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Competent closed form soliton solutions to the Riemann wave equation and the Novikov-Veselov equation

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ABSTRACT

The Riemann wave equation and the Novikov-Veselov equation are interesting nonlinear equations in the sphere of tidal and tsunami waves in ocean, river, ion and magneto-sound waves in plasmas, electromagnetic waves in transmission lines, homogeneous and stationary media etc. In this article, the generalized Kudryashov method is executed to demonstrate the applicability and effectiveness to extract travelling and solitary wave solutions of higher order nonlinear evolution equations (NLEEs) via the earlier stated equations. The technique is enucleated to extract solitary wave solutions in terms of trigonometric, hyperbolic and exponential function. We acquire bell shape soliton, consolidated bell shape soliton, compacton, singular kink soliton, flat kink shape soliton, smooth singular soliton and other types of soliton solutions by setting particular values of the embodied parameters. For the precision of the result, the solutions are graphically illustrated in 3D and 2D. The analytic solutions greatly facilitate the verification of numerical solvers on the stability analysis of the solution.

Introduction

The nonlinear evolution equations (NLEEs) are the special form of the partial differential equations (PDEs). These equations are widely used as models to describe the physical significance in various branches of mathematical and physical sciences, especially in applied and pure mathematics, physics, chemistry, biology, biochemistry and many other subjects [1]. Therefore, in order to comprehend the qualitative features of these phenomena properly analytic solutions of NLEEs play a significant role. Analytic solutions of nonlinear wave equations graphically demonstrate and allow unscrambling the mechanisms of many intricate phenomena. Since the NLLEs describe many physical and mathematical incidents, the analytic solutions of such equations are of fundamental importance [2,3]. Particularly, the soliton solutions of the NLEEs play a significant role in the dynamic of pulse propagation through optical fibers for transcontinental trans-oceanic distance.

Therefore, searching the travelling wave solutions is becoming attractive research area in nonlinear science. However, not all equations posed of these models are solvable. Consequently, many new techniques have been developed by mathematicians, engineers and physicists, like as the Sine-Gordon expansion method [4], the solitary wave ansatz method [5–7], the homogeneous balance method [8], the finite difference method [9], the tanh function method [10], the generalized Kudryashov method [11], the modified simple equation method [12], the dual mode Burgers equation [13], the F-expansion method [14,15], the rational exp-function method [16], Caputo fractional partial derivatives [17], the Durboux transformation method [18], the improved Kudryashov method [19], the first integral method [20], shehu transform [21], the (G'/G)-expansion method [22], the dual-mode Schrodinger equation [23], the tanh-method [24], the $\exp(-\varphi(\xi))$-expansion method [25], the exp-function method [26–28], local fractional homotopy analysis method [29], the trial equation method [30], the improved F-expansion method [31] etc.

The generalized Kudryashov method is an important and powerful method to accomplish analytic solutions to the NLEEs. To the best of our understanding, the Riemann wave equation and the NV equation yet have not been investigated by the generalized Kudryashov method. Therefore, in this study, we put in use the generalized Kudryashov method [32–37] to construct the soliton solutions to the Riemann wave equation and the Novikov-Veselov equation. Through implementing the aforesaid method, we found scores of solitary wave solutions. Analytic solutions permit researchers to plan and carry on experiments by

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building suitable environment to find out the parameters or functions of the NLEEs. Recently, many authors find exact solutions; general solutions; lumps solutions and interaction solutions by transformed rational function method, generalized Kudryashov method applied and extended modified direct algebraic method, extended mapping method and Seadawy techniques to find solutions for some nonlinear PDEs [38–45]. Finally, there are many methods to find solutions to nonlinear differential equations, see [46–55].

Methodology

The generalized Kudryashov method is an influential technique to constitute the analytic solutions to the NLEEs. Using this method, we examine the travelling wave solutions as a general case. We will analyze the travelling wave solutions systematically and graphically by making use of the generalized Kudryashov method.

We presume a NLEE associated with a function $v = v(x, t)$as follows

$$G\left(v, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2}, \frac{\partial^2 v}{\partial x \partial t}, \cdots\right) = 0 \tag{1}$$

wherein $v = v(x, t)$ is not a known function, G is a polynomial of the variable v and its derivatives and $\frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}\cdots$ be the partial derivatives with respect to t, x respectively in which highest order linear term and nonlinear term are engaged.

The generalized Kudryashov method arise some steps as follows:

First step

At the beginning, we assume the wave variable

$$v(x, t) = v(\xi), \xi = x + y - ct \tag{2}$$

where into μ is the wave number and c is the travelling wave velocity. When we put in use the above wave variable, the Eq. (1) is transformed into the subsequent ordinary differential equation (ODE),

$$P\left(v, \frac{dv}{d\xi}, \frac{d^2 v}{d\xi^2}, \cdots\right) = 0. \tag{3}$$

Second step

According to the generalized Kudryashov method, we look for the analytic solution of the Eq. (3) in the following form

$$v(\xi) = \frac{\sum_{r=0}^{N} a_r Q^r(\xi)}{\sum_{s=0}^{M} b_s Q^s(\xi)} \tag{4}$$

wherein $a_r(r = 0, 1, 2, \cdots N)$ and $b_s(s = 0, 1, 2, \cdots M)$ are unknown constants to be investigated such that $a_N \neq 0$ and $b_M \neq 0$.

The engaged entity $Q(\xi)$ satisfies the following ODE

$$\frac{dQ(\xi)}{d\xi} = Q^2(\xi) - Q(\xi) \tag{5}$$

The solution of Eq. (5) can be written as

$$Q(\xi) = \frac{1}{1 + De^{\xi}} \tag{6}$$

where D is a constant of integration.

Third step

Compute the positive integer numbers N and M present in the Eq. (4) by using the homogeneous balance between the highest order linear term and nonlinear term occurring in the Eq. (3).

