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Research Article

Bessel Beam Diffraction by an Aperture in an Opaque Screen

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The scattering of the Bessel beam by a circular aperture in an opaque screen is investigated by the geometrical theory of diffraction approach. The geometrical optics and diffracted and scattered fields are obtained. The effect of the aperture to the scattering process is analyzed. The uniform versions of field expressions are derived. The geometrical optics and diffracted and scattered fields are examined numerically.

1. Introduction

The Bessel beam which is a type of nondiffracting beam was first reported by Durnin [1] and Durnin et al. [2]. The nondiffraction property is related to its propagation characteristics. Such a beam does not spread on the transverse plane when it propagates. The propagation characteristic of the Bessel beam was investigated by many researchers [3–8]. The ideal Bessel beam has an infinite energy like the plane wave. For this reason, physical realization of a Bessel beam is problematic. Various forms of apertures are envisaged in order to truncate this kind of beams [9, 10]. The scattering characteristics of several types of Bessel beams by finite aperture were investigated by Jiang et al. [11]. The first experimental study of a Bessel beam that was scattered by a circular aperture was put forward by Durnin et al. [12]. In the Durnin's study, the propagation characteristics of the Bessel and Gauss beams were experimentally compared. The first theoretical study about the scattering of Bessel beams by a circular aperture was carried out in Vicari's work [3]. An analytic expression of a truncated Bessel beam diffraction by an aperture was obtained by De Nicola [4]. Also, the focusing properties of truncated Bessel beam were studied by using Huygens-Fresnel diffraction integral by De Nicola [5]. In another study, total scattered field was evaluated analytically using a finite aperture but the effect of the geometrical optics and the diffracted fields were not examined separately by Overfelt and Kenney [6]. A new analytic approximation to the integral of Bessel function and the application to the transmittance of a circular aperture were presented by Ladera and Martin [13]. Although Bessel beam is an exact solution of the Helmholtz equation, the literature also includes the paraxial form of this beam. The paraxial Bessel beam, which can also be obtained from the paraxial approximation of the wavenumbers, satisfies the paraxial wave equation [14]. The paraxial Bessel beam was firstly studied by Piestun et al. [15]. The scattering of a paraxial Bessel beam by a circular aperture in an opaque screen was examined by Umul [16]. The detailed explanations about the geometrical optics and diffracted waves were given for the paraxial Bessel beam in his examination.

The aim of this study is to investigate the effect of the Bessel beam diffraction, by a circular aperture in an opaque screen, to the scattered field. We should keep in mind that an opaque screen does not transmit or reflect the incoming waves [17]. The geometrical optics (GO) and the diffracted fields are separately examined, independent of the scattered field. In the high frequency asymptotic techniques, field expressions approach to infinity at the reflection and the shadow boundaries. These regions are called transition regions. For the elimination of these regions, uniform theories were introduced [18]. Thus, field expressions take finite values in the transition regions. Moreover to our study, the diffracted field expressions are transformed to uniform versions. To the best of our knowledge the uniform versions of

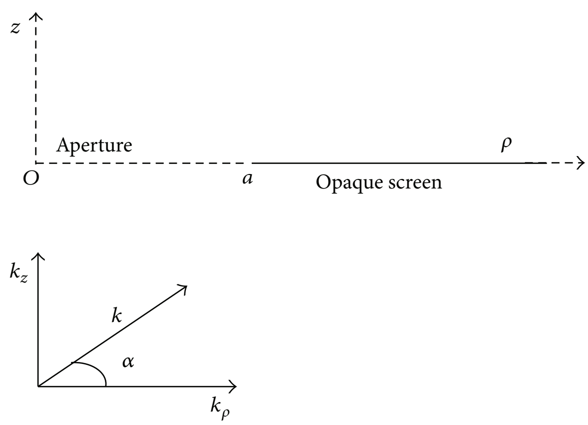

Figure 1: The geometry of the problem.

