Modified theory of physical optics

Yusuf Z. Umul

Cankaya University, Engineering Faculty, Electronic and Communication Dept., Balgat-Ankara, Turkey
yziya@cankaya.edu.tr

Abstract: A new procedure for calculating the scattered fields from a perfectly conducting body is introduced. The method is defined by considering three assumptions. The reflection angle is taken as a function of integral variables, a new unit vector, dividing the angle between incident and reflected rays into two equal parts is evaluated and the perfectly conducting (PEC) surface is considered with the aperture part, together. This integral is named as Modified Theory of Physical Optics (MTPO) integral. The method is applied to the reflection and edge diffraction from a perfectly conducting half plane problem. The reflected, reflected diffracted, incident and incident diffracted fields are evaluated by stationary phase method and edge point technique, asymptotically. MTPO integral is compared with the exact solution and PO integral for the problem of scattering from a perfectly conducting half plane, numerically. It is observed that MTPO integral gives the total field that agrees with the exact solution and the result is more reliable than that of classical PO integral.

OCIS codes: (260.0260) Physical Optics; (260.1960) Diffraction theory; (260.2110) Electromagnetic theory.

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1. Introduction

The geometrical theory of diffraction (GTD), first introduced by Keller [1] for two canonical problems of plane wave diffraction from a perfectly conducting (PEC) half plane and cylinder [2,3], is a high frequency asymptotic ray technique. The method is developed from the known solutions of simple shapes, named as canonical problems [4]. GTD has a few limitations one of which is the infinite fields which are occurring at shadow boundaries. Uniform asymptotic theory (UAT) and uniform theory of diffraction (UTD) are developed to overcome this difficulty and used with success for a wide variety of problems [5]. In spite of their usefulness, these theories fail near caustics and focal points. Another defect of GTD appears at the limited range of problems to which it can be applied. The ray techniques (GTD, UAT, UTD) need the appropriate diffraction coefficients, found from the exact solution of Helmholtz equation, but it is not able to find rigorous solutions to all canonical problems [6]. For example a GTD formulation for the impedance half plane problem could be constructed only after the works of Senior and Maliuzhinets [8-10]. Some physical optics (PO) based techniques are developed to find approximated diffraction coefficients for GTD [6,7].

PO is a high frequency technique, which determines reflected fields and uses an approximation of the induced surface current density on a perfectly conducting surface in proportion to the tangential incident magnetic field [11], but fails in evaluating the edge diffracted fields [4,12]. In order to correct the PO surface field approximation, the Physical Theory of Diffraction (PTD) was developed by Ufimtsev [13]. PTD uses additional current components, called residual or fringe current. Michaeli developed equivalent edge currents that allow the evaluation of the far diffracted field for directions not on the Keller cone [14,15]. Michaeli also introduced Extended Physical Theory of Diffraction (EPTD), as an extension to Ufimtsev's theory to aperture integration, by formulating in terms of incremental diffraction coefficients (IDC) [16]. These two methods are used frequently for the analysis of reflectors and backscattering from complex objects [17,18].

There are two deficiencies in PO theory. First of all, only the perfectly conducting surface (or the scatter surface) is considered in forming the scattering integral and the aperture part is omitted. As a result of this negligence, the reflected and reflected diffracted fields can be evaluated, but there will be no information about incident and incident diffracted waves. A second restriction is the acceptance of the discontinuous surface as a continuous surface and taking the reflection angle equal to the incidence angle, when evaluating the PO current. This acceptance fails at the edge discontinuity.

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It is the aim of this paper to correct the deficiencies of PO theory and obtain the exact solution of edge diffraction problems for various geometries. Three axioms are introduced with this purpose. The aperture and scatter surfaces are considered, the reflection and transmission angles are taken as variables of the scatter and aperture coordinates, and a new unit vector, which divides the angle between the incident and reflected (transmitted) rays into two equal parts, is defined. This new theory is named as Exact Theory of Physical Optics (MTPO). It is important to note that changing the scatterer coordinates is analogous to the method of Modified Edge Representation (MER), in which the edge of the scatterer is replaced with a modified one, defining a new unit vector that satisfies the diffraction law at each point [19,20]. This method is introduced in order to overcome the false singularities in equivalent edge currents for PO and GTD. Defining a new direction for the unit vector was also offered in the literature, in order to improve PO [21].

Asymptotic evaluation of MTPO integral gives the exact edge diffraction coefficient. One of the original points of this paper is the expression of the phase function of the MTPO integral by a new form [22]. The new method is applied to a well known canonical problem of half plane in order to examine its validity. The exact edge diffracted fields can be found without considering additional fringe waves or equivalent currents for a wide spectrum of problems for scattering from perfectly conducting bodies with MTPO integral method. The method can also be applied to the geometries where the radius of curvature is a function of angle, like parabolic and hyperbolic reflectors.

GTD and PTD need the diffraction coefficients, found from the solution of canonical problems, in order to construct high frequency asymptotic fields for more complicated geometries. For this reason their applicability is limited with the number of solved basic problems as mentioned before. In contrary, exact edge diffracted fields can be evaluated directly from the asymptotic evaluation of the MTPO integral. Half plane problem, solved in Section 3, can be considered with this aim. GTD uses directly the edge diffraction coefficient found by Sommerfeld [23] and can only define the behavior of edge diffracted fields with the existence of this coefficient, but exact edge diffracted fields and the related coefficient are found from the asymptotic evaluation of the MTPO integral, constructed by using the geometry.

