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A NOVEL *R/S* FRACTAL ANALYSIS AND WAVELET ENTROPY CHARACTERIZATION APPROACH FOR ROBUST FORECASTING BASED ON SELF-SIMILAR TIME SERIES MODELING

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Abstract

It has become vital to effectively characterize the self-similar and regular patterns in time series marked by short-term and long-term memory in various fields in the ever-changing and complex global landscape. Within this framework, attempting to find solutions with adaptive mathematical models emerges as a major endeavor in economics whose complex systems and structures are generally volatile, vulnerable and vague. Thus, analysis of the dynamics of occurrence of time section accurately, efficiently and timely is at the forefront to perform forecasting of volatile states of an economic environment which is a complex system in itself since it includes interrelated elements interacting with one another. To manage data selection effectively and attain

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robust prediction, characterizing complexity and self-similarity is critical in financial decision-making. Our study aims to obtain analyzes based on two main approaches proposed related to seven recognized indexes belonging to prominent countries (DJI, FCHI, GDAXI, GSPC, GSTPE, N225 and Bitcoin index). The first approach includes the employment of Hurst exponent (HE) as calculated by Rescaled Range (R/S) fractal analysis and Wavelet Entropy (WE) in order to enhance the prediction accuracy in the long-term trend in the financial markets. The second approach includes Artificial Neural Network (ANN) algorithms application Feed forward back propagation (FFBP), Cascade Forward Back Propagation (CFBP) and Learning Vector Quantization (LVQ) algorithm for forecasting purposes. The following steps have been administered for the two aforementioned approaches: (i) HE and WE were applied. Consequently, new indicators were calculated for each index. By obtaining the indicators, the new dataset was formed and normalized by min-max normalization method' (ii) to form the forecasting model, ANN algorithms were applied on the datasets. Based on the experimental results, it has been demonstrated that the new dataset comprised of the HE and WE indicators had a critical and determining direction with a more accurate level of forecasting modeling by the ANN algorithms. Consequently, the proposed novel method with multifarious methodology illustrates a new frontier, which could be employed in the broad field of various applied sciences to analyze pressing real-world problems and propose optimal solutions for critical decision-making processes in nonlinear, complex and dynamic environments.

Keywords: (R/S) Fractal Analysis; Wavelet Entropy; Hurst Exponent; Forecasting; Artificial Neural Network; Financial Time Series; Self-Similarity.

1. INTRODUCTION

In complex systems, constituents that interact with one another are involved in a system that is in motion. Complexity is apparent in macroeconomic environments such as financial markets in which the system does not work in equilibrium, given the conditions of ever-changing and volatile global landscape. At this point, adaptive mathematical models are employed to find optimal solutions to the problems of real-world matters. Several mathematical methods can be employed to reveal the complexity of systems and structures like economic networks and financial markets that require regularity and self-similarity characterization. As one of these related mathematical methods, Hurst exponent (HE) is used frequently in many disciplines to reveal self-similar and regular statistical patterns and structures. Based on this, as a global characteristic, HE characterizes the long-memory dependence. Estimation of the HE (with R/S analysis) using time series is significant in the investigation of processes that display self-similarity properties. In contemporary fractality hypothesis regarding financial time series, it is assumed that the market is a system which has a self-regulating characteristic; and information on past events can affect the present decisions including long-term trends as well as correlations. A time series is defined as a series of data points that are indexed in an order based on time order, some examples of which are related to solar system,[1] geological phenomena,[2] medical matters[3,4] and the daily closing value of stock exchange markets.[5–7] Another relevant study[8] introduces a new methodology for pair trading based on the HE calculation. Their results show that Hurst approach yields more accurate results.

Trend is one of the important components of time series showing trends of certain parameters in the long run. Doing analysis is essential for accurate forecasting of variables in time. When the market keeps its fractal structure, it remains stable. Yet, it is important to accurately predict unstable states of a market structure such as crises and conflict situations. Considering the unstable, transient and volatile properties of financial markets, there are certain methods other than Hurst in order to tackle this challenging situation. A study conducted concerning Brazilian market[9] examined auto correlations and cross correlations of daily price returns related to seven Brazilian market sectors. The analysis of the study shows that a multifractal analysis of cross correlations across different market sectors presents the Brazilian sectors' multifractal description.

As a measure of uncertainty of a given system, entropy is also used extensively in various applications.[10–14] Entropy is a concept that denotes the irregularity of a system. When this method is chosen as a method of classification, algorithms produce solutions that would reduce entropy. Thus, the disorder is reduced in a given system, knowledge gain increases, which shows an inverse correlation. One form of entropy, the Wavelet Entropy (WE), is a measure of the degree of order and/or disorder of a signal, which provides beneficial information on the underlying dynamical processes associated with the signal.[15] In essence, WE and wavelet decomposition are used for estimating the degree of order or disorder.[16] Generally, WE is capable of analyzing the transient features of non-stationary signals, which is a frequently encountered situation in financial markets around the world.[17] Neural networks have been employed extensively as forecasters of time series. Typical examples of the use of neural networks are meteorological phenomena,[18] network traffic forecasting[19] and market predictions,[20] to name just a few.[21,22] In addition, Artificial Neural Network (ANN) approach own powerful pattern recognition properties; and therefore, they are capable of performing more accurately than the other existing modeling techniques in numerous applications.[23,24] In financial markets, in which forecasting is an important task, ANNs provide a prominent guiding perspective to the relevant persons who have interest in the market for their decision-making processes.

Time series is a frequently encountered matter in numerous disciplines since time series enable one to explore the future values of a series based on the past values with some extent of margin of error.[25] In time series forecasting, the first issue to be addressed is whether or not the time series in question is predictable. It is aimed that the time series that have at least some degree of predictability can be analyzed and identified. Financial markets are volatile and demonstrate a myriad of parameters that can be easily affected by social, economic and political developments. Accordingly, time series analysis and forecasting have gained prominence as an active field of research over the last decades.[26–29] Therefore, various forecasting models have been generated, and researchers have based their studies on statistical techniques so that they can predict data regarding time series. Successful time series prediction is a main objective in numerous areas of application, varying from economic and business planning to inventory and production control, signal processing and control to weather forecasting. Conventional statistical methods used to be utilized in the past for forecasting time series data. However, data regarding the time series are most of the time known for their nonlinearity and irregularity.[30,31] Being one of the ways to estimate future behavior, time series models can be used for short-term and/or long-term forecasting, which appears to be an advantage particularly for investors in terms of reliability. For example, while short-term estimations may cover a 60-min prediction values, long-term forecasting includes monthly and yearly values.[32]

Stocks make up a portion of companies' capital.[33] While yielding investment for their investors and partners, they also provide resource for companies. Thus, they ensure the development of companies while allowing their recognition.[34,35] Prediction of changes of indexes in the long term seem to be a significant problem for validating strategies of investment in the financial instruments. Changes in this can allow us to make predictions of the turning point of indexes value in the long term. Related to this subject matter, several studies have been conducted using HE and WE methods. One such study was done by Ref. 33 who addressed the time-varying efficiency of stock markets by generalized HE analysis. The results comparatively revealed different degrees of long-term dependence and efficient, inefficient and developed markets. Another study by[36] focused on the local properties of the time series concerning the evolution of share prices that belong to 126 companies on the Warsaw Stock Exchange. HE (diffusion coefficient) was used to see the effects of long-term trend changes. Being employed in financial literature to study the dynamics of the stock markets, the HE was used by Refs.[37] and.[38] The authors examined the market efficiency of the Greek and the Shenzhen stock markets within the context of local market reforms. The study by Ref. 34 employed fractal geometry for self-similarity in financial time series in the stock market. Particularly dwelling on the full distributions of lag-k jumps, a scaling behavior was characterized by the HE for that study.[35] Calculated the HE $H(t)$ of a number of time series by dynamical implementation of detrending moving average (DMA) scaling technique. The accuracy of this technique was assessed by calculating the simulating monofractal Brownian paths, exponent $H(t)$ for artificial series, using assigned HE H. Concerning WE,[39] conducted a study on the forecasting of crude oil price dynamics and used an analysis based on WE, the algorithm of which was introduced for determining the optimal wavelet families and decomposition scale in order to produce enhanced forecasting performance. Regarding entropy-based (EB) methodology, the paper by Ref. 40 worked on the determination of an optimal level for multiresolution decomposition. The author focused on the wavelet analysis for revealing the complex dynamics along different timescales for the Great Britain Pound foreign exchange rate. The study explored heterogeneity regarding market agent behavior with different trading preferences. On similar subject matter, the work of Refs.[41–44] studied the financial hazards and prediction of financial risks employing EB approaches.

