Results in Physics 17 (2020) 103039

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Propagation of harmonic waves in a cylindrical rod via generalized Pochhammer-Chree dynamical wave equation

Asghar Ali[a], Aly R. Seadawy[b,*], Dumitru Baleanu[c,d,e]

[a] Mathematics Department, University of Education, Lahore, Pakistan
[b] Mathematics Department, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[c] Department of Mathematics, Cankaya University, Ankara, Turkey
[d] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan
[e] Institute of Space Sciences, 077125 Magurele, Romania

ARTICLE INFO

Keywords:
Generalized Pochhammer-Chree equation
Generalized direct algebraic method
Improved simple equation method
Modified F-expansion method
Solitary solutions

ABSTRACT

In this manuscript, three novel schemes (Generalized direct algebraic; Improved simple equation and Modified F-expansion methods) are successfully utilized to find the solitary solutions of generalized form of Pochhammer-Chree equation.The concerned wave model has been used in the scrutiny for the propagating of harmonic waves in a cylindrical rod and several problems in fluid mechanics and waves theory in physics. The obtained results have imperative role in the field of the nonlinear science.

1. Introduction

The dynamical behaviors of solitons have altered the life style of the people. For exemplar, discussions of internet phone tricks are allocated with help of soliton transmission, over transcontinental and transoceanic distances, in the itinerary of optical fibers cables. Moreover, in the previous few decades important progress has been made and many powerful and capable techniques for acquiring the analytical traveling wave solutions for nonlinear partial differential equations have been discussed in the literature [1–10]. Many efficient methods have been discovered for nonlinear wave equations such as; Homogeneous balance method [11], G/G'-expansion method [12,13], Extended tanh function method [14], Extended F-expansion method, Jacobi elliptic function method [15], Transformed rational function method [16], Weierstrass elliptic function expansion method [17], Hirota's bilinear method [18], Generalized Kudryashov method, Trial equation method [19], Solitary ansatz method [20], Auxiliary equation method [21], Modified Kudryashov method [22,23], Sine–Gordon equation expansion method [24], Modified F-expansion method [25], Exp($\varphi(-\xi)$-expansion method [26] and many more in [27–43].

Here our focus in this article is to study Pochhammer -Chree equation which is used in the propagation of harmonic waves in a cylindrical rod. Distinct methodologies to find the exact solution of generalized PC equation were used, algebraic method was imposed in [44] for explicit and exact solutions of generalized PC equation. Similarly Exp-functional technique in [45] was employed for PC model. Moreover recently authors in [46] discussed the solitary wave solutions of PC equation via $Exp(-\phi(\xi))$-expansion method. But our main focus is here successfully implementation of three novel analytical techniques, so called generalized direct algebraic, Improved form of simple equation and modified F-expansion on the generalized PC equation. The achieved results are more general as compared to discuss in [44–46]. Hence our adopted methods are general and powerful tools for nonlinear problems. The remainder of this article is arranged as; Description of proposed methods are illustrated in Section 2, application in 3. Discussion and conclusion in Sections 4 and 5.

2. Description of proposed methods

Consider

$$H_1(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, ...) = 0. \tag{1}$$

Let

$$u = V(\xi), \quad \xi = x - \omega \times t. \tag{2}$$

Substitute Eq.(2) into Eq. (1).

$$H_2(V, V', V'', ...) = 0, \tag{3}$$

* Corresponding author.
E-mail addresses: aly742001@yahoo.com (A.R. Seadawy), dumitru@cankaya.edu.tr (D. Baleanu).

https://doi.org/10.1016/j.rinp.2020.103039
Received 15 January 2020; Received in revised form 27 February 2020; Accepted 29 February 2020
Available online 07 March 2020

2.1. Generalized direct algebraic method

Let solution (3) has,

$$V=\sum^{N} i=0A_i\Psi^i+\sum^{-N} i=-1B_{-i}\Psi^i+\sum^{N} i=2C_i\Psi^{i-2}\Psi'+\sum^{N} i=1D_i\left(\frac{\Psi'}{\Psi}\right)^i. \tag{4}$$

Suppose Ψ satisfies following,

$$\Psi'=\sqrt{\beta_1\Psi^2+\beta_2\Psi^3+\beta_3\Psi^4}. \tag{5}$$

Put (4) with (5) in (3), solve for require destination of Eq. (1).

2.2. Improved simple equation method

Let (3) has solution,

$$V=\sum^{m} i=-mA_i\Psi^i. \tag{6}$$

Let Ψ gratify,

$$\Psi'=c_0+c_1\Psi+c_2\Psi^2+c_3\Psi^3. \tag{7}$$

Put (6) with (7) in (3). after solving achieved the require solution of (1).

2.3. Modified F-expansion method

Let us suppose that (3) has solution as:

$$V=a_0+\sum^{m} i=1a_iF^i(\xi)+\sum^{m} i=1b_iF^{-i}(\xi). \tag{8}$$

Let F gratifies,

$$F'=A+BF+CF^2. \tag{9}$$

Put (8) with (9) in (3), after solving obtained solution Eq. (1).

