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Chirped solitons in negative index materials generated by Kerr nonlinearity

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ABSTRACT

In this paper, we are concerned with chirped solitary wave solutions in negative indexed materials having Kerr nonlinearity and self-phase modulation term. An auxiliary equation method together with an ansatz technique are employed. New chirped dark solitons, bright solitons, and trigonometric map solutions by using the auxiliary equation technique are obtained. Both 2- and 3-dimensional graphs are provided to illustrate the obtained results. The presented research will be useful especially for scientists who are studying solitons.

Introduction

In recent years, materials with simultaneously negative electric permittivity and magnetic permeability have become quite attractive research tools due to their interesting properties. J. Pendry [1] highlighted the negative index materials (NIMs) exhibiting these properties in his experimental work. Some researchers, e.g. [2], have focused on nonlinear mediums with negative index refraction. In the last decade, a large number of studies were devoted to investigating short pulses at NIMs with different types of nonlinearities, e.g. [3–6]. Some researchers [5] achieved a suitable model that characterizes pulse propagation at non-linear NIMs. Subsequently, this model has been employed to explore W-shape and bright soliton-type solutions [7], chirped dark and bright solitons at NIMs [3], rogue wave[8], solitons reproduction [9], modulation instability [10] and discrete solitons [11,12]. These localized solitons in negative index nonlinear materials usually take the form of shift solitons, spatio-temporal soliton, spatial soliton, e.g. see [13]. More recently, a lot of research papers were announced that study high-order NSE for negative index materials. The interested readers can look at [3–5,8,13,14,20–22,24–28] for chirped dark, periodic, bright, rogue waves and G soliton solutions.

Nowadays, new exact solutions that describe a short pulse propagation in negative index nonlinear materials of nonlinear PDEs including NLSE have become a significantly important research area. Semi-inverse variational principle [15], the simplest equation approach [16], the integral technique [17], the Ansatz method [18], generalized Tanh method [18] are some of the highly efficient methods for this purpose found in the literature.

In this paper, we consider a generalized NLSE, which is an appropriate model for negative index materials having Kerr dispersion, low group velocity dispersion (GVD) and Kerr nonlinearity. The model was studied in [4,19] with third-order and fourth-order dispersions (FOD), normal-GVD and anomalous-GVD. As a consequence, an exact dipole solitary wave has been achieved. In this paper, we adopt two analytical methods: Auxiliary equation method and the Ansatz method to establish new chirped solitary waves solutions.

Generalized Nonlinear Schrödinger equation

We consider the following generalized NLSE appropriate for negative index meta-materials with self-steepening effects and Kerr nonlinearity:

$$\frac{\partial\psi}{\partial\zeta} + i\frac{b_2}{2}\frac{\partial^2\psi}{\partial\tau^2} + S\frac{\partial}{\partial\tau}(|\psi|^2\psi) - ib_1|\psi|^2\psi = 0. \tag{1}$$

where $\psi(\zeta,\tau)$ typifies the complex envelop of the electric field, $\zeta = Z/L_D$ is the distance of propagation, and $\tau = T/T_0$ illustrates the time variable. Z and T are the distance of propagation and time in some

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retarded frame, respectively. $L_D = T_0^2|\beta_2|$ stands for the length of dispersion and T_0 is the initial pulse duration. The constant $b_2 = sgn[\beta_2] = \pm 1$ takes into account the normal-or anomalous-GVD