There are different fields in which WE has been used extensively, proposing its benefits. The study by Ref. 16 is concerned with medical data, showing the importance of WE for feature extraction purposes. The authors aimed at developing an accurate pathological brain detection system through an automatic and effective classifier of magnetic resonance (MR) images. Therefore, the authors used WE and HU moment invariants (HMIs) for feature extraction, proposing the use of WE to replace conventional discrete wavelet transform (DWT) method. The experiments of the study revealed that the method had a higher performance compared to the existing methods based on accuracy. The study conducted by Ref. 45 is on atrial fibrillation in clinical practice. The results of the study show WE as the highest single predictor.

ANN approach in modeling has gained prominence in various applications where complex real-world phenomena are analyzed mathematically. ANN algorithms are proposed to enhance the results concerning prediction. This is another reason why they have grabbed researchers' attention from many fields of applications including but not limited to medical imaging, signal processing, economic and financial modeling. Over the past years, ANN algorithms have been used more often in the time series forecasting analysis which is concerned with complex real-world phenomena, pattern recognition and pattern classification capabilities.[46,47] To illustrate such studies,[48] made use of ANNs for the prediction and analysis of financial time series. They examined if it would be viable to relax the stationarity condition to non-stationary time series. Related to financial market efficiency,[49] aimed at recognizing major reversal points of long-term trend of stock market index, suggesting a computational model of financial time series analysis, with a combination of different approaches which are efficiency evaluation methods (Shannon's entropy, HE), sensitivity analysis as well as neural networks. One relevant study was done by Ref. 50, which proposed a novel approach to forecast the stock prices. For this aim, they employed Wavelet Denoising-based Back Propagation (WDBP) neural network which they deemed effective in the prediction of the stock prices. They also performed the comparison with single Back Propagation (BP) neural network on their real data set. A more recent study performed in 2018 by Ref. 51 utilized HE, fractals as well as neural networks for the aim of forecasting financial asset returns in Brazil. Their study handled the verification of a relationship existence between the prediction error of financial asset returns and long-term memory in fractal time series. They obtained such data by ANN.

While using datasets, researchers encounter a common problem, which is the existence of some missing data among the datasets they are working on. The reason for this is that it is rare to find a homogenous dataset whose values are complete. To deal with such missing data problematic, one of the methods that can be employed, as it has been utilized in this study, is the linear interpolation methods. Such methods are used to compensate the missing data. Researchers used LI methods to deal with the issue of missing data. One relevant study was done by Ref. 52. The authors worked on the interpolation methods for filling in the gaps in time series, uncertainty quantifications as well as efficiency criteria.

The purpose of this study is to achieve improvement in the forecasting accuracy in the long run by HE indicators and WE indicators. Thus, comparison has been made with regard to the forecasting accuracy rates of financial asset returns by artificial neural networks algorithms through Feed Forward BP (FFBP), Cascade Forward BP (CFBP) and Learning Vector Quantization (LVQ). The contributions of this paper can be outlined as follows: first, seven recognized indexes belonging to prominent countries in terms of financial trends (DJI, FCHI, GDAXI, GSPC, GSTPE, N225, Bitcoin) were handled in financial time series application. The time period covers April 1, 2013 and December 29, 2017 for the financial index (FI) dataset (789×28) (see Table 1). This indicates that the study handled long-time period indexes. The data for the missing days in the FI dataset (789×28), made up

Table 1 FI Dataset.

Indexes	Country	Values	Dataset Size
DJI	USA		
FCHI	France	Open	
GDAXI	Germany	High	789 × 28
GSPC	USA	Low	
GSTPE	USA	Close	
N225	Japan		
Bitcoin			

Table 2 FILI Dataset.

Indexes	Country	Values	Dataset Size
DJI	USA		
FCHI	France	Open	
GDAXI	Germany	High	977 × 28
GSPC	USA	Low	
GSTPE	USA	Close	
N225	Japan		
Bitcoin			

Table 3 FILI+HE+WE Dataset.

Indexes	Country	Values	Dataset Size
DJI	USA	Open	
FCHI	France	High	
GDAXI	Germany	Low	977 × 42
GSPC	USA	Close	
GSTPE	USA	**HE**	
N225	Japan	**WE**	
Bitcoin			

of the indexes for the daily values thereof (Open, High, Low, Close) [Currency in USD ($)], have been calculated through LI method. Thus, financial time series index dataset Generated FI with LI (FILI) dataset (977 × 28) (see Table 2) has been obtained. Second, new indicators have been identified by the HE as calculated by Rescaled Range (R/S) fractal analysis (HE) and WE methods in the for a successful forecasting modeling of financial time series index dataset FILI dataset (977 × 28). Subsequently, through the calculation and inclusion of HE (H=0.55), and WE indicators, FILI with HE +WE indicators (FILI + HE + WE) dataset (977 × 42) (see Table 3) has been obtained. HE = 0.55 with its value approximating 1 indicates that the analyzed time series is continuous and it also owns a specific trend that is not probable to change the direction in near future, which is also known as persistent trending. Normalized FILI + HE + WE dataset (977 × 42) and FI dataset (977 × 28) (see Table 2) have been obtained by the application of min-max normalization on the FILI with HE + WE indicators dataset (977 × 42) and FILI dataset (977 × 28). Afterwards, in the normalized FILI + HE + WE dataset (977 × 42) and FILI dataset (977 × 28), the previous 20-day indices have been calculated and a new dataset has been formed for the forecasting of the closing value of any day for each index. Based on the daily close value of each index in the new datasets formed, ANN algorithms (namely FFBP, CFBP and LVQ) have been employed for the forecasting modeling. As a next step, 90% of the FILI+HE+WE dataset (977 × 42) and FILI dataset (977 × 28) have been allocated for the training datasets and 10% for the test datasets as to chronological time order. Finally, the forecasting performances of these algorithms have been compared based on their forecasting accuracy rates results.

Even though the results obtained from the earlier studies[9–16,32–39] yielded promising results, it has been observed that their results were constrained by a certain limitation, that is, Hurst indicator was used as a single method or merely WE was used in various fields (not limited to finance). In our proposed method in the present study, HE + WE have been used in conjunction with each other in the FILI dataset. The experimental results, consequently, demonstrate that FILI + HE + WE dataset yielded higher accuracy results in terms of forecasting based on the ANN methods.

1.1. The Motivation of the Proposed Method

In complex dynamics and structure, like the world of finance within the ever-changing global economic landscape, the characterization of self-similar and regular patterns in time series is vital. The proposed model in this study has attempted to provide optimal solutions with adaptive mathematical framework. The originality of this work is based on the fact that no earlier work exists in the literature in which HE as calculated by R/S fractal analysis and WE methods are employed by obtaining the indicators to identify self-similarity and regularity of financial time-series whilst performing the forecasting by ANN algorithms in the field of finance. The significance of HE and WE has been revealed for critical decision making regarding forecasting purposes. Regarding the indicators, most studies in the literature prefer to omit the missing data in their practice. Yet, this study has opted for generating the missing data by using LI method. All these can be stated as the methodological novelty of this paper.