A time factor e^{jwt} is assumed and suppressed throughout the paper.

2. Exact theory of physical optics

The geometry in Fig. 1 is considered. S_1 is the perfectly conducting surface and S_2 is the aperture part.

A general procedure will be given in order to find the total diffracted fields by taking into account these two surfaces. Three axioms can be introduced as

1. Scattering fields from S_1 and S_2 surfaces are considered. The incident waves induce a surface current on S_1 and integration of this current gives the reflected and reflected diffracted fields as in classical PO theory, but this solution will not include information about incident diffracted fields. For this reason, S_2 surface must be considered. Equivalent currents can be defined on the aperture according to the Equivalent Source Theorem and radiated field can be obtained by integrating the related currents on S_2. Radiated fields contain the data about incident and incident diffracted waves. This approach is analogous to the solution of aperture antenna problems with Equivalent Source Theorem. A surface current can be defined for S_1 as

$$\vec{J}_{es} = \vec{n}_1 \times \vec{H}_t \Big|_{S_1} \tag{1}$$

where $\vec{H}_t$ is the total magnetic field on the perfectly conducting surface. Equivalent Source Theorem can be applied to S_2 and equivalent surface currents can be defined as

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$$\vec{J}_{es} = \vec{n}_2 \times \vec{H}_i \Big|_{S_2} \quad , \quad \vec{J}_{ms} = -\vec{n}_2 \times \vec{E}_i \Big|_{S_2} \tag{2}$$

for $\vec{E}_i$, $\vec{H}_i$ are the incident fields on the aperture.

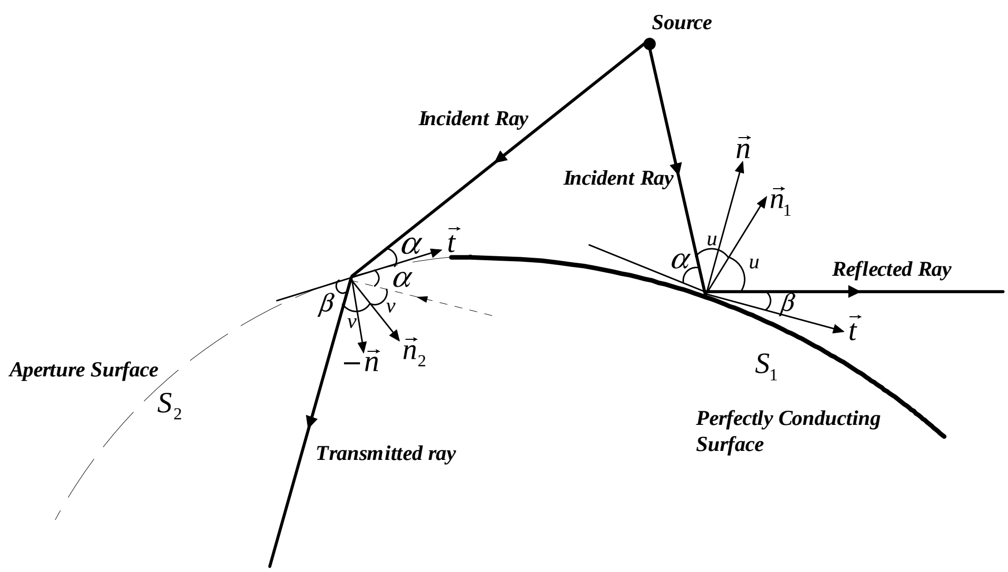

Fig. 1. Scattered fields from a perfectly conducting surface and an aperture continuation

2. The reflection and transmission angles (β) are variables which depend on the surface ($S_1 + S_2$) coordinates.

3. A new unit vector ($\vec{n}_1$, $\vec{n}_2$), which divides the angle between the reflected (or transmitted) and the incident rays into two equal parts, can be defined. $\vec{n}_1$ can be written as

$$\vec{n}_1 = \cos(u+\alpha)\vec{t} + \sin(u+\alpha)\vec{n} \tag{3}$$

for S_1 and

$$\vec{n}_2 = \cos(v+\alpha)\vec{t} - \sin(v+\alpha)\vec{n} \tag{4}$$

for S_2 where α is the angle of incidence, $\vec{t}$ and $\vec{n}$ are the actual tangential and normal unit vectors of the surface, respectively. The boundary conditions in Eqs. (1) and (2) will be evaluated according these new unit vectors. u and v are equal to $\frac{\pi}{2} - \frac{\alpha+\beta}{2}$. The total scattered electric field can be defined as

$$\vec{E}_t = \vec{E}_{is} + \vec{E}_{rs} \tag{5}$$

where

$$\vec{E}_{is} = -\frac{j\omega\mu_0}{4\pi} \iint_{S_2} \vec{n}_2 \times \vec{H}_i \Big|_{S_2} \frac{e^{-jkR_2}}{R_2} dS' + \frac{1}{4\pi} \iint_{S_2} \nabla \times \left(\vec{n}_2 \times \vec{E}_i \Big|_{S_2} \frac{e^{-jkR_2}}{R_2} \right) dS' \tag{6}$$

