Research Article

A Paradox of the Average Waiting Time for the Case of a Single Bottleneck on the Commuters' Route

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Average waiting time is considered as one of the basic performance indicators for a bottleneck zone on a route for commuter traffic. It turns out that the average waiting time in a queue remains paradoxically unchanged regardless of how fast the queue dissolves for a single bottleneck problem. In this study, the paradox is verified theoretically for the deterministic case with constant arrival and departure rates. Consistent results with the deterministic case have also been obtained by simulation runs for which vehicle interarrival time is a random variable. Results are tabulated for interarrival times which have uniform, triangular, normal, and exponential distributions along with a statistical verification of the average waiting time paradox.

1. Introduction

Roads and transportation facilities have the very unfavorable characteristic of economical imbalance given the overutilization during rush hours and the underutilization for the remaining long hours of the day. The highly nonuniform consumer demand throughout the day and the inflexible supply of the commodities make this fact inevitable resulting with a challenge for economists [1–5]. Restrictions, tolls, and taxes have been partial remedies to reduce the nonuniformity of the demand. Congestion management through application of tolls has become a popular field in transportation engineering literature during the recent years. However, actual implementation examples are limited due to reluctance of political decision makers even though traffic problems are severe impediments to social and economic activities in many metropolitan areas [6, 7]. Reasonably successful congestion toll management experience should be noted especially for the examples of Singapore [8, 9] and London [10]. A brief overview of tolling implementations (including terminated road pricing schemes) throughout the cities of the world is available at [11]. Besides toll implementation staggered work hours or flexible time schedules for employees implemented by central business district firms can also be a remedy for traffic congestion during morning and evening rush hours [12, 13]. Due to global pandemic of 2020–2021, renewed attention should be expected for implementation of staggered work hours to reduce crowding at work places, malls, and social gathering places.

An early treatment of traffic congestion management based on marginal cost pricing is due to Walters [14], and the single bottleneck system on a commuters' route joining the central business district of the town with the residential areas has been constructed by the pioneering work of Vickrey [15] in which the effects of time-varying tolls have been analyzed [16]. The relative simplicity and applicability of this approach have made it a useful tool for transportation engineers. In the absence of tolls, private vehicle owners using a commuters' route make a choice of leaving their house very early or arrive at their office late [17]. Application of tolls encourages some drivers to shift to public transportation or share their cars with their coworkers; meanwhile, some drivers may prefer payment of tolls instead of losing valuable time due to traffic congestion. Therefore, the effect of tolling offsets the equilibrium point to reduce peak-time demand [15, 18, 19]. Vickrey's approach has been a source of increasing attention with hundreds of relevant scholarly publications during the last 50 years [20].
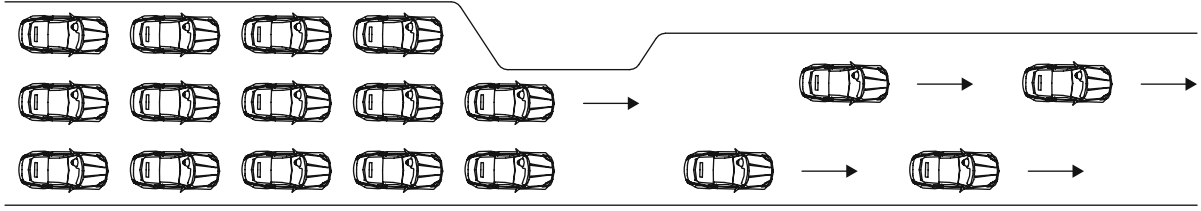
Figure 1: Sketch of a bottleneck on a commuters' route during rush hour traffic causing the formation of a queue.

In this study, the focus is on the queue dynamics of the single bottleneck on the commuters' route rather than the effects of tolling on consumer behavior for a scarce commodity. The emphasis will be on the average waiting time at the bottleneck of a congested route which is one of the reasonable metrics for queueing systems. The average waiting time paradox which is the primary issue of this paper was encountered during an earlier study for a deterministic queueing system of a single bottleneck traffic congestion problem [21]. The cited paradox is that the average waiting time for this system depends only on the parameters of queue buildup (the arrival rate, departure rate, and the duration of the queue buildup phase) irrespective of the depletion rate of the queue. An analytic verification has been made apart from the simulation output indicating the paradox, and in the present work, consistent results are also obtained for the case of a stochastic queue buildup of a bottleneck zone on the commuters' route.