the scattered Bessel beams by a circular aperture in an opaque screen have not been studied yet. The effect of the aperture on the Bessel beam is investigated numerically in terms of the uniform diffracted and scattered fields. The circular aperture which exists in the opaque screen is symmetric around the z-axis. The circular aperture problems are important for the microwave antenna systems and also the optic devices such as lenses, microscopes. Imaging performance of the Bessel beam microscopy was analyzed by Snoeyink in terms of the relative spatial resolution increase and relative brightness of images [19]. The light used in microscopy illumination is also important. The scattered light causes the wrong interpretation of microscopic data by generating ghost images [20]. In this point of view, using nondiffracting beams in the illumination process has advantages. It was reported in an experimental study that scanned Bessel beam reduces the scattering effects and increases the image quality and penetration depth in dense media [21]. In another study, it was shown that Bessel beams provide the highest image resolution [22]. As can be seen from the studies, Bessel beam illumination is widely used proper source for the imaging. The scattering of light affects the quality of images so analysis of the scattering and diffraction of the Bessel beam by an aperture is an important problem.

The time factor $\exp(j\omega t)$ is suppressed throughout the paper and ω is the angular frequency.

2. Theory

The aperture of the opaque screen is illuminated by the Bessel beam of

$$u_i = J_0\left(k_\rho \rho\right) e^{-jk_z z}, \tag{1}$$

where k_ρ and k_z are the wavenumbers which are in the direction of ρ and z, respectively. The geometry of the problem is given in Figure 1. The aperture is quite large according to the wavelength.

The asymptotic expression of the Bessel function can be written as

$$J_\chi(x) \cong \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{1}{2}\chi\pi - \frac{1}{4}\pi\right), \tag{2}$$

where x is sufficiently greater than one [23]. Using (2), the incident field expression in (1) can be rearranged as

$$u_i = \left[\cos\left(k_\rho \rho\right) + \sin\left(k_\rho \rho\right)\right] \frac{e^{-jk_z z}}{\sqrt{\pi k_\rho \rho}} \tag{3}$$

for $k_\rho \rho \gg 1$. It can be seen from (3) that beam is divided into two terms, which will bring simplicity in evaluation. The geometrical theory of diffraction (GTD) coefficient for an aperture opaque screen can be described as

$$D = -\frac{e^{-j(\pi/4)}}{2\sqrt{2\pi}} \frac{1}{\cos\left(\left(\eta - \phi_0\right)/2\right)}, \tag{4}$$

where ϕ_0 is the angle between incidence field and half plane and η is the angle between diffracted field and half plane, respectively [24]. Because of the nonreflection property of opaque screen, only the incident diffracted term, D, is taken into account. The diffracted field expression is found from the expression

$$u_d = u_i\left(Q_e\right) D, \tag{5}$$

where Q_e is the diffraction point where $z = 0$ and $\rho = a$. Bessel beam is a standing wave. In order to obtain the GTD diffracted fields, incident angles must be known. Because of this reason, Bessel wave is split to Hankel functions, which are traveling waves. Incident field expression in the diffraction point is then written as

$$u_i\left(Q_e\right) = \frac{1}{\sqrt{\pi k_\rho a}} \left[\cos\left(k_\rho a\right) + \sin\left(k_\rho a\right)\right] \tag{6}$$

and using the trigonometric relations, the field expression is divided into two ray components and can be found as

$$u_i\left(Q_e\right) = \frac{1}{2j\sqrt{\pi k_\rho a}} \left[(j+1) e^{jk_\rho a} + (j-1) e^{-jk_\rho a}\right], \tag{7}$$

where according to the time factor $\exp(j\omega t)$ the first and the second terms show the waves' values at the edge point ($\rho = a, z = 0$), which are propagating in the decreasing and the increasing ρ directions, respectively. As a contrast the second part represents a wave which is converging and the second term is the diverging cylindrical wave. Substituting (7) in (5), the diffracted field expression is obtained as

$$\begin{aligned} u_d = &-\frac{1}{2j\sqrt{\pi k_\rho a}} \left[(j+1) e^{jk_\rho a} + (j-1) e^{-jk_\rho a}\right] \\ &\cdot \frac{e^{-j(\pi/4)}}{2\sqrt{2\pi}} \frac{1}{\cos\left(\left(\gamma - \phi_{01,2}\right)/2\right)} \frac{e^{-jkR_s}}{\sqrt{kR_s}}, \end{aligned} \tag{8}$$