and

$$\vec{E}_{rs} = -\frac{j\omega\mu_0}{4\pi}\iint_{S_1} \vec{n}_1 \times \vec{H}_t\Big|_{S_1} \frac{e^{-jkR_1}}{R_1} dS' \tag{7}$$

for an electric polarized incident wave. The total scattered magnetic field can be written as

$$\vec{H}_t = \vec{H}_{is} + \vec{H}_{rs} \tag{8}$$

where

$$\vec{H}_{is} = \frac{1}{4\pi}\iint_{S_2} \nabla \times \left(\vec{n}_2 \times \vec{H}_i\Big|_{S_2} \frac{e^{-jkR_2}}{R_2} \right) dS' + \frac{j\omega\varepsilon}{4\pi}\iint_{S_2} \vec{n}_2 \times \vec{E}_i\Big|_{S_2} \frac{e^{-jkR_2}}{R_2} dS' \tag{9}$$

and

$$\vec{H}_{rs} = \frac{1}{4\pi}\iint_{S_1} \nabla \times \left(\vec{n}_1 \times \vec{H}_t\Big|_{S_1} \frac{e^{-jkR_1}}{R_1} \right) dS' \tag{10}$$

for a magnetic polarized incident wave. It is important to note that the rotational operation in Eqs. (6), (9) and (10) is applied according to the coordinates of the observation point. Here $\vec{E}_{rs}$ ($\vec{H}_{rs}$) and $\vec{E}_{is}$ ($\vec{H}_{is}$) denote the reflected scattered and incident scattered fields, respectively.

3. Scattering from a perfectly conducting half plane: MTPO approach

The geometry in Fig. 2 is considered. An electric polarized (electrical field is parallel to the surface) plane wave is illuminating the half plane.

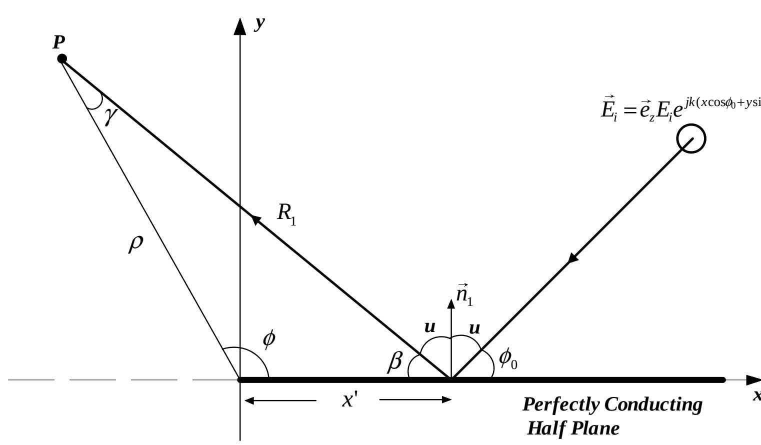

Fig. 2 Reflection geometry from a perfectly conducting half plane

Exact Theory of Physical Optics will be applied to this problem in order to evaluate scattered fields. The magnetic field of the plane wave can be written as

$$\vec{H}_i = -\frac{E_i}{Z_0}\left(\vec{e}_x \sin\phi_0 - \vec{e}_y \cos\phi_0\right) e^{jk(x\cos\phi_0 + y\sin\phi_0)} \tag{11}$$

where ϕ_0 is the angle of incidence. The method, given in the previous part, will be applied to this problem. As can be seen from Fig. 2, there are two scattered ray paths. One belongs to the

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reflected (for $0 \le \phi \le \pi - \phi_0$) and reflected diffracted (for $0 \le \phi \le 2\pi$) rays and the second is the incident (for $\pi \le \phi \le \pi + \phi_0$) and the incident diffracted (for $0 \le \phi \le 2\pi$) rays. The integrals of Eqs. (6) and (7) will be considered. Reflected plane wave can be written as

$$\vec{E}_r = \vec{e}_z E_r e^{jk(x\cos\beta - y\sin\beta)} \tag{12}$$

where β is a variable angle which is the function of surface coordinates. The amplitude of the electric field can be found as

$$E_r = -E_i e^{jkx'(\cos\phi_0 - \cos\beta)} \tag{13}$$

by using the boundary condition of $\vec{n}_1 \times \left(\vec{E}_i + \vec{E}_r\right)\Big|_{S_1} = 0$, where $\vec{E}_i = \vec{e}_z E_i e^{jk(x\cos\phi_0 + y\sin\phi_0)}$ and $\vec{n}_1 = \cos(u+\phi_0)\vec{e}_x + \sin(u+\phi_0)\vec{e}_y$. For this problem, u is equal to $\frac{\pi}{2} - \frac{\phi_0 + \beta}{2}$. The MTPO surface current can be evaluated as

$$\vec{J}_{MTPO} = \frac{E_i}{Z_0}\left[\cos u - \cos(u + \beta + \phi_0)\right] e^{jkx'\cos\phi_0} \vec{e}_z \tag{14}$$