In the next section, a description of the single bottleneck model is given. Simulation results for this model to compute the average waiting time are discussed in the following section along with a statement of the average waiting time paradox for single bottleneck systems. The paradox is supported by an analytic derivation of the average waiting time for the deterministic queue buildup case. In the final section, simulation results for stochastic interarrivals are tabulated. The simulation results are used to make a statistical verification of the average waiting time paradox for the case of stochastic interarrivals.

2. The Single Bottleneck Model and the Average Waiting Time Paradox

The single bottleneck on a unidirectional route is a deterministic model which assigns differing payoffs to individuals for their productive time in office and unproductive or less productive time either lost in traffic or due to early arrivals in office [15, 18]. The argument is that individuals are willing to pay a toll to avoid valuable time lost due to traffic congestion [22]. Since this paper is solely aimed at investigating the average waiting time paradox, the supply demand equilibrium in the presence of congestion tolls will be disregarded and a fixed time interval in which the arrival rate to a bottleneck zone exceeds the departure rate will be considered instead.

Assume a major single route used by commuters and traffic congestion occurs at a single zone of this route. Let the congestion be in one direction only, meaning a possible rush hour situation in the morning (or in the other direction

during the evening). There is free flow traffic with no congestion until 7:30 A.M. but after that for a duration of 60 minutes, the arrival rate at the bottleneck zone exceeds the departure rate resulting with a queue along the commuting direction (see Figure 1). After 8:30 A.M., the arrival rate falls below the capacity of the bottleneck point and the queue will disappear in a while to be followed by free flow traffic again.

In real life, changes in the arrival rates are gradual but for simplicity, it will be assumed that the arrival rate immediately increases to a_1 vehicles per minute at 7:30 A.M. The bottleneck zone can handle up to a maximum of d vehicles per minute (which is denoted as the departure rate). Right after 8:30 A.M., the arrival rate suddenly drops to a value below the departure rate (given as a_2). Therefore, the preliminary assumption of the model is as follows:

$$a_1 > d > a_2. \tag{1}$$

For the sake of modeling convenience, two phases of traffic congestion will be distinguished: (i) phase-1: between 7:30 A.M. and 08:30 A.M. for which a linear queue buildup is observed and (ii) phase-2: after 8:30 A.M. during which the queue diminishes at a linear rate. Duration of the second phase naturally depends on the value of a_2. The queue structure is given in Figure 2 as a graph which is the continuous version approximation of what should actually display discrete arrivals.

Quantifying the severity of traffic congestion during morning rush hour is an issue of scholarly debate [23]. However, for the purpose of assessment of the defined single bottleneck problem, a performance indicator which one would naturally consider is the average waiting time of vehicles arriving at the bottleneck zone during phase-1 and phase-2. Given fixed values for d and a_1, one would expect that the longer the duration of phase-2, the longer the average waiting time should be. It will be verified however that the average waiting time primarily depends on d and a_1; the arrival rate during phase-2 (as long as the assumption given by Equation (1) is satisfied) does not alter the average waiting time.

3. Deterministic Simulation for Constant Rate Arrivals

Discrete-event simulation can be used for analysis of various traffic-related problems (see for example [24]). For an application of the single bottleneck queuing system, arrivals and departures at discrete time instants are required [25]; therefore, finer intervals than seconds will be necessary (hence, the use of one-sixtieth of a second as the tertia, with
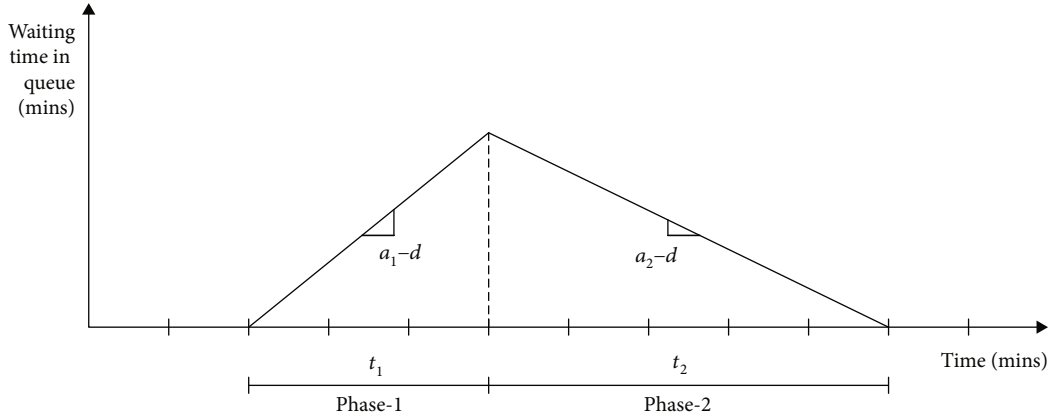
FIGURE 2: Waiting time of vehicles given the arrival time to bottleneck zone during the rush hour traffic.