3. Applications

3.1. Application of generalized direct algebraic method

Consider the generalized form of Pochhammer-Chree equation in [46]

$$u_{tt}-u_{ttxx}-(au-bu^3)_{xx}=0. \tag{10}$$

Put (2) into (10), integrate twice with neglecting constant of integration, yields following ODE form as;

$$-aV+bV^3-\omega^2V''+V\omega^2=0 \tag{11}$$

Let Eq. (11) has solution form as,

$$V=A_1\Psi+A_0+\frac{B_1}{\Psi}+\frac{D_1\Psi'}{\Psi} \tag{12}$$

Put (12) along with (5) in (11) (See Fig. 1);

$$A_0=0,\quad A_1=\frac{\sqrt{a\beta_3}}{\sqrt{\beta_1b+2b}},\quad D_1=\frac{\sqrt{a}}{\sqrt{b(\beta_1+2)}},\quad B_1=0,$$
$$\omega=-\frac{\sqrt{2a}}{\sqrt{\beta_1+2}} \tag{13}$$

Put (13) in (12) (See Fig. 2),

Case- I

$$V_1=\frac{\sqrt{a}\left(\beta_1^{3/2}\in\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)\right)}{\frac{\sqrt{b(\beta_1+2)}\left((2\beta_2)\left(-\beta_1\left(\in\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)\right)}{\beta_2}}-\frac{\sqrt{a\beta_3}\left(\beta_1\left(\in\coth\left(\frac{1}{2}\sqrt{\beta_1}(\xi+\xi_0)\right)+1\right)\right)}{\beta_2\sqrt{\beta_1b+2b}}$$
$$\beta_1>0,\quad \beta_2^2-4\beta_1\beta_3=0,\quad a>0,\quad b>0. \tag{14}$$

Case- II

$$V_2=-\frac{\sqrt{a}\left(\sqrt{\frac{\beta_1}{\beta_3}}\left(\frac{\sqrt{\beta_1}\in\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}-\frac{\sqrt{\alpha_1}\in\sinh^2(\sqrt{\beta_1}(\xi+\xi_0))}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta)^2}\right)\right)}{\sqrt{b(\beta_1+2)}\left(2\left(-\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\in\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)\right)}$$
$$-\frac{\sqrt{a\beta_3}\left(\sqrt{\frac{\beta_1}{4\beta_3}}\left(\frac{\in\sinh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta}+1\right)\right)}{\sqrt{\beta_1b+2b}},$$
$$\beta_1>0,\quad \beta_3>0,\quad \beta_2=\sqrt{4\beta_1\beta_3},\quad a>0,\quad b>0. \tag{15}$$

Case- III

$$V_3=\frac{\sqrt{a}\left(-\left(\beta_1\left(\frac{\sqrt{\beta_1}\in\cosh(\sqrt{\beta_1}(\xi+\xi_0))}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p^2+1}}-\frac{\sqrt{\beta_1}\in\sinh(\sqrt{\beta_1}(\xi+\xi_0))(\sinh(\sqrt{\beta_1}(\xi+\xi_0))+p)}{(\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p^2+1})^2}\right)\right)\right)}{\frac{\sqrt{b(\beta_1+2)}\left(\beta_2\left(-\beta_1\left(\frac{\in(\sinh(\sqrt{\beta_1})+p)}{\cosh(\sqrt{\beta_1})+\eta\sqrt{p^2+1}}+1\right)\right)\right)}{\beta_2}}+$$
$$\frac{\sqrt{a\beta_3}\left(-\beta_1\left(\frac{\in(\sinh(\sqrt{\beta_1})+p)}{\cosh(\sqrt{\beta_1}(\xi+\xi_0))+\eta\sqrt{p^2+1}}+1\right)\right)}{\beta_2\sqrt{\beta_1b+2b}},\quad \beta_1>0,\quad a>0,\quad b>0. \tag{16}$$

3.2. Applications of improved simple equation method

Suppose solution of (11) is;

$$V=A_1\Psi+\frac{A_{-1}}{\Psi}+A_0 \tag{17}$$

Put (17) into (11) along with (7) (See Fig. 3),
Case I: $c_3=0$,
Family-I

$$A_1=\frac{2\sqrt{a}c_2}{\sqrt{b(2-(4c_0c_2-c_1^2))}},\quad A_0=\frac{\sqrt{a}c_1}{\sqrt{b(2-(4c_0c_2-c_1^2))}},$$
$$A_{-1}=0,\quad \omega=\frac{\sqrt{2a}}{\sqrt{2-(4c_0c_2-c_1^2)}} \tag{18}$$

Substitute (18) in (17) (See Fig. 4),

$$V_4=\frac{\sqrt{a}\sqrt{4c_0c_2-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_0c_2-c_1^2}(\xi+\xi_0)\right)}{\sqrt{-b(4c_0c_2-c_1^2-2)}},\quad 4c_0c_2>c_1^2,$$
$$a>0,\quad b>0. \tag{19}$$

Family-II

$$A_1=0,\quad A_0=-\frac{\sqrt{a}c_1}{\sqrt{bc_1^2-4bc_0c_2+2b}},$$
$$A_{-1}=-\frac{2\sqrt{a}c_0}{\sqrt{b(c_1^2-4c_0c_2+2)}},\quad \omega=\frac{\sqrt{2}\sqrt{a}}{\sqrt{c_1^2-4c_0c_2+2}} \tag{20}$$

Put (20) in (17),

$$V_5=\frac{\sqrt{a}\left(c_0\left(c_1-\sqrt{4c_0c_2-c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_0c_2-c_1^2}(\xi+\xi_0)\right)\right)-c_1c_2\right)}{c_2\sqrt{b(c_1^2-4c_0c_2+2)}},$$
$$4c_0c_2>c_1^2,\quad a>0,\quad b>0. \tag{21}$$

