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Research Paper

Exact Solution for Nonlinear Local Fractional Partial Differential Equations

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Abstract. In this work, we extend the existing local fractional Sumudu decomposition method to solve the nonlinear local fractional partial differential equations. Then, we apply this new algorithm to resolve the nonlinear local fractional gas dynamics equation and nonlinear local fractional Klein-Gordon equation, so we get the desired non-differentiable exact solutions. The steps to solve the examples and the results obtained, showed the flexibility of applying this algorithm, and therefore, it can be applied to similar examples.

Keywords: Adomian decomposition method, Sumudu transform method, Local fractional derivative operator, Local fractional, Nonlinear local fractional gas dynamics equation, Nonlinear local fractional Klein-Gordon equation.

1. Introduction

The perception of a derivative of order any real number is no longer a stunning fact but quite the contrary. Its contribution has allowed and still allows us to glimpse natural phenomena which surround us in a different way and to model them by differential equations or systems of non-integer order and which better reflect the passage from real to known. Given the importance of these fractional differential equations and others, many researchers have worked to improve existing methods, or discover new methods to solve them, or find approximate solutions for them. These efforts have affected this area in several methods, including the Adomian decomposition method and in the abbreviation (ADM), which is among the most famous methods developed recently, where it was developed by George Adomian [1,2].

With the advent of fractional differential equations, researchers used this method to solve this new type of equations [3-8], and it was also used to solve another fractional equations which include, local fractional differential equations [9-11], local fractional partial differential equations [12-18], and local fractional integro-differential equations [19-21], or we find them benefit from the combined with some known transforms, such as: Laplace transform and Sumudu transform, in order to facilitate the solution of this type of equations, especially nonlinear ones. Among these works, we find local fractional laplace decomposition method [22] and local fractional Sumudu decomposition method [17].

The idea of this article is to work on the method proposed in [17] to solve linear local fractional partial differential equations in order to extend it to solve nonlinear partial differential equations within local fractional derivative. The value of the LFSDM is to enable us to combine two powerful methods to obtain exact solutions for nonlinear local fractional gas dynamics equation and nonlinear local fractional Klein-Gordon equation.

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2. Basic Definitions

In this section, we will present the basic concepts on fractional local calculus, and in particular the local fractional derivative, local fractional integral and local fractional Sumudu transform.

2.1 Local fractional derivative

Definition 2.1

Setting $g(\rho)\in C_\sigma(\alpha,\beta)$, the local fractional derivative of $g(\rho)$ with order σ at $\rho=\rho_0$ is defined as [23,24]:

$$g^{(\sigma)}(\rho)=\left.\frac{d^\sigma g}{d\rho^\sigma}\right|_{\rho=\rho_0}=\lim_{\rho\to\rho_0}\frac{\Delta^\sigma(g(\rho)-g(\rho_0))}{(\rho-\rho_0)^\sigma}, \tag{1}$$

where

$$\Delta^\sigma(g(\rho)-g(\rho_0))\cong\Gamma(1+\rho)(g(\rho)-g(\rho_0)), \tag{2}$$

and $C_\sigma(\alpha,\beta)$, designates the class of functions called local fractional continuous on the interval (α,β).

2.2 Local fractional integral

Definition 2.2

The local fractional integral of $g(\rho)$ of order σ in the interval $[\alpha,\beta]$ is defined as [23,24]:

$${}_\alpha I_\beta^{(\sigma)}g(\rho)=\frac{1}{\Gamma(1+\sigma)}\int_\alpha^\beta g(\tau)(d\tau)^\sigma=\frac{1}{\Gamma(1+\alpha)}\lim_{\Delta\tau\to 0}\sum_{j=0}^{N-1}g(\tau_j)(\Delta\tau_j)^\sigma, \tag{3}$$

where $\Delta\tau_j=\tau_{j+1}-\tau_j,\Delta\tau=\max(\Delta\tau_0,\Delta\tau_1,\Delta\tau_2,...)$, and $[\tau_{j+1},\tau_j],\tau_0=\alpha,\tau_N=\beta$ is a partition of the interval $[\alpha,\beta]$.