Another contribution of the paper is concerned with its interdisciplinary approach since there is the combination of the proposed model with adaptive mathematical methods to provide optimal solutions for financial real-world matters. Within this framework, accurate, efficient and timely forecasting appears at the forefront since vagueness, uncertainty and volatility are common characteristics of financial markets in the current risk society[53–55] marked by challenging matters. Based on this complicated matter within a complex system, this study along with its multifarious methodology comprised of various stages has aimed at resolving the issue of accurate forecasting with the adaptive mathematical model we have proposed.

ANN approach in modeling is also significant in various applications in which complex real-world phenomena are mathematically analyzed. Therefore, the multifarious methods including the ANN algorithms used in this study have attempted to fulfill the accurate and reliable forecasting of market indicators by integrating perspectives from different disciplines to contribute a novel and productive approach so that uncertainty and other unpredictable aspects of finance markets can be mitigated.

The experimental results obtained by multifarious methodology in this study demonstrate that HE and WE indicators have a critical and determining direction, while emphasizing self-similarity and complexity characterization for more accurate and robust forecasting modeling by the ANN algorithms. Thus, the study aims to open a new frontier to be used in various applied sciences in order to assess real-world problems and to propose global optimal solutions in critical decision-making processes.

This paper is organized as follows: Sec. 2 deals with Materials and Methods with explanations of the methods applied in the respective order. The relevant methods explained are LI, HE as calculated by Rescaled range (R/S) fractal analysis, WE and ANN Algorithms (FFBP algorithm, CFBP algorithm, LVQ algorithm). Section 3 presents Experimental results along with the forecasting models with the ANN algorithms and their comparative analyzes. Finally, Sec. 4 provides ther conclusion.

2. MATERIALS AND METHODS

2.1. Data

The daily closing values (Open, High, Low, Close) [Currency in USD ($)] from stock market indices DJI index (USA), FCHI index (France), GDAXI index (Germany), GSPC index (USA), GSTPE index (USA), N225 index (Japan)[56] and Bitcoin[57] for five working days have been used in this study. The time period covers April 1, 2013 up until December 29, 2017 (see Table 1). This FI dataset will be referred to as FI dataset in the following parts of this paper.

2.1.1. *Linear Interpolation*

LI is a method for curve fitting that uses linear polynomials in order to form new data points within the range of a discrete set of known data points. The simplest interpolation form is to combine two data points within a straight line. Equation (1) for the LI is[58,59] as follows:

$$f_1(x) = b_0 + b_1(x - x_0). \tag{1}$$

Here, x denotes the independent variable and x_0 is the known value of the independent variable. $f_1(x)$ shows the dependent variable value for a value x of the independent variable. The following equations (Eq. (1)) are to be employed:

$$b_0 = f(x_0) \tag{2}$$

and

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}. \tag{3}$$

The dataset in this study also contains missing values which are specified as follows: DJI (data for 31 days are missing), FCHI (17 days of data are missing), GDAXI (24 days of data are missing), GSPC (data belonging to 31 days are missing), GSTPE (data concerning 36 days are missing), N225 (data regarding 49 days are missing) and Bitcoin (no missing data exist)).

Through the application of Eq. (1) to the FI dataset (see Table 1) indexes, generated FILI dataset has been obtained that belong to the indexes which do not have daily data (see Table 2).

2.2. Methods

In this study, we provided two contributions based on two main approaches. The first approach includes HE as calculated by Rescaled Range (R/S)

fractal analysis and WE to improve the predictive power in the long run in financial markets. The second approach consists of the application of ANN algorithms FFBP. CFBP and LVQ algorithm) for forecasting purposes.

The multiple steps of the approach in our study are specified in detail as follows:

(i) FILI dataset (977 × 28) (see Table 2) has been obtained by using the LI method for the generation of the missing index values. FI dataset (789 × 28) (see Table 1) includes the daily closing values (Open, High, Low, Close) [Currency in USD ($)] from stock market indices DJI (USA) index (China), FCHI index (France), GDAXI index (Germany), GSPC index (USA), GSTPE index (USA), N225 index (Japan)[56] and Bitcoin.[57]

(ii) The HE as calculated by Rescaled Range (R/S) fractal analysis, HE (HE = 0.55 value), is approximated to 1, which shows that the analyzed time series has a specific trend that is not expected to change its direction in near future (persistent trending). It has been applied on the values of FILI dataset and seven more indicators have been identified and included into the FILI dataset.

(iii) WE has been applied on the values of the FILI dataset and seven more indicators have been identified and included in the FILI dataset.

With the combination of steps (ii) and (iii), FILI + HE + WE dataset (977 × 42) (see Table 3) has been generated with HE and WE indicators being included into FILI dataset (977 × 28).

(iv) Min-max normalization method has been applied on the FILI+HE+WE dataset (977 × 42) (see Table 3), which refers to FILI with HE (seven indicators) and WE (seven indicators) [Currency in USD ($)]. The predictive value for any day concerning each index in the normalized dataset has been formed based on the previous 20 days (Open, Close, High, Low) (20 × 42 = 840). There are 977 days between the 21st day (April 21, 2013) and 977th day (December 29, 2017), and for each day 840 indicators have been obtained (20 days × 42 daily parameters = 840 indicators). The dataset (977 × 840) has been modeled through FFBP, CFBP and LVQ algorithms and the Daily Closing Values of the indexes have been predicted (Prediction Result for Current Index). The daily values (Open, Close, High, Low) of the indexes in the new dataset obtained with this dimension (977 × 840) have been modeled by these three algorithms. The Daily Closing Values of the indexes modeled have been predicted as well (Forecasting Result for Current Index).

(v) Min-max normalization method has been applied on the FILI dataset (977 × 28) (see Table 2). The predictive value for any day concerning each index in the normalized dataset has been formed based on the previous 20 days (Open, Close, High, Low) (20 × 28 = 560). The dataset (977 × 560) has been modeled through FFBP, CFBP and LVQ algorithms, and the Daily Closing Values of the indexes have been predicted.

(vi) FILI + HE + WE dataset (977 × 840) and FILI dataset (977 × 560) have been allocated for the training of the dataset (90%) and the testing of the dataset (10%).

(vii) The Daily Closing Values of the indexes in the test datasets have been predicted by FFBP, CFBP and LVQ algorithms in line with the training/forecast models, and the prediction performances of the algorithms (FFBP, CFBP and LVQ) have been compared in line with the accuracy rates results regarding the datasets mentioned in step (vi).

Through the integration of the two financial measures of information efficiency, HE and WE, aggregated EB indicators were designed with FFBP, CFBP, LVQ algorithms and then its ability to predict the turning point in the trend of the financial time series and to calibrate the index trading strategy was explored. According to experimental results, HE and WE indicators are the most successful prediction descriptor in each financial time series indexes dataset as regards the accuracy rate outcomes of the aforementioned algorithms.