Case II: $c_0=c_3=0$,

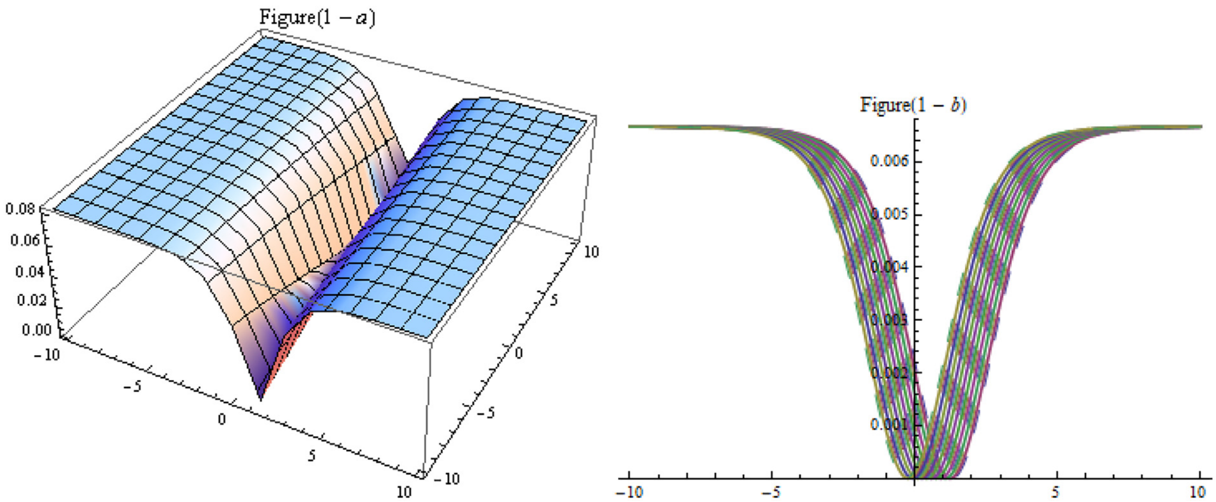

Fig. 1. Profile of solution (15) with $\beta_1 = \beta_3 = 1,\ \beta_2 = 2.0,\ \eta = 1,\ \xi_0 = -.5,\ \epsilon = 1,\ a = 0.01,\ b = 0.4.$

$$A_1 = \frac{2\sqrt{a}\,c_2}{\sqrt{b(c_1^2+2)}},\quad A_0 = \frac{\sqrt{a}\,c_1}{\sqrt{bc_1^2+2b}},\quad A_{-1} = 0,\quad \omega = \frac{\sqrt{2a}}{\sqrt{c_1^2+2}} \tag{22}$$

Put (22) in (17),

$$V_6 = \frac{(2\sqrt{a}\,c_2)(c_1\exp(c_1(\xi+\epsilon)))}{\sqrt{b(c_1^2+2)}\,(1-c_2\exp(c_1(\xi+\epsilon)))} + \frac{\sqrt{a}\,c_1}{\sqrt{bc_1^2+2b}},\quad c_1 > 0. \tag{23}$$

$$V_7 = \frac{\sqrt{a}\,c_1}{\sqrt{bc_1^2+2b}} - \frac{(2\sqrt{a}\,c_2)(c_1\exp(c_1(\xi+\epsilon)))}{\sqrt{b(c_1^2+2)}\,(c_2\exp(c_1(\xi+\epsilon))+1)},\quad c_1 < 0. \tag{24}$$

Case III: $c_1 = c_3 = 0$,
Family-I

$$A_1 = 0,\quad A_0 = 0,\quad A_{-1} = -\frac{\sqrt{2}\sqrt{a}\,c_0}{\sqrt{b-2bc_0c_2}},\quad \omega = -\frac{\sqrt{a}}{\sqrt{1-2c_0c_2}} \tag{25}$$

Put (25) in (17),

$$V_8 = \frac{\sqrt{2}\sqrt{a}\sqrt{c_0c_2}\cot(\sqrt{c_0c_2}(\xi+\xi_0))}{\sqrt{b-2bc_0c_2}},\quad c_2c_0 > 0. \tag{26}$$

$$V_9 = -\frac{\sqrt{2}\sqrt{a}\sqrt{-c_0c_2}\coth(\sqrt{-c_0c_2}(\xi+\xi_0))}{\sqrt{b-2bc_0c_2}},\quad c_2c_0 < 0. \tag{27}$$

Family-II

$$A_1 = -\frac{\sqrt{2}\sqrt{a}\,c_2}{\sqrt{b-2bc_0c_2}},\quad A_0 = 0,\quad A_{-1} = 0,\quad \omega = -\frac{\sqrt{a}}{\sqrt{1-2c_0c_2}} \tag{28}$$

Put (27) in (17),

$$V_{10} = -\frac{\sqrt{2}\sqrt{a}\sqrt{c_0c_2}\tan(\sqrt{c_0c_2}(\xi+\xi_0))}{\sqrt{b-2bc_0c_2}},\quad c_2c_0 > 0. \tag{29}$$

$$V_{11} = \frac{\sqrt{2}\sqrt{a}\sqrt{-c_0c_2}\tanh(\sqrt{-c_0c_2}(\xi+\xi_0))}{\sqrt{b-2bc_0c_2}},\quad c_2c_0 < 0. \tag{30}$$

Family-III

$$A_1 = -\frac{\sqrt{2}\sqrt{a}\,c_2}{\sqrt{4bc_0c_2+b}},\quad A_0 = 0,\quad A_{-1} = -\frac{\sqrt{2}\sqrt{a}\,c_0}{\sqrt{4bc_0c_2+b}},$$
$$\omega = -\frac{\sqrt{a}}{\sqrt{4c_0c_2+1}} \tag{31}$$

Put (31) in (17),

$$V_{12} = -\frac{2\sqrt{2}\sqrt{a}\sqrt{c_0c_2}\csc(2\sqrt{c_0c_2}(\xi+\xi_0))}{\sqrt{4bc_0c_2+b}},\quad c_2c_0 > 0. \tag{32}$$

