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Dynamical Behavior and Sensitivity Analysis of a Delayed Coronavirus Epidemic Model

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Abstract: Mathematical delay modelling has a significant role in the different disciplines such as behavioural, social, physical, biological engineering, and bio-mathematical sciences. The present work describes mathematical formulation for the transmission mechanism of a novel coronavirus (COVID-19). Due to the unavailability of vaccines for the coronavirus worldwide, delay factors such as social distance, quarantine, travel restrictions, extended holidays, hospitalization, and isolation have contributed to controlling the coronavirus epidemic. We have analysed the reproduction number and its sensitivity to parameters. If, $\mathbf{R_{covid}} < \mathbf{1}$ then this situation will help to eradicate the disease and if, $\mathbf{R_{covid}} > \mathbf{1}$ the virus will spread rapidly in the human beings. Well-known theorems such as Routh Hurwitz criteria and Lasalle invariance principle have presented for stability. The local and global stabilizes for both equilibria of the model have also been presented. Also, we have analysed the effect of delay reason on the reproduction number. In the last, some very useful numerical consequences have presented in support of hypothetical analysis.

Keywords: Coronavirus (COVID-19), delay mathematical model, reproduction number, sensitive analysis, stability analysis.

1 Literature survey

Human beings are mostly given the taste to master the environment of which they are apart. To control the environment, he has devised many ways and tools. However, he has restrained to some extent. These restrictions have saved him from demolishing and

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violating the laws of nature. The most powerful forces among these are water, air, and soil. Abiding by the laws of nature, he can make his environment worth living for all people and other species on earth. For this purpose, man has introduced many deadly tools, weapons, and technical instruments. He also has discovered diseases, their origin, and their cure. He has been successful in coping with these disasters so far, but violating the laws of nature has paved way too many fatal disasters and epidemics. The first and second World Wars are vivid examples of killing and destruction. Self-supremacy and Xenophobia are the reasons behind this disaster. Fear, hatred, and insecurity have compelled them to violate the idea of cohabitation and dwelling in societies. Nationalism is a great step in this respect. With the spread of this idea, people have overcome Xenophobia and other such disasters. They have provided strength and security under the title of nationality. But they have failed in conceiving the idea that man is mortal. He can no longer stay with any concept in this word. He simply has to follow the laws of nature. Briefly speaking, in past decades, a vast amount of people died who were overindulged in sexual activities. Their violation from nature destroyed them thoroughly. It is a fact that nature does not allow us to eat and drink all types of fruit, vegetables, and drinks. It doesn't even allow us to eat all types of animals and insects because they are harmful to human beings. We cannot interact with every creature. Nature has its lawful course for interaction. When we try to use nature against its course, we become prey to many deadly diseases like HIV, Ebola, Congo fever, Lassa fever, and Dengue, etc. All these diseases spread due to eating rats, bats, and interacting with other animals and insects. Presently, the world is suffering from coronavirus (COVID-19). This deadly virus has killed a massive amount of people in China, Italy, America, Iran, Pakistan, and many other countries in the world. It transmits from man to man and directly hits the respiratory system. People have tried to develop techniques and cure to save from this deadly virus but not to enough level. This virus is still causing deaths and destruction in Europe, Asia, and other continents. In the present paper, we are trying to give its mathematical data. In the last month of 2019, there were some cases that appeared of pneumonia with unknown origin in the capital city of province Hubei Wuhan, China. The number of people has died due to this fatal coronavirus all over the world. More than two hundred and six countries have convicted of the new coronavirus. Now the pandemic of coronavirus is a global issue announced by the World Health Organization (WHO). It was like pneumonia without clear symptoms. All techniques and measures proved ineffective. Further, it transmitted from man to man. It appears within ten days and is causing panic in Europe, North Africa, America, and Asia till now. The roundabout figure of casualties has reached up to 50,000. Tahir et al. [Tahir, Shah, Zaman et al. (2019)] have given an analysis of the deterministic model of the middle east respiratory syndrome (MERS) coronavirus. Zhao et al. [Zhao and Chen (2020)] have discussed the mathematical model of the outbreak of coronavirus in China. Shim et al. [Shim, Tariq, Choi et al. (2020)] have found the transmission of coronavirus in South Korea by using static analysis. Kucharski et al. [Kucharski, Russell, Diamond et al. (2020)] have presented optimal control strategies in the modeling of coronavirus. Jiang et al. [Jiang, Coffee, Bari et al. (2020)] have found data prediction of coronavirus by using artificial intelligence structure. Li et al. [Li, Chao and Zhang (2019)] have presented the modeling of emotion classification based on brain wave. Wang et al. [Wang, Li, Zou et al. (2020)] have found the classical approaches in the net modeling of images. Shereen et al. [Shereen, Khan, Kazmi et al. (2020)] have presented features, origin, and transmission of coronaviruses. Yang et al. [Yang and Wang (2020)] have found compartments modeling for the novel coronavirus epidemic in Wuhan, China. Lin et al. [Lin, Zhao, Gao et al. (2020)] have presented the conceptual model of coronavirus in people and government reactions Mathematical modeling has an effective tool to study the dynamics of coronavirus model. Raza et al. [Raza, Rafiq, Baleanu et al. (2019)] have found a computational analysis of the stochastic HIV/AIDS model in the two sex populations. Arif et al. [Arif, Raza, Rafiq et al. (2019)] have presented the stochastic analysis of the hepatitis B virus with the migration effect of humans. Abodayeh et al. [Abodayeh, Arif, Raza et al. (2020)] investigated the dynamics of the stochastic foot and mouth disease in the animal population. In this analysis, we have derived reproduction several COVID-19, the given reproduction number has a significant role in the nonlinear dynamics, biological engineering, and many more. If the reproduction number is less than one, than its mean COVID-19 has controlled or controlling strategies are effective. Otherwise, if the reproduction number is greater than one than its mean COVID-19 has fluently increased and the virus is endemic. Actually, in this model, we have introduced delay reason. The role of delay reason is quarantine or place of isolation or vaccination etc. In common epidemiological models, if we controlled infection rate then disease becomes stable or control. In the current situation of COVID-19, we can't control infection or transmission rates of viruses. So, we just used the delay tactics to overcome the pandemic of corona virus-like as social distancing, quarantine, isolation, etc. Fortunately, the delay factors or delay tactics in the modeling are independent of all other types of transmissions rates. Overall world, the only control strategies of COVID-19 are social distancing and isolations, etc. For the importance or impact of delay reason, we have introduced the delay differential equations model from the biological engineering and nonlinear dynamical problems.

The strategy of our paper is as follows: In Section 2, we have discussed the formulation, the equilibrium of the model. In Section 3, we discussed the local stability of the model. In Section 4, we have discussed the global stability of the model. In Section 5, we discussed numerical consequences for the support of the theoretical analysis of the model. In Section 6, conclusion and future guidance have presented.