Fourth step

Embedding the solution (4) into Eq. (3) and taking the assistance of the Eq. (5), we receive a polynomial in $Q(\xi)$. Locating all the terms of the same power and then equalizing the coefficient of $Q(\xi)$ to be zero, we accomplish a system of algebraic equations for $a_r(r = 0, 1, 2, \cdots N)$, $b_s(s = 0, 1, 2, \cdots M)$, c and μ.

We can solve these algebraic equations with the help of Mathematica software to compute the values of the unknown constants $a_r(r = 0, 1, 2, \cdots N)$, $b_s(s = 0, 1, 2, \cdots M)$, c and μ. Having investigated these unknown constants, the desired solutions of the NLEE (1) can be found.

Determination of solutions

In this section, we implement the generalized Kudryashov method for finding the broad-ranging solitary wave solutions to the Riemann wave equation and the Novikov-Veselov (NV) equation.

The Riemann wave equation

Suppose that the Riemann wave equation [35] is in the following form

$$\frac{\partial v}{\partial t} + l\frac{\partial^3 v}{\partial x^2 \partial y} + mv\frac{\partial w}{\partial x} + nw\frac{\partial u}{\partial x} = 0 \tag{7}$$

$$\frac{\partial v}{\partial y} = \frac{\partial w}{\partial x} \tag{8}$$

Consider the following wave variable

$$v(x, y, t) = v(\xi), \xi = \mu x + \alpha y - ct \tag{9}$$

wherein c is the speed of the travelling wave which is determined later, μ and α be the wave numbers. Implementing the wave variable (9), the Riemann wave Eqs. (7) and (8) transformed into the following ODEs

$$\alpha l\mu^2\frac{d^3 v}{d\xi^3} + mv\mu\frac{dw}{d\xi} + nw\frac{dv}{d\xi} = 0. \tag{10}$$

$$\alpha\frac{dv}{d\xi} = \mu\frac{dw}{d\xi}. \tag{11}$$

Integrating the Eq. (11) and omitting the constant of integration, we attain

$$w = \frac{\alpha}{\mu}v \tag{12}$$

Eliminating w and $\frac{dw}{d\xi}$ from the Eq. (10), we secure the following ODE

$$\alpha l\mu^2\frac{d^3 v(\xi)}{d\xi^3} + \alpha(m + n)v\frac{dv}{d\xi} - 2cv(\xi) = 0 \tag{13}$$

Again, integrating once and omitting the constant of integration, we acquire

$$2\alpha l\mu^2\frac{d^2 v(\xi)}{d\xi^2} + \alpha(m + n)v^2 - 2cv(\xi) = 0 \tag{14}$$

Taking the homogeneous balance between the highest order linear term and nonlinear term into the Eq. (14), we obtain

$$N = M + 2 \tag{15}$$

If we set $M = 1$, this yields $N = 3$.

Then the solution (4) can be written as

$$v(\xi) = \frac{a_0 + a_1 Q(\xi) + a_2 Q^2(\xi) + a_3 Q^3(\xi)}{b_0 + b_1 Q(\xi)} \tag{16}$$

wherein a_0, a_1, a_2, a_3, b_0 and b_1 are the arbitrary constants to be calculated.

Inserting the Eq. (16) into the Eq. (14) and using the Eq. (5), we obtain a polynomial in $Q(\xi)$. Equalizing the coefficient of the same power of $Q(\xi)$, we accomplish a system of algebraic equations as follows:

$$\alpha m a_0^2 b_0 + \alpha n a_0^2 b_0 + 2c a_0 b_0^2 = 0 \tag{17}$$

$$2\alpha l\mu^2 a_1 b_0^2 + 2\alpha n a_0 a_1 b_0 + 2\alpha m a_0 a_1 b_0 - 2c a_1 b_0^2 - 2\alpha l\mu^2 b_1 a_0 b_0 + \alpha m a_0^2 b_1 + \alpha n a_0^2 b_1 - 4c b_1 a_0 b_0 = 0 \tag{18}$$

$$8\alpha l\mu^2 a_2 b_0^2 - 6\alpha l\mu^2 a_1 b_0^2 + 2\alpha l\mu^2 a_0 b_1^2 + 2\alpha m a_0 a_1 b_1 + 2\alpha m a_0 a_2 b_0 + 2\alpha n a_0 a_1 b_1 + 2\alpha n a_0 a_2 b_0 - 2c a_0 b_1^2 - 2c a_2 b_0^2 + 6\alpha l\mu^2 a_0 b_0 b_1 - 2\alpha l\mu^2 a_1 b_0 b_1 + \alpha m a_1^2 b_0 + \alpha n a_1^2 b_0 - 4c a_1 b_0 b_1 = 0. \tag{19}$$

$$18\alpha l\mu^2 a_3 b_0^2 - 2\alpha l\mu^2 a_0 b_1^2 - 20\alpha l\mu^2 a_2 b_0^2 + 2\alpha l\mu^2 a_1 b_0^2 + 2\alpha m a_0 a_2 b_1 + 2\alpha m a_0 a_3 b_0 + 2\alpha m a_1 a_2 b_0 + 2\alpha n a_0 a_2 b_1 + 2\alpha n a_0 a_3 b_0 + 2\alpha n a_1 a_2 b_0 - 2c a_1 b_1^2 - 2c a_3 b_0^2 + 2\alpha l\mu^2 a_1 b_0 b_1 + 6\alpha l\mu^2 a_2 b_0 b_1 - 4\alpha l\mu^2 a_0 b_0 b_1 + \alpha m a_1^2 b_1 + \alpha n a_1^2 b_1 - 4c a_3 b_0 b_1 = 0. \tag{20}$$