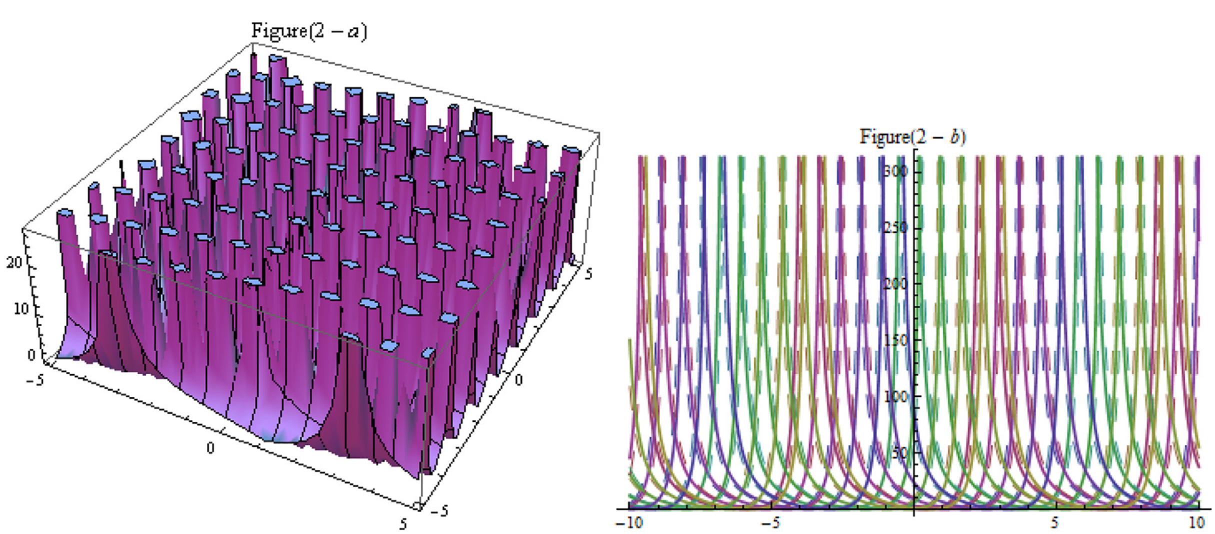

Fig. 2. Profile of solution (21) with $a = 12,\ b = 5,\ c_1 = 1,\ c_2 = 0.5,\ c_0 = 1,\ \xi_0 = 0.5.$

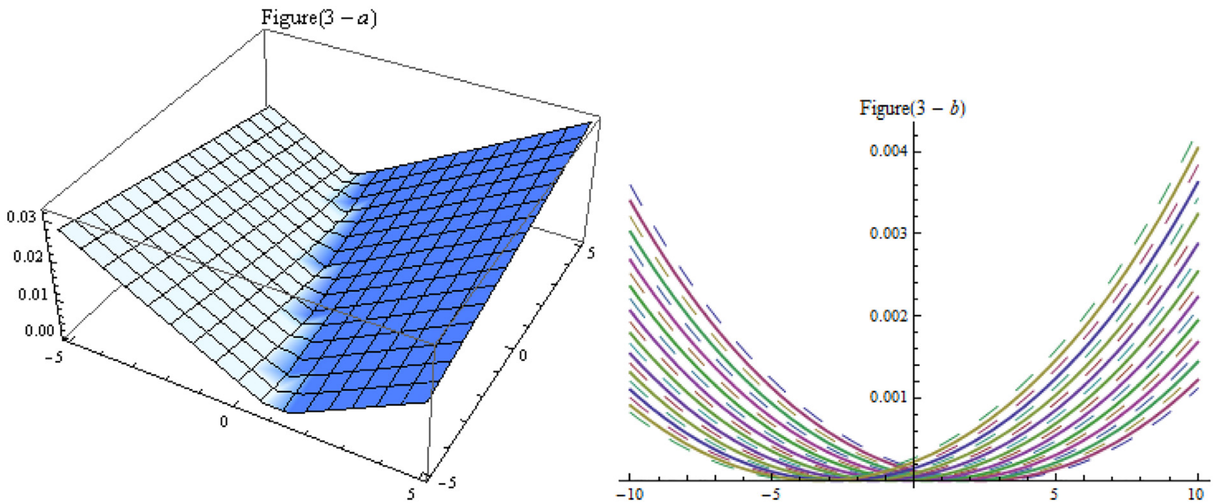

Fig. 3. Profile of solution (29) with $a = 0.1$, $b = 0.01$, $c_2 = 0.1$, $c_0 = 0.01$, $\xi_0 = 0.5$.

$$V_{13} = -\frac{2\sqrt{2}\sqrt{a}\sqrt{-c_0c_2}\,\text{csch}(2\sqrt{-c_0c_2}(\xi+\xi_0))}{\sqrt{4bc_0c_2+b}}, \quad c_2c_0 < 0. \tag{33}$$

3.3. Applications of modified F-expansion method

Let solution of (11) is;

$$V = a_0 + a_1F + \frac{b_1}{F} \tag{34}$$

Put (31) in (11),
For $A = 0$, $B = 1$, $C = -1$, we have,

$$a_1 = -\frac{\sqrt{2a}}{\sqrt{3b}}, \quad a_0 = \frac{\sqrt{a}}{\sqrt{3b}}, \quad b_1 = 0, \quad \omega = \sqrt{\frac{2a}{3}} \tag{35}$$

Put (35) in (34),

$$V_{14} = \frac{\sqrt{a}}{\sqrt{3b}} - \frac{\sqrt{2a}}{2\sqrt{3b}}\left(1+\tanh\left(\frac{1}{2}\xi\right)\right) \tag{36}$$

$A = 0$, $C = 1$, $B = -1$,

$$a_1 = -\frac{2\sqrt{a}}{\sqrt{3}\sqrt{b}}, \quad a_0 = \frac{\sqrt{a}}{\sqrt{3}\sqrt{b}}, \quad b_1 = 0, \quad \omega = \sqrt{\frac{2a}{3}} \tag{37}$$

Put (37) into (34),

$$V_{15} = \frac{\sqrt{a}}{\sqrt{3b}} - \frac{\sqrt{a}}{\sqrt{3b}}\left(1-\coth\left(\frac{1}{2}\xi\right)\right) \tag{38}$$

