Research Article

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Exact optical solitons of the perturbed nonlinear Schrödinger–Hirota equation with Kerr law nonlinearity in nonlinear fiber optics

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Abstract: This article studies dark, bright, trigonometric and rational optical soliton solutions to the perturbed nonlinear Schrödinger–Hirota equation (PNLSHE). Hence, we have examined two cases: first, restrictions have been done to the third-order (TOD) (γ) as constraint relation, and the coupling coefficients (σ) is obtained as well as the velocity of the soliton by adopting the traveling wave hypothesis. Second, the TOD and the coupling coefficients are non-zero value, sending back to the PNLSHE, which has been studied in refs. [4,10,16] recently. By employing two relevant integration technics such as the auxiliary equation and the modified auxiliary equation method, miscellaneous optical solitary wave is obtianed, which is in agreement with the outcomes collected by the previous studies [4,16]. These results help in obtaining nonlinear optical fibers in the future.

Keywords: optical solitons, perturbed nonlinear Schrödinger-Hirota equation.

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1 Introduction

Investigation of optical solitons have decidedly gained momentum in the field of the solitary waves. Review of various solutions have been made to the nonlinear Schrödinger equations with low group velocity dispersion, dispersion terms, Kerr nonlinearities, spatiotemporal dispersion, self-steepening, etc. Habitually, these results are qualifying combo solitons, chirped free and chirped solitons, and dark combo soliton [1–10,16–24]. If the applications of these results are numerous, but communication by optic fibers is one of them. Moreover, solitons have revolutionized the communication system through the wave guides more recently. It is undoubtedly that the soliton constitutes the pillar of data transfer and communication at unimaginable distances.

However, all the strength of the optical system lies on well-known effects, which at the same time constitute conditions restrictions. Most of the time, pulse propagation in optical fibers can be concerned by group velocity dispersion (GVD), nonlinearity and polarization mode dispersion. Regarding nonlinearity effect, it is observed a wide class such Kerr effect, Raman scattering, Brilouin scattering just to a few.

To date, we find in the literature a variety of mathematical methods that have facilitated the construction of traveling wave solutions, such as the auxiliary equation method [2,3], the sine-Gordon expansion method

[4], the simplest equation approach [5], the modified auxiliary equation [6], the sine-cosine method [7], new the (G'/G)-expansion method [8,9], the sine-Gordon expansion [10], Homotopy perturbation Sumudu transform method [11–14], computational algorithm [15] and so on.

Recently, a wide class of model have been used to investigate optical solitons in optic fibers, such as Chen-Lee-Liu model [27], Fokas–Lenells equation [28] and Klein–Gordon–Zakharov equations [29,30]. The various fibers are usually monomode, multimode, twin-core and multiple-core couplers with different types of nonlinearities (i.e., Kerr, power law, parabolic law and dual-power law). Alongside these models, the famous nonlinear Schrödinger equation has also experienced an ascent in the search for optical solitons. The nonlinear Schrödinger's equation is also known for its virtue in the study of elementary and specific propagation of dispersive and nonlinear waves. In the following section, the dimensionless form of the PNLSHE with spatiotemporal dispersion and Kerr law nonlinearity will be presented as well as the physical terms and coefficients that it abounds.

2 Nonlinear Schrödinger–Hirota equation with Kerr law nonlinearity

The PNLSHE that reflect pulse propagation in a dispersive optical fibers [18] was treated analytically by refs. [4,16,17]. As a result, dark, dark-bright, new type of jacobian elliptic function solutions and inclosed optical solitons have been retrieved. It is expressed in the following form:

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + cq|q|^2 + i[\gamma q_{xxx} + \sigma|q|^2 q_x] \\ &\quad = i[q_x + \lambda(|q|^2 q)_x + \nu(|q|^2)_x q]. \end{aligned} \tag{1}$$

In view of the real involvement of optical solitons in the transport of information, the challenge is to build reliable and stable exact optical solitons to perform the transcontinental transportation of data. The challenge in this article is to seek analytical solutions that can lead to direct application in optical fibers. Hence, the model is the dimensionless couple of the dispersive nonlinear Schrodinger–Hirota equation that was recently used by Inc et al. [4].

$$\begin{aligned} &i\psi_t + a\psi_{xx} + b\psi_{xt} + cq|q|^2 + i[\gamma\psi_{xxx} + \sigma|q|^2\psi_x] \\ &\quad = i[\psi_x + \lambda(|q|^2 q)_x + \nu(|q|^2)_x q], \end{aligned} \tag{2}$$

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + c\psi|\psi|^2 + i[\gamma q_{xxx} + \sigma|\psi|^2 q_x] \\ &\quad = i[q_x + \lambda(|\psi|^2\psi)_x + \nu(|\psi|^2)_x\psi], \end{aligned} \tag{3}$$

where $q(x, t)$ and $\psi(x, t)$ are the complex envelop of the the electric field. x and t are the propagation distance and time depends variables, respectively. The parameters a accounts for normal or anomalous GVD at the carrier frequency, b and c are spatiotemporal coefficient and nonlinearity term, respectively. γ accounts for the third-order dispersion (TOD), λ and ν denotes the self-steepening and self frequency shift, respectively, and σ is a coupling parameter. Therewith, the Kerr nonlinearity term in the set of equations (2) and (3) comes from the fact that a light wave in an optical fiber fronts nonlinear effects owing to the nonharmonic movement of electrons coming from an external electric field.