$$2\alpha l\mu^2 a_2 b_1^2 - 42\alpha l\mu^2 a_3 b_0^2 + 12\alpha l\mu^2 a_2 b_0^2 + 2\alpha m a_0 a_3 b_1 + 2\alpha m a_1 a_2 + 2\alpha m a_1 a_3 b_0 + 2\alpha n a_0 a_3 b_1 + 2\alpha n a_1 a_2 b_1 + 2\alpha n a_1 a_3 b_0 - 2c a_0 b_0^2 - 18\alpha l\mu^2 a_2 b_0 b_1 + 22\alpha l\mu^2 a_3 b_0 b_1 + \alpha m a_2^2 b_0 + \alpha n a_2^2 b_0 - 4c a_3 b_0 b_1 = 0 \tag{21}$$

$$8\alpha l\mu^2 a_3 b_1^2 - 6\alpha l\mu^2 a_2 b_1^2 + 24\alpha l\mu^2 a_3 b_0^2 + 2\alpha m a_1 a_3 b_1 + 2\alpha m a_2 a_3 b_0 + 2\alpha n a_1 a_3 b_1 + 2\alpha n a_2 a_3 b_0 - 2c a_3 b_1^2 - 54\alpha l\mu^2 a_3 b_0 b_1 + 2\alpha l\mu^2 a_2 b_0 b_1 + \alpha m a_2^2 b_1 + \alpha n a_2^2 b_1 = 0 \tag{22}$$

$$4\alpha l\mu^2 a_2 b_1^2 - 20\alpha l\mu^2 a_3 b_1^2 + 2\alpha m a_2 a_3 b_1 + 2\alpha n a_2 a_3 b_1 + 32\alpha l\mu^2 a_3 b_0 b_1 + \alpha m a_3^2 b_0 + \alpha n a_3^2 b_0 = 0 \tag{23}$$

$$12\alpha l\mu^2 a_3 b_1^2 + \alpha m a_3^2 b_1 + \alpha n a_3^2 b_1 = 0 \tag{24}$$

Solving all these algebraic equations, we attain the solutions as follows:

Set-1:

$$c = \alpha l\mu^2,\ a_0 = 0,\ a_1 = 0,\ a_2 = -a_3,\ a_3 = a_3,\ b_0 = 0,\ b_1 = -\frac{1}{12}\frac{a_3(m+n)}{l\mu^2}$$

Set-2:

$$c = -\alpha l\mu^2,\ a_0 = 0,\ a_1 = -\frac{1}{6}a_2,\ a_2 = a_2,\ a_3 = -a_2,\ b_0 = 0,\ b_1 = \frac{1}{12}\frac{a_2(m+n)}{l\mu^2}$$

Set-3:

$$c = -\alpha l\mu^2,\ a_0 = a_0,\ a_1 = \frac{-2(3a_0 m + 3a_0 n + l\mu^2 b_1)}{m+n},\ a_2 = \frac{6(a_0 m + a_0 n + 2l\mu^2 b_1)}{m+n},\ b_1 = b_1$$

$$a_3 = \frac{12l\mu^2 b_1}{m+n},\ b_0 = -\frac{1}{2}\frac{a_0(m+n)}{l\mu^2}$$

Set-4:

$$c = \alpha l\mu^2,\ a_0 = 0,\ a_1 = a_1,\ a_2 = \frac{12l\mu^2 b_1 - m a_1 - n a_1}{m+n},\ a_3 = -\frac{12l\mu^2 b_1}{m+n},\ b_0 = \frac{1}{12}\frac{a_1(m+n)}{l\mu^2}$$

For evaluating the analytic solutions of the Eq. (14), we will use the values of a_0, a_1, a_2, a_3, b_0 and b_1 into the solution (16). At first, we substitute the values presented in set-1 and the value of $Q(\xi)$ into the solution (16) and obtained the subsequent solutions:

$$v(\xi) = \frac{12Dl\mu^2}{(m+n)(D^2\exp(\xi) + \exp(-\xi) + 2\text{D})} \tag{25}$$

Converting the solution (25) from exponential function into hyperbolic function, yields

$$v(\xi) = \frac{24Dl\mu^2}{(m+n)((D^2+1)\cosh(\xi) + (D^2-1)\sinh(\xi) + 4D)} \tag{26}$$

Since D is an integral constant, so we can pick the values of B arbitrarily. Therefore, if we pick $D = 1$, the solution (26) becomes

$$v(\xi) = \frac{12l\mu^2}{(m+n)(\cosh(\xi) + 2)} \tag{27}$$

Again, if we pick $D = -1$, the solution (26) becomes

$$v(\xi) = \frac{-12l\mu^2}{(m+n)(\cosh(\xi) - 2)} \tag{28}$$

From the above discussion, there are two cases arise:

Case-1:

For $v(\xi) = \frac{12l\mu^2}{(m+n)(\cosh(\xi)+2)}$, the solution (12) reduces to the following form

$$w(\xi) = \frac{12\alpha\mu l}{(m+n)(\cosh(\xi) + 2)} \tag{29}$$

Case-2:

For $v(\xi) = \frac{-12l\mu^2}{(m+n)(\cosh(\xi)-2)}$, the solution (12) reduces to the following form

$$w(\xi) = \frac{-12\alpha\mu l}{(m+n)(\cosh(\xi) - 2)} \tag{30}$$

Here, we observed that the set-1 contains the solutions (26)–(30). These solutions involve several parameters. The solution shape depends on the parameters. When we choose the values of the parameters arbitrarily, the several types of graphs are acquired.