2 Formulation of model

In this paper, we have considered the dynamics of coronavirus pandemic model with the seafood market versus humans. The whole population has represented with $N_p(t)$ and divided into the five compartments as follows: For any time t, the susceptible humans presented with $S_p(t)$, exposed humans presented with $E_p(t)$, symptomatic infected humans presented with $I_p(t)$, asymptomatic infected presented with $A_p(t)$ and recovered humans presented with $R_p(t)$. The seafood market (reservoir) has represented with $M(t)$. Simply the dynamics of humans and reservoir have described through the nonlinear delay differential equations as shown in Fig. 1.

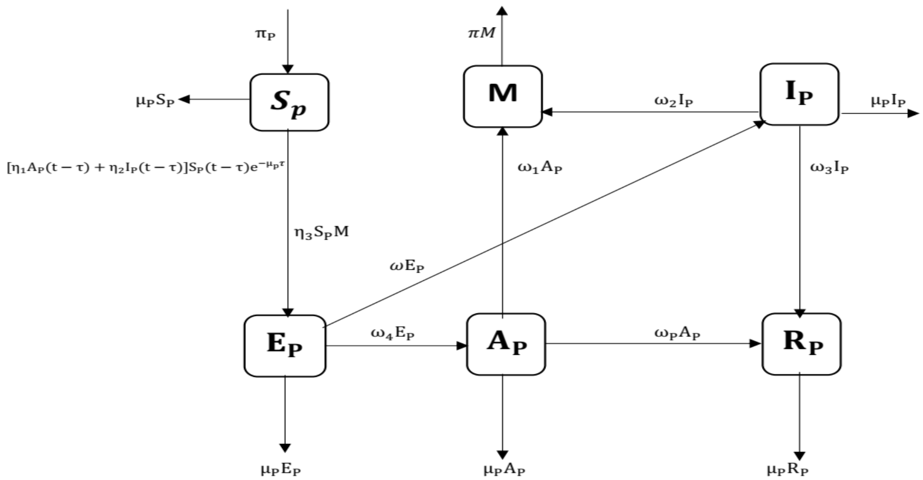


Figure 1: Flow map of COVID-19 delay model

The limits of the delay model have described as follows: π_p is the recruitment rate of humans, μ_p is the mortality rate with natural incidences or due to virus infection, η_1 is the virus getting a rate of susceptible humans from asymptomatic humans, η_2 is the virus getting a rate of susceptible humans from symptomatic humans, η_3 is interaction rate of susceptible humans with reservoir or seafood place or market, ω is the interaction rate of symptomatic infected and exposed humans, ω_4 is the interaction rate in which exposed humans becomes asymptomatic infected humans, ω_1 is the rate of asymptotic carriers who visit the seafood market, ω_2 is the rate of symptomatic carriers who visit the seafood market, ω_p is the rate of quarantine or isolation or vaccination of asymptomatic infected humans, ω_3 is the rate of quarantine or isolation or vaccination of symptomatic infected humans and π is the rate at which virus removed from the seafood market. The coronavirus pandemic model has based on the following assumptions: the seafood market is enough source of the virus, considering two ways of dispersion of virus as symptomatic and asymptomatic carriers who visit the seafood place or market and the interaction rate of susceptible humans with the seafood market. Without loss of generality, all types of other interactions with the seafood market have ignored. After getting the virus from the seafood market, asymptomatic and symptomatic carriers, the susceptible humans can have interaction with other human compartments. The system of delay differential equations of the model as follows:

$$\frac{dS_p}{dt} = \pi_p - \left(\eta_1 A_p(t-\tau) + \eta_2 I_p(t-\tau)\right) S_p(t-\tau)e^{-\mu_p\tau} - \eta_3 S_p(t)M(t) - \mu_p S_p(t). \quad (1)$$

$$\frac{dE_p}{dt} = \left(\eta_1 A_p(t-\tau) + \eta_2 I_p(t-\tau)\right) S_p(t-\tau)e^{-\mu_p\tau} + \eta_3 S_p(t)M(t) - \omega E_p(t) - \omega_4 E_p(t) - \mu_p E_p(t). \quad (2)$$

$$\frac{dI_p}{dt} = \omega E_p(t) - \omega_3 I_p(t) - \mu_p I_p(t). \quad (3)$$

$$\frac{dA_p}{dt} = \omega_4 E_p(t) - \omega_p A_p(t) - \mu_p A_p(t). \quad (4)$$

$$\frac{dR_p}{dt} = \omega_p A_p(t) + \omega_3 I_p(t) - \mu_p R_p(t). \quad (5)$$

$$\frac{dM}{dt} = \omega_2 I_p(t) + \omega_1 A_p(t) - \pi M(t). \quad (6)$$

The initial conditions $\phi = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$ of Eqs. (1) to (5) are defined in the Banach space as $C_+ = \{\phi \epsilon C[-\tau, 0],\ R_+^5: \phi_1(0) = S_p(0), \phi_2(0) = E_p(0), \phi_3(0) = I_p(0), \phi_4(0) = A_p(0), \phi_5(0) = R_p(0)\}$,

where, $R_+^5 = \{S_p, E_p, I_p, A_p, R_p \epsilon R^5: S_p \geq 0, E_p \geq 0, I_p \geq 0, A_p \geq 0, R_p \geq 0\}$.

We assume $\phi_i(0) > 0, (i = 1,2,3,4,5)$ due to biological meanings. The total dynamics of Eqs. (1) to (6) has obtained by adding the first five equations as follows:

$\frac{dS_p}{dt} + \frac{dE_p}{dt} + \frac{dI_p}{dt} + \frac{dA_p}{dt} + \frac{dR_p}{dt} \leq \pi_p - \mu_p N_p$ and $S_p + E_p + I_p + A_p + R_p = N_p$.

$\frac{dN_p}{dt} \leq \pi_p - \mu_p N_p$.

The feasible region of Eqs. (1) to (6) as follows:

$\Gamma = \{S_p(t), E_p(t), I_p(t), A_p(t), R_p(t) \epsilon R_+^5: N_p(t) \leq \frac{\pi_p}{\mu_p}, M \epsilon R_+\}$.

The initial value problem, $\phi' = \pi_p - \mu_p \phi$, with $\phi(0) = N_p(0)$ has solution $\phi(t) = k_1 e^{-\mu_p t} + \frac{\pi_p}{\mu_p}$ and $\lim_{t \to \infty} \phi(t) = \frac{\pi_p}{\mu_p}$. Therefore, $N_p(t) \leq \phi(t)$ which shows that $\lim_{t \to \infty} Sup\ N_p(t) \leq \frac{\pi_p}{\mu_p}$. Thus, all solutions of Eqs. (1) to (6) lies in the feasible region Γ. The feasible region is positive and bounded for Eqs. (1) to (6). Hence, the region Γ is positive invariant.

2.1 Equilibrium points

The Eqs. (1) to (6) admit two equilibrium states in the feasible region Γ. A COVID free equilibrium of the models (1-6) as follows:

$C_1 = (S_p{}^1, E_p{}^1, I_p{}^1, A_p{}^1, R_p{}^1) = (\frac{\pi_p}{\mu_p}, 0,0,0,0)$.