The model of PNLSHE will help to obtain an optical pulse through an optical fiber and will provoke the nonlinear birefringence. Moreover, this event will be used to eliminate low-intensity socle occurring when pulses are squeezed by utilizing a fiber-grating supercharger [25]. The nonlinear birefringence produced by an intense pulse can aid to modify the shape of the resulting pulse, even in lake of a pump pulse. This is justified in view of that during its transmission via a combination of fiber and polarizer mostly belong on the intensity. It is important that fibers deliver light without modifying their condition of polarization. Those fibers are called polarization-preserving or polarization-retention fibers. Nowadays, mixed polarization solitons have gained a lot of attention in nonlinear fiber optics. It became possible to look for closely exact solutions to the couple of PNLHSE, which describes wave propagation along an optical fiber.

To achieve the main goal of this study, Section 3 employs a transformation hypothesis to equations (2) and (3). Also, two integration algorithms such as the auxiliary equation and the modified auxiliary equation methods are apllied, which will drive to optical solitons.

3 Soliton-like solutions

To unearth soliton-like solutions to the set of equations (2) and (3), this section presents traveling-wave hypothesis to obtain nonlinear ordinary equation of the perturbed NLSHE. The followings are the expressions of the wave solution:

$$\psi(x, t) = \phi_1(\xi)e^{i[\theta(x,t)]}, \tag{4}$$

$$q(x,t) = \phi_2(\xi)e^{i[\theta(x,t)]}, \tag{5}$$

where $\xi = x - vt$ and $\theta(x,t) = -\kappa x + \omega t + \theta_0$. v is a speed of the wave, κ is the wave number, ω represents the frequency of the soliton and θ_0 is the phase constant.

Inserting equations (4) and (5) into equations (2) and (3) gives the real parts:

$$\begin{aligned}&(b\kappa\omega - \omega - \kappa - a\kappa^2 - \kappa^3\gamma)\phi_2 + (a - bv + 3\kappa\gamma)\phi_2''\\&+ (c - 3\kappa\lambda - 2v)\phi_1^3 + \kappa\sigma\phi_1^2\phi_2 = 0,\end{aligned} \tag{6}$$

$$\begin{aligned}&(b\kappa\omega - \omega - \kappa - a\kappa^2 - \kappa^3\gamma)\phi_1 + (a - bv + 3\kappa\gamma)\phi_1''\\&+ (c - 3\kappa\lambda - 2v)\phi_2^3 + \kappa\sigma\phi_2^2\phi_1 = 0,\end{aligned} \tag{7}$$

thence, the imaginary parts are as follows:

$$\begin{aligned}&(-v - 2a\kappa + b\kappa v + b\omega - 3\kappa^2\sigma - 1)\phi_2'\\&- (3\lambda + 2v)\phi_1^2\phi_1' + \sigma\phi_1^2\phi_2' + \gamma\phi_1''' = 0.\end{aligned} \tag{8}$$

$$\begin{aligned}&(-v - 2a\kappa + b\kappa v + b\omega - 3\kappa^2\sigma - 1)\phi_1'\\&- (3\lambda + 2v)\phi_2^2\phi_2' + \sigma\phi_2^2\phi_1' + \gamma\phi_2''' = 0.\end{aligned} \tag{9}$$

To unify the expressions of equations (6)–(9), we set $i = 1, 2$ and $j = 1, 2$. So, equations (6)–(9) drop to the following expression:

$$\begin{aligned}&(b\kappa\omega - \omega - \kappa - a\kappa^2 - \kappa^3\gamma)\phi_i + (a - bv + 3\kappa\gamma)\phi_i''\\&+ (c - 3\kappa\lambda - 2v)\phi_j^3 + \kappa\sigma\phi_j^2\phi_i = 0,\end{aligned} \tag{10}$$

and the imaginary parts are as follows:

$$\begin{aligned}&(-v - 2a\kappa + b\kappa v + b\omega - 3\kappa^2\sigma - 1)\phi_i'\\&- (3\lambda + 2v)\phi_j^2\phi_1' + \sigma\phi_i^2\phi_j' + \gamma\phi_j''' = 0.\end{aligned} \tag{11}$$

Then, the two cases are presented here.

Case 1. Suppose $i \neq j \Rightarrow \phi_i \neq \phi_j$, later from equation (11), the restrictions are made as follows:

$$\lambda = -\frac{2}{3}v, \tag{12}$$

$$\gamma = 0, \tag{13}$$

$$\sigma = 0, \tag{14}$$

$$v = \frac{b\omega - 2a\kappa - 1}{1 - b\kappa}, \quad b\kappa \neq 1. \tag{15}$$

Consequently, equation (10) becomes

$$l_0\phi_i + l_1\phi_i'' + l_2\phi_j^3 = 0, \tag{16}$$

$l_0 = (b\kappa\omega - \omega - \kappa - a\kappa^2)$, $l_1 = (a - bv)$, $l_2 = c + 2v(\kappa - 1)$.