On the contrary, placing the solution set-2 and the value of $Q(\xi)$ into the solution (16) we obtain the solution $v(\xi)$ as follows:

$$v(\xi) = \frac{2l\mu^2(D^2\exp(\xi) - \exp(-\xi) + 4\text{D})}{(m+n)(D^2\exp(\xi) + \exp(-\xi) + 2\text{D})} \tag{31}$$

Transforming the solution (32) from exponential function into hyperbolic function, yields

$$v(\xi) = \frac{2l\mu^2((D^2-1)\cosh(\xi) + (D^2+1)\sinh(\xi) + 4D)}{(m+n)((D^2+1)\cosh(\xi) + (D^2-1)\sinh(\xi) + 2D)} \tag{32}$$

Since D is an integral constant, so we can pick the values of D arbitrarily. Therefore, if we pick $D = 1$, the solution (32) becomes

$$v(\xi) = \frac{2l\mu^2(\sinh(\xi) + 2)}{(m+n)(\cosh(\xi) + 1)} \tag{33}$$

Again, if we pick $D = -1$ the solution (32) becomes

$$v(\xi) = \frac{2l\mu^2(\sinh(\xi) - 2)}{(m+n)(\cosh(\xi) - 1)} \tag{34}$$

Here also two cases arise:

Case-1:

For $v(\xi) = \frac{2l\mu^2(\sinh(\xi)+2)}{(m+n)(\cosh(\xi)+1)}$, the solution (12) is simplified as

$$w(\xi) = \frac{2\alpha\mu l(\sinh(\xi) + 2)}{(m+n)(\cosh(\xi) + 1)} \tag{35}$$

Case-2:

For $v(\xi) = \frac{2l\mu^2(\sinh(\xi)-2)}{(m+n)(\cosh(\xi)-1)}$, the solution (12) is simplified as

$$w(\xi) = \frac{2\alpha\mu l(\sinh(\xi) - 2)}{(m+n)(\cosh(\xi) - 1)} \tag{36}$$

Again, we observed that the set-2 contains the solutions (32)–(36).

These solutions also contain several parameters. Since the solution shape depends on the parameters, so when we choose the values of the parameters arbitrarily, the several types of graphs are secured. From the desired graph, we can discuss the nature of the solitary wave.

Similarly, for the other selection of the values of D, we obtain the other types of solutions. But for simplicity, the other solutions are not written here.

Alternatively, the values of the constants assembled in set-3 and set-4, provide adequate new solutions but for minimalism, the solutions are not documented here.

The Novikov-Veselov (NV) equation

We assume the family of Novikov-Veselov (NV) equations [36] with constant coefficients is in the following form

$$\frac{\partial v}{\partial t}+a\frac{\partial^3 v}{\partial x^3}+b\frac{\partial^3 v}{\partial y^3}+h\frac{\partial}{\partial x}(vw)+d\frac{\partial}{\partial y}(vz)=0 \tag{37}$$

$$\frac{\partial w}{\partial y}=\frac{\partial v}{\partial x} \tag{38}$$

$$\frac{\partial z}{\partial x}=\frac{\partial v}{\partial y} \tag{39}$$

where into a, b, h and d be the nonzero parameters and $v(x, y, t)$ be an unknown function which depends on the space coordinates $x$and$y$and the temporal variable t.

Let us choose the wave transformation

$$v(x, y, t)=v(\xi),\ \xi=x+y-ct, \tag{40}$$

wherein c is the travelling wave velocity which is to be estimated. Embedding the wave transformation (40), the wave Eqs. (38) and (39) are changed into the ODEs as bellows:

$$\frac{dw}{dy}=\frac{dv}{dx}. \tag{41}$$

$$\frac{dz}{dx}=\frac{dv}{dy}. \tag{42}$$

Integrating once and taking the integral constant, we achieve

$$w=v+k_1 \tag{43}$$

$$z=v+k_2 \tag{44}$$

where $k_1$and $k_2$are integral constants.

At first, we neglect the integral constants and then inserting the solutions (40), (43) and (44) into the Eq. (37), we attain the following ODE

$$(a+b)\frac{d^3 v}{d\xi^3}+(h+d)\frac{d}{d\xi}(v^2)-c\frac{dv}{d\xi}=0 \tag{45}$$

Again, integrating once and omitting the constant of integration, we achieve

$$(a+b)\frac{d^2 v}{d\xi^2}+(h+d)v^2-cv=0 \tag{46}$$

On account of the homogeneous balance between the highest order linear term and nonlinear term occurring in the Eq. (46), we secure

$$N=M+2, \tag{47}$$

wherein M is the free parameter.

If we set $M=1$, yields $N=3$. For $M=1$ and $N=3$, the solution shape is identical to the solution (16) and therefore the solution has not been written in this section.

Setting the solution (16) into the Eq. (46) and taking the assistance of the Eq. (5), we accomplish a polynomial in $Q(\xi)$. Equalizing the coefficient of $Q(\xi)$ to be zero, we obtain the subsequent algebraic equations:

$$h{a_0}^2b_0+d{a_0}^2b_0-c{b_0}^2a_0=0. \tag{48}$$

$$a{b_0}^2a_1+b{b_0}^2a_1+h{a_0}^2b_0+d{a_0}^2b_1-c{b_0}^2a_1+2da_0a_1b_0-aa_0b_1b_0-ba_0 b_1b_0-2ca_0b_1b_0+2ha_0a_1b_0=0. \tag{49}$$

$$4a{b_0}^2a_2+a{b_1}^2a_0-3b{b_0}^2a_1+4b{b_0}^2a_2+b{b_1}^2a_0+h{a_1}^2b_0+d{a_1}^2b_0-c{b_1}^2 a_0-c{b_0}^2a_2+2ha_0a_1b_1+2ha_0a_2b_0+2da_0a_1b_1+2da_0a_2b_0-3aa_0 b_0b_1-aa_1b_0b_1+3ba_0b_0b_1-ba_1b_0b_1-3a{b_0}^2a_1-2ca_1b_0b_1=0. \tag{50}$$