$S = 1/\omega_0 - 1/(k_0 v_g) + \partial[\partial\omega\mu(\omega)]/(\partial\omega)/[\omega\mu(\omega)]|_{\omega=\omega_0}$

and controls the self-steepening; b_1 illustrates Kerr nonlinearity and corresponds to self-focusing and self-defocusing non-linearity; and N represents the order of soliton. $\varepsilon(\omega)$ is the dispersive permittivity and $\mu(\omega)$ is permeability. In NIMs, refractive index $n^2(\omega_0)$ in carrier frequency ω_0 of medium is $\varepsilon(\omega_0)\mu(\omega_0)$. GVD β_2 in units of c is described by $\lambda_2 - 1/(k_0 v_g^2)$ and

$$\lambda_m = \frac{m! \sum_{j=0}^{N} K_j L_{m-j}}{2k_0}$$

where

$$K_m = \partial^m[\omega\varepsilon(\omega)]/(m!\partial\omega^m)|_{\omega=\omega_0}$$

and

$$L_m = \partial^m[\omega\mu(\omega)]/(m!\partial\omega^m)|_{\omega=\omega_0}$$

The group velocity of the pulse propagation can be expressed as: $v_g = 2k_0/[K_0L_1 + K_1L_0]$ and the wave number is $k_0 = \pm\sqrt{\varepsilon(\omega_0)\mu(\omega_0)}\,\omega_0/c$. Next, we will derive generalized NLSE via traveling wave Ansatz.

Traveling wave solution

We obtain chirp soliton solutions to Eq. (1) via the Ansatz method as follows:

$$\psi(\zeta,\tau) = \phi(\xi)\exp[ig(\xi)], \quad \xi = \zeta - v\tau, \tag{2}$$

where v is the frame of velocity. The corresponding chirp equation is written as

$$\delta\omega(\zeta,\tau) = -\frac{\partial}{\partial\tau}[g(\xi)]$$

Substituting Eq. (2) in Eq. (1), we get

$$\phi g' + \frac{b_2}{2}v^2\phi'' - \frac{b_2}{2}v^2\phi g'^2 - Sv\phi^3 g' - b_1\phi^3 = 0, \tag{3}$$

and

$$\phi' - b_2 v^2\phi' g' - \frac{b_2}{2}v^2\phi g'' - 3Sv\phi^2\phi'^2 = 0. \tag{4}$$

Eq. (4) yields

$$g' = -\frac{3S}{2b_2 v}\phi^2 + \frac{1}{b_2 v^2}. \tag{5}$$

and the corresponding chirp is

$$\delta\omega(\tau) = \frac{3S}{2b_2}\phi^2 - \frac{1}{b_2 v} \tag{6}$$

Hence, by substituting Eq. (5) in Eq. (3), we get

$$\frac{1}{2}b_2 v^2\phi'' + \frac{1}{2b_2 v^2}\phi - \left(\frac{S}{b_2 v} + b_1\right)\phi^3 + \frac{3}{8}\frac{S^2}{b_2}\phi^5 = 0. \tag{7}$$

To reduce ODE (7), we multiply (7) with ϕ', and integrate it for ξ assuming a zero integration constant. Finally, we get the following equation:

$$\frac{1}{4}b_2 v^2\phi'^2 + \frac{1}{4b_2 v^2}\phi^2 - \frac{1}{4}\left(\frac{S}{b_2 v} + b_1\right)\phi^4 + \frac{3}{48}\frac{S^2}{b_2}\phi^6 = 0. \tag{8}$$

Next, we set $\phi^2 = f$, hence:

$$f'^2 = c_0 f^2 + c_1 f^3 + c_2 f^4, \tag{9}$$

where

$$c_0 = -\frac{4}{b_2^2 v^4} \quad c_1 = \frac{4}{b_2 v^2}\left(\frac{S}{b_2 v} + b_1\right), \quad c_2 = -\frac{S^2}{b_2^2 v^2}$$

The ODE (refer to Eq. (9)) is an auxiliary equation that may be solved by an algebraic technique presented in [23]. Thus, in order to construct chirp and the chirped soliton solutions, the sign of the parameters in Eq. (9) plays a key role. Therefore, by using [23], we construct chirp and chirped solitons as follows:

Case 1: $c_0 > 0$, $\Delta > 0$, where $\Delta = c_1^2 - 4c_0c_2$

$$f(\xi) = \frac{2c_0 sech(\sqrt{c_0}\xi)}{\sqrt{\Delta} - c_1 sech(\sqrt{c_0}\xi)} \tag{10}$$

hence the chirp:

$$delta\omega_{1,1}(\tau) = \frac{3S}{2b_2}\left[\frac{2c_0 sech(\sqrt{c_0}\xi)}{\sqrt{\Delta} - c_1 sech(\sqrt{c_0}\xi)}\right] - \frac{1}{b_2 v} \tag{11}$$

and the complex envelope:

$$psi_{1,1}(\xi) = \sqrt{\frac{2c_0 sech(\sqrt{c_0}\xi)}{\sqrt{\Delta} - c_1 sech(\sqrt{c_0}\xi)}}\, e^{ig(\zeta - v\tau)} \tag{12}$$

Case 2: $c_0 > 0$, $\Delta < 0$

$$f(\xi) = -\frac{2c_0 csch(\sqrt{c_0}\xi)}{\sqrt{-\Delta} - c_1 sech(\sqrt{c_0}\xi)} \tag{13}$$

hence the chirp:

$$\delta\omega_{2,1}(\tau) = \frac{3S}{2b_2}\left[-\frac{2c_0 csch(\sqrt{c_0}\xi)}{\sqrt{-\Delta} - c_1 sech(\sqrt{c_0}\xi)}\right] - \frac{1}{b_2 v}, \tag{14}$$

and the complex envelope:

$$\psi_{1,1}(\xi) = \sqrt{-\frac{2c_0 csch(\sqrt{c_0}\xi)}{\sqrt{-\Delta} - c_1 sech(\sqrt{c_0}\xi)}}\, e^{ig(\zeta - v\tau)}. \tag{15}$$

Case 3: $c_0 > 0$, $\Delta_1 < 0$, where $\Delta_1 = c_1^2 - 4c_0c_2 - 4c_0^2$

$$f_{3,1}(\xi) = -\frac{2c_0 sech(\sqrt{c_0}\xi)}{c_1 sech(\sqrt{c_0}\xi) + \sqrt{-\Delta_1}\tanh(\sqrt{c_0}\xi) - 2c_0} \tag{16}$$

$$f_{3,2}(\xi) = -\frac{2c_0 csch(\sqrt{c_0}\xi)}{c_1 csch(\sqrt{c_1}\xi) - \sqrt{\Delta_1}\coth(\sqrt{c_0}\xi) + 2c_0} \tag{17}$$

From $f_{3,1}(\xi)$ and $f_{3,2}(\xi)$, the chirp soliton turns out to be:

$$\delta\omega_{3,1,1}(\tau) = \frac{3S}{2b_2}\left[-\frac{2c_0 sech(\sqrt{c_0}\xi)}{c_1 sech(\sqrt{c_0}\xi) + \sqrt{-\Delta_1}\tanh(\sqrt{c_0}\xi) - 2c_0}\right] - \frac{1}{b_2 v} \tag{18}$$

$$\delta\omega_{3,1,2}(\tau) = \frac{3S}{2b_2}\left[-\frac{2c_0 csch(\sqrt{c_0}\xi)}{c_1 csch(\sqrt{c_1}\xi) - \sqrt{\Delta_1}\coth(\sqrt{c_0}\xi) + 2c_0}\right] - \frac{1}{b_2 v} \tag{19}$$

and the complex envelope:

$$\psi_{3,1,1}(\xi) = \sqrt{-\frac{2c_0 sech(\sqrt{c_0}\xi)}{c_1 sech(\sqrt{c_0}\xi) + \sqrt{-\Delta_1}\tanh(\sqrt{c_0}\xi) - 2c_0}}\, e^{ig(\zeta - v\tau)} \tag{20}$$

$$\psi_{3,1,2}(\xi) = \sqrt{-\frac{2c_0 csch(\sqrt{c_0}\xi)}{c_1 csch(\sqrt{c_1}\xi) - \sqrt{\Delta_1}\coth(\sqrt{c_0}\xi) + 2c_0}}\, e^{ig(\zeta - v\tau)} \tag{21}$$