which is equal to $\vec{n}_1 \times \left(\vec{H}_i + \vec{H}_r\right)\Big|_{S_2} = \vec{J}_{ETPO}$ at the perfectly conducting half plane. The classical PO surface current component is equal to

$$\vec{J}_{PO} = \vec{e}_z 2\frac{E_i}{Z_0}\sin\phi_0 e^{jkx'\cos\phi_0} \tag{15}$$

for this problem. The trigonometric expression in Eq. (14) can be evaluated by using the geometry in Fig. 2. As a result one obtains

$$\vec{J}_{MTPO} = \frac{2E_i}{Z_0}\sin\left(\frac{\beta+\phi_0}{2}\right) e^{jkx'\cos\phi_0}\vec{e}_z \tag{16}$$

for the MTPO surface current. The current that will flow on the surface for a magnetic polarized incident wave is calculated and compared with PO surface current in the Appendix section. The scattered electrical field can be written as

$$\vec{E}_{rs} = -\vec{e}_z \frac{jkE_i}{2\pi}\int_{x'=0}^{\infty}\int_{z'=-\infty}^{\infty} e^{jkx'\cos\phi_0}\frac{e^{-jkR_1}}{R_1}\sin\left(\frac{\beta+\phi_0}{2}\right)dx'dz' \tag{17}$$

by using Eq. (7). Here R_1 is equal to

$$R_1 = \sqrt{(x-x')^2 + y^2 + (z-z')^2}\ . \tag{18}$$

z' part of Eq. (17) gives a Hankel function a

$$\int_C e^{-jkch\alpha} d\alpha = \frac{\pi}{j} H_0^{(2)}(kR) \tag{19}$$

by using the variable change of $(z - z') = R\, sh\alpha$ where R is equal to $\sqrt{(x-x')^2 + y^2}$. As a result one obtains

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$$\vec{E}_{rs} = -\vec{e}_z \frac{kE_i}{2} \int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} H_0^{(2)}(kR) \sin\left(\frac{\beta+\phi_0}{2}\right) dx' \tag{20}$$

for the reflected scattered wave. The incident scattered field can be found by considering Fig. 3 and Eq. (7). The image of the incident field is considered. The reflected waves for $x' \in (-\infty, 0]$ is equal to the transmitted incident field for this region. The equivalent image field can be written as

$$\vec{E}_{ieq} = -\vec{e}_z E_i e^{jk(x\cos\phi_0 - y\sin\phi_0)} \tag{21}$$

and magnetic field is found to be

$$\vec{H}_{ieq} = -\frac{E_i}{Z_0}\left(\vec{e}_x \sin\phi_0 + \vec{e}_y \cos\phi_0\right) e^{jk(x\cos\phi_0 - y\sin\phi_0)} \tag{22}$$

for the plane waves illuminating the upper part of the plane.

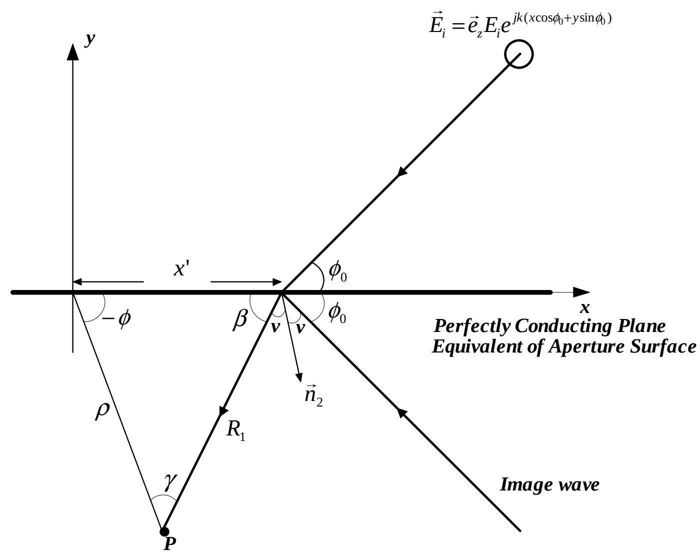

Fig. 3. Transmission geometry for the modified theory of physical optics

The reflected electric and magnetic fields can be evaluated by following the steps between Eqs. (12) and (13). The equivalent surface current is found to be

$$\vec{J}_{MTPO} = -\frac{2E_i}{Z_0} \sin\left(\frac{\beta+\phi_0}{2}\right) e^{jkx'\cos\phi_0} \vec{e}_z \tag{23}$$

where $\vec{n}_2$ is equal to $\cos(v+\phi_0)\vec{e}_x - \sin(v+\phi_0)\vec{e}_y$ for the aperture surface and $v = \frac{\pi}{2} - \frac{\phi_0+\beta}{2}$. The operations from Eqs. (17) to (20) are also valid for this case. As a result one obtains

$$\vec{E}_{is} \approx \vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{x'=-\infty}^{0} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\frac{\phi_0+\beta}{2} dx' \tag{24}$$

for the incident scattered wave. The total scattered electric field can be written as

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$$\vec{E}_s = \vec{E}_{rs} + \vec{E}_{is} \tag{25}$$

for

$$\vec{E}_{rs} \approx -\vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\left(\frac{\beta+\phi_0}{2}\right) dx' \tag{26}$$

denoting the reflected scattered field found from Eq. (20) by using the Debye asymptotic expansion of $H_0^{(2)}(kR_1)$. The integrals of Eqs. (24) and (26) denote the scattered fields from the half plane. Eq. (26) gives the reflected and reflected diffracted waves. Eq. (24) consists of incident field for $\phi \ge \pi$ and incident diffracted wave for $\phi \in [0, 2\pi]$.