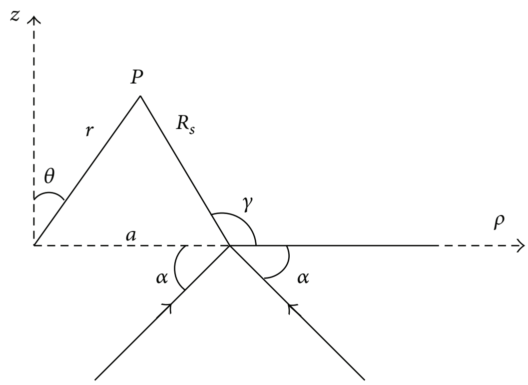

Figure 2: Related angle geometry for the ray components.

where R_s is the ray-path of diffracted wave and equal to $[r^2+a^2-2ra\sin\theta]^{1/2}$. The related angles are given in Figure 2. ϕ_{01}, ϕ_{02} angles are equal to $2\pi-\alpha$ and $\pi+\alpha$, respectively. At this stage diffracted field expression given in (8) can be rearranged according to the ray configurations along the positive and negative ρ directions.

The first diffracted field component is written as

$$u_{d1} = -\frac{1}{\sqrt{\pi}}\frac{j+1}{2j}\frac{e^{jk_\rho a}}{\sqrt{k_\rho a}}\frac{e^{-j(\pi/4)}}{2\sqrt{2\pi}}\frac{1}{\cos((\gamma-\phi_{01})/2)}\frac{e^{-jkR_s}}{\sqrt{kR_s}} \quad (9)$$

and the second component is written as

$$u_{d2} = -\frac{1}{\sqrt{\pi}}\frac{j-1}{2j}\frac{e^{-jk_\rho a}}{\sqrt{k_\rho a}}\frac{e^{-j(\pi/4)}}{2\sqrt{2\pi}}\frac{1}{\cos((\gamma-\phi_{02})/2)}\frac{e^{-jkR_s}}{\sqrt{kR_s}}. \quad (10)$$

Because of the circular aperture the geometry of the problem is symmetric around ϕ where it is the angle cylindrical coordinates in the x-y plane. In the shadow boundaries, diffracted field expressions approach to infinity so the field expressions have to be transformed to uniform versions. The uniform theory is based on the relation of

$$\operatorname{sign}(x)\, F[|x|] \cong \frac{e^{-j(\pi/4)}}{2\sqrt{\pi}}\frac{e^{-jx^2}}{x} \quad (11)$$

when $x \gg 1$ [25]. The sign(x) is the signum function which is equal to 1 for $x > 0$ and equal to -1 for $x < 1$. The $F[x]$ is the Fresnel function and is defined as

$$F[x] = \frac{e^{j(\pi/4)}}{\sqrt{\pi}}\int_x^\infty e^{-jy^2}\,dy. \quad (12)$$

In order to obtain the uniform version of the fields, detour parameters are introduced as

$$\xi_{\phi_{01,2}} = -\sqrt{2kR_s}\cos\frac{\gamma-\phi_{01,2}}{2} \quad (13)$$

which will replace x in (11). Hence, the uniform version of the first diffracted field expression is found as

$$u_{d1} = \frac{1}{\sqrt{\pi}}\frac{j+1}{2j}\frac{e^{jk_\rho a}}{\sqrt{k_\rho a}}e^{jkR_s\cos(\gamma-\phi_{01})}\operatorname{sign}(\phi_{01})\, F[|\phi_{01}|] \quad (14)$$

and the second diffracted field becomes

$$u_{d2} = \frac{1}{\sqrt{\pi}}\frac{j-1}{2j}\frac{e^{-jk_\rho a}}{\sqrt{k_\rho a}}e^{jkR_s\cos(\gamma-\phi_{02})}\operatorname{sign}(\phi_{02})\, F[|\phi_{02}|]. \quad (15)$$

