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Analysis of fractional model of guava for biological pest control with memory effect

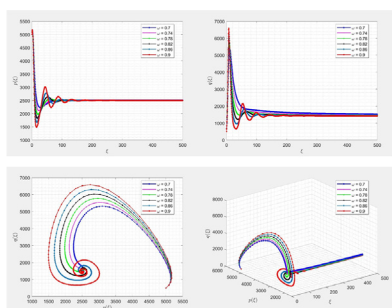
Jagdev Singh [a], Behzad Ganbari [b], Devendra Kumar [c,*], Dumitru Baleanu [d,e]

[a] Department of Mathematics, JECRC University, Jaipur 303905, Rajasthan, India
[b] Department of Basic Science, Kermanshah University of Technology, Kermanshah, Iran
[c] Department of Mathematics, University of Rajasthan, Jaipur 302004, Rajasthan, India
[d] Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Eskisehir Yolu 29. Km, Yukarıyurtcu Mahallesi Mimar Sinan Caddesi No: 4 06790, Etimesgut, Turkey
[e] Institute of Space Sciences, Magurele-Bucharest, Romania

HIGHLIGHTS

- A fractional guava fruit model with memory effect is developed.
- Stability of the fractional model is discussed.
- Existence and uniqueness of the solution is examined.
- A numerical method is applied for numerical simulation.
- Effect of various parameters is analyzed on guava borers and natural enemies.

GRAPHICAL ABSTRACT



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ABSTRACT

Introduction: Fractional operators find their applications in several scientific and engineering processes. We consider a fractional guava fruit model involving a non-local additionally non-singular fractional derivative for the interaction into guava pests and natural enemies. The fractional guava fruit model is considered as a Lotka-Volterra nature.
Objectives: The main objective of this work is to study a guava fruit model associated with a non-local additionally non-singular fractional derivative for the interaction into guava pests and natural enemies.
Methods: Existence and uniqueness analysis of the solution is evaluated effectively by using Picard Lindelof approach. An approximate numerical solution of the fractional guava fruit problem is obtained via a numerical scheme.
Results: The positivity analysis and equilibrium analysis for the fractional guava fruit model is discussed. The numerical results are demonstrated to prove our theoretical results. The graphical behavior of solution of the fractional guava problem at the distinct fractional order values and at various parameters is discussed.
Conclusion: The graphical behavior of solution of the fractional guava problem at the distinct fractional order values and at various parameters shows new vista and interesting phenomena of the model. The results are indicating that the fractional approach with non-singular kernel plays an important role in the study of different scientific problems. The suggested numerical scheme is very efficient for solving nonlinear fractional models of physical importance.

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* Corresponding author.
E-mail addresses: jagdevsinghrathore@gmail.com (J. Singh), b.ghanbary@yahoo.com (B. Ganbari), devendra.maths@gmail.com (D. Kumar), dumitru@cankaya.edu.tr (D. Baleanu).

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Introduction, background, and preliminaries

A mathematical model in biology describes mathematical description, treatment in addition modeling of the biological processes applying mathematical concepts and logics. To construct science of ecology, mathematical models have played a crucial role. In nineteenth century, a prey-predator equation of Lotka-Volterra was very helpful to develop the ecological research work of the population dynamics [1]. A prey-predator (PP) model exists when the prey performs as a resource for other organism (we say predator). This PP interrelation has useful consequence on the population of predators, since predators get extra nutriment and they are capable to develop new predators. In the opposite manner, there is injurious consequence on the population of preys as the predator eradicates them [2]. For instance we can study the PP model for guava. Guava belongs to the myrtle family. Guava cultivated in India by farmers in addition it is cultivated in people's home gardens. It has become familiar due to its easily availability nearly throughout the year. Guava helps a person to manage the need of vitamins and minerals in human's body. The availability of guava is very easy compared to several another fruits. One can enhance his/her vitamins and nutrition level, if he/she can use this fruit in right the process. Moreover, a person economically earns profit from this special fruit [3]. To develop the prey-predator model on sugarcane a Lotka-Volterra dynamical system was applied [4]. It demonstrates the interaction into pest in addition natural enemies via biological control strategy. Pest is very dangerous for all the crops. In the present paper, we discuss how to manage the population of pest at the equilibrium level lower the economical vandalism by using biological pest control strategy.

The department of agriculture and cooperation, ministry of agriculture, India, in Aisa based integrated pest management package represents the facts well about nutritional deficiencies, insecticide resistance of management, safety measures pest, insect and diseases, etc. The details study also provides natural control, mechanical control additionally cultural control moreover it describes how to utilize pesticides [3]. Authors of a research article [5] discussed a model with mathematical logic and concept for biological pest control (BPC). To get the optimal pest control approach a linear assessment control problem has been development for the system by proposing natural enemies. Numerical outcomes of the BPC focused on the Lotka-volterra system are discussed to demonstrate the efficacy of the fractional order mathematical problem. In another work, the optimal pest control problem (OPCP) arising in population dynamical processes is discussed [6]. The OPCP is bifurcated in two portions. Karmaker et al. [7] studied a model which is expressed for the interaction into natural enemies and guava pests.