$$2b{b_0}^2a_1-10b{b_0}^2a_2-b{b_1}^2a_0+h{a_1}^2b_1+9a{b_0}^2a_3-c{b_1}^2a_1-c{b_0}^2a_3+a{b_1}^2 a_0+9b{b_0}^2a_3-10a{b_0}^2a_2+2a{b_0}^2a_1+d{a_1}^2b_1-2ba_0b_0b_1+ba_1b_0 b_1+2ha_0a_2b_1+2ha_0a_3b_0+2ha_2a_1b_0+2da_0a_2b_1+2da_0a_3 b_0+2da_1a_2b_0+2da_0a_3b_0+3aa_2b_0b_1-2aa_0b_0b_1+aa_1b_0b_1+3ba_2 b_0b_1-2ca_2b_0b_1=0. \tag{51}$$

$$a{b_1}^2a_2-21a{b_0}^2a_3-21b{b_0}^2a_3+b{b_1}^2a_2+6a{b_0}^2a_2+6b{b_0}^2a_2+h{a_2}^2b_0+d {a_2}^2b_0-c{b_1}^2a_2+2ha_0a_3b_1+2ha_1a_2b_1+2ha_1a_3b_0+2da_0a_3b_1+2da_1 a_2b_1+2da_1a_3b_0-9aa_2b_0b_1+11aa_3b_0b_1-9ba_2b_0b_1+11ba_3b_0 b_1-2ca_3b_0b_1=0. \tag{52}$$

$$4a{b_1}^2a_3-3a{b_1}^2a_2-3b{b_1}^2a_2+4b{b_1}^2a_3+2a{b_0}^2a_3+12b{b_0}^2a_3+h{a_2}^2b_1+d {a_2}^2b_1-c{b_1}^2a_3+2ha_1a_3b_1+2ha_2a_3b_0+2da_1a_3b_1+2da_2a_3 b_0-27aa_3b_0b_1-27ba_3b_0b_1+6aa_2b_0b_1+6ba_2b_0b_1=0. \tag{53}$$

$$2a{b_1}^2a_2+2b{b_1}^2a_2-10a{b_1}^2a_3-10b{b_1}^2a_3+h{a_3}^2b_0+d{a_3}^2b_0+2ha_2a_3 b_1+2da_2a_3b_1+16aa_3b_0b_1+16ba_3b_0b_1=0. \tag{54}$$

$$6a{b_1}^2a_3+6b{b_1}^2a_3+h{a_3}^2b_1+d{a_3}^2b_1=0. \tag{55}$$

Resolving all these algebraic equations by using Mathematica, we obtain the solutions as follows:

Set-1:

$$c=(a+b),\ a_0=0,\ a_1=0,\ a_2=a_2,\ a_3=-a_2,\ b_0=0,\ b_1=-\frac{1}{6}\frac{a_2(h+d)}{a+b}$$

Set-2:

$$c=-a-b,\ a_0=-\frac{b_0(a+b)}{h+d},\ a_1=\frac{-6ab_0-ab_1+6bb_0-bb_1}{h+d},\ a_2=\frac{-6(ab_0-ab_1+bb_0-bb_1)}{h+d}$$

$$a_3=-\frac{6b_1(a+b)}{h+d},\ b_0=b_0,\ b_1=b_1$$

Set-3:

$$c=a+b,\ a_0=0,\ a_1=a_1,\ a_2=\frac{6ab_1-ha_1+6bb_1-da_1}{h+d},\ a_3=-\frac{6b_1(a+b)}{h+d},\ b_0=\frac{1}{6}\frac{a_1(h+d)}{a+b}$$

For investigating the analytic solutions of the Eq. (46), we will use the values of a_0, a_1, a_2, a_3, b_0 and b_1 into the solution (16). At first, we use the values presented in set-1 and the value of $Q(\xi)$ into the solution (16) and obtained the subsequent solutions:

$$v(\xi)=\frac{6D(a+b)}{(h+d)(D^2\exp(\xi)+\exp(-\xi)+2D)} \tag{56}$$

Exchanging the solution (56) from exponential function into hyperbolic function, yields

$$v(\xi)=\frac{12D(a+b)}{(h+d)((D^2+1)\cosh(\xi)+(D^2-1)\sinh(\xi)+4D)} \tag{57}$$

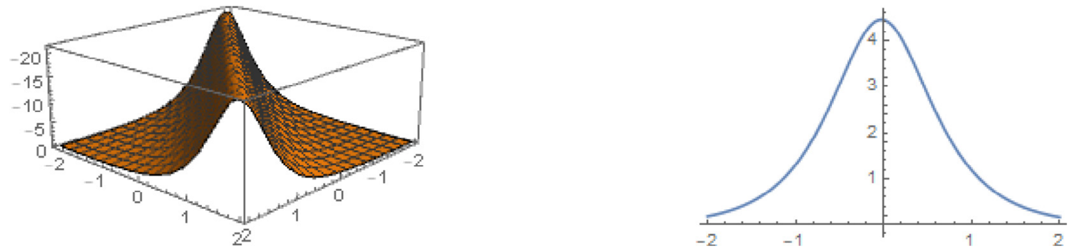

Fig. 1. 3D and 2D plot of the solution (26) for the values $D = -0.76$, $l = -0.37$, $m = -0.40$, $n = -0.41$, $\alpha = 2.00$, $\mu = -1.96$ of the parameters.