4. Asymptotic evaluation of scattering integrals

The incident, reflected and diffracted fields will be evaluated asymptotically by the method of stationary phase and edge point technique [4,23] for k→∞. The total electric field can be obtained as

$$\vec{E}_t \approx \vec{e}_z \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \left(\int_{x'=-\infty}^{0} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\frac{\phi_0+\beta}{2} dx' - \int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\left(\frac{\beta+\phi_0}{2}\right) dx' \right) \tag{27}$$

by combining Eqs. (24) and (26). The phase functions of the integrals in Eq. (27) can be written as

$$g(x') = \rho\cos\gamma + x'(\cos\beta - \cos\phi_0) \tag{28}$$

for the geometry in Figs. 2 and 3. The first derivative of the Eq. (28) can be expressed as

$$\frac{\partial g}{\partial x'} = -\rho\sin\gamma\frac{d\gamma}{dx'} - x'\sin\beta\frac{d\beta}{dx'} + \cos\beta - \cos\phi_0 \tag{29}$$

where γ is equal to

$$\gamma = \pi - \beta \mp \phi \tag{30}$$

according to the geometry in Figs. 2 and 3. One obtains

$$\rho\sin\gamma\frac{d\gamma}{dx'} = -x'\sin\beta\frac{d\beta}{dx'} \tag{31}$$

by using sine relations and the derivative of Eq. (30) according to x'. As a result one obtains

$$\beta_s = \phi_0 \tag{32}$$

which denotes the reflection law by considering

$$\left.\frac{\partial g}{\partial x'}\right|_s = \cos\beta_s - \cos\phi_0 = 0 \tag{33}$$

at the stationary phase point. The second derivative of Eq. (28) can be evaluated as

$$\left.\frac{\partial^2 g}{\partial x'^2}\right|_s = \frac{\sin^2\phi_0}{l} \tag{34}$$

where l can be expressed as

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$$l = R_1\big|_s = \rho\cos\gamma_s \tag{35}$$

denoting the ray path of the reflected and incident fields. γ_s is equal to $\pi - \phi_0 + \phi$ for the incident wave and $\pi - \phi_0 - \phi$ for the reflected wave. The phase function of Eq. (28) can be expanded for the first three terms as

$$g(x') \approx l + \frac{1}{2}\frac{\sin^2\phi_0}{l}(x' - x'_s)^2 \tag{36}$$

and the amplitude function is evaluated for the first term of Taylor series as

$$f(x'_s) \approx \pm\frac{kE_i}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\frac{\sin\phi_0}{\sqrt{kl}} \tag{37}$$

where it can be seen that the trigonometric parts of the MTPO current reduces to the PO current. The integrals in Eqs. (24) and (26) can be written as

$$E_{r,i} \approx \pm\frac{kE_i\sin\phi_0}{\sqrt{2\pi}}e^{j\frac{\pi}{4}}\frac{e^{-jkl}}{\sqrt{kl}}\int_{-\infty}^{\infty}e^{-jk\frac{\sin^2\phi_0}{l}(x'-x'^2)}dx' \tag{38}$$

and can be evaluated easily by considering the integral of

$$\int_{-\infty}^{\infty}e^{-\frac{y^2}{2}}dy = \sqrt{2\pi}\ . \tag{39}$$

As a result, one obtains

$$E_r \approx -E_i e^{jk\rho\cos(\phi+\phi_0)} \tag{40}$$

for the reflected fields from the half plane. The radiated wave from the aperture can be written as

$$E_i \approx E_i e^{jk\rho\cos(\phi-\phi_0)} \tag{41}$$

from the stationary point contribution. The edge diffracted fields can be evaluated by using the formula

$$\vec{E}_d \approx \mp\vec{e}_z\frac{1}{jk}\frac{f(0)}{g'(0)}e^{-jkg(0)} \tag{42}$$

where $f(0)$ and $g(0)$ denote the values of the amplitude and the phase functions of Eqs. (24) and (26) at the edge point, respectively [4,24]. The plus sign in Eq. (42) is used when the edge point is the lower value of the integral, and minus sign is used for the upper limit. $g'(0)$ is the value of the first derivative of the phase function at the edge point. The related phase and amplitude quantities for Eq. (26) can be written as

$$g(0) = \rho\ , \tag{43}$$

$$g'(0) = -(\cos\phi + \cos\phi_0) \tag{44}$$

and

$$f(0) = -\frac{kE_i}{2\sqrt{2\pi}}e^{j\frac{\pi}{4}}\frac{\cos\left(\frac{\phi-\phi_0}{2}\right)}{\sqrt{k\rho}} \tag{45}$$