By considering the above discussed work and usefulness of guava fruit in our daily life we are motivated to carry the research work on guava fruit. In this article, we study fractional guava fruit model founded on the prey-predator model associated with the Atangana-Baleanu (AB) operator. In this work, prey indicates pest in addition predator designates natural enemies. Initiating natural enemies in the current agriculture technique to cultivate guava fruit, here we represent how pests are enhanced in addition diminished in the existence of natural enemies.

Since integer order derivative is local in nature so it does not contain the complete memory and hence it does not describe the physical behavior of the model. To overcome this challenge, we use the fractional derivative. It is well known that fractional derivative is non-local in nature and due to this characteristic it contains the whole memory and physical nature of the model. Fractional order model are used to determine the real world problems with a strategic solution. There are many mathematicians working in this field, some related work is hereby expressed for more details and outcomes one can see the published books and papers [8,9,10,11,12,13,14,15,16,17,18,19]. Srivastava et al. [20] suggested a reliable analytical scheme for a fractional biological population problem having carrying capacity. Dubey et al. [21] studied the fractional extension of biochemical reaction problem. Yang et al. [22] investigated a fractal nonlinear Burgers' model describing acoustic signals propagation. In the same sequence, Atangana and Baleanu (AB) [11] studied an advanced and useful derivative of fractional order. Cădariu and Radu [23] discussed fixed point techniques for the generalized stability of differential equations in a single variable. In this attempt, we use this advanced fractional operator to guava fruit model. The key objective of this research is to discuss the new characteristic of guava fruit model associated with Mitag-Leffler kernel. Further, we discuss the new characteristic between guava pest and natural enemies.

Here, we provide some key definitions of AB operator [11] as follows:

Definition 1.1. Suppose $h \in H^1(c, d), d > c, \omega \in (0, 1]$ and it is also differentiable in nature, hence AB operator in Caputo' kind is expressed in the following way

$$cABCD_{\xi}^{\omega}(h(\xi)) = \frac{B(\omega)}{1-\omega}\int_c^{\xi} h'(g)E_{\omega}\left[-\frac{\omega}{1-\omega}(\xi-g)^{\omega}\right]dg. \tag{1}$$

where E_{ω} is Mittag-leffler function and $B(0) = 1 = B(1)$.

Definition 1.2. Assume $0 < \omega < 1$, then Atangan-Baleanu integral of function $h(\xi)$ of fractional order ω is represented as

$$cABI_{\xi}^{\omega}(h(\xi)) = \frac{(1-\omega)}{B(\omega)}h(\xi) + \frac{\omega}{B(\omega)\Gamma(\omega)}\int_c^{\xi} h(g)(\xi-g)^{\omega-1}dg, \xi \geqslant 0. \tag{2}$$

The foundation of this article is divided in sections as: In Section 'Fractional model of the problem', guava model pertaining to AB derivative is formulated. In Section 'Positivity analysis and equilibrium analysis of the fractional guava model', we discuss positivity analysis and equilibrium analysis for the fractional guava fruit model. Section 'Stability analysis of the fractional system', represents the stability analysis of the model at the equilibrium points. Section 'Existence and uniqueness analysis' is dedicated to existence and uniqueness (EU) examination of the solution of guava model associated with AB derivative by applying Picard Lidelof approach. In Section 'Approximate numerical solution of the model', approximate numerical solution of the fractional guava fruit problem is shown via numerical technique. Section 'Numerical experiments', demonstrates the numerical results and graphical behavior of solution of fractional guava model. Finally, the concluding statement is given in Section 'Conclusions and outcomes of the paper'.

Fractional model of the problem

The fractional guava model is based on PP model. Here guava borers denote prey while natural enemies indicates predator. The symbols and formation of the model as follows

p : No. of guava borers,
q : No. of natural enemies,
α : Rate of growth of guava borers,
αp : Term which contributes in increasing the guava borers at Malthusian process,
u : Rate of reduction of guava borers with carrying capacities,
v : Rate of reduction of natural enemies with carrying capacities,

l : Rate of growth of natural enemies,
wpq : Rate of growth of guava borers contribution to the natural enemies.

Therefore, the rate of change of the guava borer with AB fractional operator is given by (Since AB fractional operator is non-local in nature and due to this property it involves the complete memory and biological aspects of the model.)

$$cABCD_{\xi}^{\omega}p = \alpha p(\xi) - up^2(\xi) - vp(\xi)q(\xi),$$

The rate of growth of the natural enemies influenced by guava borers with AB fractional operator is given as

$$cABCD_{\xi}^{\omega}q = -lq(\xi) + wp(\xi)q(\xi).$$

The Ist equation represents the guava borers and the IInd equation designates the population of natural enemy in the guava fruit. The fractional order mathematical model of the biological process can be presented in the subsequent manner

$$\begin{aligned} cABCD_{\xi}^{\omega}p &= \alpha p(\xi) - up^2(\xi) - vp(\xi)q(\xi), \\ cABCD_{\xi}^{\omega}q &= -lq(\xi) + wp(\xi)q(\xi). \end{aligned} \tag{3}$$

Positivity analysis and equilibrium analysis of the fractional guava model

In this section, we discuss positivity analysis and equilibrium analysis for the fractional guava model.

Theorem 3.1:. *The guaua borers and natural enemies for the fractional order problem (3) are regularly non-negative* $\forall$ *given* $\xi \geqslant 0$.