For $A = \frac{1}{2}$, $B = 0$, $C = -\frac{1}{2}$,
Family-I

$$a_1 = 0, \quad a_0 = 0, \quad b_1 = -\frac{\sqrt{a}}{\sqrt{3}\sqrt{b}}, \quad \omega = \sqrt{\frac{2a}{3}} \tag{39}$$

Put (39) in (34),

$$V_{16} = \left(\frac{-\sqrt{a}}{\sqrt{3}\sqrt{b}(\coth(\xi) \pm csch(\xi))}\right) \tag{40}$$

Family-II

$$a_1 = \frac{\sqrt{a}}{\sqrt{3}\sqrt{b}}, \quad a_0 = 0, \quad b_1 = 0, \quad \omega = -\sqrt{\frac{2a}{3}} \tag{41}$$

Put (41) in (34),

$$V_{17} = \left(-\frac{\sqrt{a}(\pm csch(\xi) + \coth(\xi))}{\sqrt{3}\sqrt{b}}\right), \quad a > 0, \quad b > 0. \tag{42}$$

Family-III

$$a_1 = \frac{\sqrt{a}}{\sqrt{6}\sqrt{b}}, \quad a_0 = 0, \quad b_1 = \frac{\sqrt{a}}{\sqrt{6}\sqrt{b}}, \quad \omega = -\frac{\sqrt{a}}{\sqrt{3}} \tag{43}$$

Put (43) in (34),

$$V_{18} = \frac{\sqrt{a}}{\sqrt{6}\sqrt{b}}\left(\frac{1}{(\coth + csch)} + (csch + \coth)\right) \tag{44}$$

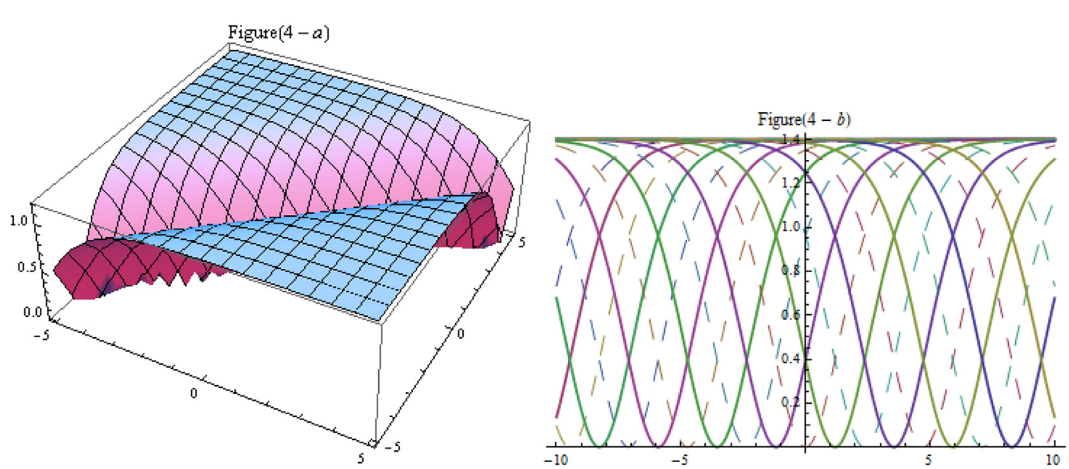

Fig. 4. Profile of solution (40) with $a = 2.1$, $b = 0.5$, $\omega = 1.18$.

For $C=-1,\ B=0,\ A=1$,
Family-I

$$a_1=0,\quad a_0=0,\quad b_1=\frac{\sqrt{\frac{2a}{3}}}{\sqrt{b}},\quad \omega=-\frac{\sqrt{a}}{\sqrt{3}} \tag{45}$$

Put (45) in (34),

$$V_{19}=\frac{\sqrt{\frac{2a}{3}}}{\sqrt{b}}\left(\frac{1}{\tanh(\xi)}\right),\quad a>0,\quad b>0. \tag{46}$$

Family-II

$$a_1=-\frac{\sqrt{\frac{2}{3}}\sqrt{a}}{\sqrt{b}},\quad a_0=0,\quad b_1=0,\quad \omega=\frac{\sqrt{a}}{\sqrt{3}} \tag{47}$$

Put (47) in (34),

$$V_{20}=-\frac{\sqrt{\frac{2}{3}}\sqrt{a}}{\sqrt{b}}(\tanh(\xi)),\quad a>0,\quad b>0. \tag{48}$$

Family-III

$$a_1=\frac{\sqrt{2}\sqrt{a}}{3\sqrt{b}},\quad a_0=0,\quad b_1=\frac{\sqrt{2}\sqrt{a}}{3\sqrt{b}},\quad \omega=\frac{\sqrt{a}}{3} \tag{49}$$

Put (49) in (34),

$$V_{21}=\frac{\sqrt{2}\sqrt{a}}{3\sqrt{b}}\left(\tanh(\xi)+\frac{1}{\tanh(\xi)}\right),\quad a>0,\quad b>0. \tag{50}$$

For $A=\frac{1}{2},\ C=\frac{1}{2},\ B=0$,
Family-I

$$a_1=0,\ a_0=0,\ b_1=-\frac{\sqrt{a}}{\sqrt{b}},\ \omega=-\sqrt{2a} \tag{51}$$

Put (51) in (34),

$$V_{22}=-\frac{\sqrt{a}}{\sqrt{b}}\left(\frac{1}{\tan(\xi)+\sec(\xi)}\right),\quad a>0,\quad b>0. \tag{52}$$

Family-II

$$a_1=-\frac{\sqrt{a}}{\sqrt{b}},\quad a_0=0,\quad b_1=0,\quad \omega=-\sqrt{2a} \tag{53}$$