an inspiration from Latin). At the same time, the arrival and departure rates are given in terms of vehicles per minute to prevent the overcrowding of digits. An arrival rate of $a_1 = 80$ vehicles per minute means that after 7:30 A.M., there is a new vehicle coming to the bottleneck zone every 45 tertias assuming constant rate arrivals. A constant departure rate of $d = 60$ vehicles per minute will be assumed, meaning a vehicle leaves the bottleneck zone every 60 tertias. The first vehicle arriving after 7:30 A.M. will wait 15 tertias, the second vehicle 30 tertias, and so on. The 4800th vehicle arriving at exactly 8:30 A.M. waits 72,000 tertias (20 minutes). Based on this simple simulation run, the average waiting time for the vehicles having arrived just before this peak queue level (phase-1 average waiting time) can be computed as 36,000 tertias (10 minutes).

During phase-2, assume that the arrival rate suddenly drops to $a_2 = 48$ vehicles per minute. Since the departure rate remains the same, the queue will start to diminish right after 8:30 A.M. From this point on, vehicles arrive at every 75 tertias. The 9598th vehicle waits 30 tertias in the queue, the 9599th vehicle 15 tertias, and when the 9600th vehicle arrives the queue has just diminished. Simulation results have been obtained by the use of Visual Basic programming in MS Excel.

One would expect that the average waiting time during phase-2 would depend on the value of a_2 in the sense that a reduced arrival rate indicates a quicker depletion of the queue and hence a shorter average waiting time, whereas a relatively larger a_2 value means a slower queue depletion and hence a longer average waiting time. However, simulation results indicate that the average waiting time is the same in both phases regardless of the variations in the value of a_2. For a simulation run with $a_2 = 48$ vehicles per minute, the queue will dissolve after 100 minutes (phase-2 duration) whereas for $a_2 = 10$ vehicles per minute, it takes 24 minutes for the queue to disappear. In both cases, the phase-2 waiting time average is 36,000 tertias (10 minutes). Simulation results with different phase-1 arrival and departure parameters have revealed a similar conclusion: phase-1 and phase-2 average waiting times (hence the overall average) turn out to be the same.

4. Theoretical Derivation of the Average Waiting Time for the Case of Constant Rate Arrivals

At the peak level of the queue, a cumulative number of $a_1 t_1$ vehicles have arrived at the bottleneck zone. The number of queuing vehicles at that instant is equivalent to $(a_1 - d)t_1$. By the end of phase-2, the queue should diminish entirely, so the duration of phase-2 can be computed from the following:

$$(a_1 - d)t_1 = (d - a_2)t_2. \tag{2}$$

Hence,

$$t_2 = \frac{(a_1 - d)t_1}{d - a_2}. \tag{3}$$

During phase-1, each new arrival waits slightly more than the previous one with a difference of (measured in tertias)

$$\frac{3600}{d} - \frac{3600}{a_1} = \frac{3600(a_1 - d)}{da_1}. \tag{4}$$

The ith arriving vehicle during phase-1 has a waiting time of $(3600i(a_1 - d)/da_1)$. Therefore, the average waiting time during phase-1 is as follows:

$$\frac{1}{(a_1 t_1 - 1)} \sum_{i=1}^{a_1 t_1 - 1} \left(\frac{3600i(a_1 - d)}{da_1} \right) = \left(\frac{3600(a_1 - d)t_1}{2d} \right). \tag{5}$$

During phase-2, each new arrival waits slightly less than the previous one with a difference of

$$\frac{3600}{a_2} - \frac{3600}{d} = \frac{3600(d - a_2)}{da_2}. \tag{6}$$

TABLE 1: For constant arrival/departure rates during phase-1 ($a_1 = 80$, $d = 60$ vehicles per minute) phase-2 duration and average waiting time for all vehicles crossing the bottleneck zone are given for various values of a_2 (departure rate being unchanged).