Proof:. For proof, see [7].

To find the equilibrium points of Eq. (3), we assume

$$cABCD_{\xi}^{\omega}p = \alpha p(\xi) - up^2(\xi) - vp(\xi)q(\xi) = \varphi(p,q),$$

and

$$cABCD_{\xi}^{\omega}q = -lq(\xi) + wp(\xi)q(\xi) = \vartheta(p,q).$$

To find equilibrium point of the model, we put

$$cABCD_{\xi}^{\omega}p = 0 = \varphi(p,q),$$

$$cABCD_{\xi}^{\omega}q = 0 = \vartheta(p,q).$$

To find the equilibrium point, we solve the algebraic equations given as

$$\alpha p(\xi) - up^2(\xi) - vp(\xi)q(\xi) = 0,$$

$$-lq(\xi) + wp(\xi)q(\xi) = 0.$$

Suppose (p^*, q^*) be the equilibrium point. Therefore

$$\varphi(p^*, q^*) = \alpha p^* - u(p^*)^2 - vp^*q^* = 0, \tag{4}$$

$$\vartheta(p^*, q^*) = -lq^* + wp^*q^* = 0. \tag{5}$$

From Eq. (5), we have

$$-lq^* + wp^*q^* = 0.$$

On solving, we have

$$q^* = 0 \text{ and } p^* = \frac{l}{w}. \tag{6}$$

Now from Eq. (4), we have

$$\alpha p^* - u(p^*)^2 - vp^*q^* = 0.$$

It gives

$$p^* = 0 \text{ and } \alpha - up^* - vq^* = 0. \tag{7}$$

Substituting the values of p^* from Eq. (6) in Eq. (7), we get

$$q^* = \frac{\alpha w - ul}{vw}. \tag{8}$$

Now putting the value of q^* from Eq. (6) in Eq. (7), we obtain

$$p^* = \frac{\alpha}{u}. \tag{9}$$

Thus there are three equilibrium points given as follows

$$(0,0), \left(\frac{l}{w}, \frac{\alpha w - ul}{vw}\right) \text{ in addition } \left(\frac{\alpha}{u}, 0\right).$$

Stability analysis of the fractional system

In this part, we evaluate the stability of various EP of the fractional guava model associated with AB fractional derivative and calculate the eigen values of Jacobian matrix (JM) at equilibrium points (EP).

The JM of the fractional guava fruit problem (3) at the EP (p^*, q^*) is given as

$$J(p^*, q^*) = \begin{pmatrix} \varphi_p(p^*, q^*) & \varphi_q(p^*, q^*) \\ \vartheta_p(p^*, q^*) & \vartheta_q(p^*, q^*) \end{pmatrix}.$$

It can be written as

$$J(p^*, q^*) = \begin{pmatrix} \alpha - 2p^* - vq^* & -vp^* \\ wq^* & -l + wp^* \end{pmatrix}.$$

At the EP $(0,0)$, we have

$$J(0,0) = \begin{pmatrix} \alpha & 0 \\ 0 & -l \end{pmatrix}.$$

The characteristic equation (CE) of JM is given by

$$|J - \lambda I| = 0.$$

The above CE can be written as

$$\lambda^2 + \lambda(l - \alpha) - \alpha l = 0.$$

On solving the above CE, we get the values of λ as

$$\lambda = \alpha, -l.$$

It is observed that α is positive and $-l$ is negative. Hence, the EP (0, 0) is not stable. Now, we verify the stability of the EP $\left(\frac{l}{w}, \frac{\alpha w - ul}{vw}\right)$.

At these points the JM is expressed as

$$J\left(\frac{l}{w}, \frac{\alpha w - ul}{vw}\right) = \begin{pmatrix} \alpha - 2u\frac{l}{w} - v\frac{\alpha w - ul}{vw} & -v\frac{l}{w} \\ v\frac{\alpha w - ul}{vw} & -l + w\frac{l}{w} \end{pmatrix}.$$

Thus, we get

$$J\left(\frac{l}{w}, \frac{\alpha w - ul}{vw}\right) = \begin{pmatrix} \frac{-ul}{w} & \frac{-vl}{w} \\ \frac{\alpha w - ul}{w} & 0 \end{pmatrix}.$$

The CE is given as

$$|J - \lambda I| = 0.$$

The above CE can be presented as

$$\lambda^2 + \frac{ul}{w}\lambda + \frac{vl}{w}\left(\frac{\alpha w - ul}{v}\right) = 0.$$

The solution of this equation is given as

$$\lambda = \frac{-\frac{ul}{w} \pm \sqrt{\left(\frac{ul}{w}\right)^2 - 4\frac{vl}{w}\left(\frac{\alpha w - ul}{v}\right)}}{2}.$$

So, we have two values of λ as follows

$$\lambda_1 = \frac{-\frac{ul}{w} + \sqrt{\left(\frac{ul}{w}\right)^2 - 4\frac{vl}{w}\left(\frac{\alpha w - ul}{v}\right)}}{2} \text{ and } \lambda_2 = \frac{-\frac{ul}{w} - \sqrt{\left(\frac{ul}{w}\right)^2 - 4\frac{vl}{w}\left(\frac{\alpha w - ul}{v}\right)}}{2}.$$