Since D is an integral constant, so we can pick the values of D arbitrarily. Therefore, if we pick $D = 1$, the solution (57) is simplified as

$$v(\xi) = \frac{6(a+b)}{(h+d)(\cosh(\xi)+2)} \tag{58}$$

Again, if we pick $D = -1$, the solution (57) is simplified as

$$v(\xi) = \frac{-6(a+b)}{(h+d)(\cosh(\xi)-2)} \tag{59}$$

From the above discussion, there are two cases arise:

Case-1:

For $v(\xi) = \frac{6(a+b)}{(h+d)(cosh(\xi)+2)}$, the solutions (43) and (44) reduces to

$$w(\xi) = \frac{6(a+b)}{(h+d)(\cosh(\xi)+2)} + k_1 \tag{60}$$

and

$$z(\xi) = \frac{6(a+b)}{(h+d)(\cosh(\xi)+2)} + k_2 \tag{61}$$

Case-2:

For $v(\xi) = \frac{-6(a+b)}{(h+d)(\cosh(\xi)-2)}$, the solutions (43) and (44) reduces to

$$w(\xi) = \frac{-6(a+b)}{(h+d)(\cosh(\xi)-2)} + k_1 \tag{62}$$

and

$$z(\xi) = \frac{-6(a+b)}{(h+d)(\cosh(\xi)-2)} + k_2 \tag{63}$$

Here, we observed that the set-1 contains the solutions (57)–(63). These solutions consist of large number of parameters. Since, the solution shape depends on the parameters, so when we choose the values of the parameters randomly, the several types of graphs are acquired. From the sketched graphs, we can discuss the nature of the solitary waves.

Similarly, for the other selection of the values of D, we obtain the other types of solutions. But for simplicity, the other solutions are not written here.

On the other hand, the values of the constants assembled in set-2 and set-3, provide adequate new solutions but for minimalism, the solutions are not documented here.

Graphical representations and discussions

In this section, we have presented the 3D and 2D graphs of the determined solutions of the considered wave equations. The different types of graphs have been drawn from the wave solution. The shape of the travelling wave changes with the change of the unknown parameters associated with the solution. We have examined the nature of the solution. Now, we depict the graphs of the solutions of the subsequent nonlinear evolution equations the Riemann wave equation and the Novikov-Veselov equation. The graphs of the solutions of the above equations are illustrated as follows:

The Riemann wave equation

From the Riemann wave equation, we obtain different type of solutions consists of some unknown parameters. These unknown parameters have effect in the nature of the solutions. That is, if the parameters receive different particular values, different types of solutions are derived from a solution. In the subsequent, we have shown the effect of the parameters associated with the solutions (26).

The general solution (26) consists of the parameters D, l, m, n, α and μ. For the values of $D = -0.76$, $l = -0.37$, $m = -0.40$, $n = -0.41$, $\alpha = 2.00$ and $\mu = -1.96$, we achieve the bell shape soliton which is characterized by infinite tails or infinite wings. The 3D figure is shown within the limit $-5 \le x, y \le 5$, $t = 0$. The 2D figure is shown for $-5 \le x \le 5$, $y = 0$ and $t = 0$ (Fig. 1).

Again, for the values $D = -0.37$, $l = -0.44$, $m = 0.54$, $n = -0.10$, $\alpha = -0.88$ and $\mu = 0.01$ of the parameters, we obtain the consolidated bell shape soliton from solution (26). The shape of this figure is wide expanding on both sides. The 3D figure is depicted within the limit $-8 \le x, y \le 8$ and $t = 0$. The 2D figure is depicted for $-8 \le x \le 8$, $y = 0$ and $t = 0$ (Fig. 2).

On the other hand, the particular solution (27) involves the parameters l, m, n, α and μ. For the values $l = -0.63$, $m = -1.35$, $n = -1.26$, $\alpha = 1.85$, $\mu = -2.00$ of the parameters, we acquire the smooth bell shape soliton. In this figure, the shape is characterized by the infinite tails. The 3D figure is illustrated within the limit $0 \le x, y \le 8$ and $t = 0$. The 2D figure is illustrated for $0 \le x \le 8$, $y = 0$ and $t = 0$ (Fig. 3).

Alternatively, for the values $l = 0.96$, $m = -0.50$, $n = -0.10$, $\alpha = -0.33$ and $\mu = -0.35$ of the parameters, we obtain the compacton soliton. A compacton is a solitary wave with compact support in which the nonlinear dispersion confines it to a finite core, therefore the exponential wings vanish. The 3D figure is shown within the limit

Fig. 2. 3D and 2D plot of the solution (26) for the values $D = -0.37$, $l = -0.44$, $m = 0.54$, $n = -0.10$, $\alpha = -0.88$, $\mu = 0.01$ of the parameters.

Fig. 3. 3D and 2D plot of the solution (27) for the values $l = -0.63$, $m = -1.35$, $n = -1.26$, $\alpha = 1.85$, $\mu = -2.00$ of the parameters.

Fig. 4. 3D and 2D plot of the solution (27) for the values $l = 0.96$, $m = -0.50$, $n = -0.10$, $\alpha = -0.33$, $\mu = -0.35$ of the parameters.

Fig. 5. 3D and 2D plot of the solution (28) for the values $l = 0.09$, $m = -1.10$, $n = -0.40$, $\alpha = -0.01$ and $\mu = 1.23$ of the parameters.

Fig. 6. 3D and 2D plot of the solution (29) for the values $l = 0.35$, $m = -0.69$, $n = -0.52$, $\alpha = -0.42$ and $\mu = -0.43$ of the parameters.

Fig. 7. 3D and 2D plot of the solution (30) for the values $l = 0.45$, $m = -0.09$, $n = -0.62$, $\alpha = 0.02$ and $\mu = 0.37$ of the parameters.

Fig. 8. 3D and 2D plot of the solution (33) for the values $l = -1.96$, $m = 0.39$, $n = -1.86$, $\alpha = -2.00$ and $\mu = -1.90$ of the parameters.

Fig. 9. 3D and 2D plot of the solution (34) for the values $l = -1.10$, $m = -0.56$, $n = -0.57$, $\alpha = -1.79$ and $\mu = -1.96$ of the parameters.