Case 4: $c_0 < 0$, and $c_1^2 - 4c_0c_2 - 4c_0^2$

$$f_{4,1}(\xi) = -\frac{2c_0\sec(\sqrt{-c_0}\xi)}{c_1\sec(\sqrt{-c_0}\xi) - \sqrt{-\Delta}} \tag{22}$$

The chirped soliton and corresponding chirp:

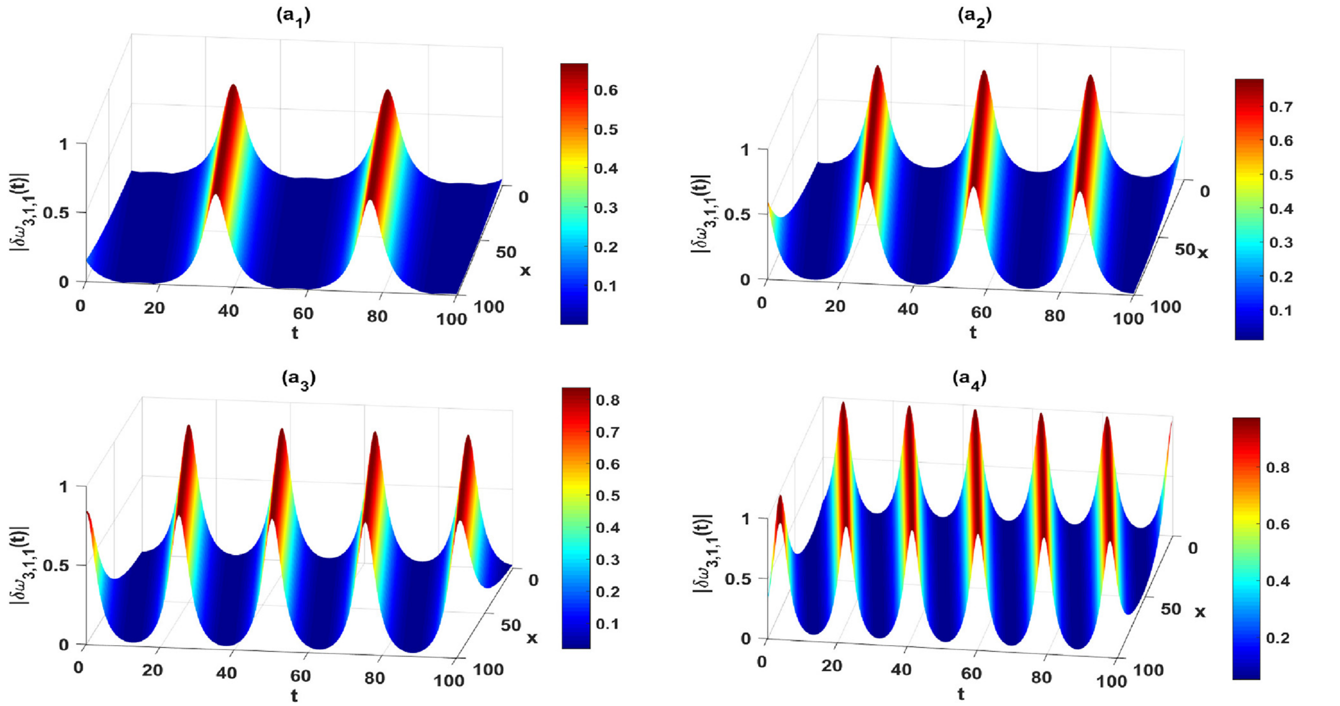

Fig. 1. Spatio-temporal plot of chirp solitary waves $|\delta\omega_{3,1,1}|$ (18) for $b_2 = 1$, $b_1 = 1.5$, $S = 25.5$, (a_1) $v = 13.25$, (a_2) $v = 9.25$, (a_3) $v = 8.005$, and (a_4) $v = 6.005$ respectively.