In this study, all the computations and analyzes were conducted and figures were obtained by M*atlab* R2018b.[60]

2.2.1. *The Hurst Exponent as Calculated by Rescaled Range (R/S) Fractal Analysis*

HE is a statistical measure that is utilized in classifying time series. It provides a measure for fractality of a time series and long-term memory.[61,62] Long memory concerns conditions that have been observed along a long period of time. Also called long-range dependence, long memory is a phenomenon which emerges in the analysis of temporal and spatial data. Long memory processes studies are normalized through the HE that utilizes Rescaled Range analysis (R/S) as one of the methods. The financial markets have fractal nature, particularly concerning the tendency of time series to return strongly to its mean or cluster in a nonlinear direction, the term generalized as HE[62] was introduced to fractal geometry. The HE and R/S analysis are employed for measuring the long memory of a time series. Long-term correlation memory indicates that some interdependences exist among observation periods of time series even though such periods are separated. Such supposition is in contradiction with the efficient market hypothesis. It, in other words, expresses that periods of temporary inefficiency exist with respect to the financial markets. When the long-term dependencies of the indexes are observed, it is likely that the investors can have better opportunities for decisions on the Daily value (Open, Low, High, Close). The long-term correlation memory and level of indexes inefficiency can be measured through the application of several financial indicators and statistical means. Such indicators are designed in a way that the level of efficiency of the indices information is measured even if there is not any knowledge regarding the causal factors that underlie. The researches which utilize such indicators generally analyze the indices. The HE is possible to be calculated by Rescaled Range analysis (R/S analysis).[33,61]

For a time series $X = X_1, X_2, \ldots, X_n$, R/S analysis method steps are provided as follows:

Step (1): The mean of the m value is calculated in line with the following:

$$m = \frac{1}{n}\sum_{i=1}^{n} X_t. \qquad (4)$$

Step (2): Mean adjusted series Y is calculated according to (5).

$$Y_t = X_t - m, \quad t = 1, 2, \ldots, n. \qquad (5)$$

Step (3): The cumulative deviation of the Z series is calculated in line with the following:

$$Z_t = \sum_{i=1}^{t} Y_i, \quad t = 1, 2, \ldots, n. \qquad (6)$$

Step (4): Range series is calculated according to the following equation:

$$R_t = \max(Z_1, Z_2, \ldots, Z_t) - \min(Z_1, Z_2, \ldots, Z_t), \quad t = 1, 2, \ldots, n. \qquad (7)$$

Step (5): The standard deviation of the S series is calculated based on the following equation:

$$S_t = \sqrt{\frac{1}{t}\sum_{i=1}^{t}(X_i - u)^2}, \quad t = 1, 2, \ldots, n. \qquad (8)$$

Step (6): Compute the Rescaled Range series (R/S)

$$(R/S)_t = R_t/S_t, \quad t = 1, 2, \ldots, n. \qquad (9)$$

Note $(R/S)_t$ is averaged across the regions. The mean till $[X_1, X_t]$, $[X_{t+1}, X_{2t}]$, $[X_{(m-1)t+1}, X_{mt}]$ is found where $m = \text{floor}(n/t)$. In reality, to use all data for calculation, a value of t is selected that can be divided by Refs. 63 and 64.

Hurst revealed that (R/S) scales by power-law as time goes up, which shows

$$(R/S)_t = c^* t^H.$$

At this point, c is a constant, t refers to the number of observations and H is termed the HE Refs. 65 and 66.

$$\log(R/S) = \log c + H \log t.$$

Concerning the HE, there are three different classifications:

(1) $H = 0.50$ means the related time series is random with random and uncorrelated events at stake. The present time does not have influence on the future direction.
(2) $0 \leq H < 0.50$ shows that the time series is ergodic or anti-persistent. The strength of such an anti-persistent behavior is based on how close H is to zero value. Such time series is more volatile than a random series.
(3) $0.50 \leq H < 1.00$ shows that the time series has a characteristic that is persistent or trend reinforcing. If the time series fluctuated (up or down) in the last period, the possibility is that it remains to be positive or negative in the upcoming future time

period, which shows that the trend is apparent. The strength of the persistence goes up when H comes closer to 1.0. If the H value is high, it will show more persistence, clearer trends and ultimately less noise, indicating less risk.

In this study, $H = 0.55$ indicates that the time series analyzed has a specific trend which is continuous and persistent. It also refers that the direction of the trend is not likely to change in the near future time period.

2.2.2. *Wavelet Entropy*

WE is an innovative tool which has the capability of analyzing the transient features regarding non-stationary signals. It merges entropy and wavelet decomposition for the purpose of estimating the degree of order or disorder[16,67] pertaining to a signal that has a high time–frequency resolution. The wavelet is a smooth swiftly vanishing oscillating function that has the benefit of a good localization in terms of time and frequency. A wavelet family $\psi_{a,b}(t)$ is a set of elementary functions that are generated by translations and dilations of a unique admissible mother wavelet $\psi(t)$.[67,68]

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right). \tag{10}$$

Here, $a, b \in Randa \neq 0$, are the scale and translation parameters, respectively, and t represents the time. The wavelet gets wider as the scale parameter a rises. Hence, a unique analytic pattern is formed with its replications being at differing scales and with variable time localization. The continuous wavelet transform (CWT) of a signal $S(t) \in L^2(R)$ (the space of real square integrable functions) is defined as the correlation the function *S(t)* has with the family wavelet $\psi_{a,b}(t)$ for each a and b as shown in the following:[45,68]

$$(W_S)(a,b) = \int_{-\infty}^{\infty} S(t)\psi_{a,b}{}^*(t)dt = \langle S, \psi_{a,b}\rangle$$
$$= |a|^{1/2}\psi\left(\frac{t-b}{a}\right)dt. \tag{11}$$

For a specific selection of the mother wavelet function $\psi(t)$ and for the discrete set of parameters, $a_j = 2^{-j}$ and $b_{j,k} = 2^{-j}k$ with $j, k \in Z$ (the set of integers), the wavelet family could be presented as indicated in the following:

$$\psi_{j,k}(t) = 2^{i/2}\psi(2^j t - k), \quad j,k \in Z. \tag{12}$$

The correlated DWT of signal $S(t)$ could be gained by

$$\mathrm{DWT}_s(j,k) = \int_{-\infty}^{+\infty} S(t)\psi_{j,k}(t)dt$$
$$= 2^{-j/2}\int_{-\infty}^{+\infty} S(t)\psi_{j,k}(2^{-j}t-k)dt. \tag{13}$$

The signal is supposed to be given the sampled values for the sake of practical signal processing, which is illustrated in the following equation:

$$S = \{s_0(n), \quad n = 1, \ldots, M\}. \tag{14}$$

In line with the wavelet theories, the signal could be stated as

$$S(t) = \sum_{j=-N}^{-1}\sum_k C_j(k)\ \psi_{j,k}(t) \tag{15}$$

in which $j = -1, -2, \ldots, -N$ refers to the number of resolution levels. If the decomposition is performed over all resolutions levels, its maximum values are $N = \log_2(M)$ and $C_1(k), C_2(k), \ldots, C_N(k)$ are the wavelet coefficients.[68,69]

Orthogonal wavelet bases were employed for the decomposition of the signal. Thus, the decomposed signals could be considered as a direct estimation of local energies at different scales. On the other hand, the wavelet coefficients could be given by $C_N(k) = \langle S, \psi_{a,b}\rangle$. In this way, it would be possible to define the wavelet energy at resolution j as

$$E_j = \sum_k |C_j(k)|^2. \tag{16}$$

The radar signal is split among non-overlapping temporal windows of length L to study the temporal evolution of the quantifiers whose definitions have been provided above. Appropriate signal values are allocated to the central point of the time window for each interval ($i = 1, \ldots, N_T$, with $N_T = M/L$). Concerning dyadic wavelet decomposition, wavelet coefficients' number from all resolution levels is two-fold smaller than in the previous level. At this point, at each level the minimum length of the temporal window contains at least one coefficient.[68,69] Through the consideration of the mean wavelet energy rather than the total wavelet energy, the mean energy at each resolution level $j = -1, -2, \ldots, -N$ pertaining to the time window that utilizes the wavelet coefficient is

$$E_j^{(i)} = \frac{1}{N}\sum_{k=(i-1)L+1}^{i.L} |C_j(k)|^2 \tag{17}$$

in which N_j denotes the number of wavelet coefficients at resolution j contained in the time window i. Consequently, wavelet coefficients' total energy at time window i can be attained by

$$E_{\text{total}}^{(i)} = \sum_{j<0} E_j^{(i)}. \tag{18}$$

Thus, the relative wavelet energy, demonstrating probability distribution of energy in scales, shall be assumed based on the following:

$$p_j^{(i)} = E_j^{(i)}/E_{\text{total}}^{(i)}. \tag{19}$$

Evidently, $\sum_j p_j^{(i)} = 1$ and the distribution $\left\{p_j^{(i)}\right\}$ is accepted as a time scale density.