Fig. 10. 3D and 2D plot of the solution (57) for the values $D = 0.51$, $a = 0.49$, $b = -0.33$, $h = 0.39$ and $d = -0.29$ of the parameters.

Fig. 11. 3D and 2D plot of the solution (58) for the values $a = -1.79$, $b = -1.96$, $h = -1.81$ and $d = -1.88$ of the parameters.

Fig. 12. 3D and 2D plot of the solution (58) for the values $a = -0.37$, $b = -0.40$, $h = -0.41$ and $d = -1.96$ of the parameters.

Fig. 13. 3D and 2D plot of the solution (58) for the values $a = -0.90$, b = −0.73, h = −0.61 and d = −0.86 of the parameters.

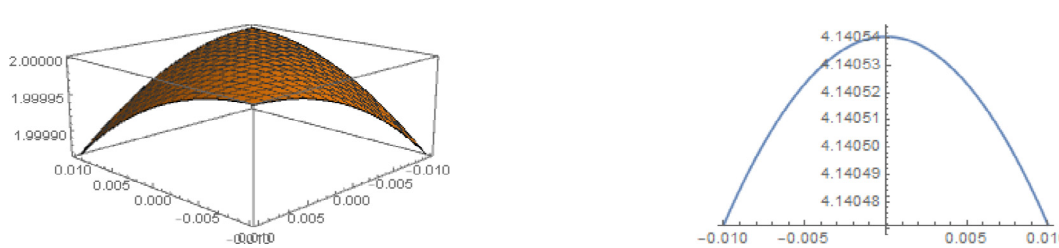

Fig. 14. 3D and 2D plot of the solution (59) for the values $a = -0.16$, $b = 0.01$, $h = -0.16$, $d = 1.48$ of the parameters.

Fig. 15. 3D and 2D plot of the solution (59) for the values $a = -0.69$, $b = -1.164$, $h = -0.51$, $d = 1.18$ of the parameters.

Fig. 16. 3D and 2D plot of the solution (60) for the values $a = -0.73$, $b = 1.43$, $h = -1.31$, $d = 0.28$, $k_1 = 0.52$ of the parameters.

Fig. 17. 3D and 2D plot of the solution (61) for the values $a = -1.39$, $b = 0.96$, $h = -1.45$, $d = -1.78$, $k_2 = -0.43$ of the parameters.

Fig. 18. 3D and 2D plot of the solution (62) for the values $a = -0.53$, $b = -1.73$, $h = -0.31$, $d = 0.28$, $k_1 = -0.43$ of the parameters.

Fig. 19. 3D and 2D plot of the solution (63) for the values $a = -0.04$, $b = -0.33$, $h = -0.48$, $d = -0.63$, $k_2 = -0.41$ of the parameters.

$-6 \le x, y \le 6$ and $t = 0$. The 2D figure is shown for $-6 \le x \le 6$, $y = 0$ and $t = 0$ (Fig. 4).

Moreover, for the values $l = 0.09$, $m = -1.10$, $n = -0.40$, $\alpha = -0.01$ and $\mu = 1.23$ of the parameters, from the solution (28), we secure the smooth soliton solution. The 3D figure is shown within the limit $-8 \le x, y \le 8$ and $t = 0$. The 2D figure is depicted for $-8 \le x \le 8$, $y = 0$ and $t = 0$ (Fig. 5).

On the contrary, for the values $l = 0.35$, $m = -0.69$, $n = -0.52$, $\alpha = -0.42$ and $\mu = -0.43$ of the parameters, from solution (29), we attain the singular kink soliton. In this wave structure, there are infinite wings on both sides and there is a small gap which makes the wave singular. The 3D figure is shown within the limit $-5 \le x, y \le 5$ and $t = 0$. The 2D figure is plotted for $-5 \le x \le 5$, $y = 0$ and $t = 0$ (Fig. 6).

Moreover, for the values $l = 0.45$, $m = -0.09$, $n = -0.62$, $\alpha = 0.02$ and $\mu = 0.37$ of the parameters, from solution (30), we receive the smooth kink soliton. The shape of this figure is upward from right to left. The 3D figure is shown within the limit $0 \le x \le 8$, $-8 \le y \le 8$ and $t = 0$. The 2D figure is sketched for $y = 0$ and $t = 0$ (Fig. 7).

Similarly, for the values $l = -1.96$, $m = 0.39$, $n = -1.86$, $\alpha = -2.00$ and $\mu = -1.90$ of the parameters, from solution (33), we find out the flat kink soliton. In this figure, the shape rise from left to right. The 3D figure is portrayed within the limit $-5 \le x, y \le 5$ and $t = 0$. The 2D figure is sketched for $-5 \le x \le 5$, $y = 0$ and $t = 0$ (Fig. 8).

Additionally, for the values $l = -1.10$, $m = -0.56$, $n = -0.57$, $\alpha = -1.79$ and $\mu = -1.96$ of the parameters, from solution (34), we reach the singular kink soliton. The 3D figure is outlined within the limit $-5 \le x, y \le 5$ and $t = 0$. The 2D figure is delineated for $-5 \le x \le 5$, $y = 0$ and $t = 0$ (Fig. 9).

Since the shape of the figure of the solutions (35) and (36) are similar to the shape of the solutions (33) and (34), therefore, we have not depicted the figure of these solutions.

The Novikov-Veselov (NV) equation

For the Novikov-Veselov (NV) equation, we have ascertained different type of solutions comprised with some unknown parameters. These unknown parameters have effect in the nature of the solutions. That is, if the parameters receive different particular values, different type of solutions are derived from the broad-ranging solution. In the subsequent, we have illustrated the effect of the parameters associated with the solution (57).

