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Analysis and dynamics of fractional order Covid-19 model with memory effect

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ABSTRACT

The present article attempts to examine fractional order Covid-19 model by employing an efficient and powerful analytical scheme termed as *q*-homotopy analysis Sumudu transform method (*q*-HASTM). The *q*-HASTM is the hybrid scheme based on *q*-HAM and Sumudu transform technique. Liouville-Caputo approach of the fractional operator has been employed. The proposed modelis also examined numerically via generalized Adams-Bashforth-Moulton method. We determined model equilibria and also give their stability analysis by employing next generation matrix and fractional Routh-Hurwitz stability criterion.

Introduction

As the beginning of the last centenary, the continuous development in mathematical models has used to analyze the blowout of transmissible disease in the epidemiology [1–3]. The investigators obtain precious information for diverse infectious diseases by study the stochastic and deterministic models. Kermack with McKendrick (1927) proposed a model beneficial for executing and evolving intricate epidemic models which considered as basic model in field of epidemiology till now [4].

Numerous communicable diseases can transmit in vertical and horizontal in both directions. Some examples of such human diseases are Hepatitis B, Herpes Simplex, HIV/AIDS and Rubella, etc. These diseases are horizontally transmitted in humans and animals through proximity amidst hosts or by disease carriers, e.g., flies and mosquitoes etc.

Epidemiological models are valuable in comprising, forecasting, employing, prevention, assessing various detection, therapy and control programs [5–7].

In year 2003, the first-time outburst termed as Severe Acute Respiratory Syndrome (SARS) occurred in mainland China ([8,9]) and other outbreak named as MERS occurred in South Korea in year 2015 ([10,11]). The recent outbreak is occurred in China and spread worldwide in the form of COVID-19. It is a transmissible disease due to a new virus called as novel corona virus. Since November 2019 to till date, cases of corona virus detected in several countries.

In the end of December 2019, first time it was recognized in Wuhan city (China). In February 2020, WHO announced it as pandemic and named it “COVID-19” and the ICTV declared “severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2)” [12]. Some studies state that it may be originate from bat or pangolins [13], and also, the spread of the virus may be connected to a seafood market ([14,15]) but no report confirms about the intermediate host. The patients of COVID-19 shown early identical symptoms as MERS-CoV and SARS-CoV infections like cough, tiredness, sore throat, fever, conjunctivitis and also in severe cases bilateral lung penetration [16]. In addition, some patients may suffer from diarrhea, loss of taste or smell, without any signs of breathing disorder ([17,18]).

From a few decades, significant development has made within the field of FDEs due to its applicability in the miscellaneous area of science and technology (Oldham and Spanier [19], Podlubny [20], Miller and Ross [21]). In 2014; El-Shahed et al. [22] considered childhood diseases model with fractional derivatives. Atangana et al. [23] discussed fractional order model for spread-ness of river blindness disease. Salman et al. [24] studied HBV infection model with fractional order derivative and Area et al. analysed fractional order Ebola epidemic Model [25], Sardar et al. developed Dengue Model with memory effect [26],

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mathematical model of HIV discussed by Babaei et al. and Baleanu et al. [27,28] and Baleanu et al. [29] examined a rubella disease problem pertaining to Caputo–Fabrizio operator.

There are numerous epidemiological models such as SIR model [30-32], SIS model [33], SEIR model [34,35], SIRC model [36,37], etc. which generally depends on the compartments.

In the literature of numerous mathematical models which study existence and uniqueness properties, stability analysis and control theory of biological and epidemic models with fractional order derivative [38–40]. Gao et al., in [41] studied of COVID-19 mathematical model in the edge of fractional calculus in Caputo sense by ADM. Atangana proposed that the virus can be transported from deceased individuals to other individuals [42]. Gao, et al. [43] reported a novel study for a replication model of the COVID-19 endemic and obtained the optimal parameters for the model to support in controlling the transmitted and spread the virus. Recently, Qureshi and Atangana [44] considered a model with the contribution of novel fractional operator to examine the diarrhea virus model.

Now a days many effective techniques are employed to get analytical and numerical and results for the epidemic models with fractional order derivative, such as Adomian decomposition method [45,46], homotopy perturbation technique [47], modified Laplace decomposition algorithm [48-50], HAM [51-53], q-HAM [54,55], q-HATM [56].

Integral transform techniques are largely employed to solve differential equations of physical importance. Sumudu transform was coined and studied by Watugala [57]. A number of important and useful results for the Sumudu transform were developed by Chaurasia and Singh [58], Belgacem et al. [59] many others.

In this work, a powerful computational method q-HASTM is involved in becoming the solutions of the time-fractional derivative of the Covid-19 model.

Singh *et al.* [60] proposed and developed the q-HASTM for examining nonlinear differential and integral equations. The q-HASTM is based on homotopy polynomials, Sumudu transform scheme and q-HAM. El-Tavil *et al.* [54,55] proposed a modified technique of HAM namely q-HAM. It is well known that HAM comprises a specific auxiliary parameter $\hbar$ for controlling the region of convergence, however, q-HAM involves $\hbar$ and n in such a manner that HAM solution is special case of q-HAM for $n = 1$. The suggested scheme is trustily useful for solving nonlinear models without considering linearization or any other restrictive suppositions and also disregards round off errors.

We have analyzed the nonlinear Covid-19 model pertaining to time-fractional operator by using a generalized Adams-Bashforth-Moulton scheme [61-64].

Preliminaries

Here, we define the required definitions and results of fractional operators and the Sumudu transform (ST).

Definition 2.1. [20] *The Liouville-Caputo* (*LC*) *fractional operator of order* α *is presented as*

$$\mathscr{D}_t^{\alpha} f(t) = \mathscr{J}_t^{m-\alpha}\mathscr{D}_t^{m} f(t) = \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-\ell)^{m-\alpha-1} f^{m}(\ell)d\ell, \quad m \in Z^+$$

we have

$$\mathscr{D}_t^{\alpha} t^{\ell} = 0, \ell < \alpha$$

$$\mathscr{D}_t^{\alpha} t^{l} = \frac{\Gamma(\ell+1)}{\Gamma(\ell-\alpha+1)} t^{\ell-\alpha}, \ell \geq \alpha.$$

Definition 2.2. *The integral operator* $\mathscr{J}_t^{\alpha}$ *of fractional order* $\alpha > 0$ *for the function* $f : \mathscr{R}^+ \to \mathscr{R}$ *in Riemann-Liouville* (*RL*) *sense is expressed as*

$$\mathscr{J}_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau$$

we have

$\mathscr{J}_t^{\alpha} t^{l} = \frac{\Gamma(l+1)}{\Gamma(l-\alpha+1)} t^{l+\alpha}, l \geq \alpha$ and $\mathscr{J}_t^{0} f(t) = f(t)$.

Definition 2.3. *Let us assume*

$$\mathscr{A} = \left\{ f(t) | \exists \mathscr{M}, \zeta_1, \zeta_2 > 0, |f(t)| < Me^{-\frac{|t|}{\zeta_i}} \quad \text{if} \quad t \in (-1)^j \times [0, \infty) \right\},$$

is a set of functions, then the ST operator over $\mathscr{A}$ is expressed by the formula [57,59],

$$\mathscr{S}[f(t)] = \mathscr{F}[u] = \int_0^{\infty} e^{-t} f(ut)dt, u \in (\zeta_1, \zeta_2)$$

Definition 2.4. *The ST of LC fractional operator* [58] *is expressed as*

$$\mathscr{S}\left[D_t^{\alpha} f(t)\right] = u^{-\alpha}\mathscr{S}[f(t)] - \sum_{k=0}^{m-1} u^{-(\alpha+k)} f^{(k)}(0+), m-1 < \alpha \leq m$$

and $\mathscr{S}[1] = 1, \mathscr{S}\left[\frac{t^{n-1}}{\Gamma(n)}\right] = u^{n-1}, n > 0.$

Authors have mostly considered the Caputo type fractional derivatives in comparison to Riemann-Liouville derivative for the reason that Caputo approach is more suitable and convenient for handling physical problems.

Formulation of mathematical model

Chen et al. [65] introduced a model and simulated the data of spreads from the source of infection to the infection in people.

Khan and Atangana [66] describe the fractional- order mathematical modelling and dynamics of the corona virus.

We will assume that all the parameters as well as variables involved in the model are non-negative during a present study in the individual community.

The total population of individuals considered as $N(t)$ which is separated in five compartments such as susceptible $S(t)$, exposed $E(t)$, symptomatic infected $I(t)$, asymptotically infected $A(t)$ and recovered $R(t)$ populations.

The following assumptions carried out during model formulation

- The recruitment of individuals increases the class of susceptible to the susceptible group at a constant rate Π.
- The childbirth rate and natural mortality rate of the individuals is indicated by parameters μ *and* δ respectively.
- The susceptible individual has been infected over ample contacts with infected individual $I(t)$ via the given term $\beta\frac{S(t)I(t)}{N}$, where contact rate $\beta$is disease transmission coefficients.
- The transmission among the asymptotically infected individual $A(t)$ with susceptible individual $S(t)$could take place at form $\beta\psi\frac{S(t)A(t)}{N}$, where $\psi \in [0,1]$ is transmissibility multiple of $A(t)$ to the $I(t)$, if $\psi = 0$, no transmissibility multiple will exist and so vanish, if $\psi = 1$, then transmissibility multiple will exist and same like infection.
- The parameter ϕ is proportion of asymptotic infection. The parameters θ and ω respectively indicate the transmission rate after accomplishing the incubation period and reduced into infected, joining the class $I(t)$ and $A(t)$.
- The individual in the symptomatic group $I(t)$ and asymptomatic group $A(t)$are removed at recovery rates of γ and τ and are added to the recovered $R(t)$ compartment.

- The susceptible individual will be infected after the interaction the reservoir or the seafood place or market class $P(t)$ through the term given by $\eta\frac{S(t)P(t)}{N}$, where η is disease transmission coefficients from $S(t)$ to $P(t)$.
- The $I(t)$ and $A(t)$ individuals contributing the virus into reservoir or the seafood place or market class $P(t)$ at a constant rate σ and ε respectively.
- The removing rate of the virus from the reservoir or the seafood place or market class $P(t)$ is indicated via κ.

Using the above assumptions to mathematical representation of the model comprise of the fractional order dynamical system as

$$\frac{d^\alpha S}{dt^\alpha}=\Pi-\beta\frac{SI}{N}-\beta\psi\frac{SA}{N}-\eta\frac{SP}{N}-\delta S$$

$$\frac{d^\alpha E}{dt^\alpha}=\beta\frac{SI}{N}+\beta\psi\frac{SA}{N}+\eta\frac{SP}{N}-(1-\phi)\theta E-\phi\omega E-\delta E \quad (1)$$

$$\frac{d^\alpha I}{dt^\alpha}=(1-\phi)\theta E-\gamma I-\delta I$$

$$\frac{d^\alpha A}{dt^\alpha}=\phi\omega E-\tau A-\delta A$$

$$\frac{d^\alpha R}{dt^\alpha}=\gamma I+\tau A-\delta R$$

$$\frac{d^\alpha P}{dt^\alpha}=\sigma I+\varepsilon A-\kappa P$$

Subject to conditions

$$\begin{array}{l} S(0)=S_0\geq 0, E(0)=E_0\geq 0, I(0)=I_0\geq 0,\\ A(0)=A_0\geq 0, R(0)=R_0\geq 0, P(0)=P_0\geq 0. \end{array} \quad (2)$$

Analysis of the model

Lemma 4.1 (Generalized mean value theorem)

Assume that $f(t)\in\mathscr{C}[a,b]$ and LC fractional operator $\mathscr{D}_t^\alpha f(t)\in\mathscr{C}(a,b]$ for $0<\alpha\leq 1$, then we get $f(t)=f(a)+\frac{1}{\Gamma(\alpha)}\mathscr{D}_t^\alpha f(t)(t-a)^\alpha$ with $0\leq t\leq \ell$, $\forall \ell\in(a,b]$.

Remark 4.1. *If $f(t)\in\mathscr{C}[0,b]$ and LC fractional operator $\mathscr{D}_t^\alpha f(t)\in\mathscr{C}(a,b]$ for $0<\alpha\leq 1$. It can be noted from Lemma that if $\mathscr{D}_t^\alpha f(t)\geq 0 \forall t\in(0,b]$, then the function $f(t)$ is non-decreasing and if $\mathscr{D}_t^\alpha f(t)\leq 0\forall t\in(0,b]$, then the function $f(t)$ is non-increasing.*

Theorem 4.1. *For the given model (1) the biological feasible region is in $\mathscr{R}_+^6$ given by the following $X=$*

$$\left\{(S(t),E(t),I(t),A(t),R(t),P(t))\in\mathscr{R}_+^6: 0\leq S+E+I+A+R\leq\frac{\mu}{\delta}, 0\leq P\leq P^*\right\}$$

, where the boundedness, uniqueness and existence hold for the model and solution remains in X.