a_2 (vehicles per minute)	Queue formation start at	Queue reaches peak at	Queue disappears at	t_1 (min)	Number of arrivals in phase-1	Total waiting time of arrivals in phase-1 (min)	t_2 (min)	Number of arrivals in phase-2	Total waiting time of arrivals in phase-2 (min)	Average waiting time in phase-1 (min)	Average waiting time in phase-2 (min)
48	07:30:00 A.M.	08:30:00 A.M.	10:10:00 A.M.	60.00	4,799	47,990.00	100.00	4,799	47,990.00	10	10
36	07:30:00 A.M.	08:30:00 A.M.	09:20:00 A.M.	60.00	4,799	47,990.00	50.00	1,799	17,990.00	10	10
24	07:30:00 A.M.	08:30:00 A.M.	09:03:20 A.M.	60.00	4,799	47,990.00	33.33	799	7,990.00	10	10
15	07:30:00 A.M.	08:30:00 A.M.	08:56:40 A.M.	60.00	4,799	47,990.00	26.67	399	3,990.00	10	10
10	07:30:00 A.M.	08:30:00 A.M.	08:54:00 A.M.	60.00	4,799	47,990.00	24.00	239	2,390.00	10	10

TABLE 2: Comparison of the average waiting times in phase-1 and phase-2 for four different interarrival time distributions: uniform, triangular, normal, and exponential. Constant rate refers to deterministic interarrivals. Mean interarrival rates for all distributions in phase-2 are 75 tertias.

	Input parameters			Simulation output results	
	Phase-1	Phase-2	Both phases	(average of 30 replications)	
	Arrival rate	Arrival rate	Departure rate		
	80	48	60	Phase-1	Phase-2
Constant	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
	45.00	75.00	60.00	36,000.00	36,000.00
Uniform	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Min:	27.50	51.60	60.00	35,944.36	35,818.27
Max:	62.50	98.40			
Triangular	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Min:	20.25	33.60	60.00	35,933.22	35,801.87
Mode:	45.00	75.00			
Max:	69.75	116.40			
Normal	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Average:	45.00	75.00	60.00	36,025.72	36,122.18
St. deviation:	10.125	16.85			
Exponential	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Average:	45.00	75.00	60.00	36,067.42	36,714.35

TABLE 3: Comparison of the average waiting times in phase-1 and phase-2 for four different interarrival time distributions: uniform, triangular, normal, and exponential. Constant rate refers to deterministic interarrivals. Mean interarrival rates for all distributions in phase-2 are 100 tertias.

	Input parameters			Simulation output results	
	Phase-1	Phase-2	Both phases	(average of 30 replications)	
	Arrival rate	Arrival rate	Departure rate		
	80	36	60	Phase-1	Phase-2
Constant	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
	45.00	100.00	60.00	36,000.00	36,000.00
Uniform	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Min:	27.50	61.00	60.00	36,076.25	36,138.29
Max:	62.50	139.00			
Triangular	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Min:	20.25	45.00	60.00	35,808.62	35,698.49
Mode:	45.00	100.00			
Max:	69.75	155.00			
Normal	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Average:	45.00	100.00	60.00	35,876.82	35,900.90
St. deviation:	10.125	22.50			
Exponential	Interarrival time	Interarrival time	Constant interdeparture time	Average waiting time	Average waiting time
Average:	45.00	100.00	60.00	35,790.16	36,156.68

A similar derivation yields the average waiting time during phase-2 as

$$\frac{1}{(a_2 t_2 - 1)} \sum_{i=1}^{a_2 t_2 - 1} \left(\frac{3600 i (d - a_2)}{d a_2} \right) = \left(\frac{3600 (d - a_2) t_2}{2d} \right). \quad (7)$$

If the formula given by Equation (3) for t_2 is substituted in Equation (7), then one obtains an identical expression to that in Equation (5) regardless of the value of a_2 provided that the condition in Equation (1) is valid. Therefore, phase-1 and phase-2 average waiting times should be equal to each other.

TABLE 4: Results of hypothesis testing for constructed paired T-tests to compare phase-1 and phase-2 average waiting times for four interarrival distributions: uniform, triangular, normal, and exponential. Mean interarrival rates for all distributions in phase-2 are 75 tertias.