for $x'_e = 0$, $\beta_e = \pi - \phi$ as the edge coordinates for the half plane. One obtains

$$\vec{E}_{rd} = \vec{e}_z \frac{E_i}{2\sqrt{2\pi}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \frac{\cos\left(\frac{\phi - \phi_0}{2}\right)}{\cos\phi + \cos\phi_0} e^{-j\frac{\pi}{4}} \tag{46}$$

for the reflected edge diffracted field. Reflected edge diffraction coefficient can be written as

$$D_{rd} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{\cos\left(\frac{\phi - \phi_0}{2}\right)}{\cos\phi + \cos\phi_0} \tag{47}$$

for the MTPO integral. The phase and the amplitude functions of Eq. (24) can be expressed as

$$g(0) = \rho\,, \tag{48}$$

$$g'(0) = -(\cos\phi + \cos\phi_0) \tag{49}$$

and

$$f(0) = -\frac{kE_i}{2\sqrt{2\pi}} e^{j\frac{\pi}{4}} \frac{\cos\left(\frac{\phi + \phi_0}{2}\right)}{\sqrt{k\rho}} \tag{50}$$

for $x'_e = 0$, $\beta_e = \phi - \pi$ as the edge coordinates for the aperture integral. The incident edge diffracted field can be evaluated as

$$\vec{E}_{id} = -\vec{e}_z \frac{E_i}{2\sqrt{2\pi}} \frac{e^{-jk\rho}}{\sqrt{k\rho}} \frac{\cos\left(\frac{\phi + \phi_0}{2}\right)}{\cos\phi + \cos\phi_0} e^{-j\frac{\pi}{4}} \tag{51}$$

by using Eqs. (48), (49) and (50) in Eq. (42). The incident edge diffraction coefficient can be written as

$$D_{id} = -\frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}} \frac{\cos\left(\frac{\phi + \phi_0}{2}\right)}{\cos\phi + \cos\phi_0} \tag{52}$$

and one obtains

$$D_t = D_{id} + D_{rd} = \frac{e^{-j\frac{\pi}{4}}}{\sqrt{2\pi}} \frac{\cos\left(\frac{\phi - \phi_0}{2}\right) - \cos\left(\frac{\phi + \phi_0}{2}\right)}{\cos\phi + \cos\phi_0} \tag{53}$$

for the total edge diffraction coefficient of MTPO integral.

5. Numerical results

MTPO integral for scattering from a perfectly conducting half plane will be compared with the classical PO approach, MTPO integral for $\beta = \phi_0$ and exact solution. Equation (27) is valid for this case and it represents the total scattered field (incident, reflected and edge diffracted). The total field, calculated by the physical optics theory, is given by

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$$E_{TPO} \approx E_i e^{jk(x\cos\phi_0 + y\sin\phi_0)} - \frac{kE_i}{2}\sin\phi_0 \int_0^{\infty} e^{jkx'\cos\phi_0} H_0^{(2)}(kR_1)dx' \tag{54}$$

for a plane wave illuminated perfectly conducting half plane [12]. The case of $\beta = \phi_0$ will also be plotted and compared numerically as

$$E_{MTPO}\Big|_{\beta=\phi_0} \approx \frac{kE_i}{2}\left(\int_{x'=-\infty}^{0} e^{jkx'\cos\phi_0} H_0^{(2)}(kR_1)\sin\phi_0 dx' - \int_0^{\infty} e^{jkx'\cos\phi_0} H_0^{(2)}(kR_1)\sin\phi_0 dx' \right) \tag{55}$$

with the series expression of Eq. (58). The first integral in Eq. (55) represents the radiated field and the second one is the physical optics integral. The incident field will be added to the PO integral for $0 \le \phi \le 2\pi$ and the geometrical optics fields (incident and reflected) will be considered for the radiated wave in the interval of $\phi \in [0, \pi]$. The total fields, obtained by the mentioned procedure, must be divided by two, because the value of the field is doubled for all values of ϕ. The total field can be written as

$$E_{TMTPO} = \frac{1}{2}\left[E_i\left(e^{jk\rho\cos(\phi-\phi_0)} - e^{jk\rho\cos(\phi+\phi_0)}\right)u(\pi-\phi) + E_{TPO} + E_R\right] \tag{56}$$

where E_R is the radiated field. The unit step function in Eq. (56) can be defined as

$$u(\pi-\phi) = \begin{cases} 1 \quad , & \phi \le \pi \\ 0 \quad , & \phi \ge \pi \end{cases} \tag{57}$$

in order to express the addition of total geometrical optics fields (reflected and incident) for $\phi \in [0, \pi]$.