Put (53) in (34),

$$V_{23}=-\frac{\sqrt{a}}{\sqrt{b}}(\tan(\xi)+\sec(\xi)),\quad a>0,\quad b>0. \tag{54}$$

Family-III

$$a_1=-\frac{\sqrt{a}}{2\sqrt{b}},\quad a_0=0,\quad b_1=-\frac{\sqrt{a}}{2\sqrt{b}},\quad \omega=-\frac{\sqrt{a}}{\sqrt{2}} \tag{55}$$

Put (55) in (34),

$$V_{24}=-\frac{\sqrt{a}}{2\sqrt{b}}\left(\frac{1}{\sec(\xi_1)(\tan(\xi_1)+}+(\tan(\xi_1)+\sec(\xi_1)\right) \tag{56}$$

$A=C=-\frac{1}{2},\ B=0$
Family-I

$$a_1=0,\quad a_0=0,\quad b_1=\frac{\sqrt{a}}{\sqrt{b}},\quad \omega=-\sqrt{2}\sqrt{a} \tag{57}$$

Put (57) in (34),

$$V_{25}=\frac{\sqrt{a}}{\sqrt{b}}\left(\frac{1}{-\tan(\xi)+\sec(\xi)}\right),\quad a>0,\quad b>0. \tag{58}$$

Family-II

$$a_1=\frac{\sqrt{a}}{\sqrt{b}},\quad a_0=0,\quad b_1=0,\quad \omega=-\sqrt{2}\sqrt{a} \tag{59}$$

Put (59) in (34),

$$V_{26}=\frac{\sqrt{a}}{\sqrt{b}}(-\tan(\xi)+\sec(\xi)) \tag{60}$$

Family-III

$$a_1=b_1=\frac{\sqrt{a}}{2\sqrt{b}},\quad a_0=0,\quad \omega=\sqrt{\frac{+a}{2}} \tag{61}$$

Put (61) in (34),

$$V_{27}=\frac{\sqrt{a}}{2\sqrt{b}}\left((\tan(\xi_1)+\sec(\xi_1))+\frac{1}{\tan(\xi_1)+\sec(\xi_1)}\right) \tag{62}$$

For $C=-1,\ B=0,\ A=-1$,

$$\omega=\frac{\sqrt{a}}{\sqrt{5}},\quad a_1=-\frac{\sqrt{\frac{2}{5}}\sqrt{a}}{\sqrt{b}},\quad a_0=0,\quad b_1=-\frac{\sqrt{\frac{2}{5}}\sqrt{a}}{\sqrt{b}} \tag{63}$$

Put (63) in (34),

$$V_{28}=-\frac{\sqrt{\frac{2}{5}}\sqrt{a}}{\sqrt{b}}\left(\tan(\xi)+\frac{1}{\tan(\xi)}\right) \tag{64}$$

For $A=0,\ B=0$,

$$\omega=\sqrt{a},\quad a_1=\frac{\sqrt{2a}\,C}{\sqrt{b}},\quad a_0=0,\quad b_1=0 \tag{65}$$

Put (65) in (34),

$$V_{29}=\frac{\sqrt{2a}\,C}{\sqrt{b}}\left(\frac{-1}{C\xi+\eta}\right),\quad a>0,\quad b>0. \tag{66}$$

For $C=0,\ B=0$,

$$\omega=\sqrt{a},\quad a_1=0,\quad a_0=0,\quad b_1=\frac{\sqrt{2}\sqrt{a}A}{\sqrt{b}} \tag{67}$$

Put (66) in (34),

$$V_{30}=\frac{\sqrt{2}\sqrt{a}A}{\sqrt{b}}\left(\frac{1}{A\xi}\right),\quad a>0,\quad b>0. \tag{68}$$

For $C=0$,

$$\omega=-\frac{\sqrt{2}\sqrt{a}}{\sqrt{B^2+2}},\quad a_1=0,\quad a_0=\frac{\sqrt{a}B}{\sqrt{bB^2+2b}},\quad b_1=\frac{2\sqrt{a}A}{\sqrt{b(B^2+2)}} \tag{69}$$

Put (69) in (34) (See Fig. 5),

$$V_{31}=a_0=\frac{\sqrt{a}B}{\sqrt{bB^2+2b}}+\frac{2\sqrt{a}A}{\sqrt{b(B^2+2)}}\left(\frac{B}{(\exp(B\xi)-A)}\right),\quad a>0,\quad b>0. \tag{70}$$

4. Results and discussion

The achieved analytical solutions of generalized PC equation via three novel techniques, having different form solutions as compared to results derived several researchers in [47–50]. With assigning the distinct particular values to the parameters in Eqs. (4), (6), (8) have numerous solutions. However, our derived solutions have few resemblance with previously results in distinct research literature. Solutions (66) and (68) are likely similar form of solutions in Eqs. (17) and (20) in [47]. Further our solutions (23) and (24) are approximately same form of solutions in Eqs. (18) and (21) in [48]. Solutions (23) and (24) are also likely form as solutions in Eqs. (11) and (12) in [49] gradually. Furthermore Our derived solutions in (26) and (27) are also likely form as solutions in Eqs. (34) and (35) in [50] respectively. Hence from these details deliberation about our discovered solutions and previous results in different research literature, we have concluded that our solutions