Proof. *In view of Lin from the theorem 3.2 [67] and remark 3.2 [67], we obtain the uniqueness and existence results of the model. We have to demonstrate that the X is positively invariant*

$$\left.\frac{d^\alpha S}{dt^\alpha}\right|_{S=0}=\Pi>0$$

$$\left.\frac{d^\alpha E}{dt^\alpha}\right|_{E=0}=\beta\frac{SI}{N}+\beta\psi\frac{SA}{N}+\eta\frac{SP}{N}\geq 0$$

$$\left.\frac{d^\alpha I}{dt^\alpha}\right|_{I=0}=(1-\phi)\theta E\geq 0$$

$$\left.\frac{d^\alpha A}{dt^\alpha}\right|_{A=0}=\phi\omega E\geq 0$$

$$\left.\frac{d^\alpha R}{dt^\alpha}\right|_{R=0}=\gamma I+\tau A\geq 0$$

$$\left.\frac{d^\alpha P}{dt^\alpha}\right|_{P=0}=\sigma I+\varepsilon A\geq 0 \quad (3)$$

on every hyper plane bounding the non-negative orthant, the vector field points into $\mathscr{R}_+^6$.

Now, By means of the circumstance that $N=S+E+I+A+R$

$$\Rightarrow\frac{d^\alpha N}{dt^\alpha}=\frac{d^\alpha S}{dt^\alpha}+\frac{d^\alpha E}{dt^\alpha}+\frac{d^\alpha I}{dt^\alpha}+\frac{d^\alpha A}{dt^\alpha}+\frac{d^\alpha R}{dt^\alpha}$$

So $\frac{d^\alpha N}{dt^\alpha}=\mu-\delta N$

On utilizing the Laplace transform in equation (4), we achieve the result

$N(t)=\left(-\frac{\mu}{\delta}+N(0)\right)E_\alpha(-\delta t^\alpha)+\frac{\mu}{\delta}$, where $E_\alpha(-\delta t^\alpha)$is termed as the Mittag-Leffler function.

Since $0\leq E_\alpha(-\delta t^\alpha)\leq 1$, if $N(0)\leq\frac{\mu}{\delta}$ then $N(t)\leq\frac{\mu}{\delta}$, so the closed set X is the positive invariant set of the system (1).

The compartments population can be normalized by the Nadopting the current state variables

$x=\frac{S}{N}, y=\frac{E}{N}, z=\frac{I}{N}, u=\frac{A}{N}, v=\frac{R}{N}, w=\frac{P}{N}$, and $\mu=\frac{\Pi}{N}$.

Therefore, the population is now normalized and non-dimensional form as

$$\frac{d^\alpha x}{dt^\alpha}=\mu-\beta xz-\beta\psi xu-\eta xw-\delta x$$

$$\frac{d^\alpha y}{dt^\alpha}=\beta xz+\beta\psi xu+\eta xw-(1-\phi)\theta y-\phi\omega y-\delta y$$

$$\frac{d^\alpha z}{dt^\alpha}=(1-\phi)\theta y-\gamma z-\delta z$$

$$\frac{d^\alpha u}{dt^\alpha}=\phi\omega y-\tau u-\delta u$$

$$\frac{d^\alpha v}{dt^\alpha}=\gamma z+\tau u-\delta v$$

$$\frac{d^\alpha w}{dt^\alpha}=\sigma z+\varepsilon u-\kappa w \quad (5)$$

with initial conditions

$$x(0)=x_0\geq 0, y(0)=y\geq 0, z(0)=z_0\geq 0, \quad (6)$$

$$u(0)=u_0\geq 0, v(0)=v_0\geq 0, w(0)=w_0\geq 0$$

Stability of fractional order system

Consider

$$\frac{d^\alpha x}{dt^\alpha}=f_1(x,y,z,u,v,w), \frac{d^\alpha y}{dt^\alpha}=f_2(x,y,z,u,v,w), \frac{d^\alpha z}{dt^\alpha}=f_3(x,y,z,u,v,w)$$

$$\frac{d^\alpha u}{dt^\alpha}=f_4(x,y,z,u,v,w), \frac{d^\alpha v}{dt^\alpha}=f_5(x,y,z,u,v,w), \frac{d^\alpha w}{dt^\alpha}=f_6(x,y,z,u,v,w) \quad (7)$$

where $0<\alpha<1$ and $\frac{d^\alpha}{dt^\alpha}$ is LC derivative.

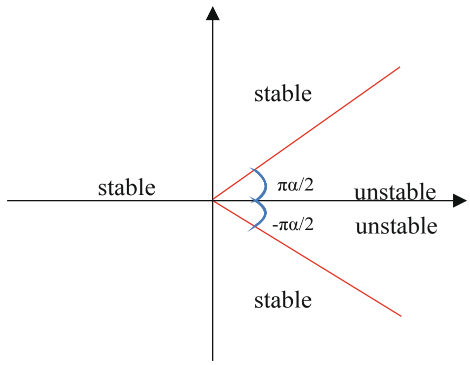

Fig. 1. Stability region for the fractional order system.

$$\text{Let } J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial w} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial v} & \frac{\partial f_4}{\partial w} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial u} & \frac{\partial f_5}{\partial v} & \frac{\partial f_5}{\partial w} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial u} & \frac{\partial f_6}{\partial v} & \frac{\partial f_6}{\partial w} \end{bmatrix} \tag{8}$$

be the Jacobian matrix of the system.

Theorem 5.1. *The model is said to be asymptotically stable (locally) if every Eigen values of the J at its fixed point satisfy* $|arg(\lambda)| > \alpha\pi/2$.

Fig. 1 demonstrates that the stability region for the model with fractional order exceeds in comparison of system with integer order. It is clearly observable from the figure that stability region for an integer order system lies only in the left part of the vertical axis while it lies also in the right part for fractional system.

Routh–Hurwitz (RH) stability criterion for system of fractional order

Let us assume a system of fractional order expressed as

$$\frac{d^\alpha}{dt^\alpha} y_i(t) = f_i(y_1, y_2, y_3, y_4, y_5, y_6), \quad i = 1,2,3,4,5,6 \; 0 < \alpha \leq 1, \tag{9}$$

with the initial conditions (IC):

$$y_1(0) = y_{10}, y_2(0) = y_{20}, y_3(0) = y_{30}, y_4(0) = y_{40}, y_5(0) = y_{50}, y_6(0) = y_{60} \tag{10}$$

To evaluate the equilibrium points of Eq. (9), taking $D_t^q Y_i(t) = 0$, this implies that $f_i(y_1, y_2, y_3, y_4, y_5, y_6) = 0$. Let $\Im^*(y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*)$ be an equilibrium point of system (9). Next a non-negative term $\epsilon(t)$i.e.$y_i(t) = y_i^* + \epsilon_i(t)$ is added to the equilibrium point for desired perturbation. Thus, we have

$$\frac{d^\alpha}{dt^\alpha}(y_i^* + \epsilon_i) = f_i(y_1^* + \epsilon_1, y_2^* + \epsilon_2, y_3^* + \epsilon_3, y_4^* + \epsilon_4, y_5^* + \epsilon_5, y_6^* + \epsilon_6),$$

$$\Rightarrow \frac{d^\alpha \epsilon_i}{dt^\alpha} = f_i(y_1^* + \epsilon_1, y_2^* + \epsilon_2, y_3^* + \epsilon_3, y_4^* + \epsilon_4, y_5^* + \epsilon_5, y_6^* + \epsilon_6), \tag{11}$$

The use of Taylor series expansion yields

$$\frac{d^\alpha \epsilon_i}{dt^\alpha} = f_i(y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*) + \frac{\partial f_i}{\partial y_1}\Big|_{eq}\epsilon_1 + \frac{\partial f_i}{\partial y_2}\Big|_{eq}\epsilon_2 + \frac{\partial f_i}{\partial y_3}\Big|_{eq}\epsilon_3 + \frac{\partial f_i}{\partial y_4}\Big|_{eq}\epsilon_4 + \frac{\partial f_i}{\partial y_5}\Big|_{eq}\epsilon_5 + \frac{\partial f_i}{\partial y_6}\Big|_{eq}\epsilon_6 + \text{higher ordered terms.} \tag{12}$$

Since $f_i(y_1^*, y_2^*, y_3^*, y_4^*, y_5^*, y_6^*) = 0$, then

$$\frac{d^\alpha \epsilon_i}{dt^\alpha} \approx \frac{\partial f_i}{\partial y_1}\Big|_{eq}\epsilon_1 + \frac{\partial f_i}{\partial y_2}\Big|_{eq}\epsilon_2 + \frac{\partial f_i}{\partial y_3}\Big|_{eq}\epsilon_3 + \frac{\partial f_i}{\partial y_4}\Big|_{eq}\epsilon_4 + \frac{\partial f_i}{\partial y_5}\Big|_{eq}\epsilon_5 + \frac{\partial f_i}{\partial y_6}\Big|_{eq}\epsilon_6 \tag{13}$$

Now Eq. (13) can be written as

$$\frac{d^\alpha \epsilon_i}{dt^\alpha} = J\epsilon, \tag{14}$$

where

$$\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6)^T, J(\Im^*) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial w} \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial v} & \frac{\partial f_4}{\partial w} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial u} & \frac{\partial f_5}{\partial v} & \frac{\partial f_5}{\partial w} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial u} & \frac{\partial f_6}{\partial v} & \frac{\partial f_6}{\partial w} \end{bmatrix}, \tag{15}$$

Here $J(\Im^*)$ satisfies the expression $C^{-1}J(\Im^*)C = D$,

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6 \end{bmatrix}, \tag{16}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$and λ_6 denotes the eigen values of J, C is the Eigen vector of J and D is a diagonal matrix.

The initial conditions (IC) for system (9) are

$$y_1(0) = y_1^* + \epsilon_1(0), y_2(0) = y_2^* + \epsilon_2(0), y_3(0) = y_3^* + \epsilon_3(0)$$

$$y_4(0) = y_4^* + \epsilon_4(0), y_5(0) = y_5^* + \epsilon_5(0), y_6(0) = y_6^* + \epsilon_6(0). \tag{17}$$

Using Eqs. (14) and (16), we obtain

$$\frac{d^\alpha}{dt^\alpha}\epsilon = (CDC^{-1})\epsilon, \frac{d^\alpha}{dt^\alpha}(C^{-1}\epsilon) = D(C^{-1}\epsilon).$$

$$\text{Hence } \frac{d^\alpha}{dt^\alpha}\zeta = D\zeta, \xi = C^{-1}\epsilon, \zeta = (\zeta_1, \zeta_2, \zeta_3)^T. \tag{18}$$

Therefore,

$$\frac{d^\alpha \zeta_i}{dt^\alpha} = \lambda_i \zeta_i, \quad i = 1,2,3,4,5,6 \tag{19}$$

The solutions of Eq. (19) are given by

$$\zeta_i(t) = E_q(\lambda_i t^q)\zeta_i(0), \quad i = 1,2,3,4,5,6$$

where $\zeta_i(0), i = 1,2,3,4,5,6$ are arbitrary constants and Mittag-Leffler function$E_\alpha(\lambda_i t^\alpha) = \sum_{n=0}^{\infty}\frac{(\lambda_i)^n t^{n\alpha}}{\Gamma(n\alpha+1)}$ satisfies the equations $\frac{d^\alpha}{dt^\alpha}\zeta_i = \lambda_i\zeta_i, i = 1,$ $2, 34,5,6$.

$$\frac{d^\alpha}{dt^\alpha}[\mathrm{E}_\alpha(\lambda_1 t^\alpha)\zeta_1(0)] = \zeta_1(0)\frac{d^\alpha}{dt^\alpha}\left[\sum_{n=0}^{\infty}\frac{(\lambda_1)^n t^{n\alpha}}{\Gamma(n\alpha+1)}\right]$$

$$= \zeta_1(0)\frac{d^\alpha}{dt^\alpha}\left[1+\frac{\lambda_1 t^\alpha}{\Gamma(\alpha+1)}+\frac{\lambda_1^2 t^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{\lambda_1^3 t^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right]$$

($\because$Caputo derivative of a constant is zero and$D_t^\alpha t^n = \frac{\Gamma(n+1)t^{n-\alpha}}{\Gamma(n-\alpha+1)}$)

$$= \zeta_1(0)\lambda_1\left[1+\frac{\lambda_1 t^\alpha}{\Gamma(\alpha+1)}+\frac{\lambda_1^2 t^{2\alpha}}{\Gamma(2\alpha+1)}+\cdots\right]$$

$$= \zeta_1(0)\lambda_1\mathrm{E}_\alpha(\lambda_1 t^\alpha)$$

$$= \lambda_1\mathrm{E}_\alpha(\lambda_1 t^\alpha)\zeta_1(0) = \lambda_1\zeta_1$$

Then $\zeta_1(t), \zeta_2(t), \zeta_3(t), \zeta_4(t), \zeta_5(t), \zeta_6(t)$ are decreasing and thus $\epsilon_1(t)$, $\epsilon_2(t), \epsilon_3(t), \epsilon_4(t), \epsilon_5(t), \epsilon_6(t)$, are decreasing.

Hence the equilibrium point $\Im^*$ is termed as the point of locally asymptotic stability if $|arg(\lambda_i)| > \alpha\frac{\pi}{2}, i = 1,2,3,4,5,6$is fulfilled [68].