Paired T-test results of the simulation replications ($\alpha = 0.05$)		
Constant	Phase-1 Average waiting time	Phase-2 Average waiting time	
	36,000.00	36,000.00	
Uniform with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	35,944.36	35,818.27	126.09
st. deviation	537.86	889.07	663.35
		T-statistics (t_0)	-1.0411
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Triangular with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	35,933.22	35,801.87	131.35
st. deviation	526.48	1,244.89	957.34
		T-statistics (t_0)	-0.7515
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Normal with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	36,025.72	36,122.18	-96.46
st. deviation	528.59	1,292.84	919.57
		T-statistics (t_0)	0.5745
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Exponential with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	36,067.42	36,714.35	-646.93
st. deviation	2,118.31	3,735.59	2,501.07
		T-statistics (t_0)	1.4167
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0

To prevent round off errors due to discontinuity, a_2 values have been chosen so that the first vehicle which does not wait in the queue witnesses the departure of the last vehicle which has waited for a positive amount of time in the queue. This can be ensured if the peak waiting time value is divisible by the quantity given by Equation (6). The maximum waiting time in the queue is experienced by the $(a_1 t_1)^{\text{th}}$ vehicle and equals to

$$\frac{3600(a_1 t_1)(a_1 - d)}{da_1} = \frac{3600(a_1 - d)t_1}{d}. \tag{8}$$

Therefore, the value below should be an integer

$$\frac{3600(a_1 - d)t_1/d}{3600(d - a_2)/da_2} = \frac{(a_1 - d)t_1 a_2}{(d - a_2)}. \tag{9}$$

Note that the quantity in Equation (9) is also equivalent to $t_2 a_2$.

A summary of the average waiting times for several phase-2 instances with various arrival rates is given in Table 1 calculated from Equations (3) and (7). As just said, the a_2 value in the table has been chosen to guarantee an integer quantity for the expression given by Equation (9). The results in the table confirm the results of the simulation runs.

Note that even if the expression given by Equation (9) (or equivalently $t_2 a_2$) is not an integer, one can still speak of a slightly revised version of the paradox by stating that average waiting times in phase-1 and phase-2 are almost identical so that there is no need to put any restriction on the choice of a_2.

5. Simulation Results for Stochastic Interarrivals

Simulation runs have also been conducted for the nondeterministic case to determine the validity of the theoretical derivations (given in Section 4) with stochastic interarrivals. For instances of stochastic interarrivals however, one should have a revised understanding of the paradox definition. Instead of stating that the average waiting time remains the same whatever the choice of a_2 is, one can say that the average waiting time in the queue of a single bottleneck traffic model for which mean arrival and departure rates fulfill the condition

TABLE 5: Results of hypothesis testing for constructed paired T-tests to compare phase-1 and phase-2 average waiting times for four interarrival distributions: uniform, triangular, normal, and exponential. Mean interarrival rates for all distributions in phase-2 are 100 tertias.

Paired T-test results of the simulation replications ($\alpha = 0.05$)		
Constant	Phase-1 Average waiting time	Phase-2 Average waiting time	
	36,000.00	36,000.00	
Uniform with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	36,076.25	36,138.29	-62.03
st. deviation	434.19	905.01	635.30
		T-statistics (t_0)	0.5348
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Triangular with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	35,808.62	35,698.49	110.14
st. deviation	424.16	1,081.58	827.89
		T-statistics (t_0)	-0.7286
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Normal with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	35,876.82	35,900.90	-24.08
st. deviation	641.01	1,174.44	788.68
		T-statistics (t_0)	0.1673
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0
Exponential with 30 replications	Phase-1 Average waiting time	Phase-2 Average waiting time	Paired differences Average waiting time
Average	35,790.16	36,156.68	-366.52
st. deviation	3,016.12	4,963.41	2,629.66
		T-statistics (t_0)	0.7634
		$t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$	Ok! Accept H_0

in Equation (1) remains almost unchanged regardless of how fast (or slow) the queue is dissolved.