Inspecting equation (16), it depends on the coefficients GVD and Kerr nonlinearity related to the self-phase modulation (SPM), which can produce an important phenomenon on the pulse propagating along the optical fibers such as cross-phase modulation. Suppose analytical solution of equation (16) can be expressed as follows [2]:

$$\phi_i = A_0 + \sum_{k=1}^{n} A_k(g(\xi))^k, \tag{17}$$

$$\phi_j = B_0 + \sum_{k=1}^{n} B_k(g(\xi))^k, \tag{18}$$

and $g(\xi)$ fulfills the auxiliary equations given as follows:

$$g_\xi = \sqrt{2(C_0 + C_1g + C_2g^2 + C_3g^3 + C_4g^4)}, \tag{19}$$

$$g_{\xi\xi} = C_1 + 2C_2g + 3C_3g^2 + 4C_4g^3. \tag{20}$$

By applying the homogeneous balance principle between ϕ_i'' and ϕ_j^3 in equation (11), it is revealed $N_i = N_j = 1$.

$$\phi_i = A_0 + A_1g(\xi), \tag{21}$$

$$\phi_j = B_0 + B_1g(\xi). \tag{22}$$

Substituting equations (16) and (17) into equation (11) and used together with equations (14) and (15), it follows the system of equation expressed in terms of $(g(\xi))^k$:

$$\begin{aligned}(g(\xi))^3 &: \quad 4l_1A_1C_4 + l_2B_1^3 = 0,\\ (g(\xi)))^2 &: \quad 3l_1A_1C_3 + 3l_2B_0B_1^2,\\ (g(\xi))^1 &: \quad l_0A_1 + 2l_1A_1C_2 + 3l_2B_0^2B_1 = 0,\\ (g(\xi))^0 &: \quad l_0A_0 + l_1A_1C_1 + l_2B_0^3 = 0.\end{aligned}$$

By using MAPLE, we obtian the following:

Result 1

$A_0 = 0,\ A_1 = A_1,\ B_0 = 0,\ B_1 = -\frac{2\kappa\sigma A_1B_0(\kappa\sigma B_0^2 + l_0)}{l_2l_0},\ C_2 = -\frac{1}{2}\frac{l_0}{l_1},$ $C_4 = -\frac{1}{4}\frac{l_2B_1^3}{l_1A_1}.$

By using Result 1, the localized solutions are constructed to equations (2) and (3), which are as follows:

(i): $C_0 = C_1 = C_3 = 0$, for $C_2 > 0$ and $C_4 < 0$

$$\psi_{1,1}(x,t) = \left\{A_1\sqrt{\frac{-C_2}{C_4}}\operatorname{sech}(\sqrt{2C_2}(x - vt))\right\}e^{i\theta(x,t)}. \tag{23}$$

$$q_{1,1}(x,t) = \left\{B_1\sqrt{\frac{-C_2}{C_4}}\operatorname{sech}(\sqrt{2C_2}(x - vt))\right\}e^{i\theta(x,t)}. \tag{24}$$

(2i): $C_0 = \frac{C_2^2}{4C_4}$, and $C_1 = C_3 = 0$, for $C_2 < 0,\ \ C_4 > 0$

$$\psi_{1,2}(x,t)=\left\{A_1\sqrt{\frac{-C_2}{2C_4}}\ \tanh\left(\sqrt{-C_2}(x-vt)\right)\right\}e^{i\theta(x,t)}. \quad (25)$$

$$q_{1,2}(x,t)=\left\{B_1\sqrt{\frac{-C_2}{2C_4}}\ \tanh\left(\sqrt{-C_2}(x-vt)\right)\right\}e^{i\theta(x,t)}. \quad (26)$$

Result 2

$A_0=-\frac{l_2B_0^3}{l_0}$, $A_1=-\frac{l_2B_0B_1^2}{l_1C_3}$, $B_0=B_0$, $B_1=B_1$, $C_2=\frac{1}{2}\frac{-l_0B_1+3B_0l_1C_3}{B_1l_1}$, $C_4=\frac{1}{4}\frac{B_1C_3}{B_0}$.

(3i): $C_0=C_1=0$, for $C_2>0$ and $C_4>0$, combined bright-dark optical solitons to SHE is revealed as follows:

$$\psi_{1,3}(x,t)=\left\{A_0+A_1\frac{C_2\mathrm{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{2\sqrt{C_2C_4}\ \tanh\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)-C_3}\right\}\times e^{i\theta(x,t)}, \quad (27)$$

$$q_{1,3}(x,t)=\left\{B_0+B_1\frac{C_2\mathrm{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{2\sqrt{C_2C_4}\ \tanh\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)-C_3}\right\}e^{i\theta(x,t)}. \quad (28)$$

(4i): $C_0=C_1=0$, for $C_2>0$ and $C_3^2-4C_2C_4>0$

$$\psi_{1,4}(x,t)=\left\{A_0+A_1\frac{2C_2\mathrm{sech}\left(\sqrt{2C_2}(x-vt)\right)}{\sqrt{C_3^2-4C_2C_4}-C_3\mathrm{sech}\left(\sqrt{2C_2}(x-vt)\right)}\right\}\times e^{i\theta(x,t)}, \quad (29)$$

$$q_{1,4}(x,t)=\left\{B_0+B_1\frac{2C_2\mathrm{sech}\left(\sqrt{2C_2}(x-vt)\right)}{\sqrt{C_3^2-4C_2C_4}-C_3\mathrm{sech}\left(\sqrt{2C_2}(x-vt)\right)}\right\}\times e^{i\theta(x,t)}. \quad (30)$$