The temporal behavior of WE is defined as Ref. 69 in line with the Shannon entropy affords a measure of the information of a given distribution to compare and analyze probability distribution[69] as presented

$$H_{\text{WT}}^{(i)}(p) = -\sum_{j<0} p_j^{(i)} \cdot \ln\left[p_j^{(i)}\right]. \tag{20}$$

The mean of the radar signal WE could be defined as in Eq. (21) in order to attain a quantifier for the whole time period:

$$H_{\text{WT}} = \frac{1}{N_T}\sum_{i=1}^{N_T} H_{\text{WT}}^{(i)}. \tag{21}$$

The definition of entropy may be stated as follows: it is a measure of uncertainty regarding the quantitative information in a system. A signal's entropy value shows the complexity degree that the signal owns. If the signal is more disordered, this will indicate a higher level of entropy.[68,69] Linked with the WT, defined from a time–frequency representation of the signal, the EB on the WT, called WE, can offer further information regarding the signal's underlying dynamical process.[69] A periodic mono-frequency signal that has a narrow band spectrum can be regarded as an ordered process. Accordingly, its wavelet energy will be in one unique wavelet resolution. As a result, its WE shall be near zero or of a very low value. A totally random signal could reflect a highly disordered behaviour and it will hold a wavelet demonstration with significant contributions from all of the frequency bands. Furthermore, the wavelet energies will be virtually equal for all of the resolution levels, which will generate WE that will have maximal values.

WE method has been applied in this study to the FILI dataset and FILI + HE + WE dataset.

2.2.3. *Artificial Neural Network Algorithms*

ANNs have powerful pattern recognition properties which could perform better than the other existing modeling techniques in numerous applications. ANNs have grabbed researchers' attention from many fields of applications including but not limited to medical imaging, economic and financial modeling as well as signal processing, a variety of supervised or unsupervised learning rules are currently available to train a network from data.[23,24] In this study, the Closing value of the indexes in the new dataset (977×840) to which ANN algorithms have been applied predicted [both unsupervised learning (FFBP, CFBP) and supervised learning (LVQ)] for any day.

FFBP Algorithm

Neural networks simulate a parallel and highly interconnected computational structure with many relatively simple individual units, as inspired by the brain and nerve system studies. Such individual units are organized in layers, which are known as input, hidden layer and output. Feedforward networks plot inputs into outputs with signals that flow in one direction, that is to say from the input layer to the hidden one and later the output one. In the hidden and output layers, each unit features a transfer function transferring the signal received. The input layer units are utilized to distribute input signals to the network although they do not have a transfer function. A numerical weight is assigned to each connection and changes the signals passing througit. A feedforward network can be thought to have three layers and a single output unit, k hidden layer units and n input units (see Fig. 1). The input layer can be denoted by a vector $X = (x_1, x_2, \ldots, x_n)'$ the hidden layer by a vector $M = (m_1, m_2, \ldots, m_k)'$ and y is the output. Any hidden layer unit receives the weighted sum of all

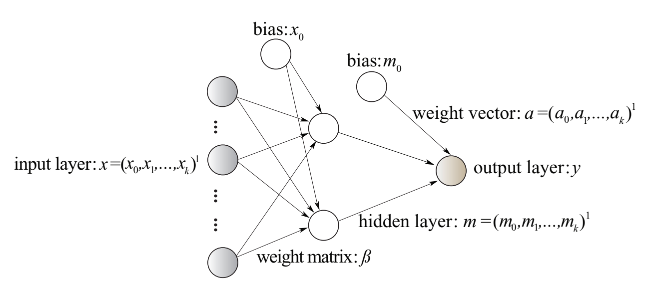

Fig. 1 The general structure of the MLP structure.

inputs and a bias term (indicated by always equaling to one), and yields an output signal as indicated by the following:[23,24,51,70–73]

$$m_j = F\left(\sum \beta_{ij} x_i\right) = F(X'B_j),\quad j = 1, 2, \ldots, k, \quad i = 0, 1, 2, \ldots, n. \tag{22}$$

Here, F is the transfer function, x_i is the ith input signal and β_{ij} is the weight of the connection from the ith input unit to the m hidden layer unit. Likewise, the weighted sum of the output signals from the hidden layer units is received by the output unit which then yields a signal,

$$y = G\left(\sum a_j m_j\right),\ j = 0,\ 1,\ 2, \ldots,\ k. \tag{23}$$

Here, G is the transfer function, e_j is the weight of the connection from the jth hidden layer unit to the output unit, and $j = 0$ indexes a bias unit x_0 which is all the time equal to one. Equation (24) can show this

$$y = G\left(a_0 + \sum_{j=1}^{k} a_j F\left(\sum \beta_{ij} x_i\right)\right) = f(X, \theta). \tag{24}$$

Here, X represents the vector of inputs, and $\theta = (a_0, a_1, \ldots, a_k, \beta_{01}, \beta_{02}, \ldots, \beta_{0k}, \beta_{11}, \beta_{12}, \ldots, \beta_{1k}, \beta_{n1}, \beta_{n2}, \ldots, \beta_{nk})$ is the vector of network weights. F and G can assume several functional forms, for example, the threshold function that generates binary (0/1) output, the sigmoid (or logistic) function that yields the output between 0 and 1. Equation (24) can be understood as a nonlinear function, representing the described feedforward neural network with three layers.

CFBP Algorithm

CFBP models resemble feed-forward networks, except that they have a weight connection from the input to each layer and from each layer to the succeeding layers. Neurons which succeed each other, respectively, are connected, and the training process is executed in this fashion. One can apply the training process at two or more levels.[72–74] A general idea regarding the methodology employed in the process of learning will be presented as follows: initialization of the weights with small random values; and propagation of the entries p_q forward through the neural network layers for all the combinations (p_q, d_q) in

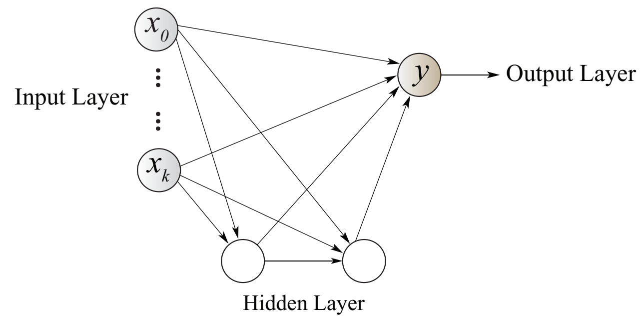

Fig. 2 The general structure of the CFBP algorithm.

the learning sample as indicated in the following:

$$a^0 = p_q; \quad a^k = f^k(W^k a^{k-1} - b^k),\quad k = 1, \ldots, m. \tag{25}$$

Back propagation of the sensitivities through the neural network layers following:

$$\begin{aligned} \delta^M &= -2F'^M(n^M)(d_q - a^M),\\ \delta^k &= F'^k(n^k)(W^{k+1})^T \delta^{k+1},\\ K &= M-1, \ldots, 1. \end{aligned} \tag{26}$$

Change of the weight and biases as follows (see Fig. 2):

$$\begin{aligned} \Delta W^k &= -\eta \delta^k (a^{k-1})^T, \quad k = 1, \ldots, M,\\ \Delta b^k &= -\eta \delta^k, \quad k = 1, \ldots, M. \end{aligned} \tag{27}$$

If the criteria for stopping are achieved, then stop; if not, which means they are not, one may apply permutation to the arrangement in which the combination created from the learning input data is presented and start Eqs.(25)–(27) over.