The purpose of the simulation runs with stochastic interarrivals has been to find out if consistent results are obtained in terms of the average waiting time similar to the deterministic case with constant rate arrivals. Departure rate is still kept at a constant rate (60 vehicles per minute or vehicles leaving the bottleneck zone every 60 tertias). Arrival rates in phase-1 and phase-2 are assumed to be stochastic (simulation runs performed for uniform, triangular, normal, and exponential interarrival times). In order to have a basis for comparison, parameters for the random variables have been chosen to have identical mean interarrival times (45 tertias in phase-1 and 75 tertias in phase-2). Variance of distributions have been adjusted so that coefficient of variation is tried to be kept constant (randomly selected to be 0.225) with the exception of the case of exponential interarrival times. Table 2 summarizes the average waiting times (in tertias) during phase-1 and phase-2 for stochastic interarrivals to be compared with the case of constant rate arrivals ($a_1 = 80$ and $a_2 = 48$ vehicles per minute). 30 simulation runs with

stochastic interarrival times have been conducted for the four mentioned distributions. Unsurprisingly for all interarrival distributions, the average waiting times in phase-1 and phase-2 are very close to 36,000 tertias (10 minutes).

Table 3 gives similar results for the case where phase-2 mean arrival rate is equal to 36 vehicles per minute. Similar conclusions have been reached for the tested various phase-2 mean arrival rates (not tabulated in this paper).

A statistical comparison of phase-1 and phase-2 average waiting times has been made to verify the validity of the paradox for stochastic interarrivals. If X is the random variable which denotes the difference between the average waiting times in phase-1 and phase-2, then the null hypothesis will be

$$H_0 : E[X] = 0, \tag{10}$$

and the alternative hypothesis can be stated as

$$H_1 : E[X] \neq 0. \tag{11}$$

One has to reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where t_0 is obtained from

$$t_0 = \frac{\bar{X}}{s/\sqrt{n}}. \tag{12}$$

The computed t value based on 30 replications for the case of uniformly distributed vehicle interarrival times (phase-2 mean arrival rate of 48 vehicles per minute) is -1.0411, and with $\alpha = 0.05$, the T-table threshold value is $t_{\alpha/2,n-1} = t_{0.025,29} = 2.04523$. Since in absolute terms the computed t value is smaller than the table value, one fails to reject the null hypothesis (meaning that there is not a significant difference between the average waiting times of phase-1 and phase-2 with 95% confidence). Similar test of hypothesis experiments have been conducted for various distributions of interarrival times (including the cases of phase-2 mean arrival rate being 36 vehicles per minute) with identical conclusions. The summaries of test of hypothesis experiments are given in Table 4 (for a phase-2 mean arrival rate of 48 vehicles per minute) and Table 5 (for a phase-2 mean arrival rate of 36 vehicles per minute).

6. Conclusions and Further Research Directions

Apart from the commuters' route, single bottleneck systems can be encountered in many queueing systems. Typical examples are banks and public offices serving customers through queue ticketing machines in which arrival rates are nonuniform throughout the day whereas the departure rate (in queueing theory better known as the service rate) is almost constant assuming an unchanging number of clerks. Modeling the queueing behavior for hourly intervals with differing customer arrival rates, one would have a similar implication of the paradox in which, for instance, a busy 13:00–14:00 interval is followed by the 14:00–15:00 interval for which the service rate exceeds the customer arrival rate. The consequence of the average waiting time paradox can be directly observed in such queueing systems. The basic difference from a bottleneck traffic model will be the additional service duration needed in banks and public offices. So an immediate further direction of research would be to experiment with simulation of queuing systems with consecutive intervals of varying customer arrival rates.

An interesting extension would be to consider the case of stochastic departure rates for the single bottleneck traffic system (equivalently, stochastic service rates in banks and public offices). Although the assumption of constant rate departures makes sense, accidents, road maintenance, or temporary checkpoint barriers are probabilistic factors which might cause variations of the departure rates. For the case of banks and public offices, the assumption of stochastic service rates can also be assumed to be more representative of situations in real life.

It should be noted that the individual waiting time at the bottleneck zone depends on the time of arrival; therefore, the vehicle owners would try to choose their arrival time to minimize their own waiting duration in the queue rather than improving an overall performance measure collectively. Nevertheless, the average waiting time as a performance indicator is significant especially for capacity readjustment policies such as dedicating an extra lane for the commuters' route or employment of an additional clerk in a bank. The verified paradox has interesting implications not only for the single bottleneck model of traffic congestion management but also for queueing systems in general. Therefore, further simulation experiments with several extensions of the given models (possibly with random departure/service rates) are foreseen as a future direction of research.

Data Availability

The authors would like to confirm that the simulation results described in Section 3 and Section 5 will be made available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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