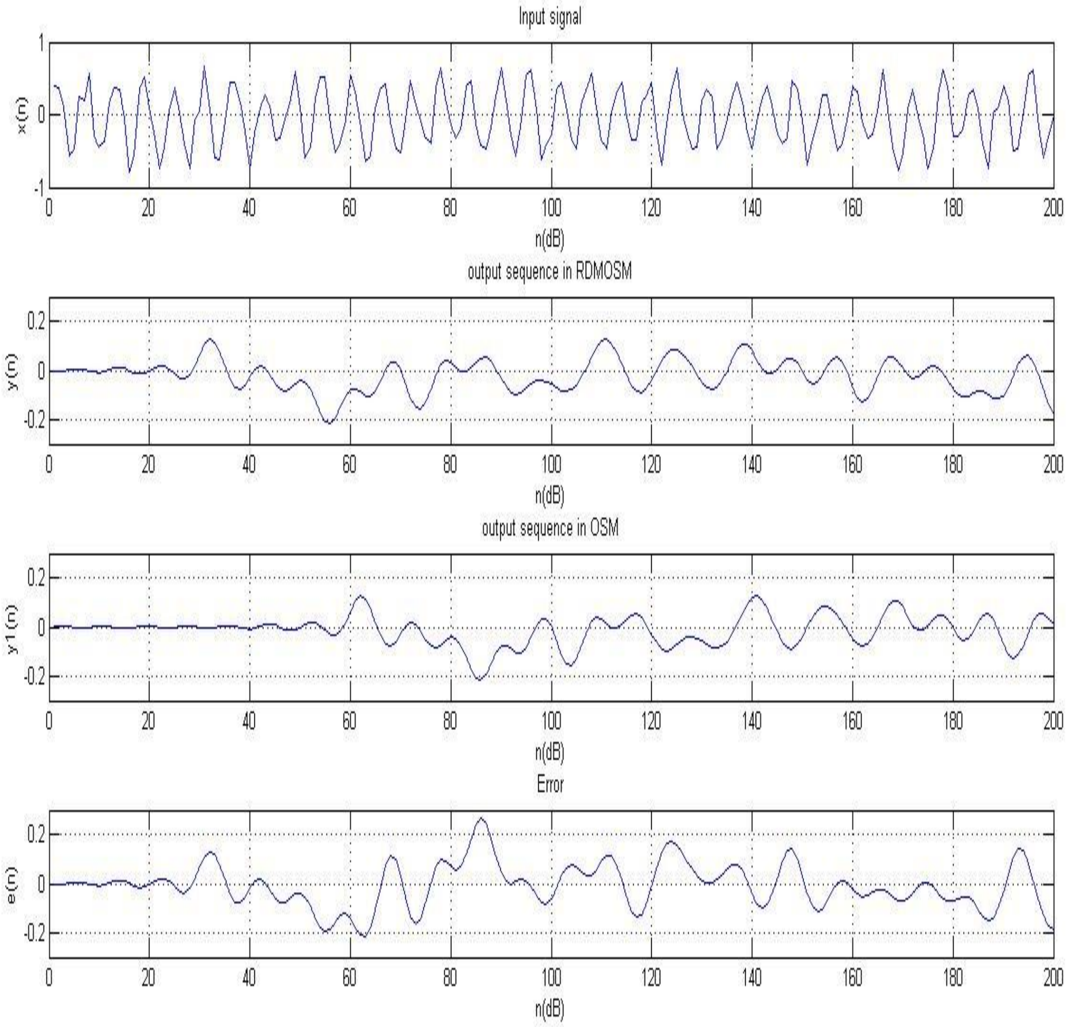

Figure 31 Comparison between RDMOSM and OSM methods.

Fig. 31 shows the output of the filter obtained by OSM and RDMOSM methods for the same input shown in Fig. 30. If the output signals obtained by ZDMOSM in Fig. 30 and by RDMOSM in Fig. 31 are compared, one can easily see that the group delay has been reduced by a factor of 1/2. Also, since the outputs obtained by OSM and RDMOSM are same, the resulting error signal becomes zero. The linear convolution can be approximated by using the MOSM method with the reduced group delay. However, this needs a zero phase filter which should be obtained by circularly shifting a linear phase impulse response.