To obtain the fixed points of the model (5) equating the RHS of the model (5) to zero i.e.

$$\frac{d^\alpha x}{dt^\alpha} = 0, \frac{d^\alpha y}{dt^\alpha} = 0, \frac{d^\alpha z}{dt^\alpha} = 0, \frac{d^\alpha u}{dt^\alpha} = 0, \frac{d^\alpha v}{dt^\alpha} = 0, \frac{d^\alpha w}{dt^\alpha} = 0$$

i.e.$f_i(x,y,z,u,v,w) = 0$, we get

$$\mu - \beta xz - \beta\psi xu - \eta xw - \delta x = 0$$

$$\beta xz + \beta\psi xu + \eta xw - (1-\phi)\theta y - \phi\omega y - \delta y = 0$$

$$(1-\phi)\theta y - \gamma z - \delta z = 0$$

$$\phi\omega y - \tau u - \delta u = 0$$

$$\gamma z + \tau u - \delta v = 0$$

$$\sigma z + \varepsilon u - \kappa w = 0$$

When $y = 0$, then $\Im_0 = \left(\frac{\mu}{\delta}, 0,0,0,0,0\right)$ which denotes adisease-free equilibrium (DFE) point of the system (5).

Theorem 5.3. *$\Im_0 = \left(\frac{\mu}{\delta}, 0,0,0,0,0\right)$ is asymptotically stable (locally) if $R_0 < 1$ and became unstable if $R_0 > 1$.*

Proof. *As considered model has DEF $\Im_0 = \left(\frac{\mu}{\delta}, 0,0,0,0,0\right)$, Using next generation matrix (NGM) method, the reproduction numberR_0 for the COVID-19 model given by (5) can be calculated from the relation $R_0 = \wp(F\cdot V^{-1})$, $\wp$ stands for spectral radius of the NGM$F\cdot V^{-1}$ [69-71]*

Assume $\mathrm{Y} = (y,z,u,w)$ then the system can be rewritten as $\frac{d\mathrm{Y}}{dx} = \mathscr{F}(\mathrm{Y}) - \mho(\mathrm{Y})$, where $\mathscr{F}(\mathrm{Y})$ is transmission part which enunciates the generation of novel infection and $\mho(\mathrm{Y})$ is transition part i.e. transfer of infection from one compartment to another.

The Jacobian matrices of $\mathscr{F}(\mathrm{Y}) = \begin{bmatrix}\beta xz + \beta\psi xu + \eta xw \\ 0 \\ 0 \\ 0\end{bmatrix}$ and $\mho(\mathrm{Y}) = \begin{bmatrix}(1-\phi)\theta y + \phi\omega y + \delta y \\ -(1-\phi\phi)\theta y + \gamma z + \delta z \\ -\phi\omega y + \tau u + \delta u \\ -\sigma z - \varepsilon u + \kappa w\end{bmatrix}$ at DFE point $\Im_0 = \left(\frac{\mu}{\delta}, 0,0,0,0,0\right)$are given by

$$F = \left.\frac{\partial\mathscr{F}_k}{\partial\mathrm{Y}_j}\right|_{\Im_0} = \begin{bmatrix}0 & \beta\frac{\mu}{\delta} & \beta\psi\frac{\mu}{\delta} & \eta\frac{\mu}{\delta} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix} \text{ and}$$

$$V = \left.\frac{\partial\mho_k}{\partial\mathrm{Y}_j}\right|_{\Im_0} = \begin{bmatrix}(1-\phi)\theta + \phi\omega + \delta & 0 & 0 & 0 \\ (1-\phi)\theta & \gamma+\delta & 0 & 0 \\ -\phi\omega & 0 & \tau+\delta & 0 \\ 0 & -\sigma & -\varepsilon & \kappa\end{bmatrix}, j,k = 1,2,3,4$$

Then the inverse of the transition matrix is also computed as

$$V^{-1} = \begin{bmatrix}\frac{1}{(1-\phi)\theta+\phi\omega+\delta} & 0 & 0 & 0 \\ \frac{\delta\theta\kappa+\theta\kappa\tau-\delta\theta\kappa\varphi-\theta\kappa\tau\varphi}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)} & \frac{1}{\gamma+\delta} & 0 & 0 \\ \frac{\gamma\kappa\phi\omega+\delta\kappa\phi\omega}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)} & 0 & \frac{1}{\tau+\delta} & 0 \\ \frac{\delta\theta\sigma+\theta\sigma\tau-\theta\sigma\tau\varphi+\gamma\varepsilon\phi\omega+\delta\varepsilon\varphi\omega}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)} & \frac{\sigma}{(\gamma+\delta)\kappa} & \frac{\varepsilon}{(\tau+\delta)\kappa} & \frac{1}{\kappa}\end{bmatrix}$$

So

$$F.V^{-1} = \begin{bmatrix}A_1 & A_2 & A_3 & A_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix}$$

Hence,

$$R_0 = \frac{\delta\theta\kappa+\theta\kappa\tau-\delta\theta\kappa\phi-\theta\kappa\tau\phi}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)}\beta\frac{\mu}{\delta} + \frac{\gamma\kappa\phi\omega+\delta\kappa\phi\omega}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)}\beta\psi\frac{\mu}{\delta} + \frac{\delta\theta\sigma+\theta\sigma\tau-\theta\sigma\tau\varphi+\gamma\varepsilon\phi\omega+\delta\varepsilon\phi\omega}{(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)}\eta\frac{\mu}{\delta}$$

$$\Rightarrow R_0 = \frac{\beta\mu\theta\delta\kappa(1-\phi)+\theta\kappa\tau\beta\mu(1-\phi)+\theta\sigma\tau\eta\mu(1-\phi)+(\gamma+\delta)(\kappa\phi\omega\beta\psi\mu+\delta\theta\sigma\eta\mu+\varepsilon\phi\omega\eta\mu)+\delta\theta\sigma\eta\mu}{\delta(\gamma+\delta)\kappa(\delta+\tau)(\delta+\theta(1-\phi)+\phi\omega)}$$

The variational matrix J for the system determined at $\Im_0$

$$J(\Im_0) = \begin{bmatrix}-\delta & 0 & -\beta\frac{\mu}{\delta} & -\beta\psi\frac{\mu}{\delta} & 0 & -\eta\frac{\mu}{\delta} \\ 0 & -(1-\phi)\theta-\phi\omega-\delta & \beta\frac{\mu}{\delta} & \beta\psi\frac{\mu}{\delta} & 0 & \eta\frac{\mu}{\delta} \\ 0 & (1-\phi)\theta & -\gamma-\delta & 0 & 0 & 0 \\ 0 & \phi\omega & 0 & -\tau-\delta & 0 & 0 \\ 0 & 0 & \gamma & \tau & -\delta & 0 \\ 0 & 0 & \sigma & \varepsilon & 0 & -\kappa\end{bmatrix}$$

has the two Eigen values are negative i.e. $-\delta$ (twice) and other can evaluated by given polynomial equation

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \quad (20)$$

$$\mathrm{a}_1 = (\gamma + 3\delta + \kappa + \tau + (1-\phi)\theta + \phi\omega);$$

$$\mathrm{a}_2 = \frac{1}{\delta}(2\gamma\delta^2 + 3\delta^3 + \gamma\delta\theta + 2\delta^2\theta + \gamma\delta\kappa + 3\delta^2\kappa + \delta\theta\kappa + \gamma\delta\tau + 2\delta^2\tau + \delta\theta\tau$$
$$+ \delta\kappa\tau - \beta\theta\mu - \gamma\delta\theta\phi - 2\delta^2\theta\phi - \delta\theta\kappa\phi + \beta\theta\mu\phi - \delta\theta\tau\phi + \gamma\delta\phi\omega + 2\delta^2\phi\omega$$
$$+ \delta\kappa\phi\omega + \delta\tau\phi\omega - \beta\mu\phi\psi\omega);$$

$$a_3 = \frac{1}{\delta}(\gamma\delta^3 + \delta^4 + \gamma\delta^2\theta + \delta^3\theta + 2\gamma\delta^2\kappa + 3\delta^3\kappa + \gamma\delta\theta\kappa + 2\delta^2\theta\kappa + \gamma\delta^2\tau + \delta^3\tau$$
$$+ \gamma\delta\theta\tau + \delta^2\theta\tau + \gamma\delta\kappa\tau + 2\delta^2\kappa\tau + \delta\theta\kappa\tau + \eta\theta\mu\sigma - \beta\theta\mu\tau - \gamma\delta^2\theta\phi - \delta^3\theta\phi$$
$$- \gamma\delta\theta\kappa\phi - 2\delta^2\theta\kappa\phi + \beta\delta\theta\mu\phi + \beta\theta\kappa\mu\phi - \eta\theta\mu\sigma\phi - \gamma\delta\theta\tau\phi - \delta^2\theta\tau\phi$$
$$- \delta\theta\kappa\tau\phi + \beta\theta\mu\tau\phi + \gamma\delta^2\phi\omega + \delta^3\phi\omega + \gamma\delta\kappa\phi\omega + 2\delta^2\kappa\phi\omega + \varepsilon\eta\mu\phi\omega$$
$$+ \gamma\delta\tau\phi\omega + \delta^2\tau\phi\omega + \delta\kappa\tau\phi\omega - \beta\gamma\mu\phi\psi\omega - \beta\delta\mu\phi\psi\omega - \beta\kappa\mu\phi\psi\omega - \beta\delta\theta\mu$$
$$- \beta\theta\kappa\mu);$$

$$\mathrm{a}_4 = \frac{1}{\delta}(\gamma\delta^3\kappa + \delta^4\kappa + \gamma\delta^2\theta\kappa + \delta^3\theta\kappa - \beta\delta\theta\kappa\mu - \beta\delta\theta\lambda\mu - \beta\theta\kappa\lambda\mu + \delta\eta\theta\mu\sigma$$
$$+ \gamma\delta^2\kappa\tau + \delta^3\kappa\tau + \gamma\delta\theta\kappa\tau + \delta^2\theta\kappa\tau - \beta\theta\kappa\mu\tau + \eta\theta\mu\sigma\tau - \gamma\delta^2\theta\kappa\phi - \delta^3\theta\kappa\phi$$
$$+ \beta\delta\theta\kappa\mu\phi - \delta\eta\theta\mu\sigma\phi - \gamma\delta\theta\kappa\tau\phi - \delta^2\theta\kappa\tau\phi + \beta\theta\kappa\mu\tau\phi - \eta\theta\mu\sigma\tau\phi$$
$$+ \gamma\delta^2\kappa\phi\omega + \delta^3\kappa\phi\omega + \gamma\varepsilon\eta\mu\phi\omega + \delta\varepsilon\eta\mu\phi\omega + \gamma\delta\kappa\tau\phi\omega + \delta^2\kappa\tau\phi\omega$$
$$- \beta\gamma\kappa\mu\phi\psi\omega - \beta\delta\kappa\mu\phi\psi\omega);$$

Using RH criteria [72,73] for the polynomial () is $a_i > 0$ for $i = 1, 2, 3, 4$ and $a_1, a_2, a_3 > a_1^2 a_4 + a_3^2$ can be easily satisfied.

Theorem 5.4. *The endemic equilibrium point* $\Im^*(x^*, y^*, z^*, u^*, v^*, w^*)$ *of the fractional model is asymptotically stable if* $R_0 > 1$.

Proof. *The variational matrix of the system* (5) *at* $\Im^*$ *written as*

$$b_3 = (3\gamma\delta^2 + 4\delta^3 + 2\gamma\delta\theta + 3\delta^2\theta + 3\gamma\delta\kappa + 6\delta^2\kappa + \gamma\theta\kappa + 3\delta\theta\kappa + 2\gamma\delta\tau + 3\delta^2\tau$$
$$+ \gamma\theta\tau + 2\delta\theta\tau + \gamma\kappa\tau + 3\delta\kappa\tau + \theta\kappa\tau - 2\gamma\delta\theta\phi - 3\delta^2\theta\phi - \gamma\theta\kappa\phi - 3\delta\theta\kappa\phi$$
$$- \gamma\theta\tau\phi - 2\delta\theta\tau\phi - \theta\kappa\tau\phi + 2\gamma\delta\phi\omega + 3\delta^2\phi\omega + \gamma\kappa\phi\omega + 3\delta\kappa\phi\omega + \gamma\tau\phi\omega$$
$$+ 2\delta\tau\phi\omega + \kappa\tau\phi\omega + \beta\psi(3\delta^2 + \theta\kappa + \theta\tau + \kappa\tau - \theta\kappa\phi - \theta\tau\phi + \kappa\phi\omega + \tau\phi\omega$$
$$+ \gamma(2\delta + \theta + \kappa + \tau - \theta\phi + \phi\omega) + \delta(2\theta + 3\kappa + 2\tau - 2\theta\phi + 2\phi\omega))u^*$$
$$+ \eta(3\delta^2 + \theta\kappa - \theta\kappa\phi - \theta\tau\phi + \kappa\phi\omega + \tau\phi\omega + \gamma(2\delta + \theta + \kappa - \theta\phi + \phi\omega)$$
$$+ \delta(2\theta + 3\kappa - 2\theta\phi + 2\phi\omega))w^* - 2\beta\delta\theta x^* - \beta\theta\kappa x^* - \eta\theta\sigma x^* - \beta\theta\tau x^*$$
$$+ 2\beta\delta\theta\phi x^* + \beta\theta\kappa\phi x^* + \eta\theta\sigma\phi x^* + \beta\theta\tau\phi x^* - \varepsilon\eta\phi\omega x^* - \beta\gamma\phi\psi\omega x^*$$
$$- 2\beta\delta\phi\psi\omega x^* - \beta\kappa\phi\psi\omega x^* + 2\beta\gamma\delta z^* + 3\beta\delta^2 z^* + \beta\gamma\theta z^* + 2\beta\delta\theta z^* + \beta\gamma\kappa z^*$$
$$+ 3\beta\delta\kappa z^* + \beta\theta\kappa z^* + \beta\gamma\tau z^* + 2\beta\delta\tau z^* + \beta\theta\tau z^* + \beta\kappa\tau z^* - \beta\gamma\theta\phi z^*$$
$$- 2\beta\delta\theta\phi z^* - \beta\theta\kappa\phi z^* - \beta\theta\tau\phi z^* + \beta\gamma\phi\omega z^* + 2\beta\delta\phi\omega z^* + \beta\kappa\phi\omega z^*$$
$$+ \beta\tau\phi\omega z^*)$$