Since here the eigen values λ_1 and λ_2 have negative real parts. Hence, the EP $\left(\frac{l}{w}, \frac{\alpha w - ul}{vw}\right)$ is asymptotically stable. Now, we verify stability at the EP $\left(\frac{\alpha}{u}, 0\right)$, at these points the JM reduces to

$$J\left(\frac{\alpha}{u}, 0\right) = \begin{pmatrix} \alpha - 2\frac{\alpha}{u} & -\frac{v\alpha}{u} \\ 0 & -l + \frac{\alpha w}{u} \end{pmatrix}.$$

Further, the CE of JM is given as

$$|J - \lambda I| = 0.$$

The above CE can be expressed as

$$\left| \begin{pmatrix} \frac{\alpha u - 2\alpha}{u} & -\frac{\alpha v}{u} \\ 0 & \frac{w\alpha - \alpha u}{u} \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0.$$

From the above equation, we get

$$\left(\frac{\alpha u - 2\alpha}{u} - \lambda \quad \frac{w\alpha - \alpha u}{u} - \lambda \right) = 0.$$

Thus, we get

$$\lambda = \frac{w\alpha - \alpha u}{u}, \frac{\alpha u - 2\alpha}{u}. \quad (10)$$

The eigen values $\lambda = \frac{w\alpha - \alpha u}{u}, \frac{\alpha u - 2\alpha}{u}$ are determinedly real numbers.

Limitations for stability of eigen values $\lambda = \frac{w\alpha - \alpha u}{u}, \frac{\alpha u - 2\alpha}{u}$.

When $w\alpha < \alpha u$ and $u < 2$ both values are negative, in this situation the EP $\left(\frac{\alpha}{u}, 0\right)$ is asymptotically stable.

When $w\alpha < \alpha u$ and $u < 2$ both values are positive, in this case the EP $\left(\frac{\alpha}{u}, 0\right)$ is unstable.

When $w\alpha < \alpha u$ is negative and $u < 2$ is positive, in this case the EP $\left(\frac{\alpha}{u}, 0\right)$ is unstable.

When $w\alpha < \alpha u$ is positive and $u < 2$ is negative, in this case the EP $\left(\frac{\alpha}{u}, 0\right)$ is unstable.

As the JM at EP (p^*, q^*) is represented as

$$J(p^*, q^*) = \begin{pmatrix} \alpha - 2p^* - vq^* & -vp^* \\ wq^* & -l + wp^* \end{pmatrix}.$$

The CE is given by $\lambda^2 + v_1\lambda + v_2 = 0$,
where the values of v_1 and v_2 is given as

$$v_1 = 2up^* + (l - wp^*) + (vq^* - \alpha), \; v_2 = 2up^*(l - wp^*) + (vq^* - \alpha)l + wep^*.$$

For the values of $v_1 > 0$ and $v_2 > 0$, as per statement of Hurwitz's postulate, the real parts of all the roots of the CE should be negative in other words asymptotically stable.