Total diffracted field expression is obtained from the addition of (14) and (15); thus

$$u_d = u_{d1} + u_{d2}. \quad (16)$$

The GO field expressions are written as

$$u_{GO1} = \frac{1}{\sqrt{\pi}}\frac{j+1}{2j}\frac{e^{jk_\rho \rho}}{\sqrt{k_\rho \rho}}e^{-jk_z z}U(-\xi_{\phi_{01}}),$$
$$u_{GO2} = \frac{1}{\sqrt{\pi}}\frac{j-1}{2j}\frac{e^{-jk_\rho \rho}}{\sqrt{k_\rho \rho}}e^{-jk_z z}U(-\xi_{\phi_{02}}), \quad (17)$$

where $U(x)$ is the unit step function indicating the GO fields boundary in the medium. Total GO field expression is obtained from addition of (17) as

$$u_{GO} = u_{GO1} + u_{GO2}. \quad (18)$$

3. Numerical Analysis

In this part of the analysis, numerical results for the scattered fields will be found against the variations in the aperture size a and the observation distance r. All quantities given in units of wavelength are proportional to the wavelength λ. In this study, the interpretation is based on the high frequency asymptotic techniques so the solutions are valid for $k_\rho \rho \gg 1$. In Figure 3 the observation distance r is taken as 3λ and the aperture size a is 2λ. The angle between the direction of propagation wavenumber and the ρ axis, α, is equal to 45°. The detailed explanation for the wavenumbers is given in Figure 1.

Figure 3 shows the total scattered GO and diffracted fields. Total scattered field includes the diffracted field and also GO field. Amplitude variations approach zero for increasing values of θ which is the expected behavior of Bessel functions. The GO field's amplitude is continuous throughout the aperture size except for the transition point. At that point, amplitude of GO field is zero. Then there is no amplitude variation that can be observed for GO field. However in the absence of the GO field in the shadow region, the amplitude variation can be observed from the total scattered field. It can be seen from Figure 3 that amplitude of the total

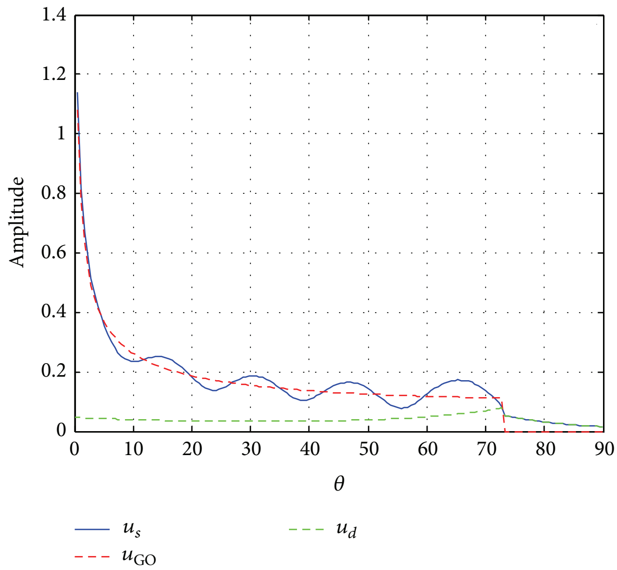

Figure 3: Scattered represented by u_s, geometric optics represented by u_{GO}, and diffracted represented by u_d fields. The observation distance r is 3λ and the aperture size a is 2λ.

