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A novel method for analysing the fractal fractional integrator circuit

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Abstract In this article, we propose the integrator circuit model by the fractal-fractional operator in which fractional-order has taken in the Atangana-Baleanu sense. Through Schauder's fixed point theorem, we establish existence theory to ensure that the model posses at least one solution and via Banach fixed theorem, we guarantee that the proposed model has a unique solution. We derive the results for Ulam-Hyres stability by mean of non-linear functional analysis which shows that the proposed model is Ulam-Hyres stable under the new fractal-fractional derivative. We establish the numerical results of the model under consideration through Atanaga-Toufik method. We simulate the numerical results for different sets of fractional order and fractal dimension.

1. Introduction and background materials

Mathematical modeling plays a key role in the various field of applied sciences. The modeling of circuit problems has become a potential area of research in recent years to study various properties of circuit problems. Researchers and engineers are interested in the study of the complex behavior of circuit problems through different mathematical methods and models. But in most cases, ordinary differential operators fail to analyze the desired behavior of non-linear problems. Due to memory and non-locality, different fractional-order operators have been used for the more critical study of physical problems [1–6]. It is seen that model involving fractional order integral and differential equations are more accurate instead of classical model [12–18].

Numerous indigenous events can be described precisely by the fractional calculus [7]. In the fractional controllers systems, the difficulty is the choice of the fractional values to fulfil the variables of the fractional order [8]. The fractional operator s^{γ} is the main constructing any fractional order systems block. There are two main problems in the effectuation of this circuit block that need to be fixed: efficiency problem in realization and reducing the number of components. In electronic, the

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reduction of the number of constituents in the effectuation of a circuit system can give lots of benefits: the reduction of the cost of effectuation, the reduction of the heating of constituents and the effectuation of a reduced model of circuit. Approximation of fractional order integrator and differentiator has constructed many techniques [9–11]. In [8], the authors proposed a circuit integrator model as

$$\begin{cases} {}_0{}^{C}D_k^{\alpha}x(k) = y(k), \\ {}_0{}^{C}D_k^{\alpha}y(k) = \frac{-1}{3}\left(x(k) + \frac{3}{2}\left(\left(z(k)^2 - 1\right)y(k)\right)\right), \\ {}_0{}^{C}D_k^{\alpha}z(k) = -y(k) - \frac{3}{5}z(k) + y(k)z(k). \end{cases} \quad (1)$$

The idea of fractal derivative solves many problems in nature. Recently, Atangana [19] proposed a fractal derivative of the convolution of function with three major definitions, namely power law, exponential decay law, and the generalized function of Mittag-Leffer. These new types of derivatives have two parameters, first one represents the fractional-order, and the other represents the fractal dimension. These operators tend to model more complex phenomena than fractional-order operators. Chaotic systems are almost one of the most important and applicable types of nonlinear circuit problems. Therefore in many cases, the exact solution is not available for such equations. On the other hand, the use of new derivative operators in structures of chaotic systems has made significant development in this field. In some cases the researchers have obtained desirable attractors, which were not achievable by common integer-order operators. This fact highlights the importance of new derivative operators in other real-world models. Therefore, we will study the model (1) through newly Atangana-Baleanu fractal-fractional operator. We consider the system (1) under fractal-fractional derivative in Atangana-Baleanu sense with fractional order γ, and fractal order β as follows

$$\begin{cases} {}_0{}^{FFM}D_k^{\gamma,\beta}x(k) = y(k), \\ {}_0{}^{FFM}D_k^{\gamma,\beta}y(k) = \frac{-1}{3}\left(x(k) + \frac{3}{2}\left(\left(z(k)^2 - 1\right)y(k)\right)\right), \\ {}_0{}^{FFM}D_k^{\gamma,\beta}z(k) = -y(k) - \frac{3}{5}z(k) + y(k)z(k). \end{cases} \quad (2)$$

with initial conditions $x(0) = x_0, y(0) = y_0, z(0) = z_0$. In the present article, we will study the existence and stability, and Atanaga-Toufik numerical scheme of the model (2). The definition Atangana Baleanu fractal-fractional derivative and its corresponding integral is given by

Definition 1.1 [19]. Let $u(k)$ be continuous in (a, b) and fractal differentiable on (a, b) with order β. Then, the fractal-fractional derivative of u of order γ is given by:

$$^{FFM}{}_aD_k^{\gamma,\beta}u(k) = \frac{AB(\gamma)}{1-\gamma}\frac{d}{dk^{\beta}}\int_a^k u(z)E_{\gamma}\left(\frac{-\gamma}{1-\gamma}(k-z)^{\gamma}\right)dz, \quad 0 < \gamma, \beta \leqslant 1, \quad (3)$$

where $AB(\gamma)$ is the normalization function such that $AB(0) = AB(1) = 1$.

Definition 1.2 [19]. Let $u(k)$ be continuous in (a, b). Then the fractal-fractional integral of u with order γ is presented as:

$$^{FFM}{}_0I_k^{\gamma,\beta}u(k) = \frac{\beta\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^k z^{\beta-1}u(z)(k-z)^{\gamma-1}dz + \frac{\beta(1-\gamma)k^{\beta-1}}{AB(\gamma)}u(k). \quad (4)$$

We organized the paper as: Section 1 deals with the introduction of the article and basic notions of the fractal-fractional calculus. Section 2 deals with existence and uniqueness of solution of the proposed model via fixed point theory. Section 3 provides the Ulam-Hyres stability of the model. The proper numerical scheme and simulations is given in Section 4. The conclusion of the article is given in Section 5.