2.3 Some properties of the local fractional operators

The local fractional operators fulfill some fundamental equations. In particular, we have the following:

Definition 2.3

In fractal space, the Mittage-Leffler function, the hyperbolic sine and hyperbolic cosine are defined as [23,24]:

$$E_\sigma(\rho^\sigma)=\sum_{m=0}^{+\infty}\frac{\rho^{m\sigma}}{\Gamma(1+m\sigma)},0<\sigma\le 1, \tag{4}$$

$$\sin_\sigma(\rho^\sigma)=\sum_{m=0}^{+\infty}(-1)^m\frac{\rho^{(2m+1)\sigma}}{\Gamma(1+(2m+1)\sigma)},0<\sigma\le 1, \tag{5}$$

$$\cos_\sigma(\rho^\sigma)=\sum_{m=0}^{+\infty}(-1)^m\frac{\rho^{2m\sigma}}{\Gamma(1+2m\sigma)},0<\sigma\le 1, \tag{6}$$

$$\sinh_\sigma(\rho^\sigma)=\sum_{m=0}^{+\infty}\frac{\rho^{(2m+1)\sigma}}{\Gamma(1+(2m+1)\sigma)},0<\sigma\le 1, \tag{7}$$

$$\cosh_\sigma(\rho^\sigma)=\sum_{m=0}^{+\infty}\frac{\rho^{2m\sigma}}{\Gamma(1+2m\sigma)},0<\sigma\le 1. \tag{8}$$

By using the local fractional derivative (1) and the definitions (2.3) it can be easily shown that:

$$\frac{d^\sigma}{d\rho^\sigma}\frac{\rho^{m\sigma}}{\Gamma(1+m\sigma)}=\frac{\rho^{(m-1)\sigma}}{\Gamma(1+(m-1)\sigma)}, \tag{9}$$

$$\frac{d^\sigma}{d\rho^\sigma}E_\sigma(\rho^\sigma)=E_\sigma(\rho^\sigma), \tag{10}$$

$$\frac{d^\sigma}{d\rho^\sigma}\sin_\sigma(\rho^\sigma)=\cos_\sigma(\rho^\sigma), \tag{11}$$

$$\frac{d^\sigma}{d\rho^\sigma}\cos_\sigma(\rho^\sigma)=-\sin_\sigma(\rho^\sigma), \tag{12}$$

$$\frac{d^\sigma}{d\rho^\sigma}\sinh_\sigma(\rho^\sigma)=\cosh_\sigma(\rho^\sigma), \tag{13}$$

$$\frac{d^\sigma}{d\rho^\sigma}\cosh_\sigma(\rho^\sigma)=\sinh_\sigma(\rho^\sigma), \tag{14}$$

$$ {}_0I_\rho^{(\sigma)}\frac{\rho^{m\sigma}}{\Gamma(1+m\sigma)}=\frac{\rho^{(m+1)\sigma}}{\Gamma(1+(m+1)\sigma)}. \tag{15}$$

2.4 Local fractional Sumudu transform

We present here the definition of local fractional Sumudu transform (denoted in this paper by (${}^{lf}S_\sigma$) and some properties concerning this transformation [12]:

$${}^{lf}S_\sigma\left\{\sum_{m=0}^{\infty}a_m\rho^{m\sigma}\right\}=\sum_{m=0}^{\infty}\Gamma(1+m\sigma)a_m w^{m\sigma}. \tag{16}$$

For examples,

$${}^{lf}S_\sigma\left\{E_\sigma(i^\sigma\rho^\sigma)\right\}=\sum_{m=0}^{\infty}i^{m\sigma}w^{m\sigma}. \tag{17}$$

$${}^{lf}S_\sigma\left\{\frac{\rho^\sigma}{\Gamma(1+\sigma)}\right\}=w^\sigma. \tag{18}$$

Definition 2.4

The local fractional Sumudu transform of $g(\rho)$ is defined by:

$${}^{lf}S_\sigma\{g(\rho)\}=F_\sigma(w)=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-w^\sigma\rho^\sigma)\frac{g(\rho)}{w^\sigma}(d\rho)^\sigma, 0<\sigma\le 1. \tag{19}$$

The Following inverse formula of (19) is defined as:

$${}^{lf}S_\sigma^{-1}\{F_\sigma(w)\}=g(\rho), 0<\sigma\le 1. \tag{20}$$

Theorem 2.1 (linearity). If ${}^{lf}S_\sigma\{g(\rho)\}=F_\sigma(w)$ and ${}^{lf}S_\sigma\{\varphi(\rho)\}=\Psi_\sigma(w)$, then one has:

$${}^{lf}S_\sigma\{g(\rho)+\varphi(\rho)\}=F_\sigma(w)+\Psi_\sigma(w). \tag{21}$$