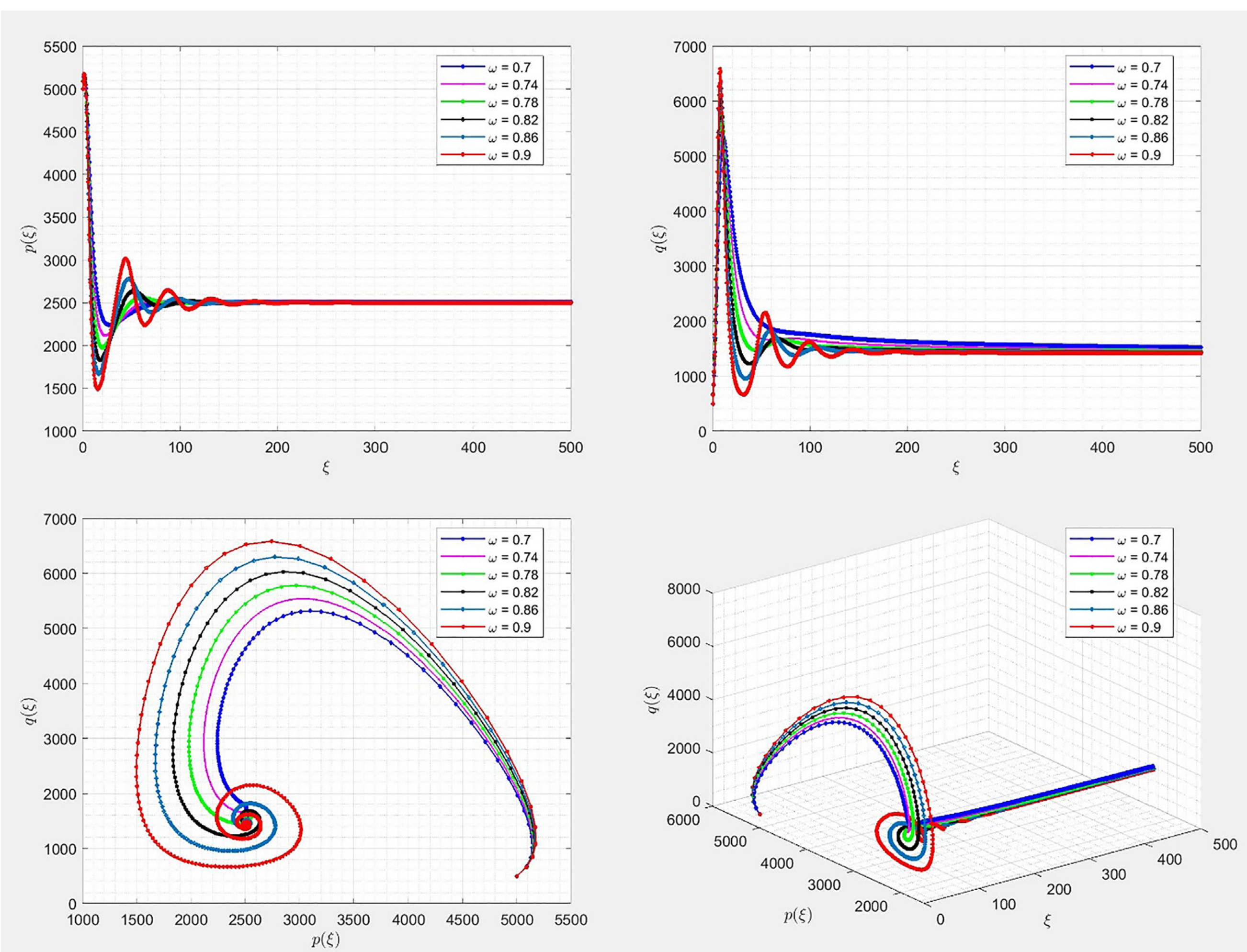

Fig. 1. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 5000$, $q_0 = 500$.

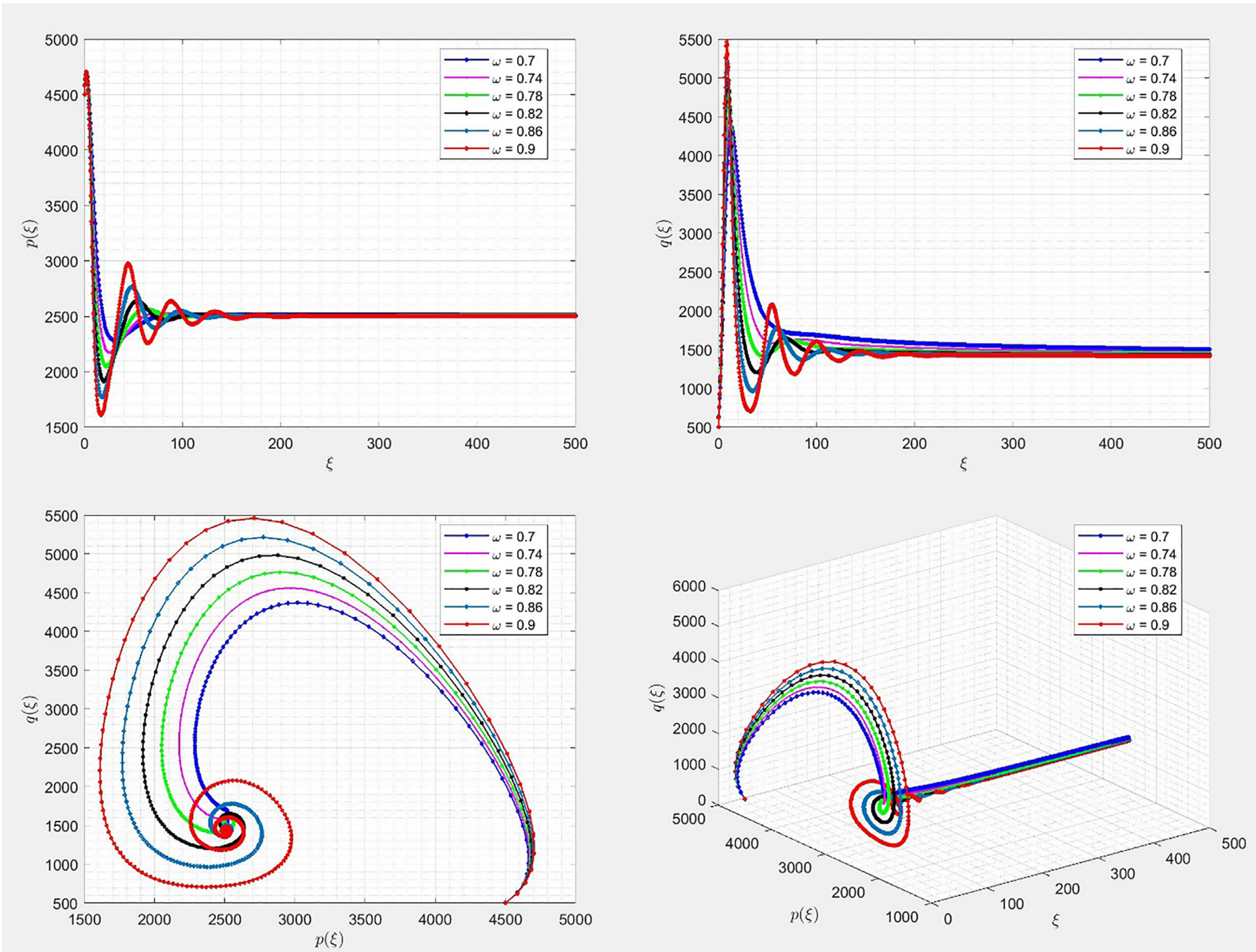

Fig. 2. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 4500$, $q_0 = 500$.