2. Existence and uniqueness results

We demonstrate the existence of least one and unique solution of the model under consideration via fixed point results. Consider the model (2) as

$$\begin{cases} {}_0{}^{ABR}D_k^{\gamma}x(k) = \beta k^{\beta-1}\mathscr{Q}(k,x,y,z), \\ {}_0{}^{ABR}D_k^{\gamma}y(k) = \beta k^{\beta-1}\mathscr{W}(k,x,y,z), \\ {}_0{}^{ABR}D_k^{\gamma}z(k) = \beta k^{\beta-1}\mathscr{E}(k,x,y,z), \end{cases} \quad (5)$$

where

$$\begin{cases} \mathscr{Q}(k,x,y,z) = y(k), \\ \mathscr{W}(k,x,y,z) = \frac{-1}{3}\left(x(k) + \frac{3}{2}\left(\left(z(k)^2 - 1\right)y(k)\right)\right), \\ \mathscr{E}(k,x,y,z) = -y(k) - \frac{3}{5}z(k) + y(k)z(k). \end{cases}$$

We can write system (5) as:

$$\begin{cases} {}_0{}^{ABR}D_k^{\gamma}\Xi(k) = \beta k^{\beta-1}\Lambda(k,\Xi(k)), \\ \Xi(0) = \Xi_0. \end{cases} \quad (6)$$

By replacing ${}_0{}^{ABR}D_k^{\gamma}$by ${}_0{}^{ABC}D_k^{\gamma,\beta}$ and applying fractional integral, we get

$$\Xi(k) = \Xi(0) + \frac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Xi(k)) + \frac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k \lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda,$$

where

$$\Xi(k) = \begin{cases} x(k) \\ y(k) \\ z(k) \end{cases}, \Xi(0) = \begin{cases} x(0) \\ y(0) \\ z(0) \end{cases}, \Lambda(k,\Xi(t)) = \begin{cases} \mathscr{Q}(k,x,y,z) \\ \mathscr{W}(k,x,y,z) \\ \mathscr{E}(k,x,y,z) \end{cases}$$

For the existence theory, we define a Banach space $\mathfrak{B} = \mathscr{Y} \times \mathscr{Y} \times \mathscr{Y}$, where $\mathscr{Y} = \mathbb{C}[0, \mathbb{T}]$ under the norm

$$\|\Xi\| = \max_{k\in[0,\mathbb{T}]}|x(k) + y(k) + z(k)|.$$

Define an operator $\mathscr{L} : \mathfrak{B} \to \mathfrak{B}$as:

$$\mathscr{L}(\Xi)(k) = \Xi(0) + \frac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Xi(k)) + \frac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k \lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda. \quad (7)$$

Now we impose growth and Lipschitz condition on nonlinear function $\Lambda(k, \Xi(k))$ as:

- For each $\Xi \in \mathfrak{B}, \exists$ constants $\mathscr{C}_{\Lambda} > 0$ and M_{Λ} such that

$$|\Lambda(k,\Xi(k))| \leqslant \mathscr{C}_\Lambda|\Xi(k)| + M_\Lambda, \tag{8}$$

- For each $\Xi, \overline{\Xi} \in \mathfrak{B}, \exists$ a constant $\mathscr{L}_\Lambda > 0$ such that

$$\left|\Lambda(k,\Xi(k)) - \Lambda\big(k,\overline{\Xi}(k)\big)\right| \leqslant \mathscr{L}_\Lambda\left|\Xi(k) - \overline{\Xi}(k)\right|. \tag{9}$$

Theorem 2.1. *Assume that the condition (8) holds. Let $\Lambda : [0,\mathbb{T}] \times \mathfrak{B} \to \mathbb{R}$be a continuous function. Then the proposed model has least one solution.*

Proof. First we have to show that the operator $\mathscr{L}$ defined by (7) is completely continuous. Since Λ is continuous, therefore, $\mathscr{L}$ is also continuous.

Let $\mathbb{H} = \{\Xi \in \mathfrak{B} : \|\Xi\| \leqslant \mathscr{R}, \mathscr{R} > 0\}$. Now for any $\Xi \in \mathfrak{B}$, we have

$$\begin{aligned}
\|\mathscr{L}(\Xi)\| = \max_{t\in[0,\mathbb{T}]}\Big|\Xi(0) &+ \tfrac{\beta t^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(t,\Xi(t)) \\
&+ \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)} \int_0^t \lambda^{\beta-1}(t-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda\Big| \\
&\leqslant \Xi(0) + \tfrac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)}(\mathscr{C}_\Lambda\|\Xi\| + M_\Lambda) \\
+\max_{t\in[0,\mathbb{T}]} &\tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)} \int_0^t \lambda^{\beta-1}(t-\lambda)^{\beta-1}|\Lambda(\lambda,\Xi(\lambda))|d\lambda \\
&\leqslant \Xi(0) + \tfrac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)}(\mathscr{C}_\Lambda\|\Xi\| + M_\Lambda) \\
&+ \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}(\mathscr{C}_\Lambda\|\Xi\| + M_\Lambda)\mathbb{T}^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta) \\
&\leqslant \mathscr{R}.
\end{aligned}$$

Thus the operator $\mathscr{L}$ is uniformly bounded, where $\mathscr{H}(\gamma,\beta)$ denote the beta function.