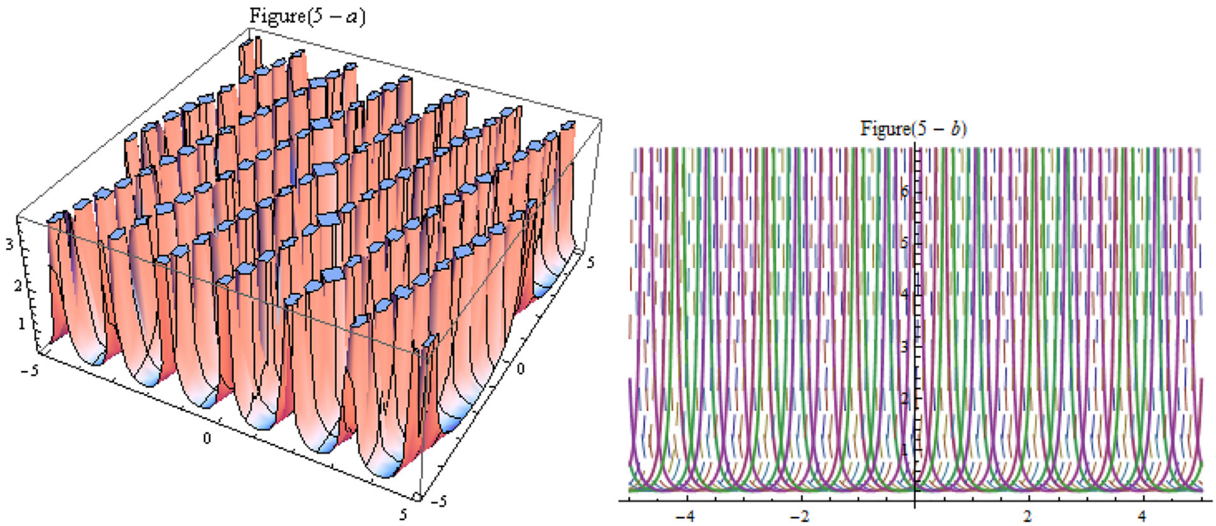

Fig. 5. Profile of solution (64) with $a = 2.1, \ b = 0.5, \ \omega = 0.28$.

are general and powerful as compared in earlier results discussed in [47–50]. From this we predict that presentation of our techniques are best mathematical tools nonlinear wave problems.

5. Conclusion

In this work, we have successfully implementation of three novel methods so called Generalized direct algebraic, Improved simple equation and modified F-expansion methods are utilized on the generalized PC equation, variety of solutions in distinct types are investigated.Our novel solutions of the mention equation are fruitful in mathematical physics.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Ehab Selima S, Seadawy Aly R, Yao Xiaohua. The nonlinear dispersive Davey-Stewartson system for surface waves propagation in shallow water and its stability. Eur Phys J Plus 2016;131:425.
[2] Kumar D, Hosseini Kamyar, Samadani F. Optik 2017;149:439–46.
[3] Hosseini Kamyar, Bekir Ahmet, Kaplan Melike. New exact traveling wave solutions of the Tzitzica-type evolution equations arising in non-linear optics. J Modern Opt 2017;64:1688–92.
[4] Hosseini Kamyar, Mayeli P, Kumar D. New exact solutions of the coupled sine-Gordon equations in nonlinear optics using the modified Kudryashov method. J Modern Opt 2018;65:361–4.
[5] Aly Seadawy, El-Rashidy K. Dispersive Solitary wave solutions of Kadomtsev-Petviashivili and modified Kadomtsev-Petviashivili dynamical equations in unmagnetized dust plasma. Results Phys. 2018;8:1216–22.
[6] Khater AH, Callebaut DK, Seadawy AR. General soliton solutions for nonlinear dispersive waves in convective type instabilities. Physica Scr 2006;74:384–93.
[7] Khater AH, Callebaut DK, Helal MA, Seadawy AR. Variational method for the nonlinear dynamics of an elliptic magnetic stagnation line. Eur Phys J D 2006;39:237–45.
[8] Seadawy AR. New exact solutions for the KdV equation with higher order nonlinearity by using the variational method. Comput Math Appl 2011;62:3741–55.
[9] Seadawy AR. Stability analysis solutions for nonlinear three-dimensional modified Korteweg-de Vries-Zakharov-Kuznetsov equation in a magnetized electron-positron plasma. Physica A 2016;455:44–51.
[10] Seadawy AR. Nonlinear wave solutions of the three-dimensional Zakharov-Kuznetsov-Burgers equation in dusty plasma. Physica A 2015;439:124–31.
[11] Wang M, Zhou Y, Li Z. Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics. Phys Lett A 1996;216(1–5):67–75.
[12] Khater AH, Callebaut DK, Seadawy AR. General soliton solutions for nonlinear dispersive waves in convective type instabilities. Physica Scr 2006;74:384–93.
[13] Ali A, Seadawy AR, Ali D. New solitary wave solutions of some nonlinear models and their applications. Adv Difference Eqs 2018;2018:232.
[14] Fan E. Extended tanh-function method and its applications to nonlinear equations. Phys Lett A 2000;277(4):212–8.
[15] Liu S, Fu Z, Liu S, Zhao Q. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations. Phys Lett A 2001;289(1):69–74.
[16] Ma WX, Lee JH. A transformed rational function method and exact solutions to the (3+1) dimensional Jimbo-Miwa equation. Chaos Solitons Fractals 2009;42(3):1356–63.
[17] Chen Y, Yan Z. The Weierstrass elliptic function expansion method and its applications in nonlinear wave equations. Chaos Solitons Fractals 2006;29(4):948–64.
[18] Wazwaz AM. Multiple-soliton solutions for the KP equation by Hirota's bilinear method and by the tanh-coth method. Appl Math Comput 2007;190(1):633–40.
[19] Liu CS. Trial equation method to nonlinear evolution equations with rank inhomogeneous: mathematical discussions and its applications. Commun Theor Phys 2006;45:219–23.
[20] Kudryashov NA, Soukharev MB. Popular Ansatz methods and Solitary wave solutions of the Kuramoto-Sivashinsky equation. Regul Chaotic Dyn 2009;14(3):407–19.
[21] Akbulut A, Kaplan M. Auxiliary equation method for timefractional differential equations with conformable derivative. Comput Math Appl 2017. https://doi.org/10.1016/j.camwa.2017.10.016.
[22] Seadawy Aly R, Ali Asghar, Albarakati Wafaa A. Analytical wave solutions of the (2 + 1)-dimensional first integro-differential Kadomtsev-Petviashivili hierarchy equation by using modified mathematical methods. Results Phys 2019;15:102775.
[23] Kumar D, Seadawy AR, Joardar AK. Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology. Chin J Phys 2018;56(1):75–85.
[24] Xie F, Yan Z, Zhang H. Explicit and exact traveling wave solutions of Whitham-Broer-Kaup shallow water equations. Phys Lett A 2001;285:76–80.
[25] Seadawy AR, Ali A, Lu D. Structure of system solutions of ion sound and Langmuir dynamical models and their applications. Pramana J Phys 2019;92:88.
[26] Lu D, Seadawy AR, Ali A. Structure of traveling wave solutions for some nonlinear models via modified mathematical method. Open Phys 2018;16:854–60.
[27] Seadawy Aly R, Alamri Sultan Z. Mathematical methods via the nonlinear two-dimensional water waves of Olver dynamical equation and its exact solitary wave solutions. Results Phys 2018;8:286–91.
[28] Qian Z, Lihua W, Lin F. Darboux transformation and explicit solutions to the generalized TD equation. Appl Math 2017;67:1–6.
[29] Seadawy AR, Lu Dianchen, Yue Chen. Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability. J Taibah Univ Sci 2017;11:623–33.
[30] Yuanfen X. Bifurcations of the exact traveling solutions for (2 + 1)-dimensional HMIS equation. Commun Theor Phys 2012;57:68–70.
[31] Helal MA, Seadawy AR. Variational method for the derivative nonlinear Schrödinger equation with computational applications. Physica Scr 2009;80:350–60.
[32] Seadawy Abdullah AR, Jun W. Three-dimensional nonlinear extended Zakharov-Kuznetsov dynamical equation in a magnetized dusty plasma via acoustic solitary wave solutions. Braz. J Phys 2018;14:9.
[33] Jumarie G. Modified Riemann-Liouville derivative and fractional Taylor series of differentiable functions further results. Comput Math Appl 2006;51(10):1367–76.
[34] Ul-Haq Tariq Kalim, Aly Seadawy. Bistable Bright-Dark solitary wave solutions of the (3+1)-Dimensional Breaking Soliton, Boussinesq equation with dual dispersion and modified Korteweg-de Vries-Kadomtsev-Petviashvili equations and their applications. Results Phys 2017;7:1143–9.
[35] Liu W, Chen K. The functional variable method for finding exact solutions of some nonlinear time-fractional differential equations. Pramana J Phys