(5i): $C_0=C_1=0$, for $C_2>0$ and it is revealed that

$$\psi_{1,5}(x,t)=\left\{A_0+A_1\frac{C_2C_3\mathrm{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{C_2C_4\left(1-\ \tanh\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)\right)^2-C_3^2}\right\}\times e^{i\theta(x,t)}. \quad (31)$$

$$q_{1,5}(x,t)=\left\{B_0+B_1\frac{C_2C_3\mathrm{sech}^2\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)}{C_2C_4\left(1-\ \tanh\left(\sqrt{2C_2}\frac{(x-vt)}{2}\right)\right)^2-C_3^2}\right\}\times e^{i\theta(x,t)}, \quad (32)$$

where $\theta(x,t)=-\kappa x+\omega t+\theta_0$.

Case 2. Suppose $i=j\Leftrightarrow\phi_i=\phi_j=\phi$, consequently equations (10) and (11) turn to

$$(b\kappa\omega-\omega-\kappa-a\kappa^2-\kappa^3\gamma)\phi+(a-bv+3\kappa\gamma)\phi''+(c-3\kappa\lambda-2v+\kappa\sigma)\phi^3=0, \quad (33)$$

$$(-v-2a\kappa+b\kappa v+b\omega-3\kappa^2\sigma-1)\phi'-(3\lambda+2v)\phi^2\phi'+\sigma\phi^2\phi'+\gamma\phi'''=0. \quad (34)$$

After integration of equation (34) and taking zero as the value of the integration constant:

$$l_0\phi+l_3\phi^3+\gamma\phi''=0, \quad (35)$$

where $l_0=-v-2a\kappa+b\kappa v+b\omega-3\kappa^2\sigma-1$; $l_3=-\frac{1}{3}(\sigma+3\lambda+2v)$.

In this study, we take into account the TOD, and then the low GVD appears without much effect in front of it. So the balance between nonlinearity and (TOD) dispersion can probably lead to stable soliton solutions.

In general, the TOD becomes more important than the GVD when the dispersion shift is considered in the fiber [26]. Without doubt, taking into account the TOD parameter could give another flavor to the results. It is certain that the expected results (solitons) can guide the dimension to be done on the optic fibers, thus for more adequate applications.

To adopt the traveling-wave solution to equations (28) and (30), the following whole series form is used:

$$\phi(\xi)=A_0+\sum_{i=1}^{n}(A_iK^{if(\xi)}+B_iK^{-if(\xi)}), \quad (36)$$

where A_1, B_i and K are arbitrary constants and $f(\xi)$ satisfies the following ODE:

$$f'(\xi)=\frac{\beta+\alpha K^{-f(\xi)}+\mu K^{f(\xi)}}{\ln(K)}, \quad (37)$$

where α, β and μ are reals constants to be determined, with $K>0$ and $K\neq1$ [6].

Taking into account the tenet on ϕ^3 and ϕ'' in equations (28) and (30), lets glean $N_1=N_2=1$. Hence,

$$\phi(\xi)=A_0+A_1K^{f(\xi)}+B_1K^{-f(\xi)}, \quad (38)$$

Substitute equations (33) and (32) into equations (28) and (30), the system of equations is obtained:

$$(K^f(\xi))^6:\ \ 2\gamma\mu^2A_1+A_1^3l_3=0,$$

$$(K^f(\xi))^5:\ \ 3\beta\gamma\mu A_1+3A_0A_1^2l_3=0,$$

$$(K^f(\xi))^4: \quad 2\alpha\gamma\mu A_1 + \beta^2\gamma A_1 + 3{A_0}^2 A_1 l_3 + 3{A_1}^2 B_1 l_3 + A_1 l_0 = 0,$$

$$(K^f(\xi))^3: \quad \alpha\beta\gamma A_1 + \beta\gamma\mu B_1 + A_0^3 l_3 + 6A_0A_1B_1l_3 + A_0 l_0 = 0,$$

$$(K^f(\xi))^2: \quad 2\alpha\gamma\mu B_1 + \beta^2\gamma B_1 + 3A_0^2 B_1 l_3 + 3A_1 B_1^2 l_3 + B_1 l_0 = 0,$$

$$(K^f(\xi))^1: \quad 3\alpha\beta\gamma B_1 + 3A_0 B_1^2 l_3 = 0,$$

$$(K^f(\xi))^0: \quad 2\gamma B_1 \alpha^2 + l_3 B_1^3 = 0.$$

Using MAPLE as a calculation tool, the following results emerge.

Set 1: $A_0 = \sqrt{\frac{l_0}{l_3(4\alpha\mu - \beta^2)}}\beta$, $A_1 = \frac{2\alpha A_0}{\beta}$, $\gamma = -\frac{2l_0}{4\alpha\mu - \beta^2}$.

Set 2: $A_0 = \sqrt{\frac{l_0}{l_3(4\alpha\sigma - \beta^2)}}\beta$, $B_1 = \frac{2\sigma A_0}{\beta}$, $\gamma = -\frac{2l_0}{4\alpha\mu - \beta^2}$.