The general solution (57) involves the parameters D, a, b, h and d. For the values $D = 0.51$, $a = 0.49$, $b = -0.33$, $h = 0.39$ and $d = -0.29$, from solution (57), we achieve the smooth soliton solution which is characterized by infinite tails or infinite wings. The 3D figure is plotted within the limit $-0.003 \le x \le 0.003$, $-2 \le y \le 2$ and $t = 0$. The 2D figure is plotted for $-0.3 \le x \le 0.3$, $y = 0$ and $t = 0$ (Fig. 10).

Again, the particular solution (58) consist of the parameters a, b, h and d. For the values $a = -1.79$, $b = -1.96$, $h = -1.81$ and $d = -1.88$ of the parameters, we achieve the bell shape soliton which is characterized by infinite tails or infinite wings. The 3D figure is sketched within the limit $-3 \le x \le 3$, $-0.1 \le y \le 0.1$ and $t = 0$. The 2D figure is sketched for $-3 \le x \le 3$, $y = 0$ and $t = 0$ (Fig. 11).

On the contrary, for the values $a = -0.37$, $b = -0.40$, $h = -0.41$ and $d = -1.96$, from the solution (58), we achieve the smooth soliton solutions. The 3D figure is portrayed within the limit $-4 \le x, y \le 4$ and $t = 0$. The 2D figure is portrayed for $-4 \le x \le 4$, $y = 0$ and $t = 0$ (Fig. 12).

Alternatively, for the values$a = -0.90$, $\mathrm{b} = -0.73$, $\mathrm{h} = -0.61$ and $\mathrm{d} = -0.86$ of the parameters, from the solution (58), we accomplish the singular kink soliton. The 3D figure is traced within the limit $-8 \le x, y \le 8$ and $t = 0$. The 2D figure is traced for $-8 \le x \le 8$, $y = 0$ and $t = 0$ (Fig. 13).

Likewise, for the values $a = -0.16$, $b = 0.01$, $h = -0.16$, $d = 1.48$ of the parameters, from solution (59), we attain the compacton soliton with finite wavelengths. When the two compactons are colliding with each other, the original shapes of the waves remain constant [37]. The 3D figure is illustrated within the limit $-1 \le x \le 1$, $-0.002 \le y \le 0.002$ and $t = 0$. The 2D figure is illustrated for $-1 \le x \le 1$, $y = 0$ and $t = 0$ (Fig. 14).

Once again, for the values $a = -0.69$, $b = -1.164$, $h = -0.51$, $d = 1.18$ of the parameters, from solution (59), we find out the soliton solution. The 3D figure is interpreted within the limit $-3 \le x \le 3$, $-2 \le y \le 2$ and $t = 0$. The 2D figure is interpreted for $-3 \le x \le 3$, $y = 0$ and $t = 0$ (Fig. 15).

Furthermore, for the values $a = -0.73$, $b = 1.43$, $h = -1.31$, $d = 0.28$, $k_1 = 0.52$ of the parameters, from solution (60), we gain the anti-bell shape soliton. The 3D figure is outlined within the limit $-0.001 \le x, y \le 0.001$ and $t = 0$. The 2D figure is outlined for $-0.001 \le x \le 0.001$, $y = 0$ and $t = 0$ (Fig. 16).

Similarly, for the values $a = -1.39$, $b = 0.96$, $h = -1.45$, $d = -1.78$, $k_2 = -0.43$ of the unknown parameters, from solution (61), we achieve the soliton solution. The 3D figure is delineated within the limit $-4 \le x \le 4$, $-1 \le y \le 1$ and $t = 0$. The 2D figure is delineated for $-4 \le x \le 4$, $y = 0$ and $t = 0$ (Fig. 17).

In a similar manner, for the values $a = -0.53$, $b = -1.73$, $h = -0.31$, $d = 0.28$, $k_1 = -0.43$ of the parameters, from solution (62), we carry out the smooth soliton solution. The 3D figure is drawn within the limit $-0.1 \le x \le 0.1$, $-2 \le y \le 2$ and $t = 0$. The 2D figure is drawn for $-0.1 \le x \le 0.1$, $y = 0$ and $t = 0$ (Fig. 18).

Furthermore, for the values $a = -0.04$, $b = -0.33$, $h = -0.48$, $d = -0.63$, $k_2 = -0.41$ of the parameters, from solution (63), we found the smooth anti-bell shape soliton. In this figure, the shape rise from both sides. The 3D figure is depicted within the limit $-2 \le x \le 2$, $-2 \le y \le 2$ and $t = 0$. The 2D figure is depicted for $-2 \le x \le 2$, $y = 0$ and $t = 0$ (Fig. 19).

Conclusion

In this article, the generalized Kudryashov method has been effectively put in use to establish the analytic solutions to the Riemann wave equation and the Novikov-Veselov equation with the assist of the aforesaid method. We have established general solitary wave solutions for each of the studied equation associated with some unknown parameters and for the definite values of the parameters some accessible solutions in the literature are originated and some fresh solutions are designated. The established solutions include diverse kinds of solitary waves, videlicet bell shape soliton, shrunk bell shape soliton, compacton soliton, singular kink soliton, flat kink shape soliton, smooth singular soliton and other types of soliton solutions. The behavior of the solitary waves have displayed graphically with respect to space and time. The graphs of the obtained solutions explicitly reveal the higher efficiency and authenticity of the generalized Kudryashov method. This advantageous and effective method might be used to examine other kinds of NLEEs which frequently arise in various scientific and real world applications. The attained solutions will be helpful to further study the problems arise in mathematical physics and engineering. Through this study, the physical interpretation of the desired solutions and the actual application in reality will be investigated.

CRediT authorship contribution statement

Hemonta Kumar Barman: Data curation, Writing - original draft, Writing - review & editing. **Aly R. Seadawy:** Conceptualization, Methodology, Software, Supervision. **M. Ali Akbar:** Visualization, Investigation. **Dumitru Baleanu:** Software, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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