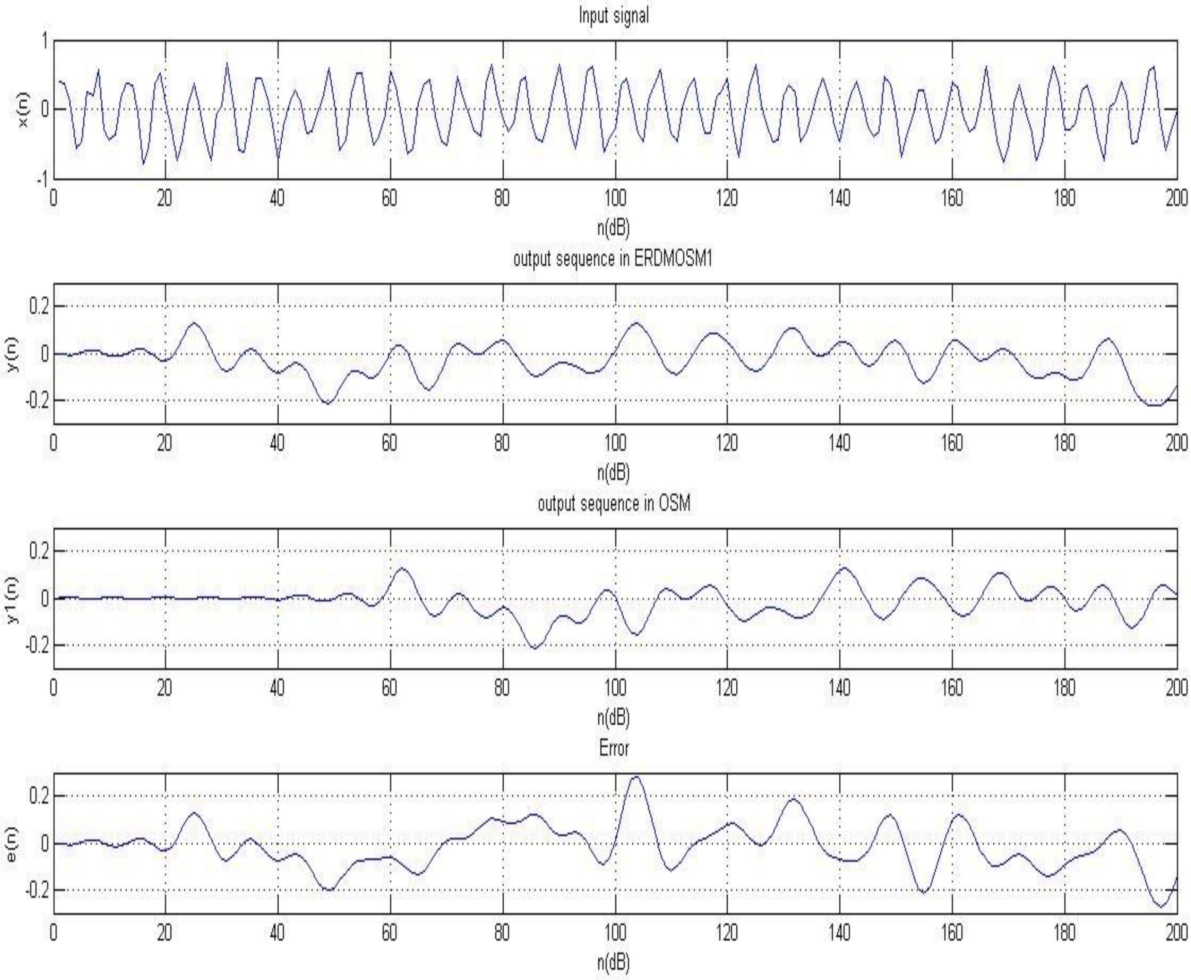

Figure 32 Comparison between ERDMOSM1 and OSM methods.

Fig. 32 and 33 show the results obtained from the OSM and enhanced-RDMOSM (ERDMOSM1 and ERDMOSM2). In Fig. 32, the results obtained by OSM and ERDMOSM1 are compared. Similarly, in Fig. 33, the results obtained from the OSM and ERDMOSM2 are compared. It can be clearly seen from these figures that ERDMOSM1 and ERDMOSM2 methods result in better delay reduction. The delay reductions obtained by the ERDMOSM 1 and ERDMOSM2 are approximately 3/5 and 4/5, respectively.