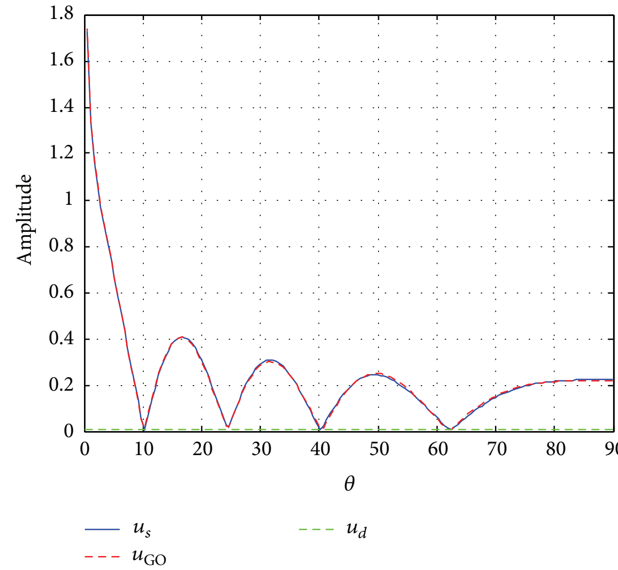

Figure 5: Scattered represented by u_s, geometric optics represented by u_{GO}, and diffracted represented by u_d fields. The observation distance r is 3λ and the aperture size a is 10λ.

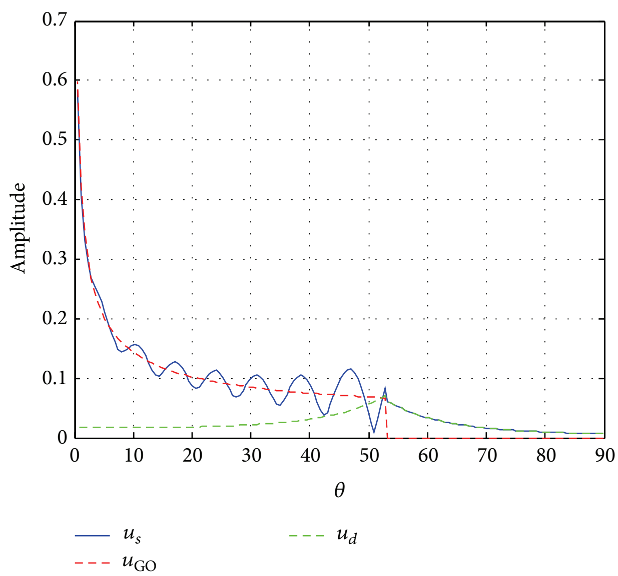

Figure 4: Scattered represented by u_s, geometric optics represented by u_{GO}, and diffracted represented by u_d fields. The observation distance r is 10λ and the aperture size a is 2λ.

scattered field is continuous for all degrees of observation. Total scattered field includes GO fields and also the diffracted fields. Diffracted field takes its maximum value at nearly 73° and compensates for deficiencies of GO field in the transition region. This region is governed by the angles γ and $\phi_{01,2}$.

Figure 4 shows the scattered fields for different values of r. In this case, r is taken as 10λ and aperture size a is the same in Figure 3. It can be seen from Figure 4 that the amplitude values decrease when the observation point is far away from the aperture. In addition, the effect of the diffracted field increases proportionally with the distance of the observation point. Moreover, depending on the angles γ and $\phi_{01,2}$, the diffraction process occurs in the smallest angles. In Figure 5, the effect of the aperture size on scattered fields can be analyzed. Unlike previous cases, the effect of diffraction cannot be observed in Figure 5. If the value of the aperture size is taken as sufficiently large according to the wavelength, the incident field passes through the aperture without any diffraction. Also, effect of the diffracted field is closer to zero. It can be seen from Figure 5 that GO field is consistent with the total scattered field; hence there is no critical point in this region.

4. Conclusion

As mentioned in Introduction, investigation of the scattering of Bessel beams is very important for practical applications such as imaging. The subfields of the scattered fields which are GO and diffracted fields were separately examined from the scattered field. Also, the uniform field expressions were obtained in this study. The effects of the various parameters on the field's behavior were examined. The uniform expressions were found to be finite in the transition regions. Hence, the field expressions do not go to infinity at the shadow boundary. All derived field expressions were analyzed for different aperture size and different distances from the aperture. The aperture size can represent the length of the lenses in the optics. The results showed that the effect of the diffracted field increases proportionally with the distance of the observation point. The aperture size also affects diffracted field. The effect of the diffracted field decreases, when the aperture size increases. Due to property of the opaque screen, reflected fields were not encountered.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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