Proof. Using formula (19), we obtain:

$$\begin{aligned}{}^{lf}S_\sigma\{g(\rho)+\varphi(\rho)\}&=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-w^\sigma\rho^\sigma)\frac{g(\rho)+\varphi(\rho)}{w^\sigma}(d\rho)^\sigma\\ &=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty\left[E_\sigma(-w^\sigma\rho^\sigma)\frac{g(\rho)}{w^\sigma}+E_\sigma(-w^\sigma\rho^\sigma)\frac{\varphi(\rho)}{w^\sigma}\right](d\rho)^\sigma \qquad (22)\\ &=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-w^\sigma\rho^\sigma)\frac{g(\rho)}{w^\sigma}(d\rho)^\sigma+\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma(-w^\sigma\rho^\sigma)\frac{\varphi(\rho)}{w^\sigma}(d\rho)^\sigma={}^{lf}S_\sigma\{g(\rho)\}+{}^{lf}S_\sigma\{\varphi(\rho)\}.\end{aligned}$$

This ends the proof.

Theorem 2.2

(1) (Sumudu transform of local fractional derivative).

If ${}^{lf}S_{\sigma}\left\{g(\rho)\right\}=F_{\sigma}(w)$, then one has:

$$ {}^{lf}S_{\sigma}\left\{\frac{d^{\sigma}g(\rho)}{d\rho^{\sigma}}\right\}=\frac{F_{\sigma}(w)-F(0)}{w^{\sigma}}. \tag{23} $$

We obtain the following results, as the direct result of (22):

$$ {}^{lf}S_{\sigma}\left\{\frac{d^{n\sigma}g(\rho)}{d\rho^{n\sigma}}\right\}=\frac{1}{w^{n\sigma}}\left[F_{\sigma}(w)-\sum_{k=0}^{n-1}w^{k\sigma}g^{(k\sigma)}(0)\right]. \tag{24} $$

When $n=2$, from (24), we get:

$$ {}^{lf}S_{\sigma}\left\{\frac{d^{2\sigma}g(\rho)}{d\rho^{2\sigma}}\right\}=\frac{1}{w^{2\sigma}}\left[F_{\sigma}(w)-g(0)-w^{\sigma}g^{(\sigma)}(0)\right]. \tag{25} $$

(2) (Sumudu transform of local fractional integral). If ${}^{lf}S_{\sigma}\left\{g(\rho)\right\}=F_{\sigma}(w)$, we have:

$$ {}^{lf}S_{\sigma}\left\{{}_{0}I_{\rho}^{(\sigma)}g(\rho)\right\}=w^{\sigma}F_{\sigma}(w). \tag{26} $$

Proof. See [12].

3. Local Fractional Sumudu Decomposition Method

Let us consider the following nonlinear operator with local fractional derivative:

$$ L_{\sigma}X(\upsilon,\nu)+R_{\sigma}X(\upsilon,\nu)+N_{\sigma}X(\upsilon,\nu)=k(\upsilon,\nu), \tag{27} $$

where $L_{\sigma}=\partial^{m\sigma}/\partial\tau^{m\sigma}(m\in\mathrm{N}^{*})$ is the linear local fractional derivative operator of order $m\sigma$, R_{σ} is the linear local fractional operator, N_{σ} represents the nonlinear local fractional operator, and $k(\upsilon,\nu)$ is given function. By applying a local fractional Sumudu transform on each side of equation (27), we obtain:

$$ {}^{lf}S_{\sigma}[L_{\sigma}X(\upsilon,\nu)]+{}^{lf}S_{\sigma}[R_{\sigma}X(\upsilon,\nu)+N_{\sigma}X(\upsilon,\nu)]={}^{lf}S_{\sigma}[k(\upsilon,\nu)]. \tag{28} $$