For equicontinuity of $\mathscr{L}$, let us take $k_1 < k_2 \leqslant \mathbb{T}$. Then consider

$$\begin{aligned}
|\mathscr{L}(\Xi)(k_2) - \mathscr{L}(\Xi)(k_1)| = \Big|&\tfrac{\beta k_2^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k_2,\Xi(k_2)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^{k_2}\lambda^{\beta-1}(k_2-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda \\
&- \tfrac{\beta k_1^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k_1,\Xi(k_1)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^{k_1}\lambda^{\beta-1}(k_1-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda\Big| \\
\leqslant &\tfrac{\beta k_2^{\beta-1}(1-\gamma)}{AB(\gamma)}(\mathscr{C}_\Lambda|\Xi(k)| + M_\Lambda) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}(\mathscr{C}_\Lambda|\Xi(k)| + M_\Lambda)k_2^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta) \\
&- \tfrac{\beta k_1^{\beta-1}(1-\gamma)}{AB(\gamma)}(\mathscr{C}_\Lambda|\Xi(t)| + M_\Lambda) - \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}(\mathscr{C}_\Lambda|\Xi(k)| + M_\Lambda)k_1^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta),
\end{aligned}$$

when $t_1 \to k_2$,then $|\mathscr{L}(\Xi)(k_2) - \mathscr{L}(\Xi)(k_1)| \to 0$. Consequently, we can say that

$$\|\mathscr{L}(\Xi)(k_2) - \mathscr{L}(\Xi)(k_1)\| \to 0, \quad \text{as } k_1 \to k_2.$$

Hence $\mathscr{L}$ is equicontinuous. So by Arzela-Ascoli theorem is completely continuous. Thus, by Schauder's fixed point result the proposed model has at least one solution. □

Theorem 2.2. *Let (9) holds. If $\rho < 1$, where*

$$\rho = \left(\frac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)} + \frac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\mathbb{T}^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta)\right)\mathscr{L}_\Lambda,$$

then the considered model has a unique solution.

Proof. For $\Xi, \overline{\Xi} \in \mathfrak{B}$,we have

$$\begin{aligned}
\left\|\mathscr{L}(\Xi) - \mathscr{L}(\overline{\Xi})\right\| = \max_{k\in[0,\mathbb{T}]}\Big|&\tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\left(\Lambda(k,\Xi(k)) - \Lambda\big(k,\overline{\Xi}(k)\big)\right) \\
&+ \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}d\lambda\left[\Lambda(\lambda,\Xi(\lambda)) - \Lambda\big(\lambda,\overline{\Xi}(\lambda)\big)\right]\Big| \\
&\leqslant \left[\tfrac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)} + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\mathbb{T}^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta)\right]\left\|\Xi - \overline{\Xi}\right\| \\
&\leqslant \rho\left\|\Xi - \overline{\Xi}\right\|.
\end{aligned}$$

Hence $\mathscr{L}$ is contraction. So by Banach contraction principle, the given model has a unique solution. □

3. Ulam-Hyres stability

Here we are going to demonstrate the Ulam-Hyres stability of the proposed model.

Definition 3.1. The proposed model is Ulam-Hyres stable if $\exists$ $\aleph_{\gamma,\beta} \geqslant 0$ such that for any $\epsilon > 0$ and for every $\Xi \in \mathbb{C}([0,\mathbb{T}],\mathbb{R})$ satisfies

$$\left|{}_0^{FFM}D_k^{\gamma,\beta}\Xi(k) - \Lambda(k,\Xi(k))\right| \leqslant \epsilon, \; k \in [0,\mathbb{T}],$$

and there exists a unique solution $\Omega \in \mathbb{C}([0,\mathbb{T}],\mathbb{R})$ such that

$$|\Xi(k) - \Omega(k)| \leqslant \aleph_{\gamma,\beta}\epsilon, \quad k \in [0,\mathbb{T}].$$

We take into consideration a small perturbation $\Phi \in \mathbb{C}[0,\mathbb{T}]$ such that $\Phi(0) = 0$. Let

- $|\Phi(k)| \leqslant \epsilon$,for $\epsilon > 0$.
- ${}_0^{FFM}D_k^{\gamma,\beta}\Xi(k) = \Lambda(k,\Xi(k)) + \Phi(k)$.

Lemma 3.2. *The solution of the perturbed model*

$$\begin{aligned}
{}_0^{FFM}D_k^{\gamma,\beta}\Xi(k) &= \Lambda(k,\Xi(k)) + \Phi(k), \\
\Xi(0) &= \Xi_0,
\end{aligned}$$

fulfills the relation given below

$$\left|\Xi(k) - \left(\Xi(0) + \tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Xi(k)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda\right)\right| \leqslant \Theta_{\gamma,\beta}\epsilon,$$

where $\Theta_{\gamma,\beta} = \dfrac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)} + \dfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\mathbb{T}^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta)$.