Also, COVID present equilibrium of the Eqs. (1) to (6) as follows:

$C_2 = (S_p{}^*, E_p{}^*, I_p{}^*, A_p{}^*, R_p{}^*)$.

where,

$S_p{}^* = \frac{\pi(\omega_p+\mu_p)(\omega_3+\mu_p)(\omega+\omega_4+\mu_p)}{\omega(\omega_p+\mu_p)(\eta_2 \pi e^{-\mu_p \tau}+\eta_3\omega_2)+\omega_4(\omega_3+\mu_p)(\eta_1 \pi e^{-\mu_p \tau}+\omega_1)}$, $E_p{}^* = \frac{\pi_p - \mu_p S_p{}^*}{\omega+\omega_4+\mu_p}$, $I_p{}^* = \frac{\omega E_p{}^*}{\omega_3+\mu_p}$, $A_p{}^* = \frac{\omega_4 E_p{}^*}{\omega_p+\mu_p}$ and $R_p{}^* = \frac{\omega_p A_p{}^* + \omega_3 I_p{}^*}{\mu_p}$.

2.2 Reproduction number

The reproduction number find the extinction and persistence of the virus in the population. If $R_{covid} < 1$, then shows the extinction of viruses in population and $R_{covid} > 1$, then shows the persistence of virus in the population. Driekmann et al.

[Driekmann, Heesterbeek and Roberts (2009)] have presented the next-generation matrix method for compartment models. We have considered the infectious and recovered compartment from the Eqs. (1) to (6) and by using COVID free equilibrium as follows:

$$\begin{bmatrix} E_p' \\ I_p' \\ A_p' \\ R_p' \end{bmatrix} = \begin{bmatrix} 0 & \frac{\eta_2 e^{-\mu_p \tau}\pi_p}{\mu_p} & \frac{\eta_1 e^{-\mu_p \tau}\pi_p}{\mu_p} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_p \\ I_p \\ A_p \\ R_p \end{bmatrix} -$$

$$\begin{bmatrix} \omega + \omega_4 + \mu_p & 0 & 0 & 0 \\ -\omega & \omega_3 + \mu_p & 0 & 0 \\ -\omega_4 & 0 & \omega_p + \mu_p & 0 \\ 0 & -\omega_p & -\omega_3 & \mu_p \end{bmatrix} \begin{bmatrix} E_p \\ I_p \\ A_p \\ R_p \end{bmatrix}.$$

where, $F = \begin{bmatrix} 0 & \frac{\eta_2 e^{-\mu_p \tau}\pi_p}{\mu_p} & \frac{\eta_1 e^{-\mu_p \tau}\pi_p}{\mu_p} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and

$$V = \begin{bmatrix} \omega + \omega_4 + \mu_p & 0 & 0 & 0 \\ -\omega & \omega_3 + \mu_p & 0 & 0 \\ -\omega_4 & 0 & \omega_p + \mu_p & 0 \\ 0 & -\omega_p & -\omega_3 & \mu_p \end{bmatrix}.$$

$$FV^{-1} = \begin{bmatrix} \frac{[\eta_2 \omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]e^{-\mu_p\tau}\pi_p}{(\omega+\omega_4+\mu_p)(\omega_3+\mu_p)\mu_p} & \frac{\eta_2 e^{-\mu_p\tau}\pi_p}{(\omega_3+\mu_p)\mu_p} & \frac{\eta_1 e^{-\mu_p\tau}\pi_p}{(\omega_p+\mu_p)\mu_p} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The spectral radius of FV^{-1} is denoted as $R_{covid} = \frac{[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]\pi_p}{(\omega+\omega_4+\mu_p)(\omega_p+\mu_p)(\omega_3+\mu_p)\mu_p} e^{-\mu_p\tau}$.

2.3 Sensitivity analysis

To test the sensitivity of the reproduction number in each of its parameters:

$$A_{\eta_1} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \eta_1}{\eta_1}} = \frac{\eta_1}{R_{covid}} \frac{\partial R_{covid}}{\partial \eta_1} = \frac{\eta_1\omega_4(\omega_2+\mu_p)}{\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)} > 0.$$

$$A_{\eta_2} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \eta_2}{\eta_2}} = \frac{\eta_2}{R_{covid}} \frac{\partial R_{covid}}{\partial \eta_2} = \frac{\eta_2\omega(\omega_p+\mu_p)}{\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)} > 0.$$

$$A_{\omega_3} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \omega_3}{\omega_3}} = \frac{\omega_3}{R_{covid}} \frac{\partial R_{covid}}{\partial \omega_3} = -\frac{\eta_2\omega\omega_3(\omega_p+\mu_p)}{(\omega_3+\mu_p)[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]} < 0.$$

$$A_{\omega_p} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \omega_p}{\omega_p}} = \frac{\omega_p}{R_{covid}} \frac{\partial R_{covid}}{\partial \omega_p} = -\frac{\eta_1 \omega_p \omega_4(\omega_3+\mu_p)}{(\omega_p+\mu_p)[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]} < 0.$$

$$A_{\pi_p} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \pi_p}{\pi_p}} = \frac{\pi_p}{R_{covid}} \frac{\partial R_{covid}}{\partial \pi_p} = 1 > 0.$$

$$A_{\omega} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \omega}{\omega}} = \frac{\omega}{R_{covid}} \frac{\partial R_{covid}}{\partial \omega} = \frac{\eta_2\omega(\omega_p+\mu_p)}{\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)} - \frac{\omega}{\omega+\omega_4+\mu_p} > 0;$$

$$\eta_2\omega(\omega_p+\mu_p) > \omega.$$

$$A_{\omega_4} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \omega_4}{\omega_4}} = \frac{\omega_4}{R_{covid}} \frac{\partial R_{covid}}{\partial \omega_4} = -\frac{\omega_4\eta_2\omega(\omega_p+\mu_p)}{(\omega+\omega_4+\mu_p)[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]} +$$
$$\frac{\eta_1\omega_4(\omega_3+\mu_p)(\omega+\mu_p)}{(\omega+\omega_4+\mu_p)[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]} < 0;$$

$$\eta_1\omega_4(\omega_3+\mu_p)(\omega+\mu_p) < \omega_4\eta_2\omega(\omega_p+\mu_p).$$

$$A_{\mu_p} = \frac{\frac{\partial R_{covid}}{R_{covid}}}{\frac{\partial \mu_p}{\mu_p}} = \frac{\mu_p}{R_{covid}} \frac{\partial R_{covid}}{\partial \mu_p} = -\frac{(\tau\alpha\beta\gamma\delta\mu_p+\alpha\beta\delta\mu_p+\alpha\beta\gamma\mu_p+\alpha\beta\gamma\delta)-(\eta_2\omega+\eta_1\omega_4)\beta\gamma\delta\mu_p}{\alpha\beta\gamma\delta} < 0;$$

$$(\tau\alpha\beta\gamma\delta\mu_p + \alpha\beta\delta\mu_p + \alpha\beta\gamma\mu_p + \alpha\beta\gamma\delta) > (\eta_2\omega + \eta_1\omega_4)\beta\gamma\delta\mu_p$$

where, $\propto = \eta_2\omega(\omega_p+\mu_p) + \eta_1\omega_4(\omega_3+\mu_p)$, $\beta = (\omega+\omega_4+\mu_p)$, $\gamma = (\omega_p + \mu_p)$ *and* $\delta = (\omega_3+\mu_p)$.