Depending on the properties of this transform, we have:

$$ \begin{aligned} {}^{lf}S_{\sigma}[X(\upsilon,\nu)]&=\sum_{k=0}^{m-1}w^{k\sigma}\frac{\partial^{k\sigma}X(\upsilon,0)}{\partial\nu^{k\sigma}}+w^{m\sigma}({}^{lf}S_{\sigma}[k(\upsilon,\nu)])\\ &-w^{m\sigma}\,{}^{lf}S_{\sigma}[R_{\sigma}X(\upsilon,\nu)+N_{\sigma}X(\upsilon,\nu)]. \end{aligned} \tag{29} $$

By taking the inverse transform on each side of the equation (28), it gives:

$$ \begin{aligned} X(\upsilon,\nu)&=\sum_{k=0}^{m-1}\frac{\partial^{k\sigma}X(\upsilon,0)}{\partial\nu^{k\sigma}}\frac{\nu^{k\sigma}}{\Gamma(1+k\sigma)}+{}^{lf}S_{\sigma}^{-1}\left(w^{m\sigma}({}^{lf}S_{\sigma}[k(\upsilon,\nu)])\right)\\ &-{}^{lf}S_{\sigma}^{-1}\left(w^{m\sigma}\,{}^{lf}S_{\sigma}[R_{\sigma}X(\upsilon,\nu)+N_{\sigma}X(\upsilon,\nu)]\right). \end{aligned} \tag{30} $$

According to the Adomian decomposition method [1], we decompose the unknown function X as an infinite series given by:

$$ X(\upsilon,\nu)=\sum_{n=0}^{\infty}X_{n}(\upsilon,\nu), \tag{31} $$

and the nonlinear term can be decomposed as:

$$N_\sigma X(\upsilon,\nu)=\sum_{n=0}^{\infty}A_n, \tag{32}$$

where A_n are Adomian polynomials [25] of X_0, X_1, X_2,... X_n and it can be calculated by:

$$A_n=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N_\sigma\left(\sum_{i=0}^{\infty}\lambda^i X_i\right)\right]_{\lambda=0},n=0,1,2,... \tag{33}$$

Substituting (31) and (32) in (30), we get:

$$\begin{aligned}\sum_{n=0}^{\infty}X_n(\upsilon,\nu)=\sum_{k=0}^{m-1}\frac{\partial^{k\sigma}X(\upsilon,0)}{\partial\nu^{k\sigma}}\frac{\nu^{k\sigma}}{\Gamma(1+k\sigma)}+{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}({}^{lf}S_\sigma[k(\upsilon,\nu)])\right)\\ -{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}\,{}^{lf}S_\sigma[R_\sigma\sum_{n=0}^{\infty}X_n(\upsilon,\nu)+N_\sigma\sum_{n=0}^{\infty}A_n]\right).\end{aligned} \tag{34}$$

On comparing both sides of (34), we have:

$$\begin{aligned}X_0(\upsilon,\nu)&=\sum_{k=0}^{m-1}\frac{\partial^{k\sigma}X(\upsilon,0)}{\partial\nu^{k\sigma}}\frac{\nu^{k\sigma}}{\Gamma(1+k\sigma)}+{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}({}^{lf}S_\sigma[k(\upsilon,\nu)])\right),\\ X_1(\upsilon,\nu)&=-{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}\,{}^{lf}S_\sigma[R_\sigma X_0(\upsilon,\nu)+A_0]\right),\\ X_2(\upsilon,\nu)&=-{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}\,{}^{lf}S_\sigma[R_\sigma X_1(\upsilon,\nu)+A_1]\right),\\ X_3(\upsilon,\nu)&=-{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}\,{}^{lf}S_\sigma[R_\sigma X_2(\upsilon,\nu)+A_2]\right),\\ &\vdots\end{aligned} \tag{35}$$

The general iteration formulas that defines the terms is given by:

$$\begin{aligned}X_0(\upsilon,\nu)&=\sum_{k=0}^{m-1}\frac{\partial^{k\sigma}X(\upsilon,0)}{\partial\nu^{k\sigma}}\frac{\nu^{k\sigma}}{\Gamma(1+k\sigma)}+{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}({}^{lf}S_\sigma[k(\upsilon,\nu)])\right),\\ X_n(\upsilon,\nu)&=-{}^{lf}S_\sigma^{-1}\left(w^{m\sigma}\,{}^{lf}S_\sigma[R_\sigma X_{n-1}(\upsilon,\nu)+A_{n-1}]\right),\end{aligned} \tag{36}$$

where $0<\sigma\le 1$ and $n,m\in\mathrm{N}^*$.