$$\delta\omega_{4,1}(\tau) = \frac{3S}{2b_2}\left[-\frac{2c_0\mathrm{sec}(\sqrt{-c_0}\xi)}{c_1\mathrm{sec}(\sqrt{-c_0}\xi) - \sqrt{-\Delta}}\right] - \frac{1}{b_2 v} \tag{23}$$

$$\psi_{4,1}(\xi) = \sqrt{-\frac{2c_0\mathrm{sec}(\sqrt{-c_0}\xi)}{c_1\mathrm{sec}(\sqrt{-c_0}\xi) - \sqrt{-\Delta}}}\, e^{ig(\zeta - v\tau)} \tag{24}$$

Graphical representation

Fig. 1 plots chirp solitary waves of $\delta\omega_{3,1,1}$ at $b_2 = 1$, $b_1 = 1.5$, $S = 25.5$, (a_1) $v = 13.25$, (a_2) $v = 9.25$, (a_3) $v = 8.005$, and (a_4) $v = 6.005$ and the corresponding chirped soliton. Fig. 2 is the evolution plot of $\psi_{3,1,1}$. Figs. 3 and 4 are analytical representations of the chirped $\delta\omega_{3,1,1}(\tau)$ at $b_2 = 1$, $b_1 = 10.5$, $S = 2.15$, $v = -1.51$, at $t = 0$, $t = 5$, $t = 10$, $t = 15$ respectively, for $-15 \leqslant x \leq 15$. One may conclude that the constants c_0, c_1, c_2 are related to normal GVD, electric permittivity, dispersive magnetic permeability, Kerr nonlinearity term, self-steepening coefficient, and the negative refraction index medium. Furthermore, the graphs illustrate the propagation of combined dark and bright solitons.