The exact solution of a half plane problem can be given as

$$E_t = 2E_i \sum_{m=1}^{\infty} e^{jm\frac{\pi}{4}} J_{m/2}(k\rho)\sin\frac{m}{2}\phi \sin\frac{m}{2}\phi_0 \tag{58}$$

for an electric polarized incident plane wave [25]. The MTPO integral can be written as

$$E_{MTPO} \approx \frac{kE_i}{\sqrt{2\pi}} e^{j\frac{\pi}{4}}\left(\int_{x'=-\infty}^{0} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\frac{\phi_0+\beta}{2} dx' - \int_{x'=0}^{\infty} e^{jkx'\cos\phi_0} \frac{e^{-jkR_1}}{\sqrt{kR_1}} \sin\left(\frac{\beta+\phi_0}{2}\right) dx' \right) \tag{59}$$

for the half plane problem. The first integral represents the incident field for $\pi \le \phi \le \pi + \phi_0$ and incident diffracted waves for $0 \le \phi \le 2\pi$. The second integral in Eq. (59) consists of the reflected and reflected diffracted fields.

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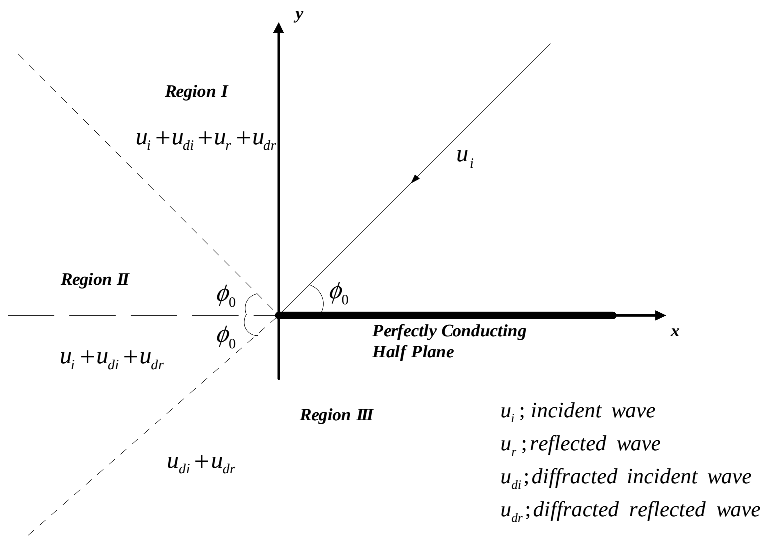

Fig. 4. Regions for scattered fields in a perfectly conducting half plane

The half plane geometry is divided into three regions as shown in Fig. 4. The geometrical optics and edge diffracted fields are plotted by considering this geometry. In Region I, it is apparent that there will be incident (u_i), reflected (u_r) and diffracted fields ($u_{di} + u_{dr}$). In Region II, there are incident and diffracted fields. There are only diffracted fields in Region III. Since PO integral consists of reflected and reflected diffracted fields, the incident field is added to Eq. (54) for all values of ϕ. The incident field is added to the integrals in Eq. (59) for $\phi \leq \pi$. The figures are plotted for $\phi_0 = \pi/4$ and $\rho = 6\lambda$.

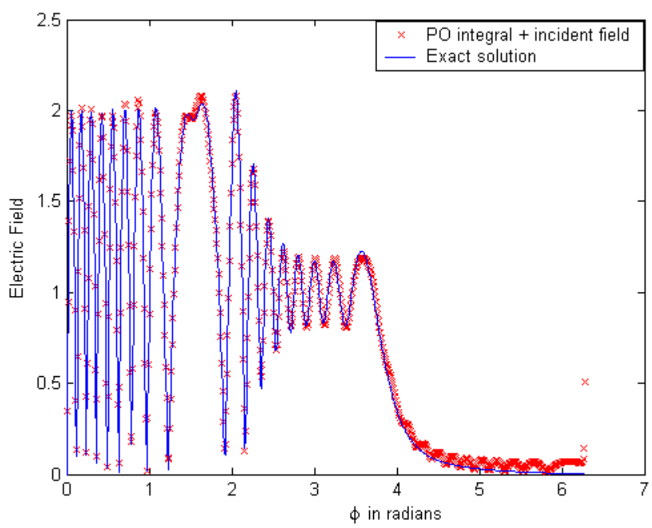

Fig. 5. Reflected and diffracted fields from perfectly conducting half plane (PO and exact solution)

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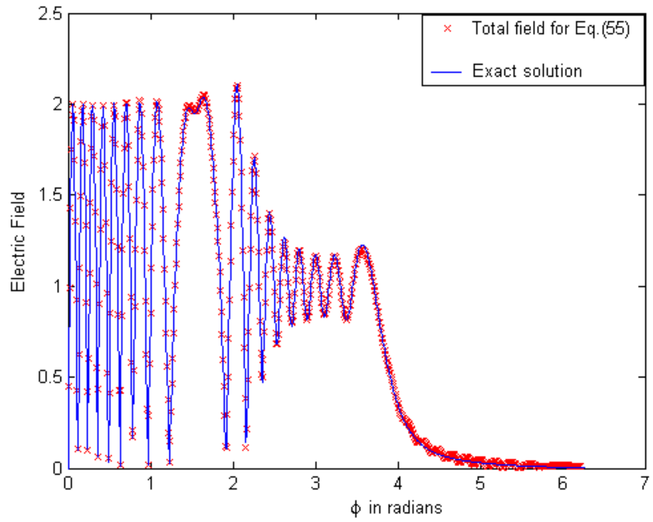