4. Applications

The work shown in this paragraph has the aim of validating the extension of the application of local fractional sumudu decomposition method (LFSDM) presented in [17] for solving the nonlinear local fractional gas dynamics equation and nonlinear local fractional Klein-Gordon equation.

Example 4.1

First, we consider that the form of the gas dynamics equation on Cantor sets becomes:

$$X_\nu^{(\sigma)}(\upsilon,\nu)+X(\upsilon,\nu)X_\upsilon^{(\sigma)}(\upsilon,\nu)+(X(\upsilon,\nu))^2-X(\upsilon,\nu)=0,0<\sigma\le 1,\upsilon\in\mathrm{R}, \tag{37}$$

under the initial condition:

$$X(\upsilon,0)=E_\sigma(-\upsilon^\sigma). \tag{38}$$

Using the formula (30), we obtain:

$$X(\upsilon,\nu)=E_\sigma(-\upsilon^\sigma)-{}^{lf}S_\sigma^{-1}\left(w^\sigma({}^{lf}S_\sigma[X(\upsilon,\nu)T_\upsilon^{(\sigma)}(\upsilon,\nu)+(X(\upsilon,\nu))^2-X(\upsilon,\nu)])\right). \tag{39}$$

Referring to the method adopted in this research [1], the solution function X can be decomposed by an infinite series defined by:

$$X(\upsilon,\nu)=\sum_{n=0}^{\infty}X_n(\upsilon,\nu), \tag{40}$$

and the nonlinear term can be decomposed as:

$$X(\upsilon,\nu)X_{\upsilon}^{(\sigma)}(\upsilon,\nu)=\sum_{n=0}^{\infty}A_n, \tag{41}$$

and

$$(X(\upsilon,\nu))^2=\sum_{n=0}^{\infty}B_n, \tag{42}$$

Substituting (40), (41) and (42) in (39), we get:

$$\sum_{n=0}^{\infty}X_n(\upsilon,\nu)=E_{\sigma}(-\upsilon^{\sigma})-{}^{lf}S_{\sigma}^{-1}\left(w^{\sigma}({}^{lf}S_{\sigma}[\sum_{n=0}^{\infty}A_n(X)+\sum_{n=0}^{\infty}B_n(X)-\sum_{n=0}^{\infty}X_n(\upsilon,\nu)])\right). \tag{43}$$

On comparing both sides of (43), we have:

$$\begin{aligned}
X_0(\upsilon,\nu)&=E_{\sigma}(-\upsilon^{\sigma}),\\
X_1(\upsilon,\nu)&=-{}^{lf}S_{\sigma}^{-1}\left(w^{\sigma}\,{}^{lf}S_{\sigma}[A_0(X)+B_0(X)-X_0(\upsilon,\nu)]\right),\\
X_2(\upsilon,\nu)&=-{}^{lf}S_{\sigma}^{-1}\left(w^{\sigma}\,{}^{lf}S_{\sigma}[A_1(X)+B_1(X)-X_1(\upsilon,\nu)]\right),\\
X_3(\upsilon,\nu)&=-{}^{lf}S_{\sigma}^{-1}\left(w^{\sigma}\,{}^{lf}S_{\sigma}[A_2(X)+B_2(X)-X_2(\upsilon,\nu)]\right),\\
&\vdots
\end{aligned} \tag{44}$$

The first few components of $A_n(X)$ and $B_n(X)$ polynomials [25], for example, are given by:

$$\begin{aligned}
A_0(X)&=X_0(\upsilon,\nu)X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu),\\
A_1(X)&=X_0(\upsilon,\nu)X_{1,\upsilon}^{(\sigma)}(\upsilon,\nu)+X_1(\upsilon,\nu)X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu),\\
A_2(X)&=X_0(\upsilon,\nu)X_{2,\upsilon}^{(\sigma)}(\upsilon,\nu)+X_2(\upsilon,\nu)X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu)+X_1(\upsilon,\nu)X_{1,\upsilon}^{(\sigma)}(\upsilon,\nu),\\
&\vdots
\end{aligned} \tag{45}$$

and

$$\begin{aligned}
B_0&=(X_0(\upsilon,\nu))^2,\\
B_1&=2X_0(\upsilon,\nu)X_1(\upsilon,\nu),\\
B_2&=(X_1(\upsilon,\nu))^2+2X_0(\upsilon,\nu)X_2(\upsilon,\nu),\\
&\vdots
\end{aligned} \tag{46}$$