Conclusion

We explored chirped solitary waves in negative indexed materials

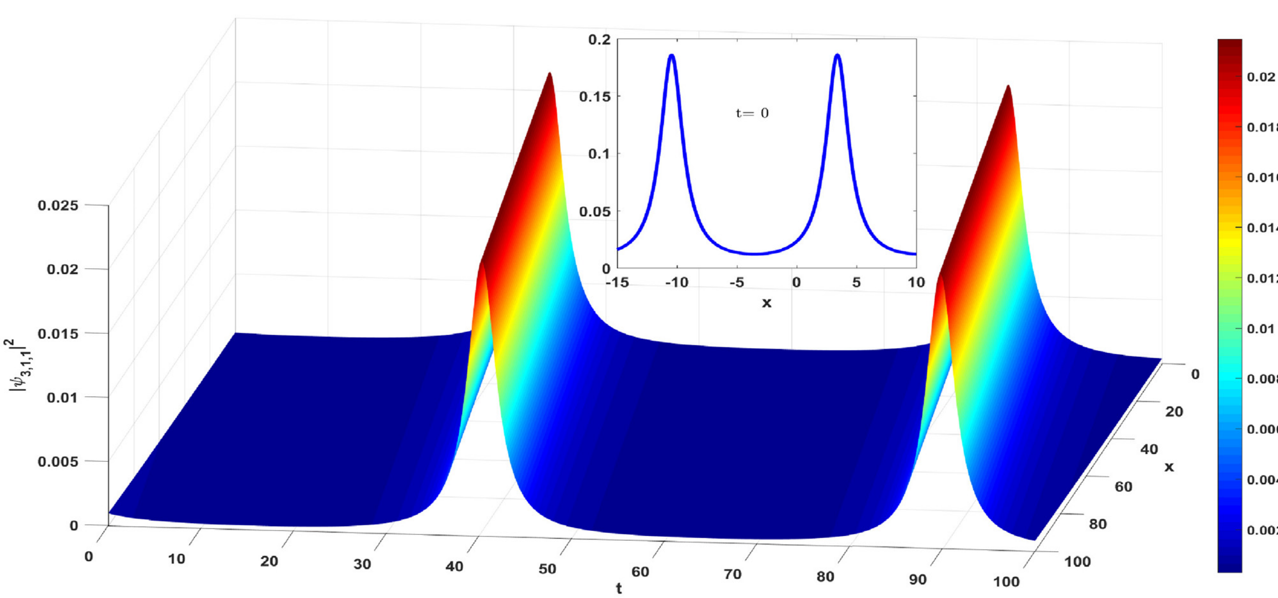

Fig. 2. Spatio-temporal plot evolution of the chirped solitons $|\psi_{3,1,1}|^2$ (20) for $b_2 = 1$, $b_1 = 10.5$, $S = 50.15$, $v = 15.75$, and the corresponding plot (2-D) (blue line) at $t = 0$ respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

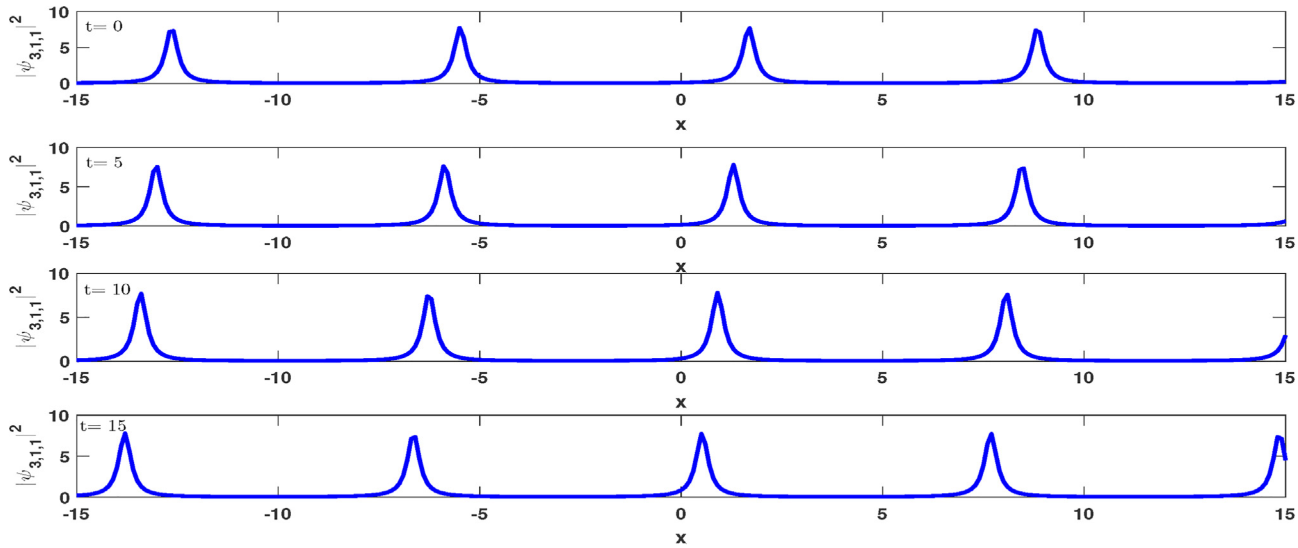

Fig. 3. Spatio-temporal plot evolution of the chirped solitons $|\psi_{3,1,1}|^2$ (20) for $b_2 = 1,\ b_1 = 10.5,\ S = 2.15,\ v = -1.51$, at $t = 0,\ t = 5,\ t = 10,\ t = 15$ respectively, for $-15 \leqslant x \leq 15$.