By using Set 1, it is gained to equations (2) and (3),

(6i): For $\beta^2 - 4\alpha\mu < 0$ and $\mu \neq 0$, we gain trigonometric functions solutions

$$\psi_{2,0}(x,t) = \left\{A_0 + A_1\left(\frac{-\beta + \sqrt{4\alpha\mu - \beta^2}\tan\left(\sqrt{4\alpha\mu - \beta^2}\,\frac{(x - vt)}{2}\right)}{2\mu}\right)\right\} \times e^{\theta(x,t)}. \tag{39}$$

or

$$\psi_{2,1}(x,t) = \left\{A_0 + A_1\left(-\frac{\sqrt{4\alpha\mu - \beta^2}\cot\left(\sqrt{4\alpha\mu - \beta^2}\,\frac{(x - vt)}{2}\right) + \beta}{2\mu}\right)\right\} \times e^{i\theta(x,t)}. \tag{40}$$

(7i): For $\beta^2 - 4\alpha\mu > 0$ and $\mu \neq 0$, we gain the following soliton solutions:

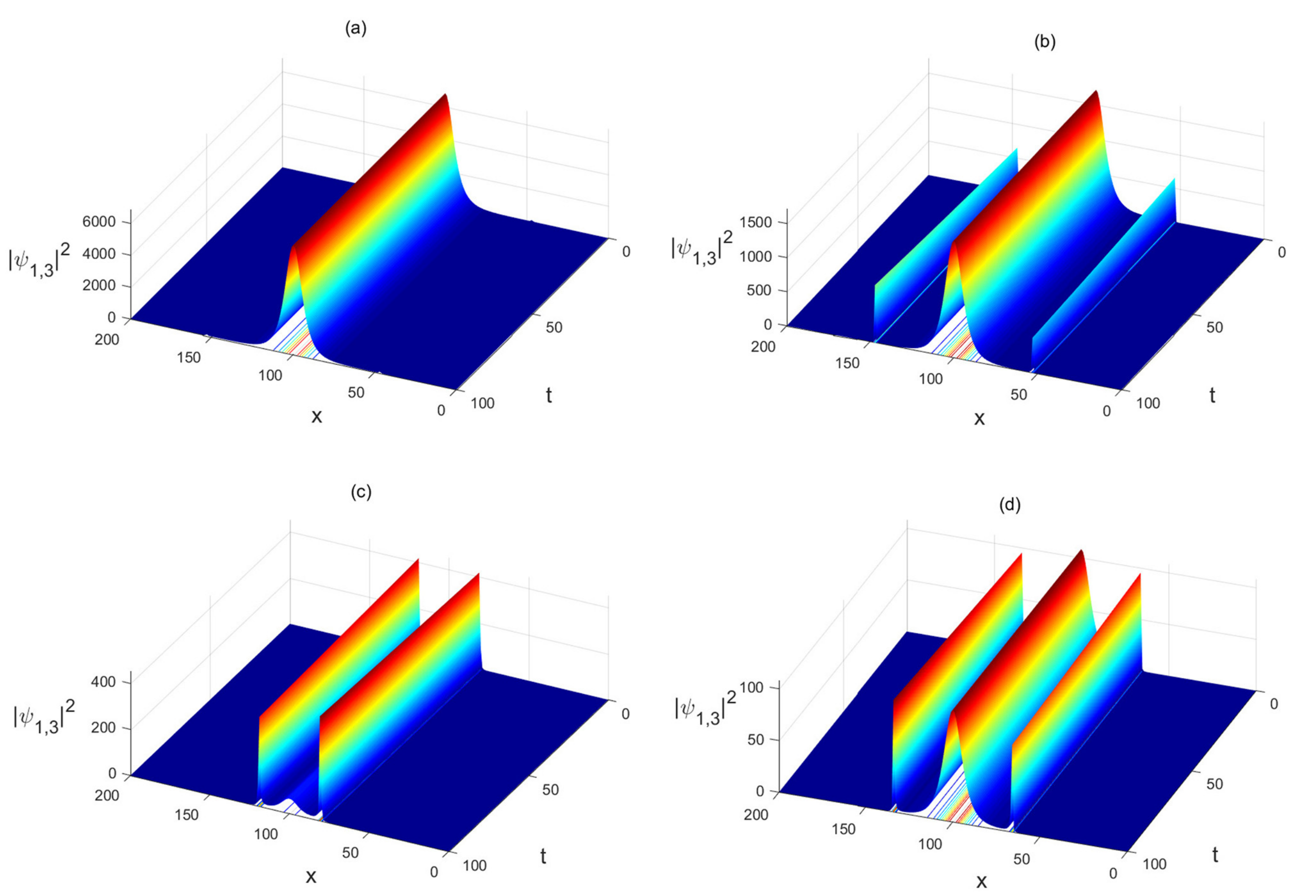

Figure 1: The plot of the bright solitons $|\psi_{1,3}(x,t)|^2$ at (a) $C_2 = 0.0448$, $C_3 = 0.2825$, $C_4 = 0.6799$, $A_0 = 1.147 \times 10^{-7}$, $A_1 = 20.0008$, $a = 1$, $b = 0.0152$, $v = 0.00015$; (b) $C_2 = 0.0478$, $C_3 = 0.0564$, $C_4 = 0.1088$, $A_0 = 5.7287 \times 10^{-7}$, $A_1 = 20.0008$, $a = 1$, $b = 0.0152$; (c) $C_2 = 0.0448$, $C_3 = 0.2825$, $C_4 = 0.6799$, $A_0 = 1.147 \times 10^{-7}$, $A_1 = 20.0008$, $a = 1$, $b = 0.0152$, $v = 0.0025$; and (d) $C_2 = 0.0447$, $C_3 = 0.0071$, $C_4 = 0.0017$, $A_0 = 1.147 \times 10^{-7}$, $A_1 = 20.0008$, $a = 1$, $b = 0.0152$, $v = 0.0075$, respectively.

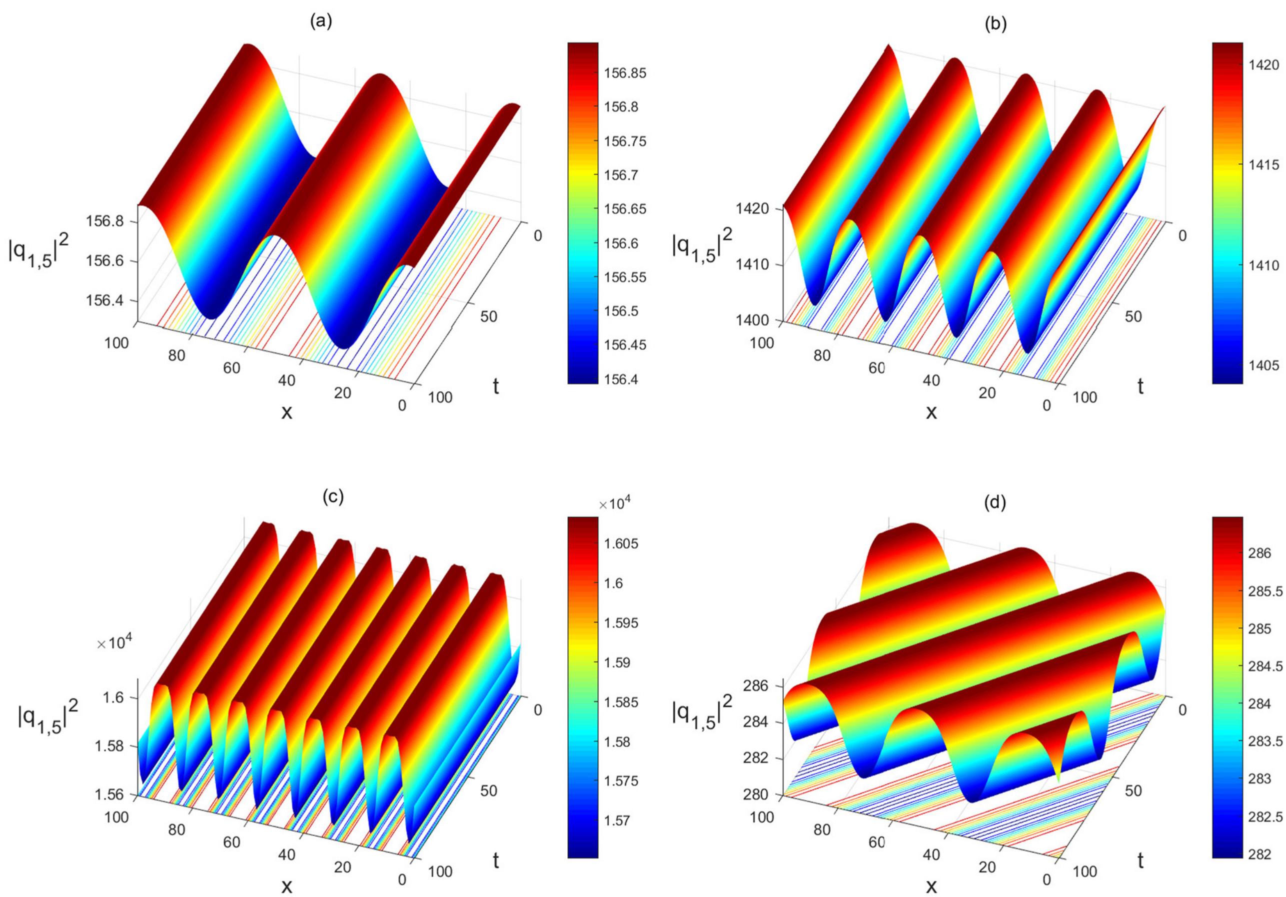

Figure 2: The plot of analytical solutions $|\psi_{1,5}(x,t)|^2$ at (a) $C_2 = 0.0075$, $C_3 = 0.0053$, $C_4 = 0.0017$, $A_0 = -5.9001\times10^{-5}$, $A_1 = 10.008$, $B_0 = 0.0021$, $B_1 = 0.8001$, $v = 0.0075$; (b) $C_2 = 0.0075$, $C_3 = 0.0053$, $C_4 = 0.0017$, $A_0 = -5.9001\times10^{-5}$, $A_1 = 10.008$, $B_0 = 0.0021$, $B_1 = 0.8001$, $v = 0.075$; (c) $C_2 = 0.0075$, $C_3 = 0.0053$, $C_4 = 0.0017$, $A_0 = -5.9001\times10^{-5}$, $A_1 = 10.008$, $B_0 = 0.0021$, $B_1 = 0.8001$, $v = 1.075$; and (d) $C_2 = 0.0075$, $C_3 = 0.0053$, $C_4 = 0.0017$, $A_0 = -5.9001\times10^{-5}$, $A_1 = 10.008$, $B_0 = 0.0021$, $B_1 = 0.8001$, $v = 1.75$, respectively.