The sensitive parameter of the model is η_1, η_2 and ω. It has concluded that the direct ratio is among η_1, η_2, ω and reproduction number R_{covid}. This means that an increase in sensitive parameters will eventually increase the number of reproduction and vice versa. Also, the rest of the parameters are insensitive. It has concluded that the inverse ratio are insensitive parameters and the reproduction number R_{covid}. This means increasing the parameters, eventually reducing the number of reproduction and vice versa.

3 Local stability

For the local stability at both equilibria of the model, we will prove the following well-known results as follows:

Theorem: For given $\tau > 0$, the Eqs. (1) to (6) is said to be locally asymptotical stable (LAS) at COVID free equilibrium $C_1 = ({S_p}^1, {E_p}^1, {I_p}^1, {A_p}^1, {R_p}^1)$, which is contained in region Γ if $R_{covid} < 1$. Otherwise the Eqs. (1) to (6) is unstable if $R_{covid} > 1$.

Proof: The Jacobean matrix for the Eqs. (1) to (6) at C_1 as follows:

$$J(C_1)=\begin{bmatrix} -\mu_p & 0 & \frac{-\eta_2\pi_p e^{-\mu_p\tau}}{\mu_p} & \frac{-\eta_1\pi_p e^{-\mu_p\tau}}{\mu_p} & 0 \\ 0 & -(\omega+\omega_4+\mu_p) & \frac{\eta_2\pi_p e^{-\mu_p\tau}}{\mu_p} & \frac{\eta_1 e^{-\mu_p\tau}}{\mu_p} & 0 \\ 0 & \omega & -(\omega_3+\mu_p) & 0 & 0 \\ 0 & \omega_4 & 0 & -(\omega_p+\mu_p) & 0 \\ 0 & 0 & \omega_3 & \omega_p & -\mu_p \end{bmatrix}$$

The following eigen values of Jacobean matrix $J(C_1)$ are obtained:

$\lambda_1=-\mu_p<0, \lambda_2=-\mu_p<0.$

$$|J(C_1)-\lambda I|=\begin{vmatrix} -(\omega+\omega_4+\mu_p)-\lambda & \frac{\eta_2\pi_p e^{-\mu_p\tau}}{\mu_p} & \frac{\eta_1 e^{-\mu_p\tau}}{\mu_p} \\ \omega & -(\omega_3+\mu_p)-\lambda & 0 \\ \omega_4 & 0 & -(\omega_p+\mu_p)-\lambda \end{vmatrix}=0.$$

Put $a_1=(\omega+\omega_4+\mu_p)$, $a_2=(\omega_3+\mu_p)$, $a_3=\frac{\eta_2\pi_p e^{-\mu_p\tau}}{\mu_p}$, $a_4=\frac{\eta_1\pi_p e^{-\mu_p\tau}}{\mu_p}$, $a_5=(\omega_p+\mu_p)$.

$$|J(C_1)-\lambda I|=\begin{vmatrix} -a_1-\lambda & a_3 & a_4 \\ \omega & -a_2-\lambda & 0 \\ \omega_4 & 0 & -a_5-\lambda \end{vmatrix}=0$$

$\lambda^3+(a_1+a_2+a_5)\lambda^2+(a_1a_2+a_1a_5+a_2a_5-\omega a_3-\omega_4a_4)\lambda+(a_1a_2a_5-\omega a_3a_5-\omega_4a_2a_4)=0.$

By using the Routh-Hurwitz Criterion of 3rd order polynomial as,

$(a_1+a_2+a_5)>0,$

$(a_1a_2a_5-\omega a_3a_5-\omega_4a_2a_4)>0$, if

$\frac{\omega a_3a_5+\omega_4a_2a_4}{a_1a_2a_5}<1$, by putting substitute values, we have $R_{covid}=\frac{[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]\pi_p}{(\omega+\omega_4+\mu_p)(\omega_p+\mu_p)(\omega_3+\mu_p)\mu_p}e^{-\mu_p\tau}<1$

and $(a_1+a_2+a_5)(a_1a_2+a_1a_5+a_2a_5-\omega a_3-\omega_4a_4)>(a_1a_2a_5-\omega a_3a_5-\omega_4a_2a_4)$, if $R_{covid}<1$.

So, all eigenvalues are negative. Hence, by Routh Hurwitz criteria C_1 is locally asymptotical stable (LAS).

If $R_{covid}>1$, that is

$\frac{[\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]\pi_p}{(\omega+\omega_4+\mu_p)(\omega_p+\mu_p)(\omega_3+\mu_p)\mu_p}e^{-\mu_p\tau}>1.$

$\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]\pi_p e^{-\mu_p\tau}>(\omega+\omega_4+\mu_p)(\omega_p+\mu_p)(\omega_3+\mu_p)\mu_p.$

$-(\omega+\omega_4+\mu_p)(\omega_p+\mu_p)(\omega_3+\mu_p)\mu_p+\eta_2\omega(\omega_p+\mu_p)+\eta_1\omega_4(\omega_3+\mu_p)]\pi_p e^{-\mu_p\tau}>0.$

Then $\lambda_3,\lambda_4,\lambda_5>0$. Hence, C_1 is unstable.

Theorem: For given $\tau > 0$, the Eqs. (1) to (6) is said to be locally asymptotical stable (LAS) at COVID present equilibrium $C_2 = (S_p^*, E_p^*, I_p^*, A_p^*, R_p^*)$, which is contained in region Γ if $R_{covid} > 1$. Otherwise, the Eqs. (1) to (6) is unstable if $R_{covid} < 1$.

Proof: The Jacobean matrix for the Eqs. (1) to (6) at C_2 as follows:

$$J(C_2) = \begin{bmatrix} -b_1 & 0 & -b_2 & -b_3 & 0 \\ b_4 & -b_6 & b_2 & b_3 & 0 \\ 0 & \omega & -b_5 & 0 & 0 \\ 0 & \omega_4 & 0 & -b_7 & 0 \\ 0 & 0 & \omega_3 & \omega_p & -\mu_p \end{bmatrix}.$$

where,

$b_1 = (\eta_1 A_p^* + \eta_2 I_p^*)e^{-\mu_p \tau} + \eta_3 M^* + \mu_p$, $b_2 = \eta_2 S_p^* e^{-\mu_p \tau}$, $b_3 = \eta_1 S_p^* e^{-\mu_p \tau}$, $b_4 = (\eta_1 A_p^* + \eta_2 I_p^*) S_p^* e^{-\mu_p \tau}$, $b_5 = \omega_3 + \mu_p$, $b_6 = \omega + \omega_4 + \mu_p$, $b_7 = \omega_p + \mu_p$.