$$J(\Im^*) = \begin{bmatrix} -\beta z^* - \beta\psi u^* - \eta w^* - \delta & 0 & -\beta x^* & -\beta\psi x^* & 0 & -\eta x^* \\ \beta z^* + \beta\psi u^* + \eta w^* & -(1-\phi)\theta - \phi\omega - \delta & \beta x^* & \beta\psi x^* & 0 & \eta x^* \\ 0 & (1-\phi)\theta & -\gamma - \delta & 0 & 0 & 0 \\ 0 & \phi\omega & 0 & -\tau - \delta & 0 & 0 \\ 0 & 0 & \gamma & \tau & -\delta & 0 \\ 0 & 0 & \sigma & \varepsilon & 0 & -\kappa \end{bmatrix}$$

has the one Eigen value is negative i.e. $-\delta$ and other can evaluated by given polynomial equation

$$\lambda^5 + \mathrm{b}_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5 = 0 \quad (21)$$

where,

$$b_1 = (\gamma + 4\delta + \theta + \kappa + \tau - \theta\phi + \phi\omega + \beta\psi u^* + \eta w^* + \beta z^*)$$

$$b_2 = (3\gamma\delta + 6\delta^2 + \gamma\theta + 3\delta\theta + \gamma\kappa + 4\delta\kappa + \theta\kappa + \gamma\tau + 3\delta\tau + \theta\tau + \kappa\tau - \gamma\theta\phi$$
$$- 3\delta\theta\phi - \theta\kappa\phi - \theta\tau\phi + \gamma\phi\omega + 3\delta\phi\omega + \kappa\phi\omega + \tau\phi\omega + \beta\psi(\gamma + 3\delta + \theta$$
$$+ \kappa + \tau - \theta\phi + \phi\omega)u^* + \eta(\gamma + 3\delta + \theta + \kappa + \tau - \theta\phi + \phi\omega)w^* - \beta\theta x^*$$
$$+ \beta\theta\phi x^* - \beta\phi\psi\omega x^* + \beta\gamma z^* + 3\beta\delta z^* + \beta\theta z^* + \beta\kappa z^* + \beta\tau z^* - \beta\theta\phi z^*$$
$$+ \beta\phi\omega z^*)$$

$$b_4 = (\gamma\delta^3 + \delta^4 + \gamma\delta^2\theta + \delta^3\theta + 3\gamma\delta^2\kappa + 4\delta^3\kappa + 2\gamma\delta\theta\kappa + 3\delta^2\theta\kappa + \gamma\delta^2\tau + \delta^3\tau$$
$$+ \gamma\delta\theta\tau + \delta^2\theta\tau + 2\gamma\delta\kappa\tau + 3\delta^2\kappa\tau + \gamma\theta\kappa\tau + 2\delta\theta\kappa\tau - \delta^2\theta\phi - \delta^3\theta\phi$$
$$- 2\gamma\delta\theta\kappa\phi - 3\delta^2\theta\kappa\phi - \gamma\delta\theta\tau\phi - \delta^2\theta\tau\phi - \gamma\theta\kappa\tau\phi - 2\delta\theta\kappa\tau\phi + \gamma\delta^2\phi\omega$$
$$+ \delta^3\phi\omega + 2\gamma\delta\kappa\phi\omega + 3\delta^2\kappa\phi\omega + \gamma\delta\tau\phi\omega + \delta^2\tau\phi\omega + \gamma\kappa\tau\phi\omega + 2\delta\kappa\tau\phi\omega$$
$$+ \beta\psi(\delta^3 + \kappa\tau(\theta + \phi\omega) + \delta^2(\theta + 3\kappa + \tau - \theta\phi + \phi\omega)$$
$$+ \delta(-\theta(2\kappa + \tau)(-1 + \phi) + \tau\phi\omega + 2\kappa(\tau + \phi\omega)) + \gamma(\delta^2 + \kappa\tau$$
$$- \theta(\kappa + \tau)(-1 + \phi) + \kappa\phi\omega + \tau\phi\omega + \delta(\theta + 2\kappa + \tau - \theta\phi + \phi\omega)))u^*$$
$$- \beta\delta^2\theta x^* - 2\beta\delta\theta\kappa x^* - 2\delta\eta\theta\sigma x^* - \beta\delta\theta\tau x^* - \beta\theta\kappa\tau x^* - \eta\theta\sigma\tau x^* + \beta\delta^2\theta\phi x^*$$
$$+ 2\beta\delta\theta\kappa\phi x^* + 2\delta\eta\theta\sigma\phi x^* + \beta\delta\theta\tau\phi x^* + \beta\theta\kappa\tau\phi x^* + \eta\theta\sigma\tau\phi x^* - \gamma\varepsilon\eta\phi\omega x^*$$
$$- 2\delta\varepsilon\eta\phi\omega x^* - \beta\gamma\delta\phi\psi\omega x^* - \beta\delta^2\phi\psi\omega x^* - \beta\gamma\kappa\phi\psi\omega x^* - 2\beta\delta\kappa\phi\psi\omega x^*$$
$$+ \beta\gamma\delta^2 z^* + \beta\delta^3 z^* + \beta\gamma\delta\theta z^* + \beta\delta^2\theta z^* + 2\beta\gamma\delta\kappa z^* + 3\beta\delta^2\kappa z^* + \beta\gamma\theta\kappa z^*$$
$$+ 2\beta\delta\theta\kappa z^* + \beta\gamma\delta\tau z^* + \beta\delta^2\tau z^* + \beta\gamma\theta\tau z^* + \beta\delta\theta\tau z^* + \beta\gamma\kappa\tau z^* + 2\beta\delta\kappa\tau z^*$$
$$+ \beta\theta\kappa\tau z^* - \beta\gamma\delta\theta\phi z^* - \beta\delta^2\theta\phi z^* - \beta\gamma\theta\kappa\phi z^* - 2\beta\delta\theta\kappa\phi z^* - \beta\gamma\theta\tau\phi z^*$$
$$- \beta\delta\theta\tau\phi z^* - \beta\theta\kappa\tau\phi z^* + \beta\gamma\delta\phi\omega z^* + \beta\delta^2\phi\omega z^* + \beta\gamma\kappa\phi\omega z^* + 2\beta\delta\kappa\phi\omega z^*$$
$$+ \beta\gamma\tau\phi\omega z^* + \beta\delta\tau\phi\omega z^* + \beta\kappa\tau\phi\omega z^* + \eta w^*(\delta^3 + \delta^2\theta + 3\delta^2\kappa + 2\delta\theta\kappa + \delta^2\tau$$
$$+ \delta\theta\tau + 2\delta\kappa\tau - \delta^2\theta\phi - 2\delta\theta\kappa\phi - \delta\theta\tau\phi - \theta\kappa\tau\phi + \delta^2\phi\omega + 2\delta\kappa\phi\omega$$
$$+ \delta\tau\phi\omega + \kappa\tau\phi\omega + \gamma(\delta^2 + \kappa\tau - \theta(\kappa + \tau)(-1 + \phi) + \kappa\phi\omega + \tau\phi\omega + \delta(\theta$$
$$+ 2\kappa + \tau - \theta\phi + \phi\omega)) + \theta\kappa\tau^*))$$

$$\begin{aligned} b_5 = & -\gamma+\gamma\delta^3\kappa+\delta^4\kappa+\gamma\delta^2\theta\kappa+\delta^3\theta\kappa+\gamma\delta^2\kappa\tau+\delta^3\kappa\tau+\gamma\delta\theta\kappa\tau+\delta^2\theta\kappa\tau \\ & -\gamma\delta^2\theta\kappa\phi-\delta^3\theta\kappa\phi-\gamma\delta\theta\kappa\tau\phi-\delta^2\theta\kappa\tau\phi+\gamma\delta^2\kappa\phi\omega+\delta^3\kappa\phi\omega+\gamma\delta\kappa\tau\phi\omega \\ & +\delta^2\kappa\tau\phi\omega+\beta(\gamma+\delta)\kappa(\delta+\tau)\psi(\delta+\theta-\theta\phi+\phi\omega)u^* \\ & +(\gamma+\delta)\eta\kappa(\delta+\tau)(\delta+\theta-\theta\phi+\phi\omega)w^*-\beta\delta^2\theta\kappa x^*-\delta^2\eta\theta\sigma x^*-\beta\delta\theta\kappa\tau x^* \\ & -\delta\eta\theta\sigma\tau x^*+\beta\delta^2\theta\kappa\phi x^*+\delta^2\eta\theta\sigma\phi x^*+\beta\delta\theta\kappa\tau\phi x^*+\delta\eta\theta\sigma\tau\phi x^*-\gamma\delta\varepsilon\eta\phi\omega x^* \\ & -\delta^2\varepsilon\eta\phi\omega x^*-\beta\gamma\delta\kappa\phi\psi\omega x^*-\beta\delta^2\kappa\phi\psi\omega x^*+\beta\gamma\delta^2\kappa z^*+\beta\delta^3\kappa z^*+\beta\gamma\delta\theta\kappa z^* \\ & +\beta\delta^2\theta\kappa z^*+\beta\gamma\delta\kappa\tau z^*+\beta\delta^2\kappa\tau z^*+\beta\gamma\theta\kappa\tau z^*+\beta\delta\theta\kappa\tau z^*-\beta\gamma\delta\theta\kappa\phi z^* \\ & -2\beta\delta^2\theta\kappa\phi z^*-\beta\gamma\theta\kappa\tau\phi z^*-\beta\delta\theta\kappa\tau\phi z^*+\beta\gamma\delta\kappa\phi\omega z^*+\beta\delta^2\kappa\phi\omega z^* \\ & +\beta\gamma\kappa\tau\phi\omega z^*+\beta\delta\kappa\tau\phi\omega z^* \end{aligned}$$

Using RH criteria for the polynomial is $b_i>0$ for $i=1,2,3,4,5$, $b_1b_2b_3>b_1^2b_4+b_3^2$ and $(b_1b_4-b_5)(b_1b_2b_3-b_1^2b_4+b_3^2)>b_5(b_1b_2-b_3)^2+b_1b_5^2$ can be easily satisfied.

The family of Voltera type Lyapunov function

$$L(\mathscr{y}_1,\mathscr{y}_2,\mathscr{y}_3,\cdots,\mathscr{y}_6)=\sum_{i=1}^{6}c_i\left(\mathscr{y}_i-\mathscr{y}_i^*-\mathscr{y}_i^*ln\frac{\mathscr{y}_i}{\mathscr{y}_i^*}\right)$$

In 2014, Aguila-Camacho and co-workers [74] showed an important result to estimate the quadratic Lyapunov functions in terms of LC fractional operator when $\alpha\in(0,1)$.

Lemma 5.1. *Consider $\mathscr{y}(t)\in\mathbb{R}^+$ be a continuous and differentiable function then*

$$\frac{d^\alpha}{dt^\alpha}\left(\mathscr{y}(t)-\mathscr{y}^*-\mathscr{y}^*ln\frac{\mathscr{y}(t)}{\mathscr{y}^*}\right)\le\left(1-\frac{\mathscr{y}^*}{\mathscr{y}(t)}\right)\frac{d^\alpha}{dt^\alpha}\mathscr{y}(t),\mathscr{y}^*\in\mathbb{R}^+,\mathscr{y}^*\in\mathbb{R}^+\forall\alpha\in(0,1)$$

Theorem 5.5. (*(Uniform Asymptotic Stability Theorem)*) [75]

Let $\mathscr{y}^*$ be a equilibrium point for the system (5) and $\Omega\subset\mathscr{R}_+^6$ be domain involving $\mathscr{y}^*$. Let $L[t,\mathscr{y}(t)]:[0,\infty]\to\mathscr{R}$ be a continuously differentiable function such that

$\mathscr{w}_1(\mathscr{y})\le L[t,\mathscr{y}(t)]\le\mathscr{w}_2(\mathscr{y})$ and $\frac{d^\alpha}{dt^\alpha}L[t,\mathscr{y}(t)]\le-\mathscr{w}_2(\mathscr{y})$, where $\mathscr{w}_1(\mathscr{y})$, $\mathscr{w}_2(\mathscr{y})$ and $\mathscr{w}_3(\mathscr{y})$ are continuous and positive definite function on Ω. Then the equilibrium opoint of the system (6) with I. C. (10) is uniformly asymptotically stable.