Existence and uniqueness analysis

Commonly a fractional differential equation pertaining to the initial conditions (ICs) is expressed as follows

$$\frac{d^{\omega}z}{d\xi^{\omega}} = h(z,\xi),\ z(\xi_0) = z_0 \tag{11}$$

As far as concern to solution of Eq. (11), there may be three cases as

(i) there may be no solution for Eq. (11),
(ii) it may be an infinite solution,
(iii) it may be a single solution.

Mathematics researchers who are interested in such types problems search for answers to following

Q1. Under which conditions, the solution for the model (3) is certain i.e. question of existence?.
Q2. Under which conditions the solution is unique i.e. question of uniqueness?

In the present section, this work explores the answers to the above questions for the guava fruit model, which was stated with the ordinary derivative. First of all, let us reorganize the system of Eq. (3) as

$$cABCD_{\xi}^{\omega}p(\xi) = h_1(\xi,p,q) = \alpha p(\xi) - up^2(\xi) - vp(\xi)q(\xi), \tag{12}$$

$$0ABCD_{\xi}^{\omega}q(\xi) = h_2(\xi,p,q) = -lq(\xi) + wp(\xi)q(\xi).$$

and $\Theta(\xi) = (p,q)$, $H(\Theta(\xi),\xi) = (h_1(\Theta(\xi)), h_2(\Theta(\xi)))$.

Suppose that solutions of system (3) are continuous functions in nature from a time interval $K$to a bounded subset $V \in R^2$, together with initial value $\Theta(\xi_0) = \Theta_0$. Now we put $K = [\xi - \xi_0, \xi + \xi_0]$, $V = R(\bar{y}, r)$ in addition

$$D_{c,r_1} = K \times [y - r_1, y + r_1] = K \times R_1,$$

$$D_{c,r_2} = K \times [y - r_2, y + r_2] = K \times R_2,$$

and $R_j$represents the closed ball in R^m around y with radius r_j. Now we assume $D_{c,r}(K,V)$indicates a set a continuously function from K to V associated with the norm