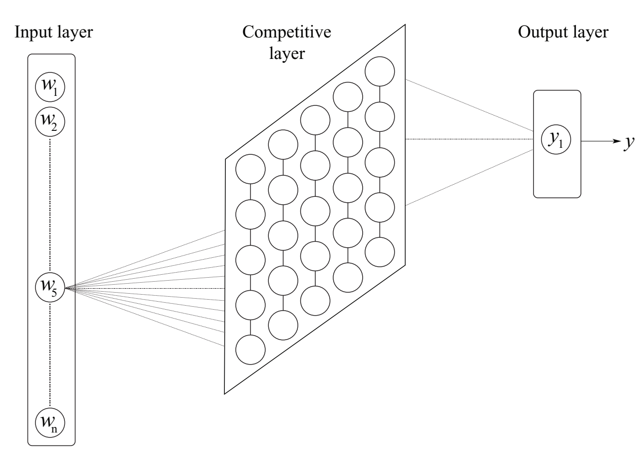

Fig. 3 The general structure of the LVQ algorithm.

Learning Vector Quantization Algorithm

Let us consider that a number of reference vectors X_n are put in the input space. In this case, each class is assigned some of such vectors. An input vector x is determined as belonging to the same class where the adjacent reference vector belongs. $X_n(t)$ represents the sequences of the X_n in the discrete-time domain.[74,76,77]

Yet, LVQ starts with initial values defined properly, and then the 1 algorithm is used to update the reference vector in following way as can be seen in the following:

$$\begin{aligned} Y_i &= w_i(t+1) = w_i(t) - \alpha(t)[x - w_i(t)], \\ Y_j &= w_j(t+1) = w_j(t) + \alpha(t)[x - w_i(t)], \end{aligned} \tag{28}$$

where $0 < \alpha(t) < 1$ and $a(t)$ may lower monotonically with time.

The two reference vectors w_i and w_j are the closest to x; x and w_j belonging to the same class, whereas x and w_i belonging to different ones. In addition, x must pass into the "window", that is outlined around the midline of w_i and w_j. This means that, where the following requirement is fulfilled (see Fig. 3), w_i and w_j undergo an update in which $d_i = |x - w_i|,\ d_j = |x - w_j|$ as can be seen from

$$\min\left(\frac{d_i}{d_j}, \frac{d_j}{d_i}\right) > s. \tag{29}$$

In this study, the dataset (977 × 840) has been trained through FFBP, CFBP and LVQ algorithms. The daily closing values (Prediction Result for Current Index, Output) have been modeled accordingly. Ninety percentage of the dataset has been allocated for the training dataset and 10% for the test dataset. The training dataset (873 × 840) has been trained by FFBP, CFBP and LVQ algorithms based on Prediction Result for Current Index. The Daily Closing values of the indices in the test dataset (104 × 840) have been predicted through FFBP, CFBP and LVQ algorithms. The prediction performance of the ANN algorithms (FFBP, CFBP and LVQ) has been compared based on the accuracy rates results.

3. EXPERIMENTAL RESULTS

Concerning this study, the procedures related to the steps that lead to the experimental results for the proposed method can be outlined in Table 4.

Table 4 The Steps Related to the Proposed Multifarious Method.

Step 1: Application of LI
Step 2: Integration of the WE and the HE as calculated by Rescaled Range (R/S) fractal analysis
Step 3: Modeling by ANN algorithms (FFBP, CFBP and LVQ)
Step 4: Comparison of modeling forecasting results

Explanations for these steps (indicated in Table 4) are elaborated below in their respective order:

Step 1: Application of LI

The time series indices (FI) dataset (789 × 28) (see Table 1) of this study is made up of the daily values of the seven indices. For the generation of the daily values that were missing in the FI dataset (789 × 28) (see Table 1) has been enabled through linear interpolation method. Thus, FILI dataset (977 × 28) has been obtained (see Table 2).

$y_i = f(x_i)$ (see Eq. (1)) of the unknown function for the daily values of the seven indices. Based on this information, in this study, it has been attempted to perform the prediction of the value $f(x)$ for all other values based on the following formula: x as Date = IndexOpen(max .value) − IndexClose(min .value) (see Table 2).

Step 2: Integration of the WE and HE as calculated by Rescaled Range (R/S) fractal analysis

HE is considered to be the most popular measure for the information efficiency. It ensures the evaluation of the long-term correlation memory as well

Table 5 ANN Algorithms Models of Network Properties.

Network Properties	Values
Training function	Levenberg–Marquardt (trainlm)
Adaption learning function	Learngdm
Performance function	Mean squared error (MSE)
Transfer function	Tansig
Epoch number	*50*
Hidden layer neuron number	*5*
Training dataset	(873 × 840)
Test dataset	(104 × 840)
Output	Closing daily value

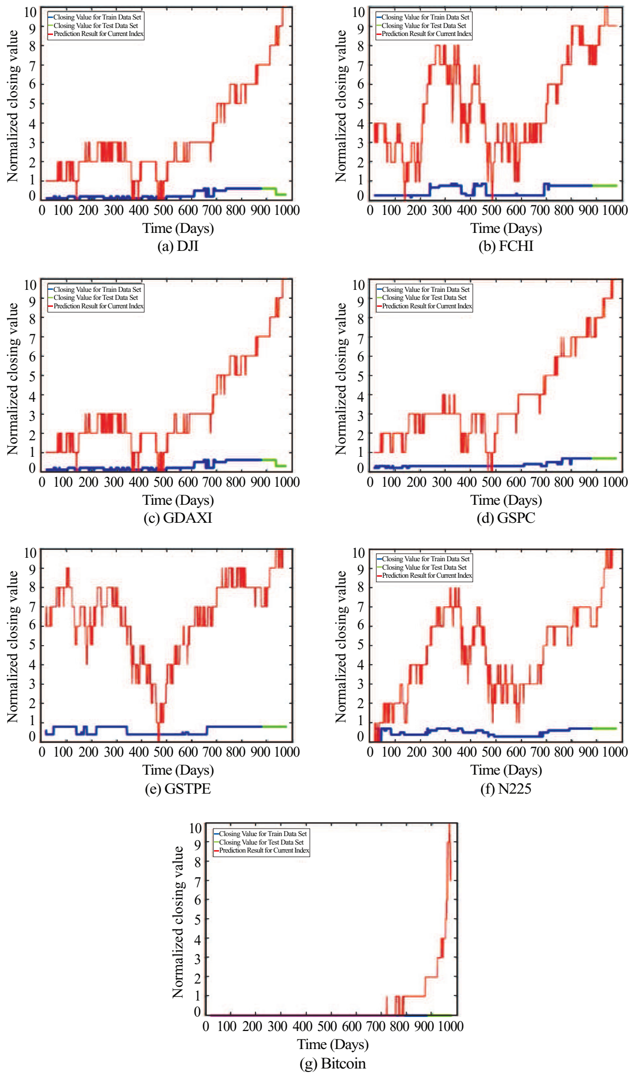

Fig. 4 Forecasting performance results obtained by FFBP algorithm for (a) DJI, (b) FCHI, (c) GDAXI, (d) GSPC, (e) GSTPE, (f) N225 and (g) Bitcoin indices.

as the classification of the time series. In this study, $H = 0.55$ with its value approximating 1 shows that the analyzed time series is continuous, and owns a specific trend that is not likely to change its direction in near future. This situation is known as persistent trending. New indicators have been calculated and added into the daily index values of the dataset.