Let us assume the subsequent Voltera type Lyapunov function

$$\begin{aligned} L(x,y,z,u,v,w)= & A_1\left(x-x^*-x^*ln\frac{x}{x^*}\right)+A_2\left(y-y^*-y^*ln\frac{y}{y^*}\right) \\ & +A_3\left(z-z^*-z^*ln\frac{z}{z^*}\right)+A_4\left(u-u^*-u^*ln\frac{u}{u^*}\right) \\ & +A_5\left(v-v^*-v^*ln\frac{v}{v^*}\right)+A_6\left(w-w^*-w^*ln\frac{w}{w^*}\right) \end{aligned}$$

then

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}L(x,y,z,u,v,w)= & A_1\frac{d^\alpha}{dt^\alpha}\left(x-x^*-x^*ln\frac{x}{x^*}\right)+A_2\frac{d^\alpha}{dt^\alpha}\left(y-y^*-y^*ln\frac{y}{y^*}\right) \\ & +A\frac{d^\alpha}{dt^\alpha_3}\left(z-z^*-z^*ln\frac{z}{z^*}\right)+A_4\frac{d^\alpha}{dt^\alpha}\left(u-u^*-u^*ln\frac{u}{u^*}\right) \\ & +A_5\frac{d^\alpha}{dt^\alpha}\left(v-v^*-v^*ln\frac{v}{v^*}\right)+A_6\frac{d^\alpha}{dt^\alpha}\left(w-w^*-w^*ln\frac{w}{w^*}\right) \end{aligned}$$

using Lemma 5.1, we get

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}L(x,y,z,u,v,w)\le & A_1\left(1-\frac{x^*}{x(t)}\right)\frac{d^\alpha x}{dt^\alpha}+A_2\left(1-\frac{y^*}{y(t)}\right)\frac{d^\alpha y}{dt^\alpha} \\ & +A_3\left(1-\frac{z^*}{z(t)}\right)\frac{d^\alpha z}{dt^\alpha}+A_4\left(1-\frac{u^*}{u(t)}\right)\frac{d^\alpha u}{dt^\alpha} \\ & +A_5\left(1-\frac{v^*}{v(t)}\right)\frac{d^\alpha v}{dt^\alpha}+A_6\left(1-\frac{w^*}{w(t)}\right)\frac{d^\alpha w}{dt^\alpha} \end{aligned}$$

$$\le A_1\left(1-\frac{x^*}{x(t)}\right)[\mu-\beta xz-\beta\psi xu-\eta xw-\delta x]$$

$$+A_2\left(1-\frac{y^*}{y(t)}\right)[\beta xz+\beta\psi xu+\eta xw-(1-\phi)\theta y-\phi\omega y-\delta y]$$

$$+A_3\left(1-\frac{z^*}{z(t)}\right)[(1-\phi)\theta y-\gamma z-\delta z]$$

$$+A_4\left(1-\frac{u^*}{u(t)}\right)[\phi\omega y-\tau u-\delta u]+A_5\left(1-\frac{v^*}{v(t)}\right)[\gamma z+\tau u-\delta v]$$

$$+A_6\left(1-\frac{w^*}{w(t)}\right)\left[\sigma z+\varepsilon u-\kappa w\right]$$

Using the relations at the steady state $\mu=\beta x^*z^*+\beta\psi x^*u^*+\eta x^*w^*+\delta x^*$, $\beta xz+\beta\psi xu+\eta xw=[(1-\phi)\theta+\phi\omega+\delta]y^*$, $(1-\phi)\theta y=(\gamma+\delta)z^*$, $\phi\omega y=(\tau+\delta)u^*$, $\gamma z=\delta v^*$, $\sigma z+\varepsilon u=\kappa w^*$.

$$\begin{aligned} \frac{d^\alpha}{dt^\alpha}L(x,y,z,u,v,w)\le & A_1\left(1-\frac{x^*}{x(t)}\right)[\beta x^*z^*+\beta\psi x^*u^*+\eta x^*w^*+\delta x^*-\beta xz \\ & -\beta\psi xu-\eta xw-\delta x] \end{aligned}$$

$$+A_2\left(1-\frac{y^*}{y(t)}\right)((1-\phi)\theta+\phi\omega+\delta)(y^*-y)$$

$$+A_3\left(1-\frac{z^*}{z(t)}\right)(\gamma+\delta)(z^*-z)$$

$$+A_4\left(1-\frac{u^*}{u(t)}\right)(\tau+\delta)(u^*-u)+A_5\left(1-\frac{v^*}{v(t)}\right)\delta(v^*-v)$$

$$+A_6\left(1-\frac{w^*}{w(t)}\right)\kappa(w^*-w)$$

$$\begin{aligned} \le A_1\left(1-\frac{x^*}{x(t)}\right)[& \beta x^*z^*-\beta xz^*+\beta xz^*+\beta\psi x^*u^*-\beta\psi xu^*+\beta\psi xu^*+\eta x^*w^* \\ & -\eta xw^*+\eta xw^*+\delta x^*-\beta xz-\beta\psi xu-\eta xw-\delta x] \end{aligned}$$

$$+A_2\left(1-\frac{y^*}{y(t)}\right)((1-\phi)\theta+\phi\omega+\delta)(y^*-y)$$

$$+A_3\left(1-\frac{z^*}{z(t)}\right)(\gamma+\delta)(z^*-z)$$

$$+A_4\left(1-\frac{u^*}{u(t)}\right)(\tau+\delta)(u^*-u)+A_5\left(1-\frac{v^*}{v(t)}\right)\delta(v^*-v)$$

$$+A_6\left(1-\frac{w^*}{w(t)}\right)\kappa(w^*-w)$$

$$\begin{aligned} \le A_1\left(1-\frac{x^*}{x(t)}\right)[& (\beta x^*+\beta x)(z^* \\ & -z)+\beta\psi u^*(x^*-x)+\beta\psi(u^*-u)+\eta w^*(x^*-x)+\eta x(w^*-w)+\delta(x^*-x)] \end{aligned}$$

$$+A_2\left(1-\frac{y^*}{y(t)}\right)((1-\phi)\theta+\phi\omega+\delta)(y^*-y)$$

$$+A_3\left(1-\frac{z^*}{z(t)}\right)(\gamma+\delta)(z^*-z)$$

$$+A_4\left(1-\frac{u^*}{u(t)}\right)(\tau+\delta)(u^*-u)+A_5\left(1-\frac{v^*}{v(t)}\right)\delta(v^*-v)$$

$$+A_6\left(1-\frac{w^*}{w(t)}\right)\kappa(w^*-w)$$

$$\leq -\frac{A_1}{x(t)}\left[(\beta x^*+\beta x)(x-x^*)(z-z^*)+(\beta\psi u^*+\eta w^*+\delta)(x-x^*)^2 + \beta\psi(x-x^*)(u-u^*)+\eta x(w-w^*)\right]$$

$$+\frac{A_2}{y(t)}((1-\phi)\theta+\phi\omega+\delta)(y^*-y)^2$$

$$+\frac{A_3}{z(t)}(\gamma+\delta)(z^*-z)^2$$

$$+\frac{A_4}{u(t)}(\tau+\delta)(u^*-u)^2+\frac{A_5}{v(t)}\delta(v^*-v)^2$$

$$+\frac{A_6}{w(t)}\kappa(w^*-w)^2$$

Clearly $\frac{d^\alpha}{dt^\alpha}L(x,y,z,u,v,w)$ is negative definite when $\alpha\in(0,1)$, in view of the Theorem 5.5, the endemic equilibrium point is uniformly asymptotically stable in the interior of Ω, when it exists.

Solution by q-HASTM

Applying the ST on the system (5), we have

$$u^{-\alpha}\mathscr{S}[x(t)]-u^{-\alpha}x(0)=\mathscr{S}[\mu-\beta xz-\beta\psi xu-\eta xw-\delta x]$$

$$u^{-\alpha}\mathscr{S}[y(t)]-u^{-\alpha}y(0)=\mathscr{S}[\beta xz+\beta\psi xu+\eta xw-(1-\phi)\theta y-\phi\omega y-\delta y]$$

$$u^{-\alpha}\mathscr{S}[z(t)]-u^{-\alpha}z(0)=\mathscr{S}[(1-\phi)\theta y-\gamma z-\delta z]$$

$$u^{-\alpha}\mathscr{S}[u(t)]-u^{-\alpha}u(0)=\mathscr{S}[\phi\omega y-\tau u-\delta u]$$

$$u^{-\alpha}\mathscr{S}[v(t)]-u^{-\alpha}v(0)=\mathscr{S}[\gamma z+\tau u-\delta v]$$

$$u^{-\alpha}\mathscr{S}[w(t)]-u^{-\alpha}w(0)=\mathscr{S}[\sigma z+\varepsilon u-\kappa w] \quad (22)$$

On simplification

$$\mathscr{S}[x(t)]-x(0)=u^{\alpha}\mathscr{S}[\mu-\beta xz-\beta\psi xu-\eta xw-\delta x]$$

$$\mathscr{S}[y(t)]-y(0)=u^{\alpha}\mathscr{S}[\beta xz+\beta\psi xu+\eta xw-(1-\phi)\theta y-\phi\omega y-\delta y]$$

$$\mathscr{S}[z(t)]-z(0)=u^{\alpha}\mathscr{S}[(1-\phi)\theta y-\gamma z-\delta z]$$

$$\mathscr{S}[u(t)]-u(0)=u^{\alpha}\mathscr{S}[\phi\omega y-\tau u-\delta u]$$

$$\mathscr{S}[v(t)]-v(0)=u^{\alpha}\mathscr{S}[\gamma z+\tau u-\delta v]$$

$$\mathscr{S}[w(t)]-w(0)=u^{\alpha}\mathscr{S}[\sigma z+\varepsilon u-\kappa w] \quad (23)$$

Consider the nonlinear operator as

$$\mathscr{N}_1[\Phi_1(t,q)]=\mathscr{S}[\Phi_1(t,q)]-x_0-u^{\alpha}\mathscr{S}[\mu-\beta\Phi_1(t,q)\Phi_3(t,q) - \beta\psi\Phi_1(t,q)\Phi_4(t,q)-\eta\Phi_1(t,q)\Phi_6(t,q)-\delta\Phi_1(t,q)]$$

$$\mathscr{N}_2[\Phi_2(t,q)]=\mathscr{S}[\Phi_2(t,q)]-y_0-u^{\alpha}\mathscr{S}[\beta\Phi_1(t,q)\Phi_3(t,q) + \beta\psi\Phi_1(t,q)\Phi_4(t,q)+\eta\Phi_1(t,q)\Phi_6(t,q)-(1-\phi)\theta\Phi_2(t,q) - \phi\omega\Phi_2(t,q)-\delta\Phi_2(t,q)]$$

$$\mathscr{N}_3[\Phi_3(t,q)]=\mathscr{S}[\Phi_3(t,q)]-z_0-u^{\alpha}\mathscr{S}[(1-\phi)\theta\Phi_2(t,q)-\gamma\Phi_3(t,q) - \delta\Phi_3(t,q)]]$$

$$\mathscr{N}_4[\Phi_4(t,q)]=\mathscr{S}[\Phi_4(t,q)]-u_0-u^{\alpha}\mathscr{S}[\phi\omega\Phi_2(t,q)-(\tau+\delta)\Phi_4(t,q)]$$

$$\mathscr{N}_5[\Phi_5(t,q)]=\mathscr{S}[\Phi_5(t,q)]-v_0-u^{\alpha}\mathscr{S}[\gamma\Phi_3(t,q)+\tau\Phi_4(t,q)u-\delta\Phi_5(t,q)]$$

$$\mathscr{N}_6[\Phi_6(t,q)]=\mathscr{S}[\Phi_6(t,q)]-w_0-u^{\alpha}\mathscr{S}[\sigma\Phi_3(t,q)+\varepsilon\Phi_4(t,q)-\kappa\Phi_6(t,q)] \quad (24)$$

Where $\Phi_i(t,q)$ are real function of t,q.

Now construct a homotopy as

$$(1-nq)\mathscr{S}[\Phi_1(t,q)-x_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_1[\Phi_1(t,q)]$$

$$(1-nq)\mathscr{S}[\Phi_2(t,q)-y_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_2[\Phi_2(t,q)]$$

$$(1-nq)\mathscr{S}[\Phi_3(t,q)-z_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_3[\Phi_3(t,q)]$$

$$(1-nq)\mathscr{S}[\Phi_4(t,q)-u_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_4[\Phi_4(t,q)]$$

$$(1-nq)\mathscr{S}[\Phi_5(t,q)-v_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_5[\Phi_5(t,q)]$$

$$(1-nq)\mathscr{S}[\Phi_6(t,q)-w_0(t)]=\hbar\mathscr{H}(t)\mathscr{N}_6[\Phi_6(t,q)] \quad (25)$$

In Eq. (25) $\mathscr{H}(t)\neq 0$ denotes the auxiliary function, $\hbar$ is auxiliary parameter.

Hence embedding parameter q enhances from zero to $\frac{1}{n}$ then solution varies from the initial guess to the needed solution.