The following eigen values of Jacobean matrix $J(C_2)$ are obtained:

$\lambda_1 = -\mu_p < 0$ and

$$|J(C_2) - \lambda I| = \begin{vmatrix} -b_1 - \lambda & 0 & -b_2 & -b_3 \\ b_4 & -b_6 - \lambda & b_2 & b_3 \\ 0 & \omega & -b_5 - \lambda & 0 \\ 0 & \omega_4 & 0 & -b_7 - \lambda \end{vmatrix} = 0.$$

$\lambda^4 + (b_1 + b_2 + b_5 + b_6 + b_7)\lambda^3 + (b_7(b_1 + b_2) + (b_1 + b_7)(b_5 + b_6) + b_5 b_6 - \omega b_2 - \omega_4 b_3)\lambda^2 + ((b_1 + b_7)(b_5 b_6 - \omega b_2) + b_2(b_1 + b_6)(1 + b_7) + b_1 b_7 (b_5 + b_6) + b_1 b_2 b_6 + \omega_4 b_3 (b_4 - b_5 - b_1))\lambda + (b_1 b_2 b_6 b_7 + b_1 b_5 b_6 b_7 - \omega b_1 b_2 b_7 + \omega_4 b_3 (b_4 b_5 - b_1 b_5)) = 0.$

So, $m_0 \lambda^4 + m_1 \lambda^3 + m_2 \lambda^2 + m_3 \lambda + m_4 = 0.$

where, $m_1 = (b_1 + b_2 + b_5 + b_6 + b_7)$,

$m_2 = (b_7(b_1 + b_2) + (b_1 + b_7)(b_5 + b_6) + b_5 b_6 - \omega b_2 - \omega_4 b_3)$,

$m_3 = ((b_1 + b_7)(b_5 b_6 - \omega b_2) + b_2(b_1 + b_6)(1 + b_7) + b_1 b_7(b_5 + b_6) + b_1 b_2 b_6 + \omega_4 b_3(b_4 - b_5 - b_1))$,

$m_4 = (b_1 b_2 b_6 b_7 + b_1 b_5 b_6 b_7 - \omega b_1 b_2 b_7 + \omega_4 b_3 (b_4 b_5 - b_1 b_5))$.

By using the Routh-Hurwitz Criterion of 4th order polynomial as,

$m_0 > 0, m_1 > 0, m_1 m_2 - m_0 m_3 > 0$,

$(m_1 m_2 - m_0 m_3) m_3 - m_1^2 m_4 > 0$ and $m_4 > 0$ only if $R_{covid} > 1$.

So, its eigenvalue is negative. Hence, by Routh Hurwitz criteria C_2 is locally asymptotical stable (LAS).

4 Global stability

For the global stability at both equilibria of the model, we will prove the following well-known results as follows:

Theorem: For given $\tau > 0$, the system Eq. (1) to Eq. (6) is said to be globally asymptotical stable (GAS) at COVID free equilibrium $C_1 = \left({S_p}^1, {E_p}^1, {I_p}^1, {A_p}^1, {R_p}^1\right)$, which is contained in region Γ if $R_{covid} < 1$. Otherwise unstable.

Proof: We have considered the Volterra- type Lyapunov function $U: \Gamma \to R$ defined as follows:

$$U = \left(S_p - {S_p}^1 - {S_p}^1 log \frac{S_p}{{S_p}^1}\right) + E_p + I_p + A_p + R_p.$$

$$\frac{dU}{dt} = (1 - \frac{{S_p}^1}{S_p})\frac{dS_p}{dt} + \frac{dE_p}{dt} + \frac{dI_p}{dt} + \frac{dA_p}{dt} + \frac{dR_p}{dt}$$

$$\frac{dU}{dt} = \left(\frac{S_p - {S_p}^1}{S_p}\right)\left(\pi_p - (\eta_1 A_p + \eta_2 I_p)S_p e^{-\mu_p \tau} - \eta_3 S_p M - \mu_p S_p\right) + \big(\eta_1 A_p + \eta_2 I_p\big)S_p e^{-\mu_p \tau} + \eta_3 S_p M - \omega E_p - \omega_4 E_p - \mu_p E_p + \omega E_p - \omega_3 I_p - \mu_p I_p + \omega_4 E_p - \omega_p A_p - \mu_p A_p + \omega_p A_p + \omega_3 I_p - \mu_p R_p.$$

$$\frac{dU}{dt} = \left(S_p - {S_p}^1\right)\left[\frac{\pi_P}{S_p} - \eta_1 A_p e^{-\mu_p \tau} - \eta_2 I_p e^{-\mu_p \tau} - \eta_3 M - \mu_p\right] + \eta_1 A_p S_p e^{-\mu_p \tau} + \eta_2 I_p S_p e^{-\mu_p \tau} + \eta_3 M S_p - \mu_p E_p - -\mu_p I_p - \mu_p A_p - \mu_p R_p.$$

Since, $C_1 = \left({S_p}^1, {E_p}^1, {I_p}^1, {A_p}^1, {R_p}^1\right)$ is an COVID free equilibrium, so for Eqs. (1) to (6), $\frac{dM^1}{dt} = \frac{d{S_p}^1}{dt} = \frac{d{E_p}^1}{dt} = \frac{d{I_p}^1}{dt} = \frac{d{A_p}^1}{dt} = \frac{d{R_p}^1}{dt} = 0$, gives

$$\mu_p = \frac{\pi_P}{{S_p}^1} - \eta_1 {A_p}^1 e^{-\mu_p \tau} - \eta_2 {I_p}^1 e^{-\mu_p \tau} - \eta_3 M^1$$

$$\frac{dU}{dt} = \left(S_p - {S_p}^1\right)\left[\frac{\pi_P}{S_p} - \eta_1 A_p e^{-\mu_p \tau} - \eta_2 I_p e^{-\mu_p \tau} - \eta_3 M - \frac{\pi_P}{{S_p}^1} + \eta_1 {A_p}^1 S_p e^{-\mu_p \tau} + \eta_2 {I_p}^1 S_p e^{-\mu_p \tau} + \eta_3 M^1\right] - \mu_p A_p\left[1 - \frac{\eta_1 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p I_p\left[1 - \frac{\eta_2 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p E_p - \mu_p R_p.$$

$$\frac{dU}{dt} = \frac{-\pi_P\left(S_p - {S_p}^1\right)}{S_p S_p^1} - \eta_1 e^{-\mu_p \tau}\left(S_p - {S_p}^1\right)\left(A_p - {A_p}^1\right) - \eta_2 {I_p}^1 e^{-\mu_p \tau}\left(S_p - {S_p}^1\right)\left(I_p - {I_p}^1\right) - \eta_3 (M - M^1)\left(S_p - {S_p}^1\right) - \mu_p A_p\left[1 - \frac{\eta_1 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p I_p\left[1 - \frac{\eta_2 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p E_p - \mu_p R_p.$$

$$\frac{dU}{dt} = \frac{-\pi_P\left(S_p - {S_p}^1\right)^2}{S_p S_p^1} - \eta_1 e^{-\mu_p \tau}\left(S_p - {S_p}^1\right)\left(A_p - {A_p}^1\right) - \eta_2 e^{-\mu_p \tau}\left(S_p - {S_p}^1\right)\left(I_p - {I_p}^1\right) - \eta_3 (M - M^1)\left(S_p - {S_p}^1\right) - \mu_p A_p\left[1 - \frac{\eta_1 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p I_p\left[1 - \frac{\eta_2 {S_p}^1 e^{-\mu_p \tau}}{\mu_p}\right] - \mu_p E_p - \mu_p R_p.$$

$\Rightarrow \frac{dU}{dt} \leq 0$ for $R_{covid} < 1$, and $\frac{dU}{dt} = 0$ only if $S_p = {S_p}^1$, $E_p = I_p = A_p = R_p = 0$. Therefore, the only trajectory of the Eqs. (1) to (6) on which $\frac{dU}{dt} = 0$ is C_1. Hence, by Lasalle's invariance principle, C_1 is globally asymptotically stable (GAS) in Γ.