In this study, WE method and $H = 0.55$ have been applied on the FILI dataset. Fourteen more indicators have been calculated and FILI with HE + WE indicators (FILI + HE + WE) dataset (977×42) have been obtained accordingly. Min-max normalization is applied to financial dataset, of A to in the range $[\text{new\_min}_A, \text{new\_max}_A]$ as presented in the following:

$$v_i' = \frac{v_i - \min_A}{\max_A - \min_A} \cdot (\text{new\_max}_A - \text{new\_min}_A) + \text{new\_min}_A. \quad (30)$$

In this study, the min-max normalization method Eq. (30) has been applied to the FILI + HE + WE dataset (977×42) (see Table 3). The values in the dataset have been standardized within [0,1] range. The 20-day values (Open, Close, High, Low) [Currency in USD ($)] have been calculated in the prediction of the closing day for each index in the normalized dataset. In this way, new dataset (977×840) has been formed in this manner. The new dataset (977×840) has been modeled through FFBP, CFBP, and LVQ algorithms (Prediction

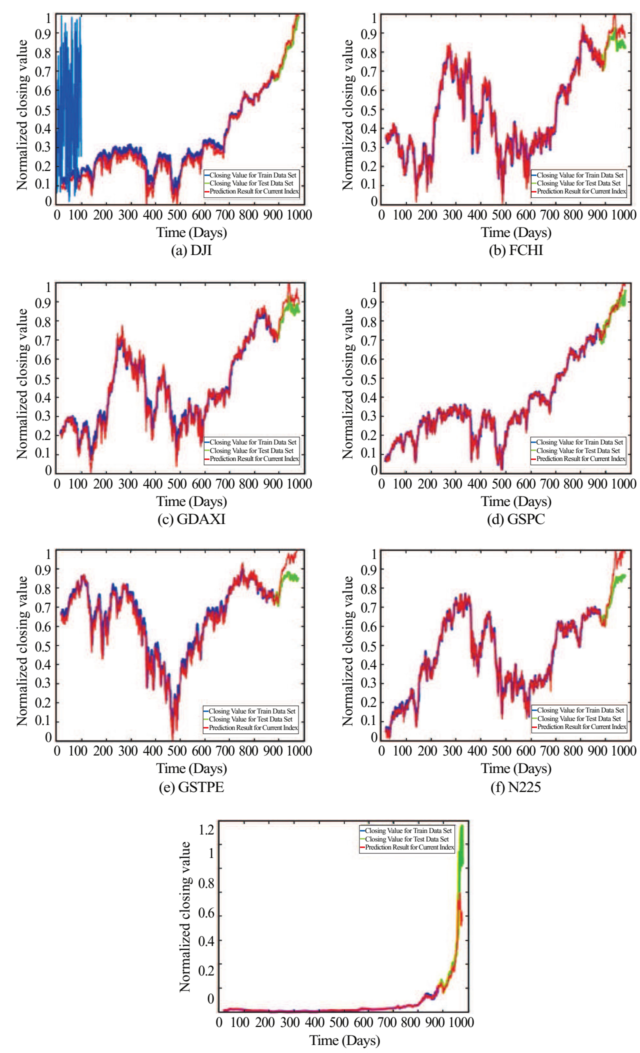

Fig. 5 Forecasting performance results obtained by CFBP algorithm for (a) DJI, (b) FCHI, (c) GDAXI, (d) GSPC, (e) GSTPE, (f) N225 and (g) Bitcoin indices.

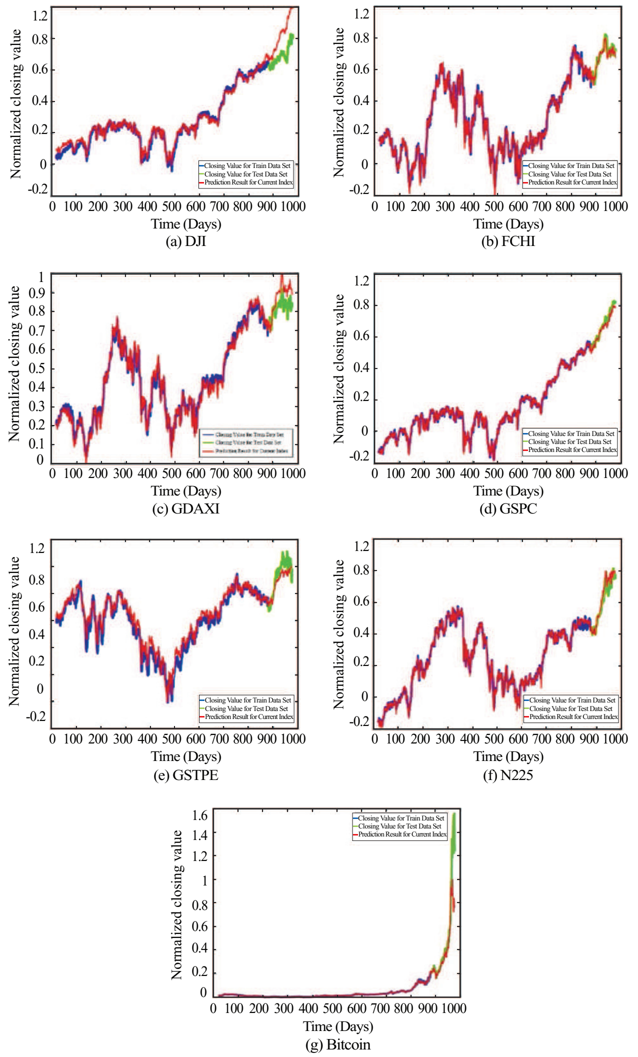

Fig. 6 Forecasting performance results obtained by LVQ algorithms for (a) DJI, (b) FCHI, (c) GDAXI, (d) GSPC, (e) GSTPE, (f) N225 and (g) Bitcoin indices.

Result for Current Index). Ninety percentage of the dataset(977 × 840) has been allocated for the train dataset (873 × 840) and 10% for the test data (104 × 840). The training dataset has been trained by FFBP, CFBP and LVQ algorithms. As for the test dataset, the Daily Closing Values of the indices in the dataset (104 × 840) have been predicted. In addition, the prediction performance of the algorithms has been compared based on the accuracy rates result.

Step 3: Forecasting modeling by ANN algorithms

Data-driven modeling approaches, ANNs, allow the data to be fully utilized, and help the time series data determine the construction and parameters of a model not restricted by parametric modeling assumptions. ANN algorithms models of network properties for this study have been presented in Table 5. For the forecasting of the daily closing values of the indices, the new dataset (977 × 840) has been modeled through the ANN algorithms (FFBP, CFBP and LVQ). Prediction Result for Current Index is represented in red colour (see Figure 4). The training dataset (873 × 840) has been trained by FFBP, CFBP and LVQ algorithms, represented in blue color (see Figs. 4–6). The prediction of the daily Closing Values pertaining to the indices has been done in the test dataset (104 × 840) (see Figs. 4–6). The prediction performance for the ANN algorithms has been compared based on the accuracy rates result (Table 6a).

The forecasting performances for the ANN algorithms (FFBP, CFBP and LVQ) have been compared based on the accuracy rates results.