Using He's polynomials (45), (46) and the iteration formulas (44), we obtain:

$$\begin{aligned}
X_0(\upsilon,\nu)&=E_{\sigma}(-\upsilon^{\sigma}),\\
X_1(\upsilon,\nu)&=E_{\sigma}(-\upsilon^{\sigma})\frac{\nu^{\sigma}}{\Gamma(1+\sigma)},\\
X_2(\upsilon,\nu)&=E_{\sigma}(-\upsilon^{\sigma})\frac{\nu^{2\sigma}}{\Gamma(1+2\sigma)},\\
X_3(\upsilon,\nu)&=E_{\sigma}(-\upsilon^{\sigma})\frac{\nu^{3\sigma}}{\Gamma(1+3\sigma)},
\end{aligned} \tag{47}$$

and so on. The first four terms of the non-differentiable approximation solution for (36), are given by:

$$X(\upsilon,\nu)=E_{\sigma}(-\upsilon^{\sigma})\left(1+\frac{\nu^{\sigma}}{\Gamma(1+\sigma)}+\frac{\nu^{2\sigma}}{\Gamma(1+2\sigma)}+\frac{\nu^{3\sigma}}{\Gamma(1+3\sigma)}+\ldots\right). \tag{48}$$

and therefore, the non-differentiable exact solution for (36) becomes:

$$X(\upsilon,\nu)=\lim_{k\to\infty}\left(E_{\sigma}(-\upsilon^{\sigma})\sum_{i=0}^{k}\frac{\nu^{i\sigma}}{\Gamma(1+i\sigma)}\right)=E_{\sigma}(-\upsilon^{\sigma})E_{\sigma}(\nu^{\sigma}). \tag{49}$$

Substituting $\sigma=1$ into (49), we obtain:

$$X(\upsilon,\nu)=e^{\nu-\upsilon}. \tag{50}$$

Note that, our solution (49) satisfies the initial conditions (37), and in the case $\sigma=1$, we obtain the same solution obtained in [26] by homotopy perturbation method, and in [27] by homotopy analysis method.

Example 4.2

We consider the following nonlinear local fractional Klein-Gordon equation:

$$X_{\nu}^{(2\sigma)}(\upsilon,\nu)-X_{\upsilon}^{(2\sigma)}(\upsilon,\nu)-(X_{\upsilon}^{(\sigma)}(\upsilon,\nu))^{2}+(X(\upsilon,\nu))^{2}=0, 0<\sigma\leq 1, \tag{51}$$

with the initial conditions:

$$X(\upsilon,0)=0, X_{\nu}^{(\sigma)}(\upsilon,0)=E_{\sigma}(-\upsilon^{\sigma}). \tag{52}$$

From the formula (29), we obtain:

$$X(\upsilon,\nu)=E_{\sigma}(\upsilon^{\sigma})\frac{\nu^{\sigma}}{\Gamma(1+\sigma)}+{}^{lf}S_{\sigma}^{-1}\left(w^{2\sigma}({}^{lf}S_{\sigma}[X_{\upsilon}^{(2\sigma)}(\upsilon,\nu)+(X_{\upsilon}^{(\sigma)}(\upsilon,\nu))^{2}-(X(\upsilon,\nu))^{2}])\right). \tag{53}$$

As the Adomian decomposition method [1] depends on the decomposed of the solution function X in an infinite series given by:

$$X(\upsilon,\nu)=\sum_{n=0}^{\infty}X_{n}(\upsilon,\nu). \tag{54}$$

The nonlinear term $(X_{\nu}^{(\sigma)}(\upsilon,\nu))^{2}$ can be decomposed as:

$$(X_{\upsilon}^{(\sigma)}(\upsilon,\nu))^{2}=\sum_{n=0}^{\infty}C_{n}(X), \tag{55}$$

and $(X(\upsilon,\nu))^{2}$ it is given in the previous form (42). Substituting (54), (55) and (42) in (53), we get:

$$X(\upsilon,\nu)=E_{\sigma}(\upsilon^{\sigma})\frac{\nu^{\sigma}}{\Gamma(1+\sigma)}+{}^{lf}S_{\sigma}^{-1}\left(w^{2\sigma}({}^{lf}S_{\sigma}[\sum_{n=0}^{\infty}X_{n,\upsilon}^{(2\sigma)}(\upsilon,\nu)+\sum_{n=0}^{\infty}C_{n}(X)-\sum_{n=0}^{\infty}B_{n}(X)])\right). \tag{56}$$