Theorem: For given $\tau > 0$, the Eqs. (1) to (6) is said to be globally asymptotical stable (GAS) at COVID present equilibrium $C_2 = \left(S_p{}^*, E_p{}^*, I_p{}^*, A_p{}^*, R_p{}^*\right)$, which is contained in region Γ if $R_{covid} > 1$. Otherwise unstable.

Proof: We have considered the Volterra-type Lyapunov function $V: \Gamma \to R$ defined as follows:

$$V= K_1\left(S_p - S_p{}^* - S_p{}^* log\frac{S_p}{S_p{}^*}\right) + K_2\left(E_p - E_p{}^* - E_p{}^* log\frac{E_p}{E_p{}^*}\right) + K_3\left(I_p - I_p{}^* - I_p{}^* log\frac{I_p{}^*}{I_p{}^*}\right) + K_4\left(A_p - A_p{}^* - A_p{}^* log\frac{A_p}{A_p{}^*}\right) + K_5\left(R_p - R_p{}^* - R_p{}^* log\frac{R_p}{R_p{}^*}\right).$$

where, K_i: $(i = 1,2,3,4,5)$ are positive constants to be chosen later.

$$\frac{dV}{dt} = K_1(1 - \frac{S_p{}^*}{S_p})\frac{dS_p}{dt} + K_2(1 - \frac{E_p{}^*}{E_p})\frac{dE_p}{dt} + K_3(1 - \frac{I_p{}^*}{I_p})\frac{dI_p}{dt} + K_4(1 - \frac{A_p{}^*}{A_p})\frac{dA_p}{dt} + K_5(1 - \frac{R_p{}^*}{R_p})\frac{dR_p}{dt}$$

$$\frac{dV}{dt} = K_1\frac{(S_p - S_p{}^*)}{S_p}\left[\pi_P - \eta_1 A_p S_p e^{-\mu_p\tau} - \eta_2 I_p S_p e^{-\mu_p\tau} - \eta_3 M S_p - \mu_p S_p\right] + K_2\frac{(E_p - E_p{}^*)}{E_p}\left[\eta_1 A_p S_p e^{-\mu_p\tau} + \eta_2 I_p S_p e^{-\mu_p\tau} + \eta_3 M S_p - (\omega + \omega_4 + \mu_p)E_p\right] + K_3\left(\frac{I_p - I_p{}^*}{I_p}\right)\left[\omega E_p - \omega_3 I_p - \mu_p I_p\right] + K_4\left(\frac{A_p - A_p{}^*}{A_p}\right)\left[\omega_4 E_p - \omega_p A_p - \mu_p A_p\right] + K_5\left(\frac{R_p - R_p{}^*}{R_p}\right)\left[\omega_3 I_p + \omega_p A_p - \mu_p R_p\right].$$

$$\frac{dV}{dt} = K_1\left(S_p - S_p{}^*\right)\left[\frac{\pi_P}{S_p} - \eta_1 A_p e^{-\mu_p\tau} - \eta_2 I_p e^{-\mu_p\tau} - \eta_3 M - \mu_p\right] + K_2\left(E_p - E_p{}^*\right)\left[\frac{\eta_1 A_p S_p e^{-\mu_p\tau}}{E_p} + \frac{\eta_2 I_p S_p e^{-\mu_p\tau}}{E_p} + \frac{\eta_3 M S_p}{E_p} - (\omega + \omega_4 + \mu_p)\right] + K_3\left(I_p - I_p{}^*\right)\left[\frac{\omega E_p}{I_p} - (\omega_3 + \mu_p)\right] + K_4\left(A_p - A_p{}^*\right)\left[\frac{\omega_4 E_p}{A_p} - (\omega_p + \mu_p)\right] + K_5\left(R_p - R_p{}^*\right)\left[\frac{\omega_p A_p}{R_p} + \frac{\omega_3 I_p}{R_p} - \mu_p\right].$$

Since, $C_2 = \left(S_p{}^*, E_p{}^*, I_p{}^*, A_p{}^*, R_p{}^*\right)$ is an COVID present equilibrium, so for Eqs. (1) to (6), $\frac{dS_p{}^*}{dt} = \frac{dE_p{}^*}{dt} = \frac{dI_p{}^*}{dt} = \frac{dA_p{}^*}{dt} = \frac{dR_p{}^*}{dt} = 0$, gives

$$\mu_p = \frac{\pi_P}{S_p} - \eta_1 A_p{}^* e^{-\mu_p\tau} - \eta_2 I_p{}^* - \eta_3 M^* \quad , \quad \omega + \omega_4 + \mu_p = \frac{\eta_1 A_p S_p e^{-\mu_p\tau}}{E_p{}^*} + \frac{\eta_2 I_p S_p e^{-\mu_p\tau}}{E_p{}^*} + \frac{\eta_3 M S_p}{E_p{}^*}, \omega_3 + \mu_p = \frac{\omega E_p}{I_p{}^*}, \omega_p + \mu_p = \frac{\omega_4 E_p}{A_p{}^*}, \mu_p = \frac{\omega_p A_p}{R_p{}^*} + \frac{\omega_3 I_p}{R_p{}^*}.$$

$$\frac{dV}{dt} = K_1\left(S_p - S_p{}^*\right)\left[\frac{\pi_P}{S_p} - \eta_1 A_p e^{-\mu_p\tau} - \eta_2 I_p e^{-\mu_p\tau} - \eta_3 M - \frac{\pi_P}{S_p} + \eta_1 A_p{}^* e^{-\mu_p\tau} + \eta_2 I_p{}^* + \eta_3 M^*\right] + K_2\left(E_p - E_p{}^*\right)\left[\frac{\eta_1 A_p S_p e^{-\mu_p\tau}}{E_p} + \frac{\eta_2 I_p S_p e^{-\mu_p\tau}}{E_p} + \frac{\eta_3 M S_p}{E_p} - \frac{\eta_1 A_p S_p e^{-\mu_p\tau}}{E_p{}^*} - \frac{\eta_2 I_p S_p e^{-\mu_p\tau}}{E_p{}^*} - \frac{\eta_3 M S_p}{E_p{}^*}\right] + K_3\left(I_p - I_p{}^*\right)\left[\frac{\omega E_p}{I_p} - \frac{\omega E_p}{I_p{}^*}\right] + K_4\left(A_p - A_p{}^*\right)\left[\frac{\omega_4 E_p}{A_p} - \frac{\omega_4 E_p}{A_p{}^*}\right] + K_5\left(R_p - R_p{}^*\right)\left[\frac{\omega_p A_p}{R_p} + \frac{\omega_3 I_p}{R_p} - \frac{\omega_p A_p}{R_p{}^*} - \frac{\omega_3 I_p}{R_p{}^*}\right].$$