Next the expansion $\Phi_i(t,q)$ in Taylor's series w.r.t. q, we have

$$\Phi_1(t,q)=x_0(t)+\sum_{m=1}^{\infty}q^m x_m(t);\ \Phi_2(t,q)=y_0(t)+\sum_{m=1}^{\infty}q^m y_m(t)$$

$$\Phi_3(t,q)=z_0(t)+\sum_{m=1}^{\infty}q^m z_m(t);\ \Phi_4(t,q)=u_0(t)+\sum_{m=1}^{\infty}q^m u(t)$$

$$\Phi_5(t,q)=v_0(t)+\sum_{m=1}^{\infty}q^m v_m(t);\ \Phi_6(t,q)=w_0(t)+\sum_{m=1}^{\infty}q^m w_m(t)$$

If the auxiliary linear operator, the initial guesses, $\hbar$ and $\mathscr{H}(t)$ are selected in appropriate manner, the above series at $q=\frac{1}{n}$, we have

$$x(t)=x_0(t)+\sum_{m=1}^{\infty}x_m(t)\left(\frac{1}{n}\right)^m,\ y(t)=y_0(t)+\sum_{m=1}^{\infty}y_m(t)\left(\frac{1}{n}\right)^m$$

$$z(t)=z_0(t)+\sum_{m=1}^{\infty}z_m(t)\left(\frac{1}{n}\right)^m,\ u(t)=u_0(t)+\sum_{m=1}^{\infty}u_m(t)\left(\frac{1}{n}\right)^m$$

$$v(t)=v(t)+\sum_{m=1}^{\infty}v_m(t)\left(\frac{1}{n}\right)^m,\ w(t)=w_0(t)+\sum_{m=1}^{\infty}w_m(t)\left(\frac{1}{n}\right)^m \quad (27)$$

Define the vectors

$$\overrightarrow{x_m}=\{x_0,x_1,x_2,\cdots,x_n\},\ \overrightarrow{y_m}=\{y_0,y_1,y_2,\cdots,y_n\},\ \overrightarrow{z_m}=\{z_0,z_1,z_2,\cdots,z_n\}$$

$$\overrightarrow{u_m}=\{u_0,u_1,u_2,\cdots,u_n\},\ \overrightarrow{v_m}=\{v_0,v_1,v_2,\cdots,v_n\},\ \overrightarrow{w_m}=\{w_0,w_1,w_2,\cdots,w_n\} \quad (28)$$

Differentiating the Eq. (25) m times w.r.t. q and take $q=0$ and divide the resultant by $m!$, we achieve the $m$thorder deformation equations

$$\mathscr{S}[x_m(t)-\chi_m x_{m-1}(t)]=\hbar\mathscr{H}(t)\mathscr{R}'_m[\overrightarrow{x_{m-1}}(t)]$$

$$\mathscr{S}[y_m(t)-\chi_m y_{m-1}(t)] = \hbar\mathscr{H}(t)\mathscr{R}''_m[\overrightarrow{y_{m-1}}(t)]$$

$$\mathscr{S}[z_m(t)-\chi_m z_{m-1}(t)] = \hbar\mathscr{H}(t)\mathscr{R}'''_m[\overrightarrow{z_{m-1}}(t)]$$

$$\mathscr{S}[u_m(t)-\chi_m u_{m-1}(t)] = \hbar\mathscr{H}(t)\mathscr{R}'^{v}_m[\overrightarrow{u_{m-1}}(t)]$$

$$\mathscr{S}[v_m(t)-\chi_m v_{m-1}(t)] = \hbar\mathscr{H}(t)\mathscr{R}^{v}_m[\overrightarrow{v_{m-1}}(t)]$$

$$\mathscr{S}[w_m(t)-\chi_m w_{m-1}(t)] = \hbar\mathscr{H}(t)\mathscr{R}^{v'}_m[\overrightarrow{w_{m-1}}(t)] \quad (29)$$

Operating inverse ST on both sides and setting $q = 1, \mathscr{H}(t) = 1$, we obtain

$$x_m(t) = \chi_m x_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}'_m[\overrightarrow{x_{m-1}}(t)]\right)$$

$$y_m(t) = \chi_m y_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}''_m[\overrightarrow{y_{m-1}}(t)]\right)$$

$$z(t) = \chi_m z_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}'''_m[\overrightarrow{z_{m-1}}(t)]\right)$$

$$u_m(t) = \chi_m u_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}'^{v}_m[\overrightarrow{u_{m-1}}(t)]\right)$$

$$v_m(t) = \chi_m v_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}^{v}_m[\overrightarrow{v_{m-1}}(t)]\right)$$

$$w_m(t) = \chi_m w_{m-1}(t) + \hbar\mathscr{S}^{-1}\left(\mathscr{R}^{v'}_m[\overrightarrow{w_{m-1}}(t)]\right)$$

Where, $\chi_m = \begin{cases} 0, m \le 1 \\ n, m > 1 \end{cases}$ and $\mathscr{R}'_m[\overrightarrow{x_{m-1}}(t)] = \frac{1}{(m-1)!}\left(\frac{\partial\Phi_1(t,q)}{\partial q^{m-1}}\right)_{q=0}$.

So,

$$\begin{aligned}\mathscr{R}'_m[\overrightarrow{x_{m-1}}(t)] &= \mathscr{S}[\mathrm{x}_{m-1}(t)] - x_0(t)\left(1-\frac{\chi_m}{n}\right)\\ &- u^{\alpha}\Bigg[\mu\mathscr{S}[1] - \beta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{z}_{m-k-1}(t)\right)\\ &- \beta\psi\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{u}_{m-k-1}(t)\right) - \eta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{w}_{m-k-1}(t)\right)\\ &- \delta\mathscr{S}[x_{m-1}(t)]\Bigg]\end{aligned}$$

$$\begin{aligned}\mathscr{R}''_m[\overrightarrow{y_{m-1}}(t)] &= \mathscr{S}[\mathrm{y}_{m-1}(t)] - y_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}\Bigg[\beta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{z}_{m-k-1}(t)\right)\\ &+ \beta\psi\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{u}_{m-k-1}(t)\right) + \eta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{w}_{m-k-1}(t)\right)\\ &- (1-\phi)\theta\mathscr{S}[\mathrm{y}_{m-1}(t)] - \phi\omega\mathscr{S}[\mathrm{y}_{m-1}(t)] - \delta\mathscr{S}[y_{m-1}(t)]\Bigg]\end{aligned}$$

$$\begin{aligned}\mathscr{R}'''_m[\overrightarrow{z_{m-1}}(t)] &= \mathscr{S}[\mathrm{z}_{m-1}(t)] - z_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[(1\\ &- \phi)\theta\mathscr{S}[\mathrm{y}_{m-1}(t)] - (\gamma+\delta)\mathscr{S}[z_{m-1}(t)]]\end{aligned}$$

$$\begin{aligned}\mathscr{R}'^{v}_m[\overrightarrow{u_{m-1}}(t)] &= \mathscr{S}[\mathrm{u}_{m-1}(t)] - u_0(t)\left(1-\frac{\chi_m}{n}\right)\\ &- u^{\alpha}[\phi\omega\mathscr{S}[\mathrm{y}_{m-1}(t) - (\tau+\delta)\mathscr{S}[u_{m-1}(t)]]\end{aligned}$$

$$\begin{aligned}\mathscr{R}^{v}_m[\overrightarrow{v_{m-1}}(t)] &= \mathscr{S}[\mathrm{v}_{m-1}(t)] - v_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[\gamma\mathscr{S}[\mathrm{z}_{m-1}(t)\\ &- \tau\mathscr{S}[u_{m-1}(t)] - \delta\mathscr{S}[v_{m-1}(t)]]\end{aligned}$$

$$\begin{aligned}\mathscr{R}^{v'}_m[\overrightarrow{w_{m-1}}(t)] &= \mathscr{S}[w_{m-1}(t)] - w_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[\sigma\mathscr{S}[\mathrm{z}_{m-1}(t)\\ &+ \varepsilon\mathscr{S}[u_{m-1}(t)] - \kappa\mathscr{S}[w_{m-1}(t)]]\end{aligned} \quad (30)$$

Therefore, we have

$$\begin{aligned}x_m(t) &= \chi_m x_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\mathscr{S}[\mathrm{x}_{m-1}(t)] - x_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}\Bigg[\mu\mathscr{S}[1]\\ &- \beta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{z}_{m-k-1}(t)\right) - \beta\psi\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{u}_{m-k-1}(t)\right)\\ &- \eta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{w}_{m-k-1}(t)\right) - \delta\mathscr{S}[x_{m-1}(t)]\Bigg]\Bigg)\end{aligned}$$

$$\begin{aligned}y_m(t) &= \chi_m y_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\mathscr{S}[\mathrm{y}_{m-1}(t)] - y_0(t)\left(1-\frac{\chi_m}{n}\right)\\ &- u^{\alpha}\Bigg[\beta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{z}_{m-k-1}(t)\right) + \beta\psi\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{u}_{m-k-1}(t)\right)\\ &+ \eta\mathscr{S}\left(\sum_{k=0}^{m-1}x_k(t)\mathrm{w}_{m-k-1}(t)\right) - (1-\phi)\theta\mathscr{S}[\mathrm{y}_{m-1}(t)]\\ &- \phi\omega\mathscr{S}[\mathrm{y}_{m-1}(t)] - \delta\mathscr{S}[y_{m-1}(t)]\Bigg]\Bigg)\end{aligned}$$

$$\begin{aligned}z(t) &= \chi_m z_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\mathscr{S}[\mathrm{z}_{m-1}(t)] - z_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[(1\\ &- \phi)\theta\mathscr{S}[\mathrm{y}_{m-1}(t)] - (\gamma+\delta)\mathscr{S}[z_{m-1}(t)]]\Bigg)\end{aligned}$$

$$\begin{aligned}u_m(t) &= \chi_m u_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\mathscr{S}[\mathrm{u}_{m-1}(t)] - u_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[\phi\omega\mathscr{S}[\mathrm{y}_{m-1}(t)\\ &- (\tau+\delta)\mathscr{S}[u_{m-1}(t)]]\Bigg)\end{aligned}$$

$$\begin{aligned}v_m(t) &= \chi_m v_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\mathscr{S}[\mathrm{v}_{m-1}(t)] - v_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[\gamma\mathscr{S}[\mathrm{z}_{m-1}(t)\\ &- \tau\mathscr{S}[u_{m-1}(t)] - \delta\mathscr{S}[v_{m-1}(t)]]\Bigg)\end{aligned}$$

$$\begin{aligned}w_m(t) &= \chi_m w_{m-1}(t) + \hbar\mathscr{S}^{-1}\Bigg(\\ &= \mathscr{S}[w_{m-1}(t)] - w_0(t)\left(1-\frac{\chi_m}{n}\right) - u^{\alpha}[\sigma\mathscr{S}[\mathrm{z}_{m-1}(t) + \varepsilon\mathscr{S}[u_{m-1}(t)]\\ &- \kappa\mathscr{S}[w_{m-1}(t)]]\Bigg)\end{aligned} \quad (31)$$

Therefore, the solution of the fractional model is given as follows (with n = 1 and $\hbar$ = -1)

Table 1
The fitted and estimated values of parameters for the COVID-19 model with fractional order derivative.

Parameter	Description	Value
μ	Birth rate	0.0018
δ	Natural mortality rate or death rate	0.01439
β	Disease transmission coefficient	0.05
ψ	Transmissibility multiple of $u(t)$ to $y(t)$, $\psi \in [0,1]$	0.02
η	Disease transmission coefficient from $w(t)$ to $x(t)$	0.000001231
φ	Proportion of asymptotic infection	0.01243
θ	Transmission rate after finishing the incubation period becomes infected and joining the class $y(t)$	0.1923
ω	Transmission rate after finishing the incubation period becomes infected and joining the class $u(t)$	0.1923
γ	Recovery rate $z(t)$ to $v(t)$	0.1724
τ	Recovery rate $u(t)$ to $v(t)$	0.07
σ	Infected symptomatic contribution on the values into $w(t)$	0.1
ε	Asymptotically infected contributing the virus into $w(t)$	0.05
κ	Rate of removing the virus from $w(t)$	0.01