On comparing both sides of (56), we have:

$$\begin{aligned} X_{0}(\upsilon,\nu)&=E_{\sigma}(\upsilon^{\sigma})\frac{\nu^{\sigma}}{\Gamma(1+\sigma)},\\ X_{1}(\upsilon,\nu)&={}^{lf}S_{\sigma}^{-1}\left(w^{2\sigma}\,{}^{lf}S_{\sigma}[X_{0,\upsilon}^{(2\sigma)}(\upsilon,\nu)+C_{0}(X)-B_{0}(X)]\right),\\ X_{2}(\upsilon,\nu)&={}^{lf}S_{\sigma}^{-1}\left(w^{2\sigma}\,{}^{lf}S_{\sigma}[X_{1,\upsilon}^{(2\sigma)}(\upsilon,\nu)+C_{1}(X)-B_{1}(X)]\right),\\ X_{3}(\upsilon,\nu)&={}^{lf}S_{\sigma}^{-1}\left(w^{2\sigma}\,{}^{lf}S_{\sigma}[X_{2,\upsilon}^{(2\sigma)}(\upsilon,\nu)+C_{2}(X)-B_{2}(X)]\right),\\ &\vdots \end{aligned} \tag{57}$$

The first few components of $C_{n}(X)$ polynomials [25], are given by:

$$C_{0}=(X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu))^{2}, \tag{58}$$

$$C_1 = 2X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu)X_{1,\upsilon}^{(\sigma)}(\upsilon,\nu),$$

$$C_2 = (X_{1,\upsilon}^{(\sigma)}(\upsilon,\nu))^2 + 2X_{0,\upsilon}^{(\sigma)}(\upsilon,\nu)X_{2,\upsilon}^{(\sigma)}(\upsilon,\nu),$$

$$\vdots$$

Using He's polynomials (42), (58) and the iteration formulas (57), we obtain:

$$\begin{aligned} X_0(\upsilon,\nu) &= E_\sigma(\upsilon^\sigma)\frac{\nu^\sigma}{\Gamma(1+\sigma)}, \\ X_1(\upsilon,\nu) &= E_\sigma(\upsilon^\sigma)\frac{\nu^{3\sigma}}{\Gamma(1+3\sigma)}, \\ X_2(\upsilon,\nu) &= E_\sigma(\upsilon^\sigma)\frac{\nu^{5\sigma}}{\Gamma(1+5\sigma)}, \\ X_3(\upsilon,\nu) &= E_\sigma(-\upsilon^\sigma)\frac{\nu^{7\sigma}}{\Gamma(1+7\sigma)}, \\ &\vdots \end{aligned} \tag{59}$$

and so on. Then, the non-differentiable solution of (51) is calculated by:

$$X(\upsilon,\nu) = \lim_{k\to\infty}\left(E_\sigma(\upsilon^\sigma)\sum_{i=0}^{k}\frac{\nu^{(2i+1)\sigma}}{\Gamma(1+(2i+1)\sigma)}\right). \tag{60}$$

Therefore, the non-differentiable exact solution of the (51) takes the form:

$$X(\upsilon,\nu) = E_\sigma(\upsilon^\sigma)\sinh_\sigma(\nu^\sigma). \tag{61}$$

Note that, our solution (61) satisfies the initial conditions (52), and in the case $\sigma = 1$, we obtain the same solution obtained in [28] by homotopy perturbation method.

5. Conclusion

The extension of the local fractional Sumudu decomposition method (LFSDM) to solve nonlinear partial differential equations leads to establish an efficient algorithm; this algorithm allows us to obtain the non-differentiable exact solution as soon as possible. Through this work, it can be assumed that this modified algorithm is suitable for solving this type of equations. The application of this method has given us precise solutions for both equations: nonlinear local fractional gas dynamics equation and nonlinear local fractional Klein-Gordon equation and can therefore be used to solve other nonlinear local fractional partial differential equations, in order to facilitate calculations and their ability to achieve the desired results.

Conflict of Interest

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