$\frac{dV}{dt} = -K_1 \frac{\pi_P(S_p - S_p{}^*)^2}{S_p S_p{}^*} - K_1\eta_1 e^{-\mu_p\tau}(S_p - S_p{}^*)(A_p - A_p^*) - K_1\eta_2 e^{-\mu_p\tau}(S_p - S_p{}^*)(I_p - I_p{}^*) - K_1\eta_3(M - M^*)(S_p - S_p{}^*) - K_2 \frac{\eta_1 A_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} - K_2 \frac{\eta_2 I_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} - K_2 \frac{\eta_3 S_p M(E_p - E_p{}^*)^2}{E_p E_p{}^*} - K_3 \frac{\omega E_p(I_p - I_p{}^*)^2}{I_p I_p{}^*} - K_4 \frac{\omega_4 E_p(A_p - A_p{}^*)^2}{A_p A_p{}^*} - K_5 \frac{\omega A_p(R_p - R_p{}^*)^2}{R_p R_p{}^*} - K_5 \frac{\omega_3 I_p(R_p - R_p{}^*)^2}{R_p R_p{}^*}$

$\frac{dV}{dt} = -K_1\left(\frac{\pi_P(S_p - S_p{}^*)^2}{S_p S_p{}^*} + \eta_1 e^{-\mu_p\tau}(S_p - S_p{}^*)(A_p - A_p^*) + \eta_2 e^{-\mu_p\tau}(S_p - S_p{}^*)(I_p - I_p{}^*) + \eta_3(M - M^*)(S_p - S_p{}^*)\right) - K_2\left(\frac{\eta_1 A_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} + \frac{\eta_2 I_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} + \frac{\eta_3 S_p M(E_p - E_p{}^*)^2}{E_p E_p{}^*}\right) - K_3 \frac{\omega E_p(I_p - I_p{}^*)^2}{I_p I_p{}^*} - K_4 \frac{\omega_4 E_p(A_p - A_p{}^*)^2}{A_p A_p{}^*} - K_5\left(\frac{\omega A_p(R_p - R_p{}^*)^2}{R_p R_p{}^*} + \frac{\omega_3 I_p(R_p - R_p{}^*)^2}{R_p R_p{}^*}\right).$

For $K_1 = K_2 = K_3 = K_4 = K_5 = 1$, we have

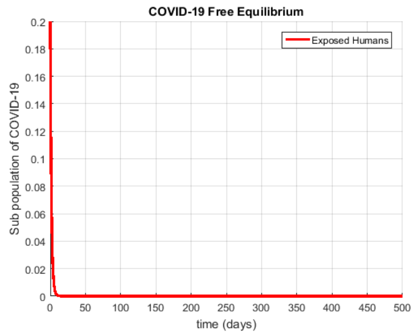
$\frac{dV}{dt} = -\left(\frac{\pi_P(S_p - S_p{}^*)^2}{S_p S_p{}^*} + \eta_1 e^{-\mu_p\tau}(S_p - S_p{}^*)(A_p - A_p^*) + \eta_2 e^{-\mu_p\tau}(S_p - S_p{}^*)(I_p - I_p{}^*) + \eta_3(M - M^*)(S_p - S_p{}^*)\right) - \left(\frac{\eta_1 A_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} + \frac{\eta_2 I_p S_p e^{-\mu_p\tau}(E_p - E_p{}^*)^2}{E_p E_p{}^*} + \frac{\eta_3 S_p M(E_p - E_p{}^*)^2}{E_p E_p{}^*}\right) - \frac{\omega E_p(I_p - I_p{}^*)^2}{I_p I_p{}^*} - \frac{\omega_4 E_p(A_p - A_p{}^*)^2}{A_p A_p{}^*} - \left(\frac{\omega A_p(R_p - R_p{}^*)^2}{R_p R_p{}^*} + \frac{\omega_3 I_p(R_p - R_p{}^*)^2}{R_p R_p{}^*}\right) \le 0$

$\Rightarrow \frac{dV}{dt} \le 0$ for $R_{covid} > 1$, and $\frac{dV}{dt} = 0$ only if $S_p = S_p{}^*, I_p = I_p{}^*, A_p = A_p{}^*, R_p = R_p{}^*$. Therefore, the only trajectory of the Eqs. (1) to (6) on which $\frac{dV}{dt} = 0$ is C_2. Hence, by Lasalle's invariance principle, C_2 is globally asymptotically stable (GAS) in Γ.

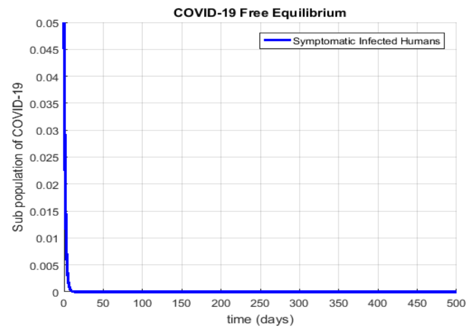
5 Numerical consequences

The numerical solution of Eqs. (1) to (5) is good agreement of the dynamical behavior of the model by using different values of the limits. Chen et al. [Chen, Rui, Wang et al. (2020)] have presented the description of limits as follows:
$\pi_s = 0.5$, $\eta_1 = 0.05$, $\eta_2 = 0.05$, $\mu_p = 0.5$, $\omega = 0.00047876$, $\eta_3 = 0.000001231$, $\omega_p = 0.854302$, $\omega_1 = 0.01$, $\omega_2 = 0.000398$, $\omega_3 = 0.09871$, $\omega_4 = 0.1243$, $\pi = 0.5$ for $R_{covid} < 1$. For $R_{covid} > 1$, $\pi_s = 0.5$, $\eta_1 = 1.05$, $\eta_2 = 1.05$, $\mu_p = 0.5$, $\omega = 1.00047876$, $\eta_3 = 0.000001231$, $\omega_p = 0.854302$, $\omega_1 = 0.01$, $\omega_2 = 0.000398$, $\omega_3 = 0.09871$, $\omega_4 = 0.1243$, $\pi = 0.5$. by using different non-negative initial conditions $S_p(0) = 0.5, E_p(0) = 0.2, I_p(0) = 0.05, A_p(0) = 0.05, R_p(0) = 0.1, M(0) = 0.1$.

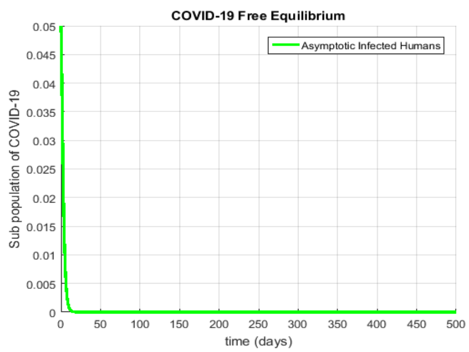
In Fig. 2, we have plotted each compartment of the delay model without time delay reason for COVID free equilibrium.