Proof. The proof is easy so we omit it. □

Lemma 3.3. *Under condition (9) along with lemma (1), the solution of the proposed model is Ulam-Hyres stable if $\rho < 1$.*

Proof. Let $\Omega \in \mathfrak{B}$ be a unique solution and $\Xi \in \mathfrak{B}$ be any solution of the proposed model, then

$$\begin{aligned}
&|\Xi(k) - \Omega(k)| \\
&= \Big|\Xi(k) - \Big[\Omega(0) + \tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Omega(k)) \\
&\quad + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Omega(\lambda))d\lambda\Big]\Big| \\
&\leqslant \Big|\Xi(k) - \Big(\Xi(0) + \tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Xi(k)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda\Big)\Big| \\
&\quad + \Big|\Xi(0) + \tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Xi(k)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Xi(\lambda))d\lambda\Big| \\
&\quad - \Big|\Omega(0) + \tfrac{\beta k^{\beta-1}(1-\gamma)}{AB(\gamma)}\Lambda(k,\Omega(k)) + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\int_0^k\lambda^{\beta-1}(k-\lambda)^{\beta-1}\Lambda(\lambda,\Omega(\lambda))d\lambda\Big| \\
&\leqslant \Theta_{\gamma,\beta}\epsilon + \left(\tfrac{\beta\mathbb{T}^{\beta-1}(1-\gamma)}{AB(\gamma)} + \tfrac{\gamma\beta}{AB(\gamma)\Gamma(\gamma)}\mathbb{T}^{\gamma+\beta-1}\mathscr{H}(\gamma,\beta)\right)\mathscr{L}_\Lambda|\Xi(t) - \Omega(t)| \\
&\leqslant \Theta_{\gamma,\beta}\epsilon + \rho|\Xi(k) - \Omega(k)|.
\end{aligned}$$

Consequently one can write

$$\|\Xi - \Omega\| \leqslant \Theta_{\gamma,\beta}\epsilon + \rho\|\Xi - \Omega\|.$$

We can write the above relation is

$$\|\Xi - \Omega\| \leqslant \aleph_{\gamma,\beta}\epsilon,$$

where $\aleph_{\gamma,\beta} = \frac{\Theta_{\gamma,\beta}}{1-\rho}$. Hence the solution of the proposed problem is Ulam-Hyres stable. □

4. Numerical results and simulations

Electronic circuits show the amazing potential to demonstrate at low cost most of nonlinear phenomena obtained in dynamical systems. We simulate the electronic circuit realization of the fractal fractional order chaotic system in this section. We consider the fractal fractional Memristor based chaotic system as [8]:

$$^{FFM}{}_aD_k^{\gamma,\beta}x(k) = y(k), \tag{10}$$

$$^{FFM}{}_aD_k^{\gamma,\beta}y(k) = -\frac{1}{3}\left(x(k) + \frac{3}{2}\left(z(k)^2 - 1\right)y(k)\right), \tag{11}$$

$$^{FFM}{}_aD_k^{\gamma,\beta}z(k) = -y - \frac{3}{5}z(k) + y(k)z(k). \tag{12}$$

The circuital equations reelated to the above system equation is obtained as [8]:

$$^{FFM}{}_aD_k^{\gamma,\beta}X = \frac{1}{C_0}\left[-\left(\frac{R_2}{R_1R_3}\right)Y\right], \tag{13}$$

$$^{FFM}{}_aD_k^{\gamma,\beta}Y = \frac{1}{C_0}\left[\left(\frac{R_7}{R_8R_5}\right)Y - \frac{1}{R_4}X - \frac{1}{R_6}Z^2Y\right], \tag{14}$$

$$^{FFM}{}_aD_k^{\gamma,\beta}Z = \frac{1}{C_0}\left[\left(\frac{R_{12}}{R_{13}R_9}\right)YZ - \frac{1}{R_{10}}Y - \frac{1}{R_{11}}Z\right]. \tag{15}$$

We define the parameters for the above system as: $R_1 = R_2 = R_3 = R_5 = R_7 = R_9 = R_{10} = R_{12} = R_{13} = 10\ \text{k}\Omega$, $R_6 = R_0 = 10.3\ \text{k}\Omega$, $R_4 = 11.62\ \text{k}\Omega$ $R_{11} = 11.49\ \text{k}\Omega$.We can write the system (10)–(12) as

$$\begin{aligned}
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k x(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= \beta k^{\beta-1}y(k),\\
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k y(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= -\frac{\beta t^{\beta-1}}{3}\left(x(k) + \frac{3}{2}\left(z(k)^2 - 1\right)y(k)\right),\\
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k z(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= \beta k^{\beta-1}\left(-y - \frac{3}{5}z(k) + y(k)z(k)\right).
\end{aligned}$$

For simplicity, we define

$$\begin{aligned}
D(k,x,y,z) &= \beta k^{\beta-1}y(k),\\
E(k,x,y,z) &= -\frac{\beta t^{\beta-1}}{3}\left(x(k) + \frac{3}{2}\left(z(k)^2 - 1\right)y(k)\right),\\
F(k,x,y,z) &= \beta t^{\beta-1}\left(-y - \frac{3}{5}z(k) + y(k)z(k)\right).
\end{aligned}$$