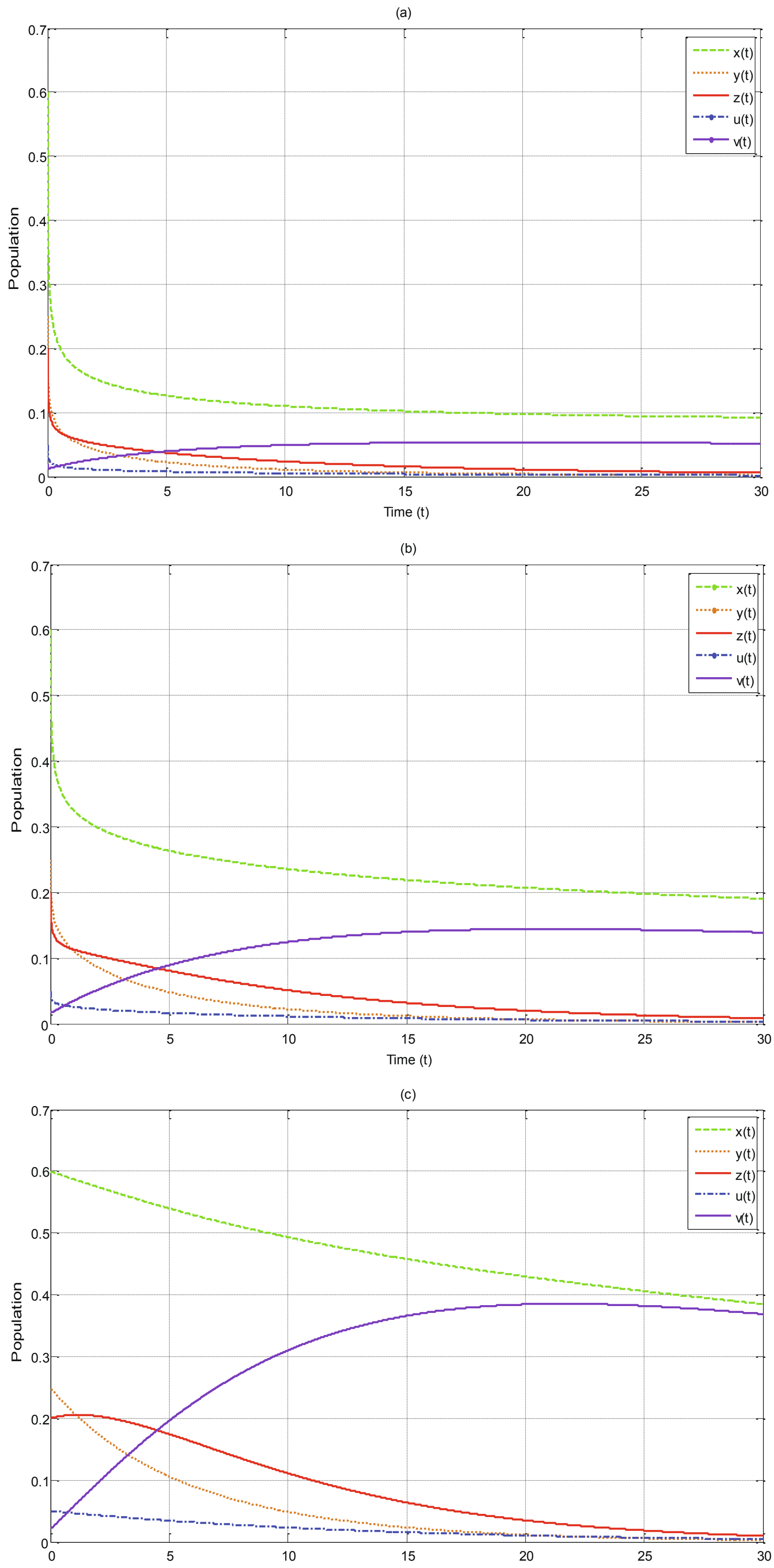

Fig. 2. Plots of $x(t), y(t), z(t), u(t), v(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table for (a) $\alpha = 0.8$ (b) $\alpha = 0.9$ (c) $\alpha = 1$.

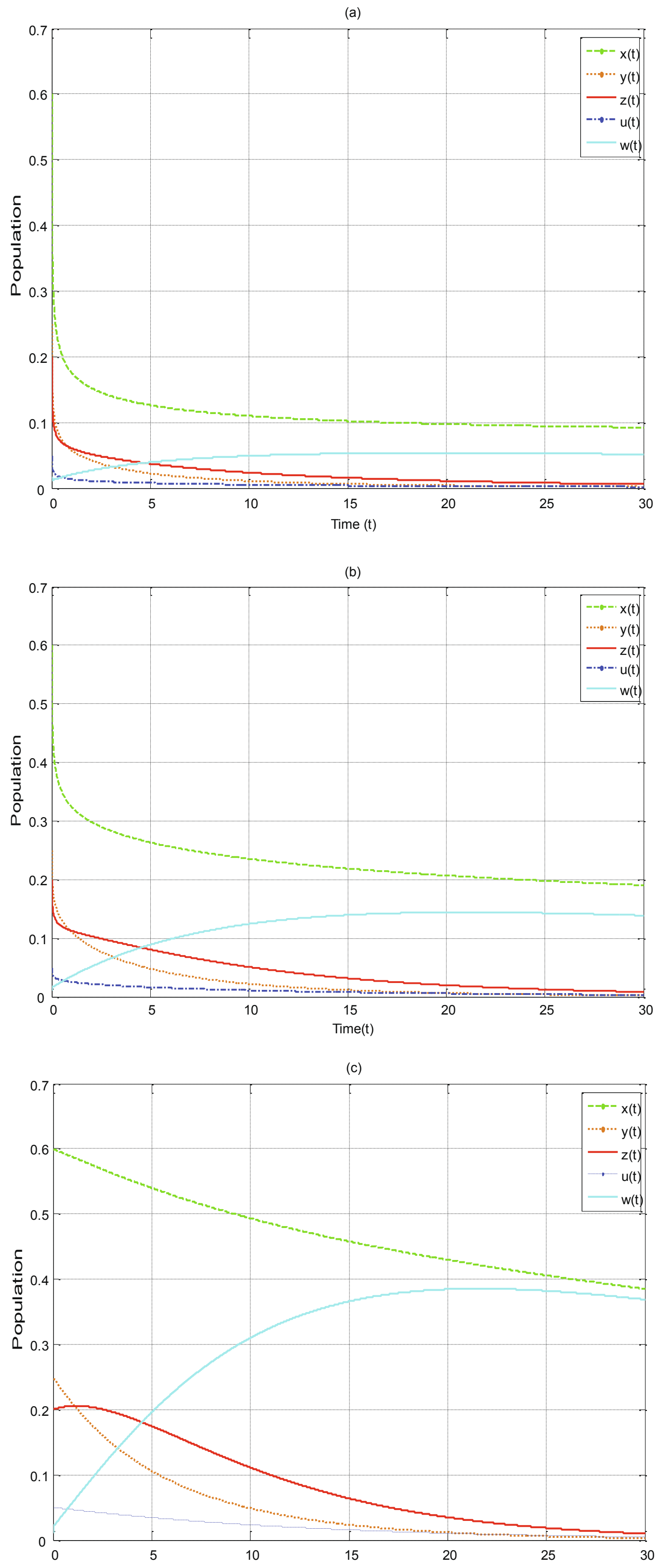

Fig. 3. Plots of $x(t), y(t), z(t), u(t), w(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table for (a) $\alpha = 0.8$ (b) $\alpha = 0.9$ (c) $\alpha = 1$.

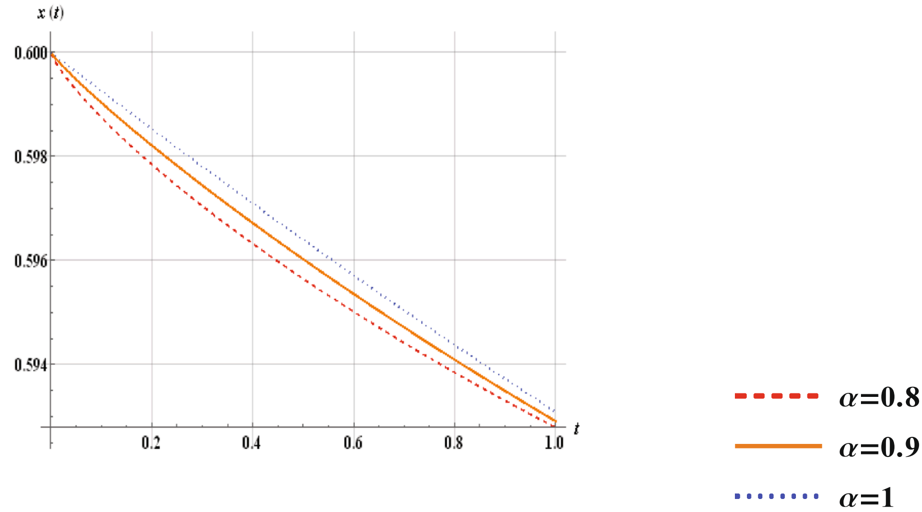

Fig. 4a. Plots of $x(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table for$\alpha = 0.8,0.9,1$.

$$x(t) = 0.6 - 0.00926\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 0.00033\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 0.00032\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 0.0000065\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$y(t) = 0.24999 - 0.13693\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 0.02897\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 0.002302\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - 0.0000065\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$z(t) = 0.2 - 0.02540\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 0.01151\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 0.00379\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$u(t) = 0.05 - 0.010866\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 0.00059\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + 0.0000064\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$v(t) = 0.02 + 0.11308\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 0.00285\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 0.00182\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}$$

$$w(t) = 0.01 + 0.0672\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 0.00182\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - 0.00105\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \tag{32}$$

Numerical approximation method

We interpret the solution of the COVID-19 model numerically, employing an extension of the ABM method with the Caputo's derivative.

By applying the generalization of Adams–Bashforth–Moulton method the system can be discretised as

$$x_{n+1} = x_0 + \frac{h^{\alpha_1}}{\Gamma(\alpha_1+2)}\left[\mu - \beta x^p_{n+1}z^p_{n+1} - \beta\psi x^p_{n+1}u^p_{n+1} - \eta x^p_{n+1}w^p_{n+1} - \delta x^p_{n+1}\right] + \frac{h^{\alpha_1}}{\Gamma(\alpha_1+2)}\sum_{j=0}^{n}\alpha_{1,j,n+1}\left[\mu - \beta x_j z_j - \beta\psi x_j u_j - \eta x_j w_j - \delta x_j\right]$$

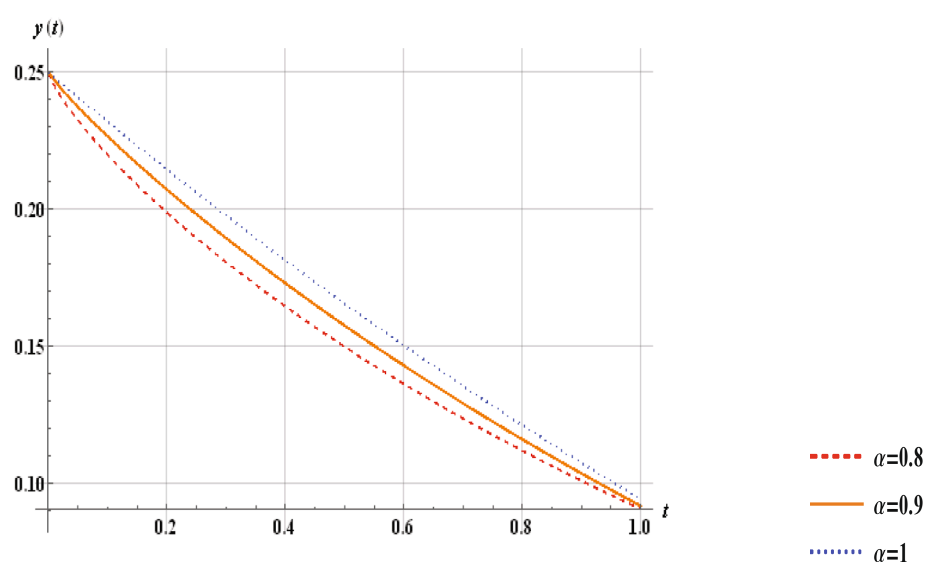

Fig. 4b. Plots of $y(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table when $\alpha = 0.8, 0.9, 1$.

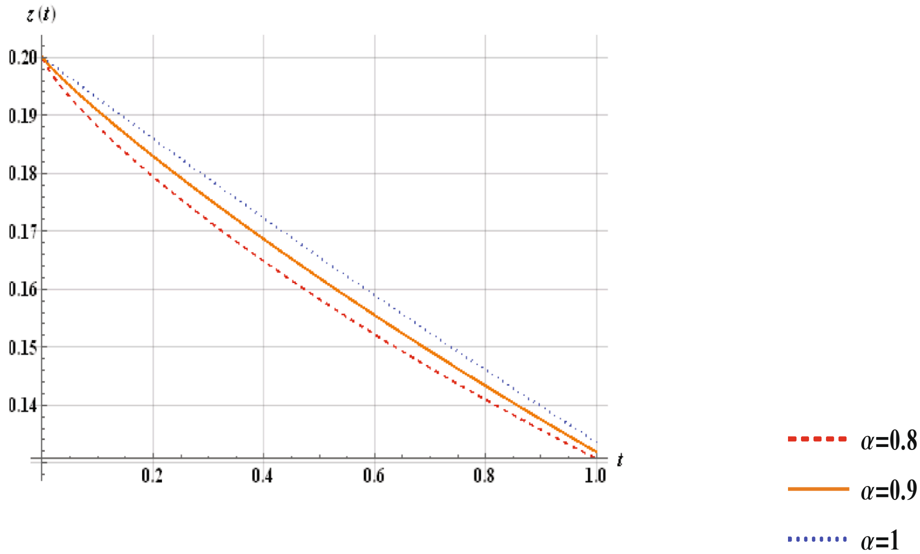

Fig. 4c. Plots of $z(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table for$\alpha = 0.8, 0.9, 1$.