$$A_1 = \sup_{D_{c,r_1}} \| h_1(\xi,\Theta(\xi)) \|,$$

$$A_2 = \sup_{D_{c,r_2}} \| h_2(\xi,\Theta(\xi)) \|.$$

Moreover, the supremum of all A_1 and A_2 is given as

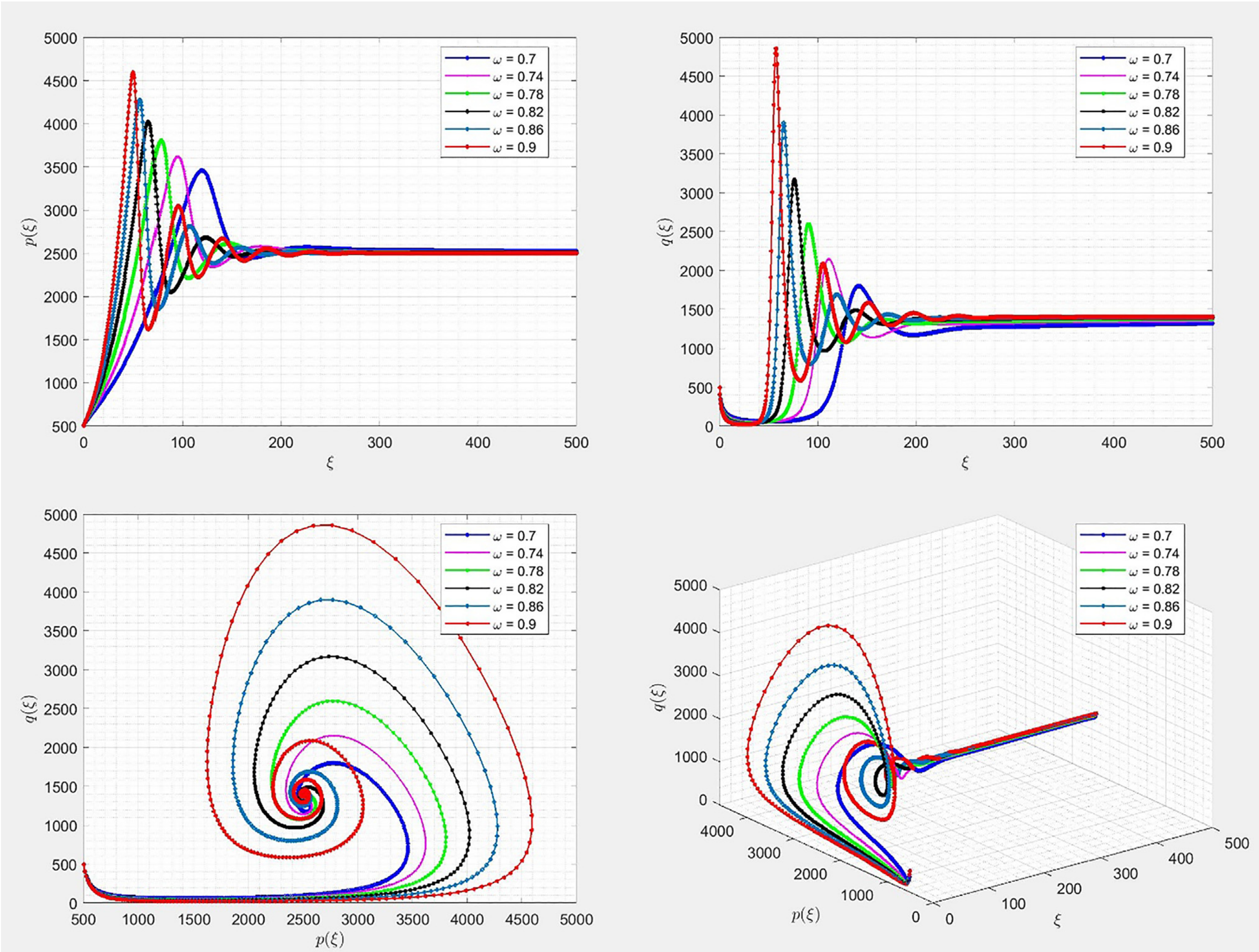

Fig. 3. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 500$, $q_0 = 500$.

$A = \sup_{D_{c,r_2}} \| h_2(\xi, \Theta(\xi)) \|.$

Now we consider a Picard operator $\Omega$from $D_{c,r}(K,V)$ to $D_{c,r}(K,V)$ as follows

$\Omega : D_{c,r}(K,V) \to D_{c,r}(K,V).$

$$\Omega(\Theta(\xi)) = \Theta(\xi_0) + \frac{1-\omega}{B(\omega)}H(\Theta(\xi)) + \frac{\omega}{B(\omega)\Gamma(\omega)} \times \int_{\xi_0}^{\xi} H(\Theta(g))(\xi-g)^{\omega-1}dg.$$

Next, we will employ the Picard-Lindelof existence and uniqueness theorem to deal the guaua model with initial value problem. Noticing the Picard- Lindelof approach with our symbols.

Theorem 5.1. *Let* $D_{c,r} : [\xi_0 - c, \xi_0 + c] \times V = R(\bar{y}, r) \to R^m$ *is continuous function and it is bounded by a constant* β. *Again let that* $D_{c,r}$ *is a Lipschitz continuous function with the Lipschitz constant* ρ *for every* $\xi \in [\xi_0 - c, \xi_0 + c]$. *Then the initial value problem (IVP) holds a unique solution* $y \in D_{c,r}(K,V)$ *as lasting as the time interval is selecting with a satisfactory value as* $0 < c < \min\left(\frac{1}{\rho}, \frac{r}{\beta}\right)$.

Proof: First of all we will prove that Picard operator Θ holds a contraction on the space $D_{c,r}(K,V)$. It is notice that h_j functions are continuous in nature because $D_{c,r}(K,V)$ is Lipschitz. Additionally V is closed and bounded in nature, hence h_j holds their minimum and maximum value on V. Therefore, $\Theta(\xi) \in D_{c,r}(K,V)$.

$$\begin{aligned}\|\Omega(\Theta(\xi)-\Theta(\xi_0))\| &= \|\frac{1-\omega}{B(\omega)}H(\Theta(\xi)) + \frac{\omega}{B(\omega)\Gamma(\omega)}\int_{\xi_0}^{\xi} H(\Theta(g))(\xi-g)^{\omega}dg\|,\\ &\leqslant \|\frac{1-\omega}{B(\omega)}H(\Theta(\xi))\| + \|\frac{\omega}{B(\omega)\Gamma(\omega)}H(\Theta(\xi))\int_{\xi_0}^{\xi} H(\Theta(g))(\xi-g)^{\omega}dg\|,\\ &\leqslant \frac{1-\omega}{B(\omega)}\|H(\Theta(\xi))\| + \frac{\omega}{B(\omega)\Gamma(\omega)}\|H(\Theta(\xi))\int_{\xi_0}^{\xi} H(\Theta(g))(\xi-g)^{\omega}dg\|,\\ &= \left(\frac{1-\omega}{B(\omega)} + \frac{\omega\Omega_{\max}^{\omega}}{B(\omega)\Gamma(\omega)}\right)A.\end{aligned}$$

Hence, it holds that Ω maps $D_{c,r}(K,V)$ into itself.

Now, let us take two elements $\Theta_1(\xi), \Theta_2(\xi) \in D_{c,r}(K,V)$, in this case, we have