Then, we will get

$$\begin{aligned}
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k x(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= D(k,x,y,z),\\
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k y(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= E(k,x,y,z),\\
\frac{AB(\gamma)}{1-\gamma}\frac{d}{dk}\int_0^k z(\tau)E_\gamma\left(\frac{-\gamma}{1-\gamma}(k-\tau)^\gamma\right)d\tau &= F(k,x,y,z).
\end{aligned}$$

Applying the AB integral yields

$$\begin{aligned}
x(k) - x(0) &= \frac{1-\gamma}{AB(\gamma)}D(k,x,y,z) + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^k (k-\tau)^{\gamma-1}D(\tau,x,y,z)d\tau,\\
y(k) - y(0) &= \frac{1-\gamma}{AB(\gamma)}E(k,x,y,z) + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^k (k-\tau)^{\gamma-1}E(\tau,x,y,z)d\tau,\\
z(k) - z(0) &= \frac{1-\gamma}{AB(\gamma)}F(k,x,y,z) + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^k (k-\tau)^{\gamma-1}F(\tau,x,y,z)d\tau.
\end{aligned}$$

We discretize these equations at t_{n+1} as

$$\begin{aligned}
x^{n+1} &= x^0 + \frac{1-\gamma}{AB(\gamma)}D(k_{n+1},x^n,y^n,z^n)\\
&\quad + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^{k_{n+1}} (k_{n+1}-\tau)^{\gamma-1}D(\tau,x,y,z)d\tau,\\
y^{n+1} &= y^0 + \frac{1-\gamma}{AB(\gamma)}E(k_{n+1},x^n,y^n,z^n)\\
&\quad + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^{k_{n+1}} (k_{n+1}-\tau)^{\gamma-1}E(\tau,x,y,z)d\tau,\\
z^{n+1} &= z^0 + \frac{1-\gamma}{AB(\gamma)}F(k_{n+1},x,y,z)\\
&\quad + \frac{\gamma}{AB(\gamma)\Gamma(\gamma)}\int_0^{k_{n+1}} (k-\tau)^{\gamma-1}F(\tau,x,y,z)d\tau.
\end{aligned}$$

Then, we obtain

$$\begin{aligned}
x^{n+1} &= x^0 + \frac{1-\gamma}{AB(\gamma)}D(k_{n+1},x^n,y^n,z^n)\\
&+ \frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[\frac{h^\gamma D(k_s,x^n,y^n,z^n)}{\Gamma(\gamma+2)}\left((n+1-s)^\gamma(n-s+2+\gamma) - (n-s)^\gamma(n-s+2+2\gamma)\right)\right]\\
&- \frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[\frac{h^\gamma D\left(k_{s-1},x^{n-1},y^{n-1},z^{n-1}\right)}{\Gamma(\gamma+2)}\left((n+1-s)^{\gamma+1} - (n-s)^\gamma(n-s+1+\gamma)\right)\right],\\
y^{n+1} &= y^0 + \frac{1-\gamma}{AB(\gamma)}E(k_{n+1},x^n,y^n,z^n)\\
&+ \frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[h^\gamma E\left(\frac{k_s,x,y^n,z}{\Gamma(\gamma+2)((n+1-s)^\gamma(n-s+2+\gamma)-(n-s)^\gamma(n-s+2+2\gamma))}\right]
\end{aligned}$$

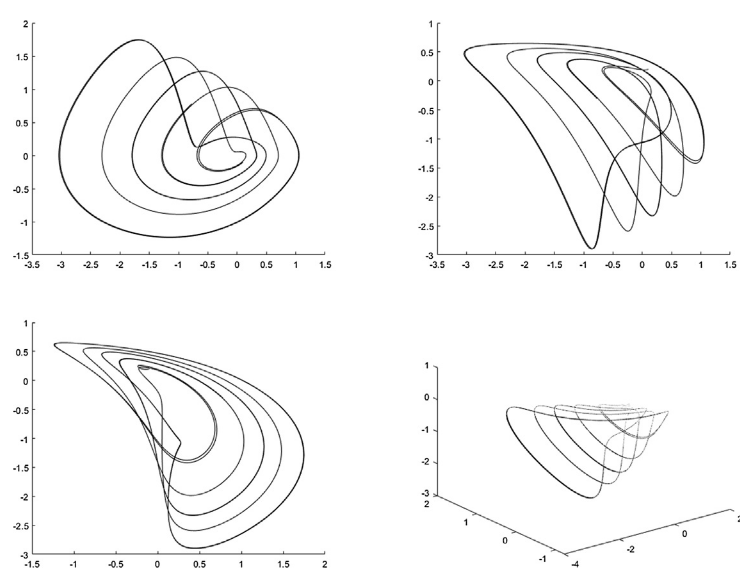

Fig. 1 The dynamical behavior of the chaotic attractor for the initial conditions $1, -1, 2, \gamma = 1$ and $\beta = 1$ using Atangana–Baleanu fractal–fractional derivative operator.