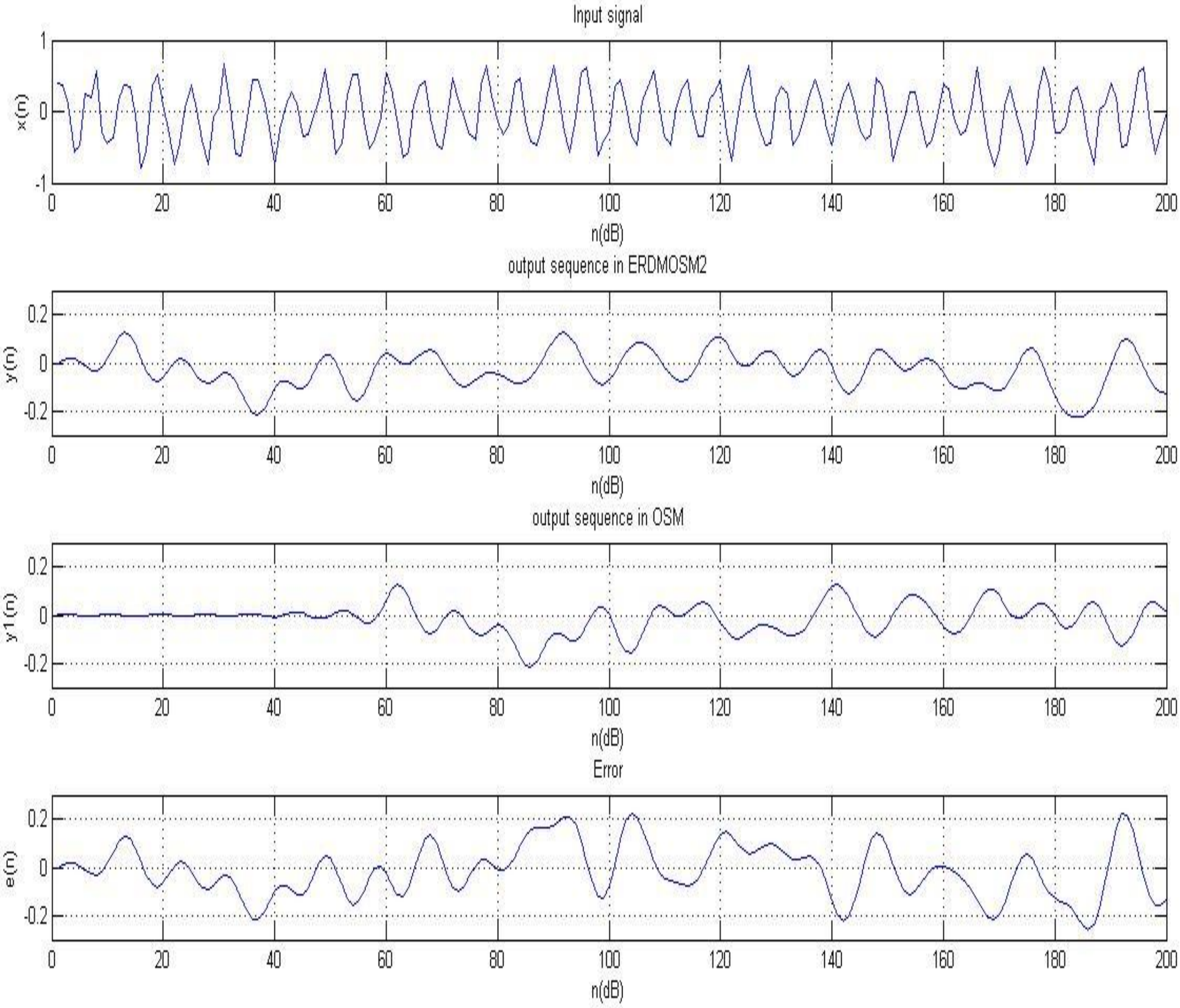

Figure 33 Comparison between ERDMOSM2 and OSM methods.

CHAPTER 5

CONCLUSION

Block involution methods like: overlap-add technique (OAM) and overlap-save technique (OSM) are usually used for a long input sequence to be filtered. In these technique, however, the output series has a finite group delay in terms of input. In this thesis, the performance of enhanced modified overlap and save method (ERDMOSM) has been investigated in reducing that group delay. In this method, the impulse response (IR) is made causal and then IR has been circularly shifted to the left by an amount of (N-1)/2 for N odd and N/2 for N even, where N is the longitude of the filter. This modified IR has been employed in OSM based block involution method. At the end, the patterns to be removed from the final convolution result have been defined. This leads to a minimized group delay. ERDMOSM1 permits a reduction of group delay by a factor of 3/5. On the other hand, ERDMOSM2 reduces the group delay by a factor of 4/5. Also, an adjustment between group delay reducing and the ripple amplitude is gotten. Despite the considerably reduction of the group delay, there is some phase troubles in passband which makes it ineffective for the actual time audio programs where group delay deviation in passband should be considered. So, the main advantage of this algorithm is in getting 83.33% group delay reducing devoid of employing whatever complex algorithm and retaining the volume responding similar to those of linear-phase filter.

Continually of this work, in future the following points can be taken in consideration:

i) Reduction of group delay deviation in pass band by employing order filters which are higher, therefore they raise to more selective filters.

ii) Generalization of the approach to all linear-phase IIR filters to obtain an effective real-time audio application filter.

iii) Application of the proposed approach to filter banks and modification of the method to reduce the overall group delay.

iv) Widening the consideration out to linear phase property of the stop band to optimize the overall delay performance of the filter.

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APPENDICES A

CURRICULUM VITAE

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