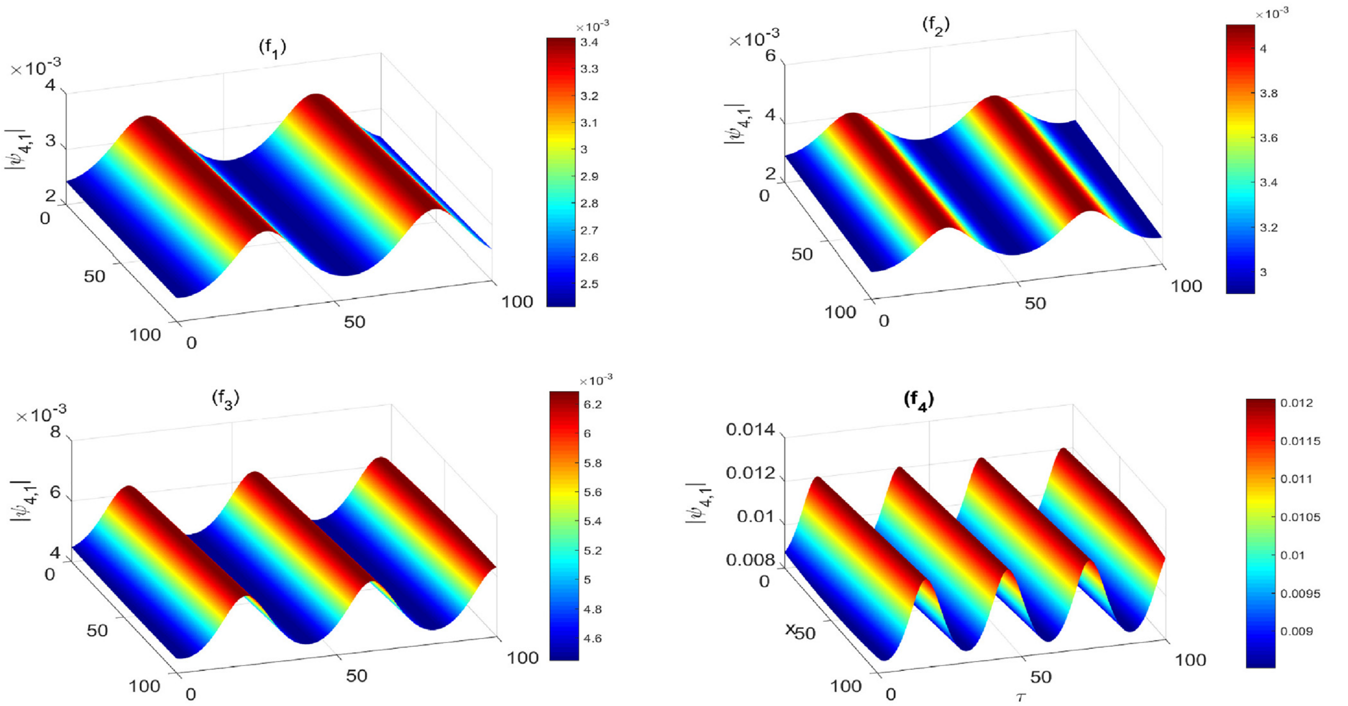

Fig. 4. Spatio-temporal evolution of chirped solitons $|\psi_{4,1}|^2$ (24) for $b_2 = 1,\ b_1 = 0.5,\ S = 0.15,$ at $(f_1)\ v = 34.075,\ (f_2)\ v = 31.075\ (f_3)\ v = 25.075$ and $(f_4)\ v = 18.075$, respectively.

having Kerr nonlinearity and self-phase modulation term. Furthermore, we established exact analytical chirped soliton-type solutions of a generalized NLSE by utilizing the auxiliary equation, which depends on the parameters of nonlinear ordinary differential equations. Results in the present paper will be quite useful for researchers who are studying solitons.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

A. Houwe: Conceptualization, Methodology, Software. **Mustafa Inc:** Data curation, Writing - original draft. **S.Y. Doka:** Software, Validation. **M.A. Akinlar:** Writing - review & editing. **D. Baleanu:** Writing - review & editing.

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