$$
\begin{aligned}
&-\frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[\frac{h^{\gamma}E(k_{s-1},x^{n-1},y^{n-1},z^{n-1})}{\Gamma(\gamma+2)}\left((n+1-s)^{\gamma+1}-(n-s)^{\gamma}(n-s+1+\gamma)\right)\right], z^{n+1}\\
&= z^{0}+\frac{1-\gamma}{AB(\gamma)}F(k_{n+1},x^{n},y^{n},z^{n})\\
&\quad+\frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[\frac{h^{\gamma}F(k_{s},x^{n},y^{n},z^{n})}{\Gamma(\gamma+2)}((n+1-s)^{\gamma}(n-s+2+\gamma)-(n-s)^{\gamma}(n-s+2+2\gamma))\right]\\
&\quad-\frac{\gamma}{AB(\gamma)}\sum_{s=0}^{n}\left[\frac{h^{\gamma}F(k_{s-1},x^{n-1},y^{n-1},z^{n-1})}{\Gamma(\gamma+2)}\left((n+1-s)^{\gamma+1}-(n-s)^{\gamma}(n-s+1+\gamma)\right)\right].
\end{aligned}
$$

by the method using in [20]. Similar things can be done for the system (13)–(15). Numerical simulations are demonstrated by the following figures.

We have simulated the obtained results for the initial conditions $x(0)=1, y(0)=1,$ and $z(0)=2$ respectively through Matlab. Here, we present the numerical results obtained for the proposed model through 2 phase and 3 phase simulations for the various fractional and fractal order sets. First of all, we simulate the proposed model at fractional and fractal dimensions equals 1 in Fig. 1. In Figs. 2, we presented the model for the fractional-order $\gamma = 0.98$ and fractal dimension $\beta = 1$. In Figs. 3–8, we have simulated the results of $x(k), y(k),$ and $z(k)$ with respect to time for different values of fractional and fractal orders. We see the dynamics of the different compartments of the model have been changed by varying fractional or fractal orders. These figures shows limits cycles behaviors and periodic orbit trajectories of the proposed

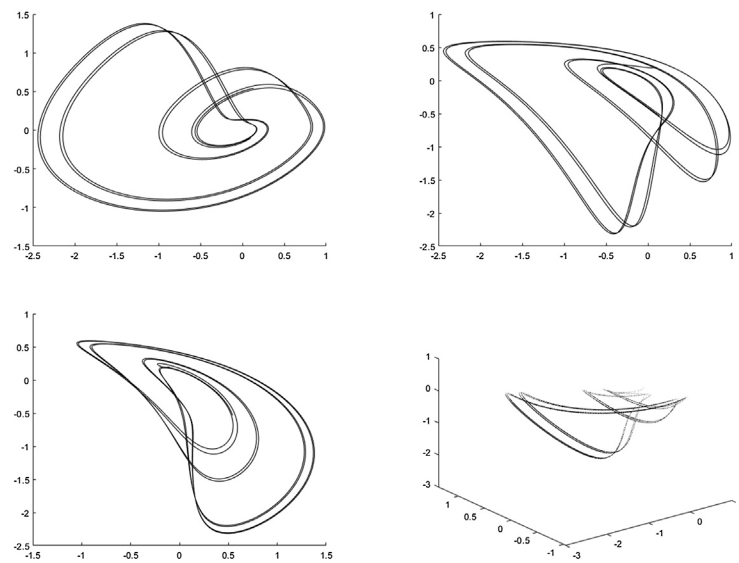

Fig. 2 The dynamical behavior of the chaotic attractor for the initial conditions $1, -1, 2, \gamma = 0.9$ and $\beta = 1$ using Atangana–Baleanu fractal–fractional derivative operator.

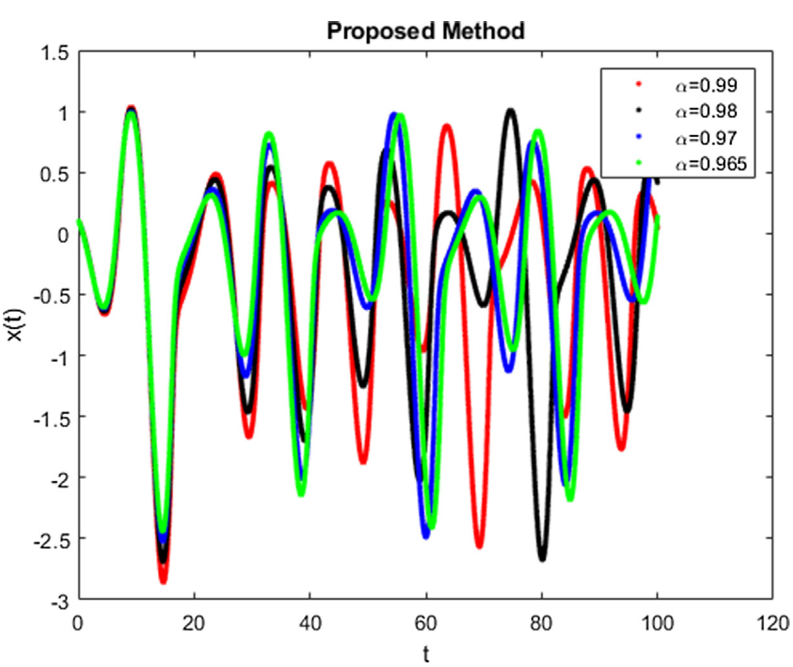

Fig. 3 Numerical simulation for $\beta = 1$ and different values of γ.