$$\psi_{2,2}(x,t) = \left\{A_0 + A_1\left(-\frac{\sqrt{-4\alpha\mu+\beta^2}\tanh\left(\sqrt{-4\alpha\mu+\beta^2}\frac{(x-vt)}{2}\right)+\beta}{\mu}\right)\right\} \times e^{i\theta(x,t)} \quad (41)$$

or bright soliton solutions

$$\psi_{2,3}(x,t) = \left\{A_0 + A_1\left(-\frac{\sqrt{-4\alpha\mu+\beta^2}\coth\left(\sqrt{-4\alpha\mu+\beta^2}\frac{(x-vt)}{2}\right)+\beta}{2\sigma}\right)\right\} \times e^{i\theta(x,t)}. \quad (42)$$

(8i): For $\beta^2 - 4\alpha\mu = 0$ and $\mu \neq 0$, the following solution is recovered:

$$\psi_{2,4}(x,t) = \left\{A_0 + A_1\left(-\frac{\sqrt{\alpha\mu}\,(x-vt)+1}{\mu(x-vt)}\right)\right\}e^{i\theta(x,t)}. \quad (43)$$

From Set 2, the following solitary wave solutions are obtained.

(9i): For $\beta^2 - 4\alpha\mu < 0$ and $\mu \neq 0$, the following trigonometric function solutions are obtained:

$$\psi_{2,11}(x,t) = \left\{A_0 + \frac{2B_1\mu}{-\beta+\sqrt{4\alpha\mu-\beta^2}\tan\left(\sqrt{4\alpha\mu-\beta^2}\frac{(x-vt)}{2}\right)}\right\} \times e^{i\theta(x,t)}, \quad (44)$$

or

$$\psi_{2,22}(x,t) = \left\{A_0 - \frac{2B_1\mu}{\beta+\sqrt{4\alpha\mu-\beta^2}\cot\left(\sqrt{4\alpha\mu-\beta^2}\frac{(x-vt)}{2}\right)}\right\} \times e^{i\theta(x,t)}. \quad (45)$$