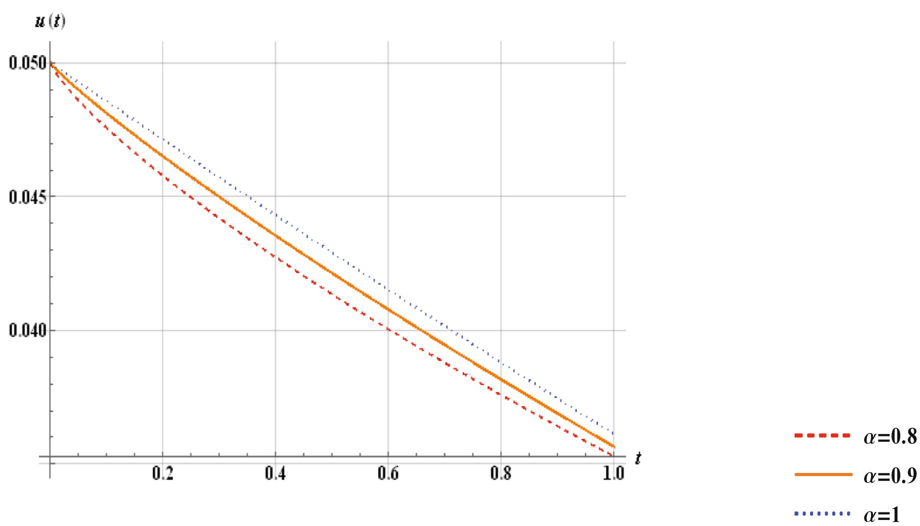

Fig. 4d. Plots of $u(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table for$\alpha = 0.8, 0.9, 1$.

$$y_{n+1} = y_0 + \frac{h^{\alpha_2}}{\Gamma(\alpha_2+2)}[\beta x_{n+1}^p z_{n+1}^p + \beta\psi x_{n+1}^p u_{n+1}^p + \eta x_{n+1}^p w_{n+1}^p - (1-\varphi)\theta y_{n+1}^p - \varphi\omega y_{n+1}^p - \delta y_{n+1}^p]$$

$$+ \frac{h^{\alpha_2}}{\Gamma(\alpha_2+2)}\sum_{j=0}^{n}\alpha_{2,j,n+1}[\beta x_j z_j + \beta\psi x_j u_j + \eta x_j w_j - (1-\varphi)\theta y_j - \varphi\omega y_j - \delta y_j]$$

$$z_{n+1} = z_0 + \frac{h^{\alpha_3}}{\Gamma(\alpha_3+2)}[(1-\varphi)\theta y_{n+1}^p - \gamma z_{n+1}^p - \delta z_{n+1}^p]$$

$$+ \frac{h^{\alpha_3}}{\Gamma(\alpha_3+2)}\sum_{j=0}^{n}\alpha_{3,j,n+1}[(1-\varphi)\theta y_j - \gamma z_j - \delta y_j]$$

$$u_{n+1} = u_0 + \frac{h^{\alpha_4}}{\Gamma(\alpha_4+2)}[\varphi\omega y_{n+1}^p - (\tau+\delta)u_{n+1}^p]$$

$$+ \frac{h^{\alpha_4}}{\Gamma(\alpha_4+2)}\sum_{j=0}^{n}\alpha_{4,j,n+1}[\varphi\omega y_j - (\tau+\delta)u_j]$$

$$v_{n+1} = v_0 + \frac{h^{\alpha_5}}{\Gamma(\alpha_5+2)}[\gamma z_{n+1}^p + \tau u_{n+1}^p - \delta v_{n+1}^p]$$

$$+ \frac{h^{\alpha_5}}{\Gamma(\alpha_5+2)}\sum_{j=0}^{n}\alpha_{5,j,n+1}[\gamma z_j + \tau u_j - \delta u_j]$$

$$w_{n+1} = w_0 + \frac{h^{\alpha_6}}{\Gamma(\alpha_6+2)}[\sigma z_{n+1}^p + \varepsilon u_{n+1}^p - \kappa w_{n+1}^p]$$

$$+ \frac{h^{\alpha_6}}{\Gamma(\alpha_6+2)}\sum_{j=0}^{n}\alpha_{6,j,n+1}[\sigma z_j + \varepsilon u_j - \kappa w_j]$$

In which

$$x_{n+1}^p = x_0 + \frac{1}{\Gamma(\alpha_1)}\sum_{j=0}^{n}B_{1,j,n+1}[\mu - \beta x_j z_j - \beta\psi x_j u_j - \eta x_j w_j - \delta x_j]$$

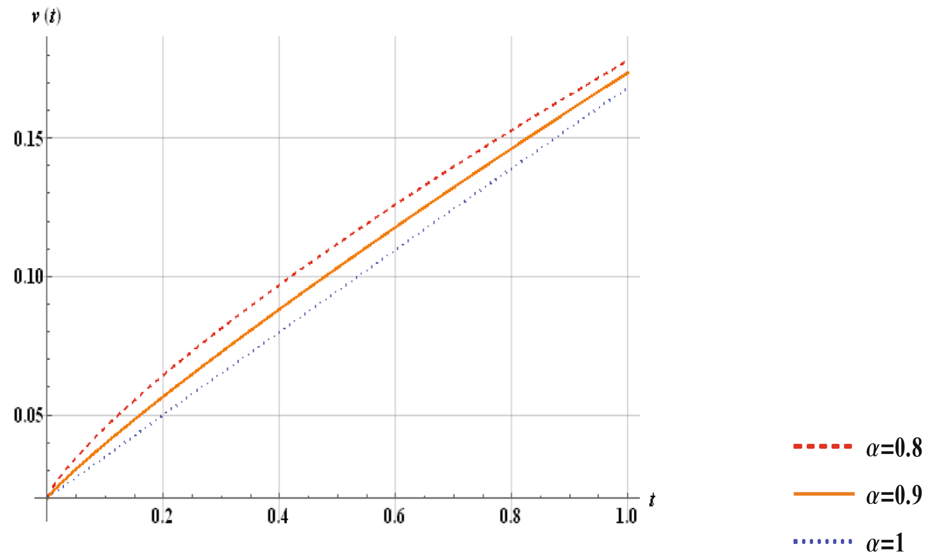

Fig. 4e. Plots of $v(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table when $\alpha = 0.8, 0.9, 1$.

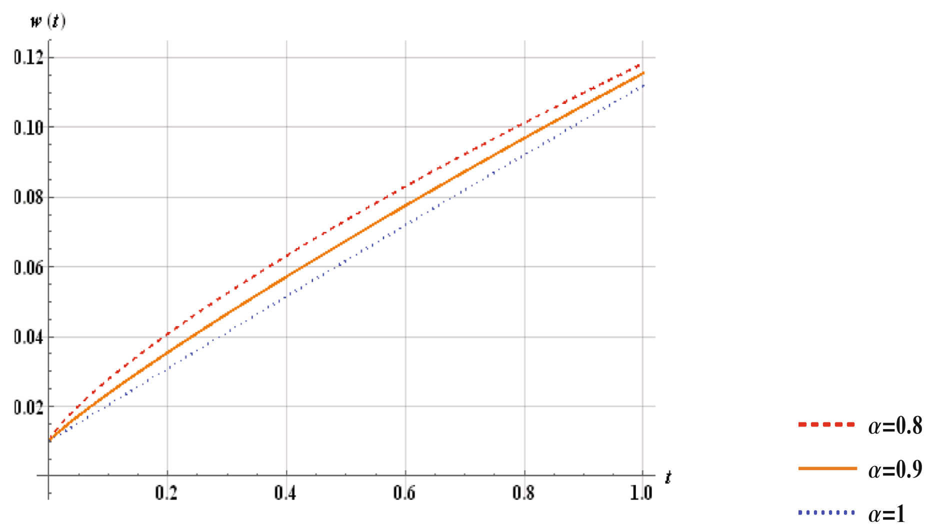

Fig. 4f. Plots of $w(t)$ vs. t with initial conditions $x_0 = 0.6, y_0 = 0.25, z_0 = 0.20, u_0 = 0.05, v_0 = 0.02, w_0 = 0.01$ and parameter values given in table when $\alpha = 0.8, 0.9, 1$.

$$y^p_{n+1} = y_0 + \frac{1}{\Gamma(\alpha_2)} \sum_{j=0}^{n} B_{2,j,n+1}[\beta x_j z_j + \beta\psi x_j u_j + \eta x_j w_j - (1-\varphi)\theta y_j - \varphi\omega y_j - \delta y_j]$$

$$z^p_{n+1} = z_0 + \frac{1}{\Gamma(\alpha_3)} \sum_{j=0}^{n} B_{3,j,n+1}[(1-\varphi)\theta y_j - \gamma z_j - \delta y_j]$$

$$u^p_{n+1} = u_0 + \frac{1}{\Gamma(\alpha_4)} \sum_{j=0}^{n} B_{4,j,n+1}[\varphi\omega y_j - (\tau+\delta)u_j]$$

$$v^p_{n+1} = v_0 + \frac{1}{\Gamma(\alpha_5)} \sum_{j=0}^{n} B_{5,j,n+1}[\gamma z_j + \tau u_j - \delta u_j]$$

$$w^p_{n+1} = w_0 + \frac{1}{\Gamma(\alpha_6)} \sum_{j=0}^{n} B_{6,j,n+1}[\sigma z_j + \varepsilon u_j - \kappa w_j]$$

$$\alpha_{i,j,n+1} = \begin{cases} n^{\alpha_{i+1}} - (n-\alpha_i)(n+1)^{\alpha_i}, ifj = 0 \\ (n-j+2)^{\alpha_{i+1}} - (n-j)^{\alpha_i+1} - 2(n-j+1)^{\alpha_i+1}, if 0 \le j \le n \\ 1, ifj = 1 \end{cases}$$

$B_{i,j,n+1} = \frac{h^{\alpha_i}}{\alpha_i}((n-j+1)^{\alpha_i} - (n-j)^{\alpha_i})$, $0 \le j \le n$ and $i = 1,2,3,4,5,6$.

Numerical results and discussion

We provide remarkable outcomes to demonstrate the usefulness of the acquired results. The values of various parameters are given in Table 1.

Case. ((*i*)) *Firstly, fix parameter values, the eigen values at DFE point are*$\lambda_1 = -0.23261$,

$\lambda_2 = -0.16087, \lambda_3 = -0.08439, \lambda_4 = -0.01439, \lambda_5 = -0.0139$

and $\lambda_6 = -0.00999$*with* $R_0 = 0.037897 < 1$,*via theorem* 5.3, *DFE point is locally asymptotically stable and disease demises out.*

Using Rough-Hurtwiz criteria for the characteristic polynomial (20) is $a_1 = 0.48787 > 0$, $a_2 = 0.075404 > 0$, $a_3 = 0.00386 > 0$ and $a_4 = 0.00032 > 0$ and $(a_1, a_2, a_3) - (a_1^2 a_4 + a_3^2) = 0.00012 > 0$ can be easily satisfied.

Case. *((ii)) fix parameter values, the Eigen values at endemic equilibrium point are* $\lambda_1 = -0.38987$

$$, \lambda_2 = -0.084432, \lambda_3 = -0.03921, \lambda_4 = 0.03520, \lambda_5 = -0.01439$$

and $\lambda_6 = -0.01$*we get one eigen value* λ_4 *is positive and using Rough-Hurtwiz criteria for the characteristic polynomial* (21) *is* $b_1 = 0.48831$,

$$b_2 = 0.03822, b_3 = -0.00019, b_4 = -5.06 \times 10^{-5}, b_5 = -4.5 \times 10^{-7}$$

and $(b_1 b_2 b_3) - \left(b_1^2 b_4 + b_3^2\right) = -1.6 \times 10^{-5} < 0$ *and* $[(b_1$

$$b_4 - b_5)(b_1 b_2 b_3 - b_1^2 b_4 + b_3^2)] - [b_5(b_1 b_2 - b_3)^2 + b_1 b_5^2] = -4.56 \times 10^{-11} < 0$$

, *so by theorem* 5.4, *the endemic equilibrium is unstable.*

Numerical results of susceptible $x(t)$, exposed $y(t)$, symptomatic infected $z(t)$, asymptotically infected $u(t)$, recovered $v(t)$ populationsand reservoir or the seafood place or market class $w(t)$ for diverse fractional order $\alpha = 0.80, 0.90$ and for the integer order $\alpha = 1$ are calculated. The outcomes are presented graphically through Figs. 2-4.

Conclusions

This article presents an application of a hybrid analytical method q-HASTM for the accomplishment of numerical solutions of COVID-19 model with fractional operator. The important goal of this method is that it can be directly used without linearization or any other restrictive conventions. Besides, two crucial aims have been achieved in the present work.

The initial one is the stability analysis of the Covid-19 model pertaining to fractional operator with the aid of the NGM and fractional RH stability criterion. It shows that the model presents two types of equilibriums, viz, DFE and the endemic equilibrium. The DFE point is asymptotically stable (locally) for $R_0 < 1$. If $R_0 > 1$, a unique endemic equilibrium point exists and is asymptotically stable under certain limitations within the feasible region.

The other one is that using a generalized ABM method for the numerical solution of the COVID-19 model involving fractional derivative. The concerned fractional Covid-19 model can also be analysed through other new numerical techniques in future with novel outcomes and conclusions in the context of numerical simulation.

Availability of data and material

Not applicable.

Author contributions

Conceptualization, S.Y., D.K. and J.S.; Formal analysis, S.Y., D.K. and D.B.; Funding acquisition, D.K. and D.B.; Investigation, S.Y. and J.S.; Methodology, D.K. and J.S.; Project administration, D.K., J.S. and D.B.; Software, S.Y., D.K. and J.S.; Supervision, J. S. and D.B.; Validation, S.Y. and J.S.; Visualization, S.Y. and D.K.; Writing-original draft, S.Y., D.K. and J.S.; Writing-review & editing, S.Y., J.S. and D.B. All authors have read and agreed to the published version of the manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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