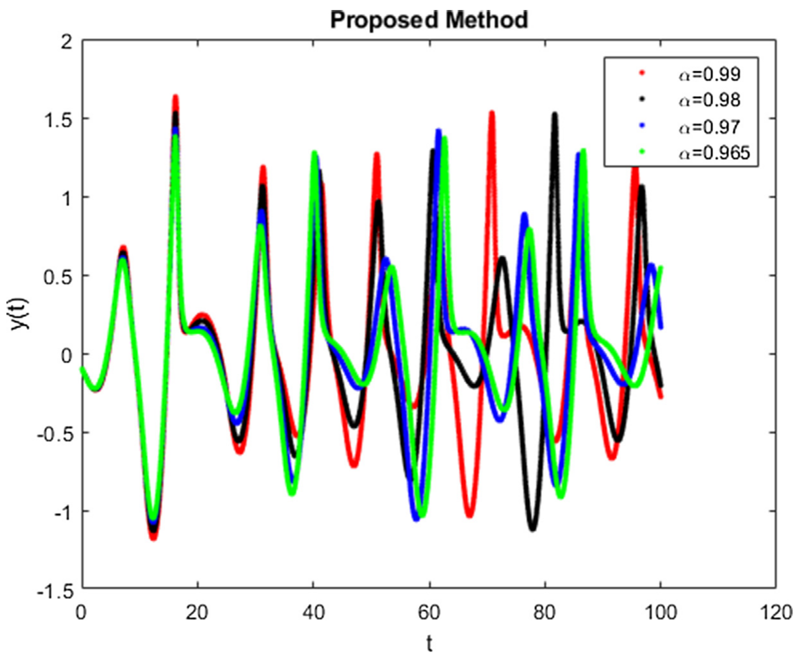

Fig. 4 Numerical simulation for $\beta = 1$ and different values of γ.

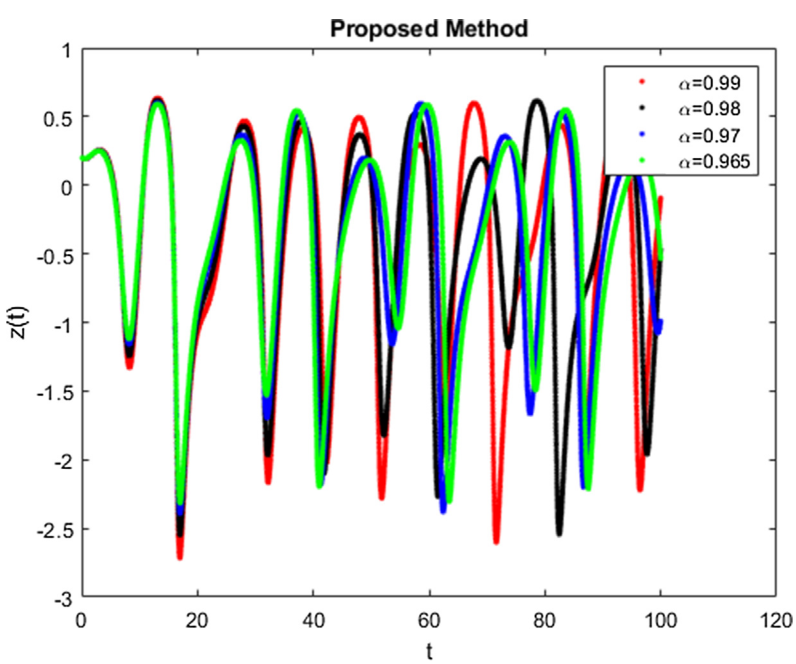

Fig. 5 Numerical simulation for $\beta = 1$ and different values of γ.

model. We have also examined the model's more complicated behavior in the form of chaotic attractors that can often not be achieved by ordinary and fractional-order operators. The fractal-fractional operators are thus better tools to examine the proposed system's more complex behavior.

5. Conclusion

In this manuscript, we formulated the integrator circuit model by the newly introduced operator, which is the combination of Atangana-Baleanu fractional operator and the fractal opera-