Table 6 **Comparative Forecasting Accuracy Rate Results by ANN Algorithms. (a) FILI + HE + WE Dataset.**

Indexes	FFBP (%)	CFBP (%)	LVQ (%)
DJI	99.24	99.73	97.44
FCHI	99.17	99.86	98.55
GDAXI	99.47	99.55	92.58
GSPC	89.02	99.89	96.94
GSTPE	96.2	99.86	98.4
N225	99.57	99.6	96.3
Bitcoin	98.06	99.03	90.4

(b) FILI Dataset

Indexes	FFBP (%)	CFBP (%)	LVQ (%)
DJI	98.25	99.7	87.48
FCHI	86.24	95.46	76.88
GDAXI	86.01	98.29	76.5
GSPC	88.2	96.06	96.9
GSTPE	94.87	96.81	98
N225	86.9	98.04	95.15
Bitcoin	95.67	97.92	78.44

Step 4: Comparison of forecasting modeling prediction results

The forecasting performance results obtained from the application of the ANN algorithms (FFBP, CFBP and LVQ) on the FILI + HE + WE dataset for each FI (DJI, FCHI, GDAXI, GSPC,

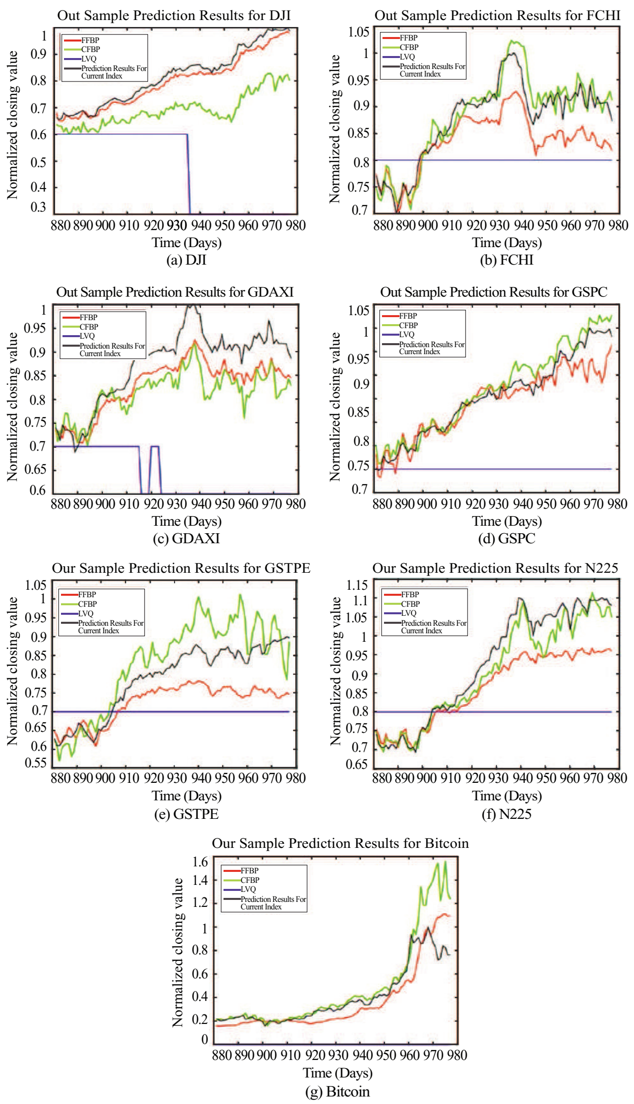

Fig. 7 Comparison of forecasting results of FFBP, CFBP, LVQ algorithms for (a) DJI, (b) FCHI, (c) GDAXI, (d) GSPC, (e) GSTPE, (f) N225, (g) Bitcoin indices Forecasting.

GSTPE, N225 and Bitcoin) are depicted in Figs. 4–6. Table 6a shows the comparative accuracy rate results as obtained by the application of the ANN algorithms (FFBP, CFBP and LVQ) on the FILI + HE + WE dataset, whereas Table 6b presents the comparative accuracy rate results as obtained by the application of the ANN algorithms (FFBP, CFBP and LVQ) on the FILI dataset (which is the dataset that does not include WE and HE indicators). Figure 7 presents the depiction of the comparative analyzes for the ANN algorithms (FFBP, CFBP and LVQ) on the FILI + HE + WE dataset.

The results in Table 6a yield more accurate forecasting results than those in Table 6b. The reason for this is the inclusion of HE and WE indicators in the FILI dataset.

The results for the test dataset (104×840) using three different models clearly reveal the efficiency of the CFBP algorithm in terms of Daily Closing Value forecasting performance results (see Table 6a). While the CFBP algorithm has performed more accurately than the FFBP and LVQ algorithms, the lowest accuracy rates has been observed for the LVQ. All in all, FILI + HE + WE dataset is the one from which the higher accuracy results have been obtained compared to the other dataset (FILI dataset).

As for the limitation of our study, it can be stated that although it illustrates forecasting capability, some missing data exist in the datasets handled (see Table 1 for the details on the missing data in the datasets). In order to manage this limitation, we used LI method. The fact that missing data exist is quite a normal situation since all kinds of datasets could contain certain levels of missing data.

4. CONCLUSION

In complex dynamics and structure in motion with their ever-changing parameters which are not in equilibrium at all, the characterization of self-similar and regular patterns is critical. Our proposed model has attempted to provide optimal solutions to real-world problems with an adaptive mathematical framework. The main purpose of this study has been to achieve enhancement in the forecasting accuracy by the characterization of the self-similar patterns in time series. Time series is important in finance field which is characterized by a volatile and unpredictable nature owing to its dynamics and structure. This study has been conducted based on two main approaches. The first one comprises HE as calculated by Rescaled Range (R/S) fractal analysis and WE so that the prediction accuracy in the long-term trend in the financial markets can be improved. HE and WE provide a new perspective to deal with self-similar patterns in time series for their prediction purposes in the financial environment. In the forecasting of time series, the first matter to address is whether or not the time series in question is predictable. The volatile financial markets display various parameters that may be easily affected by economic, social and political developments. Therefore, it is important to interpret the data accurately so that forecasting can be made properly, efficiently and timely. ANN approach in modeling has become significant in various applications in which the mathematical analyzes of complex real-world phenomena are performed. In order to form the forecasting model in an efficient way ANN algorithms (FFBP, CFBP and LVQ) were applied and this stage makes up the second approach of this study. When compared with the other studies,[9–50] this study has attempted to provide some novel contributions. It has been conducted on such an extensive dimension of the FI dataset (789×28) with prominent regions and their indices (see Table 1). When earlier works related to Hurst and Entropy[9–16,32–39] are reviewed, it is seen that HE and WE are not used in conjunction with each other. Yet, in our study, with the inclusion of the indicators, new dataset, FILI + HE + WE dataset (977×42), has been obtained with the HE and WE. In addition, as different from previous works,[9–16,32–39] it has been the first time ANN algorithms have been applied on both the financial index (FI) dataset (789×28) and FILI+HE+WE dataset (977×42) for forecasting purposes besides the comparative analyzes of each algorithm. Consequently, the experimental results indicate that the highest accuracy rate has been found for the CFBP algorithm, which reveals the significance and critical role of HE and WE in the dataset (FILI+HE+WE dataset (977×42)) with both indicators included. Consequently, the experimental results indicate that the highest accuracy rate has been found for the CFBP algorithm, which reveals the significance and critical role of HE and WE in the dataset (FILI + HE + WE dataset (977×42)) with both indicators included.

Based on these results and considerations, we may present the following directions for future research:

(1) The multifarious methodology employed and the novel model illustrate that HE and WE are obviously critical and determining for forecasting purposes. Based on this perspective, it may be possible for future research to focus and work on different datasets in various applied fields (including but not limited to economy).
(2) The proposed model of our study can open up a direction for researchers to focus on the characterization of self-similar and regular indicators to identify the significant attributes in their forecasting processes. This proposed aspect of the model can help the researchers with their multi-criteria decision-making processes so that they can construct adaptive models to assess risks, manage volatility and costs in unstable and transient environments. In addition, the model can provide a way regarding the effective, management of missing data.
(3) The method we proposed can be used as interface(s) in various applied sciences (used like the ones mentioned in Refs. 78 and 79 to address real-world matters in critical decision-making processes in the future.
(4) The results and the model can provide a new direction for long-term risk management and towards the selection of data and obtaining robust forecasting results.

All in all, this study with the interdisciplinary approach has attempted to provide adaptive mathematical methods to present optimal solutions in various areas including but not limited to mathematics, economics, finance, data science, engineering, crisis management, strategic management, medicine and genetics for accurate, applicable and efficient forecasting. The experimental results we obtained from the model validate the applicability and efficiency of the multifarious method proposed. As a consequence, we expect that the results derived from the detailed data analyses and applications will provide an alternative guidance for relevant future works and different aspects of research for accurate planning, robust forecasting and efficient decision-making.

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