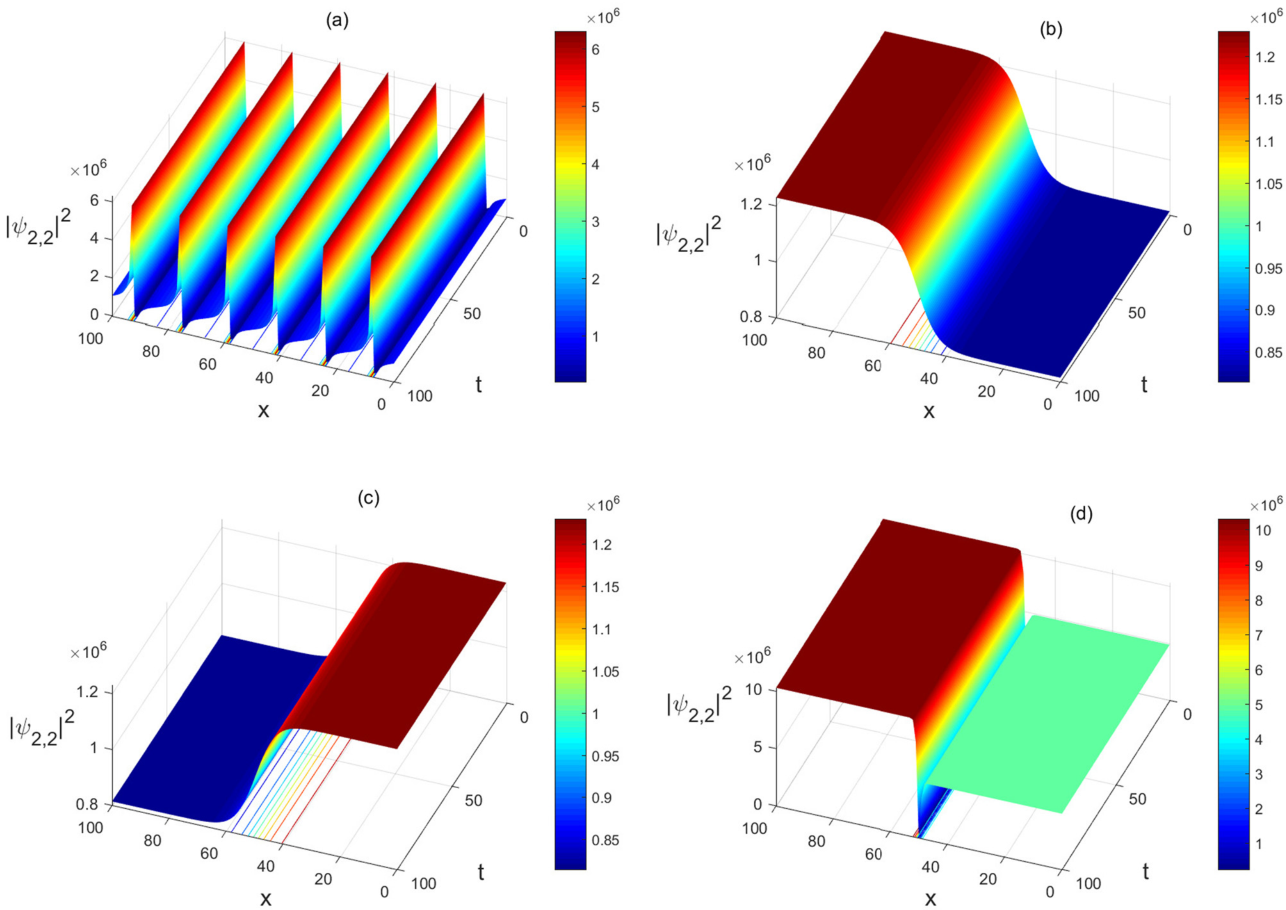

Figure 3: Depict soliton solution of $|\psi_{2,2}(x,t)|^2$ at (a) $\alpha = -2.007$, $\beta = 2.7$, $A_0 = 0.0402$, $A_1 = 370.677$; (b) $\alpha = 2.00072$, $\beta = 0.59$, $A_0 = 0.0402$, $A_1 = 370.677$; (c) $\alpha = 2.00072$, $\beta = 0.51$, $A_0 = 0.0402$, $A_1 = 370.677$; and (d) $\alpha = 2.007$, $\beta = 0.59$, $A_0 = 0.0402$, $A_1 = 370.677$, respectively.

(10i): For $\beta^2 - 4\alpha\mu > 0$ and $\mu \neq 0$, we obtian uncovered dark soliton solutions:

$$\psi_{2,33}(x,t) = \left\{A_0 - \frac{2B_1\mu}{\sqrt{-4\alpha\mu+\beta^2}\,\tanh\left(\sqrt{-4\alpha\mu+\beta^2}\,\frac{(x-vt)}{2}\right)+\beta}\right\} \times e^{i\theta(x,t)}, \tag{46}$$

or bright soliton solutions

$$\psi_{2,44}(x,t) = \left\{A_0 - \frac{2B_1\mu}{\sqrt{-4\alpha\mu+\beta^2}\,\coth\left(\sqrt{-4\alpha\mu+\beta^2}\,\frac{(x-vt)}{2}\right)+\beta}\right\} \times e^{i\theta(x,t)}. \tag{47}$$

(11i): For $\beta^2 - 4\alpha\mu = 0$ and $\mu \neq 0$, the following solutions are obtained:

$$\psi_{2,55}(x,t) = \left\{A_0 - \frac{B_1\mu(x-vt)}{\sqrt{\alpha\mu}\,(x-vt)+1}\right\}e^{i\theta(x,t)}. \tag{48}$$

4 Results and discussion

Figure 1 shows a graphical illustration of the bright soliton (Figure 1(a)) and the one that seem to be like bright solitons (Figure 1(b–d)). Figure 2 shows the fusion bright and dark solitons. The bright soliton is known as the first-order soliton, which is concerning by a balance effect producing by the second-order dispersion (GVD) and Kerr nonlinearity (SPM) in an anomalous regime. To suit this result, the third-order dispersion is negligible ($\gamma = 0$) and that circumscribes better the details obtained on constraint relating to these parameters in equation (13).

Definitively it is emerged that the predictions done on the boundary conditions of an amplitude, which must not tend toward a zero value, are illustrated through the obtained graphically solutions (Figures 1 and 2). By adopting the modified auxiliary equation, it revealed multiple kink, anti-kink, kink and double kink-like soliton solutions (Figure 3(a–d)). Moreover, the obtained analytical results plotted in Figure 1 illustrate one, two and three optical solitons like optical solitons moleculesr.

5 Conclusion

The main aim of this article is to obtain optical solitons that could satisfy the constraint conditions posed on the different parameters of the PNLSHE. Hence, the third-order dispersion term was initially considered negligible (i.e., $\gamma = 0$). Bright and dark optical solitons have been successfully obtained. However, the search for these results took into account two important factors, namely, the second-order dispersion (GVD) and the Kerr nonlinearity, which gave rise to SPM. To consolidate the results obtained, the TOD, the SPM and the GVD were taken into account. Thus, relevant results such as multiple kink-like solitons solutions, kink, anti-kink like soliton solutions and double kink-like soliton have emerged. Compared to refs. [4,16,17], the obtained results point out the behavior of an optical soliton in the absence of TOD and also the valuable effect of TOD dispersion. Besides, two and three optical solitons emerge by adopting the auxiliary equation method. Without incertitude, these results will have physical explaination in the context of soliton molecules. In the feature, we will be more interested in birefringence aspect and cross-phase modulation to build soliton pulses compression and ultrashort optical pulses. For applications of the fractional differential equations, the readers can refer to refs. [30–39].

We need to investigate some new type optical solitons and modulation instability analysis of some fractional NLSE type equations in the future.

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