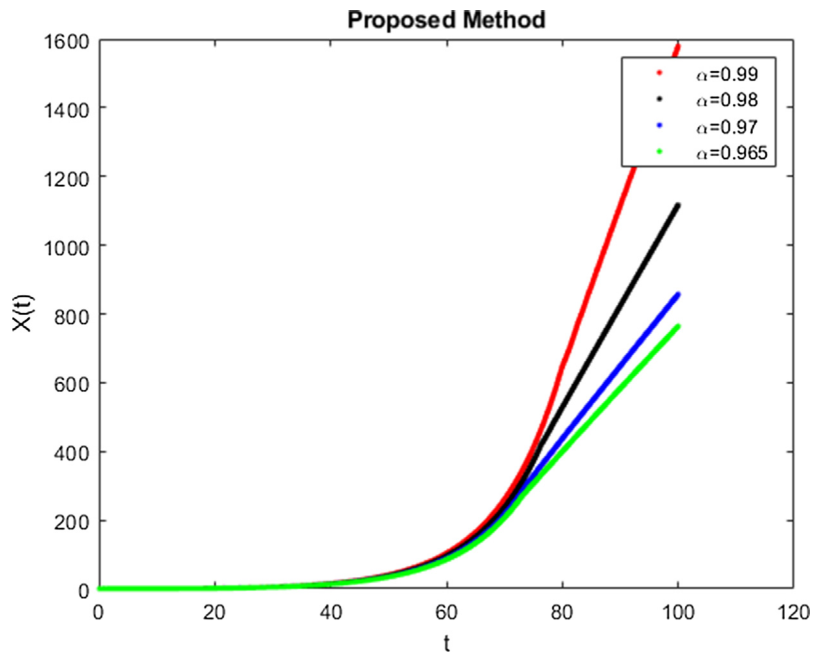

Fig. 6 Numerical simulation for different fractional orders and $\beta = 1$.

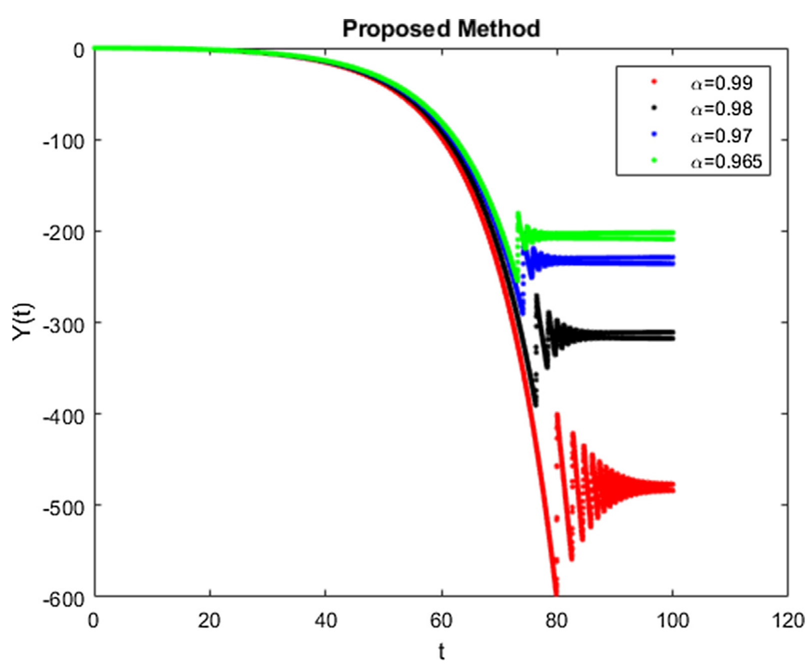

Fig. 7 Numerical simulation for different fractional orders and $\beta = 1$.

tor. We proved some results about the existence of least one solution and a unique solution of the given model by Schauder's and Banach fixed point theorems respectively. We established the results of the Ulam-Hyres stability of the model by mean of non-linear functional analysis. We constructed general numerical results of the model through Atanagan-Toufik numerical method. Lastly, we simulated the numerical results for different fractional and fractal orders. We have observed the chaotic attractors and more complex behavior in the figures. From Fig. 1, we conclude that when the fractional-order and fractal dimension is equal to 1, then we recover the integer-order simulation of the integrator circuit model.

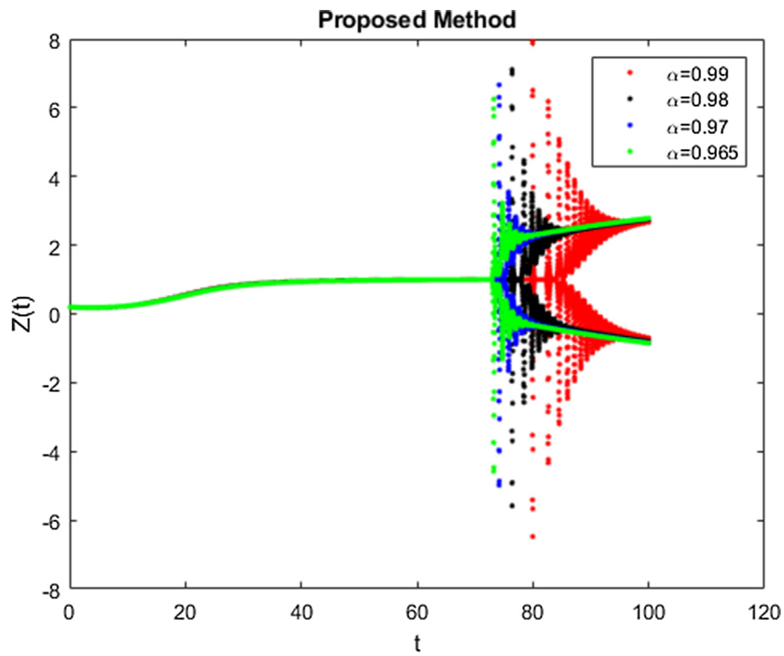

Fig. 8 Numerical simulation for different fractional orders and $\beta = 1$.

Thus, fractal-fractional operators can analyze better dynamics of the complex behaviour of physical phenomena than the fractional and fractal operators.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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