Fig. 6. Reflected and diffracted fields from perfectly conducting half plane [MTPO ($\beta = \phi_0$) and exact solution]

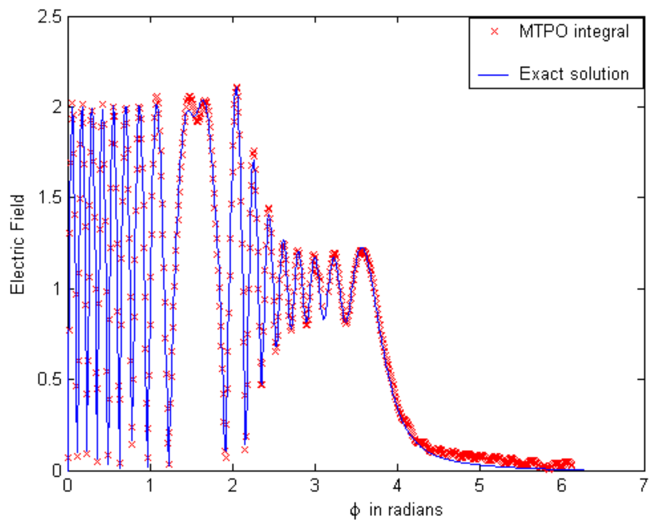

Fig. 7. Reflected and diffracted fields from perfectly conducting half plane (MTPO and exact solution)

Figure 5 shows the variation of Eqs. (54) and (58) versus observation angle. It can be seen that PO integral deviates from the exact asymptotic solution after $\phi = \pi + \phi_0$, since the edge diffraction field, found from the PO phase contribution, is not the exact field. Incident field is added to Eq. (54) for all values of ϕ.

Figure 6 depicts the variation of MTPO ($\beta = \phi_0$) field and the exact scattered fields in Eqs. (56) and Eq. (58) versus the observation angle. It is observed that the MTPO integral,

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written for a constant reflection and transmission angle, is harmonious with the exact waves for all values of ϕ.

In Fig. 7, the integral in Eq. (59) is compared with Eq. (58). It can be seen that the fields are compatible.

7. Conclusion

In this work, a new approach to Physical Optics concept is introduced by defining three axioms. This method is named as Exact Theory of Physical Optics (MTPO). The introduced procedure is applied to a well known problem of perfectly conducting half plane and exact scattered fields (reflected and edge diffracted) and also incident field for $\pi \le \phi \le \pi + \phi_0$ is evaluated by using asymptotic methods. Numerical results show that MTPO integral gives the exact fields harmonious with the exact series solution of Helmholtz equation. It is important to note that the integral, related with the reflected field, gives the reflected diffracted fields and the incident diffracted waves are evaluated from the integral, written for the aperture.

8. Appendix

MTPO current for a magnetic polarized incident wave will be evaluated. Figure 2 can also be considered for a magnetic polarized illumination. Magnetic field of the incident wave can be written as

$$\vec{H}_i = \vec{e}_z H_i e^{jk(x\cos\phi_0 + y\sin\phi_0)} \tag{60}$$

and electric field can be found as

$$\vec{E}_i = Z_0 H_i \left(\sin\phi_0 \vec{e}_x + \cos\phi_0 \vec{e}_y\right) e^{jk(x\cos\phi_0 + y\sin\phi_0)} \tag{61}$$

by using the Maxwell-Ampere equation. Reflected plane wave can be written as

$$\vec{H}_r = \vec{e}_z H_r e^{jk(x\cos\beta - y\sin\beta)} \tag{62}$$

where β is a variable angle which is the function of surface coordinates, as in Eq. (12). The reflected electric field can be found as

$$\vec{E}_r = -Z_0 H_r \left(\sin\beta \vec{e}_x + \cos\beta \vec{e}_y\right) e^{jk(x\cos\beta - y\sin\beta)} \tag{63}$$

by considering Maxwell-ampere equation. One obtains

$$H_r e^{jkx'\cos\beta} = H_i e^{jkx'\cos\phi_0} \tag{64}$$

as a result of the boundary condition of $\vec{n}_1 \times \left(\vec{E}_i + \vec{E}_r\right)\Big|_{S_1} = 0$, with Eqs. (61) and (63) for $\vec{n}_1 = \cos(u+\phi_0)\vec{e}_x + \sin(u+\phi_0)\vec{e}_y$. The MTPO surface current can be evaluated as

$$\vec{J}_{MTPO} = 2H_i \left[\vec{e}_x \cos\frac{\beta - \phi_0}{2} - \vec{e}_y \sin\frac{\beta - \phi_0}{2}\right] e^{jkx'\cos\phi_0} \tag{65}$$

which is equal to $\vec{n}_1 \times \left(\vec{H}_i + \vec{H}_r\right)\Big|_{S_2} = \vec{J}_{MTPO}$ on the perfectly conducting half plane. The classical PO surface current component is equal to

$$\vec{J}_{PO} = \vec{e}_x 2H_i e^{jkx'\cos\phi_0} \tag{66}$$

for this problem. It is apparent that, the current, given in Eq. (65) is equal to $\vec{J}_{PO}$ for $\beta = \phi_0$.

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