2013;81(3):377–84.
[36] Kutluay S, Esen A. A lumped Galerkin method for solving the Burgers equation. Int. J Comput Math 2004;81:1433–44.
[37] Khater AH, Callebaut DK, Helal MA, Seadawy AR. Variational method for the nonlinear dynamics of an elliptic magnetic stagnation line. Eur Phys J D 2006;39:237–45.
[38] Seadawy AR. Approximation solutions of derivative nonlinear Schrodinger equation with computational applications by variational method. Eur Phys J Plus 2015;130(182):1–10.
[39] Guan Xue, Liu Wenjun, Zhou Qin, Biswas Anjan. Some lump solutions for a generalized (3 + 1)-dimensional Kadomtsev?Petviashvili equation. Appl Math Comput 2020;366:124757.
[40] Fan Xiaoyu, Tianqi Qu, Huang Shucheng, Chen Xiaoxiao, Cao Menghua, Zhou Qin, Liu Wenjun. Analytic study on the influences of higher-order effects on optical solitons in fiber laser. Optik 2019;186:326–31.
[41] Liu Suzhi, Zhou Qin, Biswas Anjan, Liu Wenjun. Phase-shift controlling of three solitons in dispersion-decreasing fibers. Nonlinear Dyn 2019;98(1):395–401.
[42] Yan Yuanyuan, Liu Wenjun. Stable transmission of solitons in the complex cubic?quintic Ginzburg-Landau equation with nonlinear gain and higher-order effects. Appl Math Lett 2019;98:171–6.
[43] Liu Wenjun, Zhang Yujia, Wazwaz Abdul Majid, Qin Zhou. Analytic study on triple-S, triple-triangle structure interactions for solitons in inhomogeneous multi-mode fiber. Appl Math Comput 2019;361:325–31.
[44] El-Wakil SA, Abdou MA. New explicit and exact traveling wave solutions for two nonlinear evolution equations. Nonlinear Dyn 2008;51:585–94.
[45] Parand K, Jamil Amani AR. solitary wave solutions of generalized Pochhammer-Chree equation via Exp-function. Int J Math Comput Phys Electr Comput Eng 2010;7(4).
[46] Kamran A, Yaqub Ayub MK, Attia R, Hassan QM, Bilal A, Shakeel M. Application of the Exp (-phi (xi))-expansion method for solitary wave solutions. Arab J Basic Appl Sci Univ Bahrain 2019;1(26):376–84.
[47] Biswas A, Krishnan EV. Exact solution for ostrovsky equation. Indian J Phys 2011;10(85):1513–21.
[48] Gepreel KA, Taher A, Nofal, Ameara A. Exact solutions for nonlinear integro-partial differential equations using the generalized Kudryashov method. J Egypt Math Soc 2017;25:438–44.
[49] Jafari H, Kadkhodaa N, Khalique CM. Travelling wave solutions of nonlinear evolution equations using the simplest equation method. Comput Math Appl 2012;64:2084–8.
[50] Mohammed O. Al-Amr, Exact solutions of the generalized (2 + 1)-dimensional nonlinear evolution equations via the modified simple equation method. Comput Math Appl 2015;69:390–7.