(a)



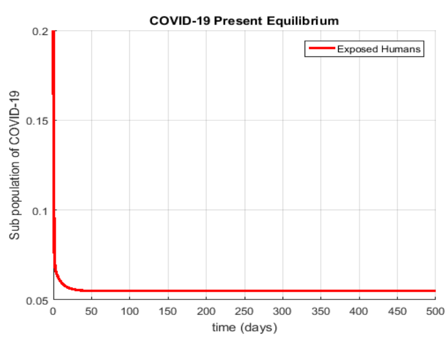
(b)



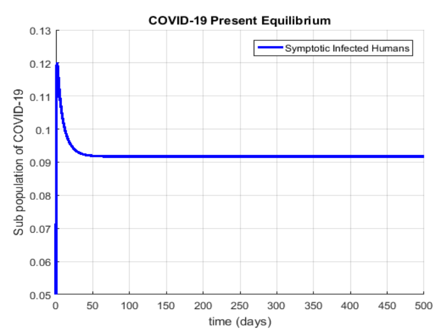
(c)

Figure 2: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\eta_1 = 0.05$, $\eta_2 = 0.05$, $\mu_p = 0.5$, $\omega = 0.00047876$, $\eta_3 = 0.000001231$, $\omega_p = 0.854302$, $\omega_1 = 0.01$, $\omega_2 = 0.000398$, $\omega_3 = 0.09871$, $\omega_4 = 0.1243$, $\pi = 0.5$, $\tau = 0$, by using initial conditions and $R_{covid} = 0.0171 < 1$

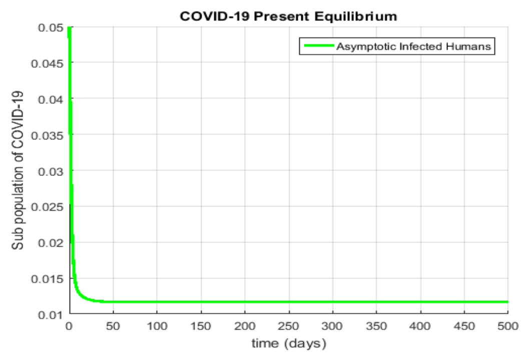
In Fig. 3, we have plotted each compartment of the delay model without time delay reason for COVID present equilibrium.



(a)



(b)



(c)

Figure 3: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\eta_1 = 1.05$, $\eta_2 = 1.05$, $\mu_p = 0.5$, $\omega = 1.00047876$, $\eta_3 = 0.000001231$, $\omega_p = 0.854302$, $\omega_1 = 0.01$, $\omega_2 = 0.000398$, $\omega_3 = 0.09871$, $\omega_4 = 0.1243$, $\pi = 0.5$, $\tau = 0$, by using the initial conditions and $R_{covid} = 1.2171 > 1$

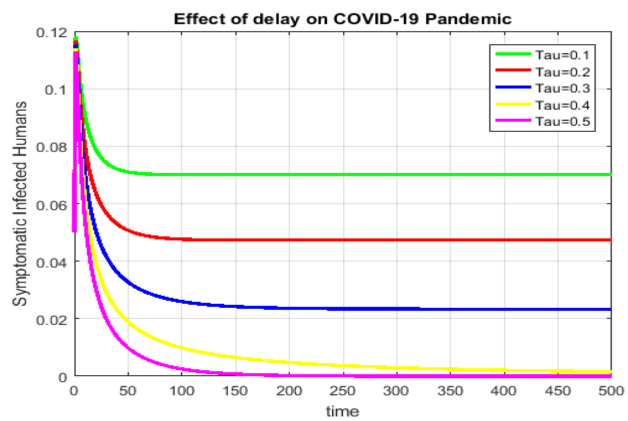


Figure 4: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\eta_1 = 1.05$, $\eta_2 = 1.05$, $\mu_p = 0.5$, $\omega = 1.00047876$, $\eta_3 = 0.000001231$, $\omega_p = 0.854302$, $\omega_1 = 0.01$, $\omega_2 = 0.000398$, $\omega_3 = 0.09871$, $\omega_4 = 0.1243$, $\pi = 0.5$, $\tau > 0$, by using the initial conditions and $R_{covid} = 1.2171 > 1$

In Fig. 4, we have observed the increase in delay tactics or delay term, the result is symptomatic infected humans reduce without any change in the transmission rate. Even though, we can see symptomatic infected humans exponentially decreases by the increase in delay tactics. Eventually, symptomatic infected humans become zero in Fig.4 when τ =0.394. So, it means humans become corona free and $R_{covid} = 0.9995 < 1$. According to given real data, if we used delay tactics like social distancing, quarantine, travel restrictions, holiday extension, hospitalization and isolation for about one hundred and forty-three days ($\tau = 0.394\ year$) then we can overcome the pandemic of coronavirus.

5.1 Effect of delay factor

In Fig. 5, we have plotted the comparison of delay reason and reproduction number of coronavirus model. We have concluded that the increase in delay tactics can change coronavirus present equilibrium to corona free equilibrium, which is quite the delay reason or delay tactics in a pandemic of coronavirus can help to control and overcome it.

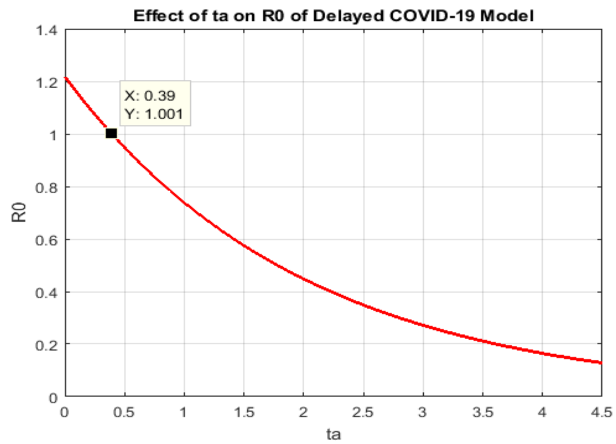


Figure 5: Comparison of delay factor and reproduction number

6 Conclusion and guidelines

The nonlinear delay dynamical modelling is a suitable tool to study any pandemic. During, the outbreak of coronavirus cure or vaccination cannot report as soon. Due to worldwide disaster, there is only delay tactics like quarantine, isolation, social distancing, etc. have used as vaccination to overcome the pandemic of coronavirus. If we can use delay tactics about one hundred and forty-three days than symptomatic infected ultimately moves to zero and eventually, susceptible humans will increase with delayed factor. The inverse relationship holds between infected and susceptible humans. As future work, we can extend this idea to all epidemic diseases and other biological problems. Also, the delay effect could be introduced in stochastic epidemic models and stochastic fractional-order dynamical systems. We shall introduce the idea of a delay in non-linearity coupled multiplex networks as presented by Zhou et al. [Zhou, Tan, Yu et al. (2019)]. Also, this analysis shall be extended in neural networking dynamics with fixed intervals as presented by Yu et al. [Yu, Liu, Xiao et al. (2019)].

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