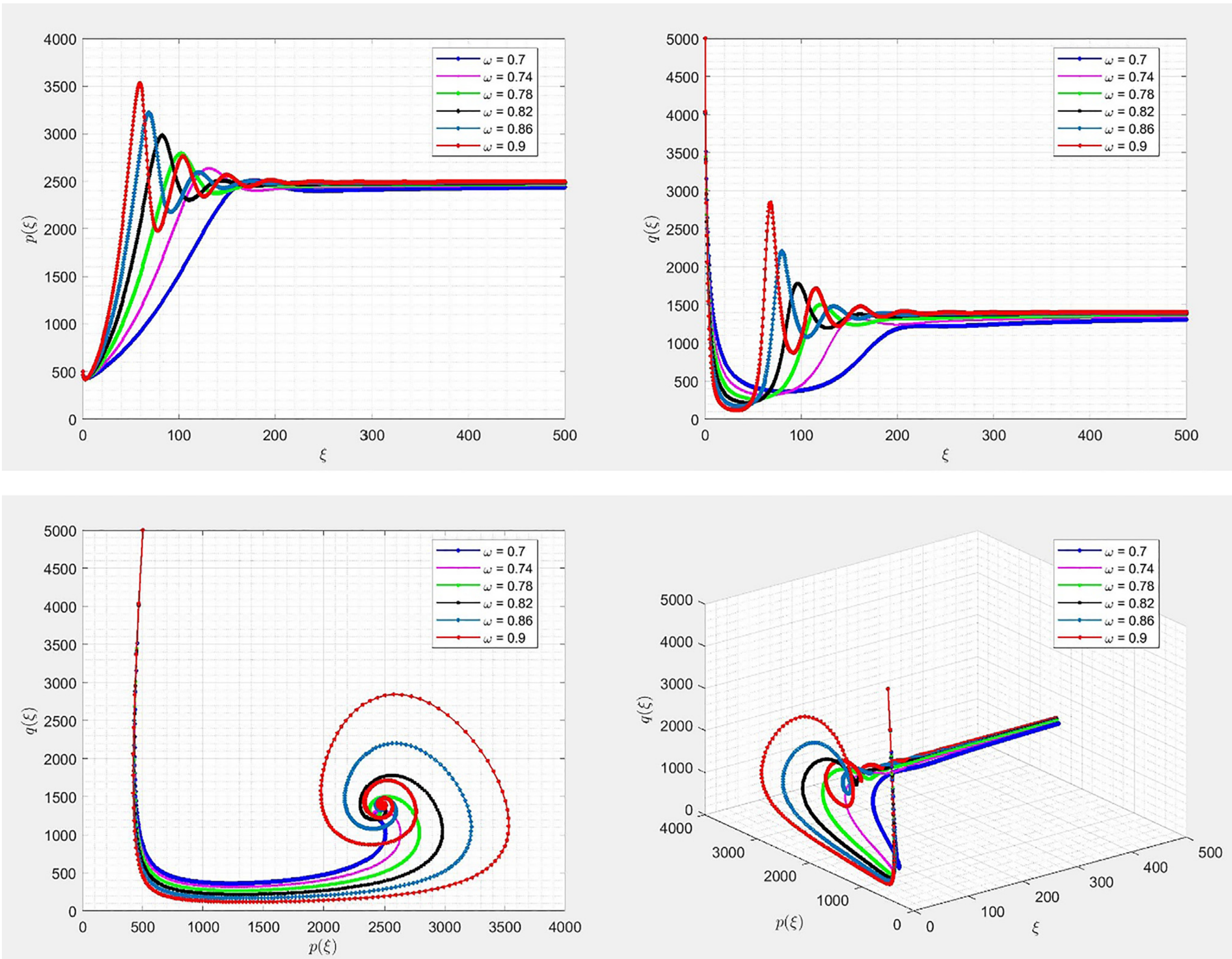

Fig. 4. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 500$, $q_0 = 5000$.

$$\| \Omega(\Theta_1(\xi)) - \Omega(\Theta_2(\xi)) \| = \| \frac{1-\omega}{B(\omega)} H(\Theta_1(\xi)) + \frac{\omega}{B(\omega)\Gamma(\omega)} \int_{\xi_0}^{\xi} H(\Theta_1(g))(\xi - g)^{\omega} dg - \frac{1-\omega}{B(\omega)} H(\Theta_2(\xi)) + \frac{\omega}{B(\omega)\Gamma(\omega)} \|, \int_{\xi_0}^{\xi} H(\Theta_2(g))(\xi - g)^{\omega} dg$$

$$\leqslant \frac{1-\omega}{B(\omega)} \| H(\Theta_1(\xi)) - H(\Theta_2(\xi)) \| + \frac{\omega}{B(\omega)\Gamma(\omega)} \times \int_{\xi_0}^{\xi} \| (H(\Theta_1(g)) - H(\Theta_2(g)))(\xi - g)^{\omega} \| dg,$$

$$\leqslant \left(\frac{1-\omega}{B(\omega)} + \frac{\omega \Omega_{\max}^{\omega}}{B(\omega)\Gamma(\omega)} \right) \rho.$$

Hence, existence and uniqueness (EU) of the solution of model (3) are verified.

Approximate numerical solution of the model

In this portion, we will investigate how to use the numerical procedure to obtain the approximate solution of the problem. To this end, let us take into account the following equation

$$0ABC\mathscr{D}_{\omega}\mathscr{Y}(\xi) = \mathscr{S}(\xi, \mathscr{Y}(\xi)). \tag{13}$$

By using the integral operator one can write

$$\mathscr{Y}(\xi) - \mathscr{Y}(0) = \frac{1-\omega}{B(\omega)} \mathscr{S}(\xi, \mathscr{Y}(\xi)) + \frac{\omega}{\Gamma(\omega)B(\omega)} \times \int_0^{\xi} \mathscr{S}(\tau, \mathscr{Y}(\tau))(\xi - \tau)^{\omega - 1} d\tau. \tag{14}$$

Taking $\xi = \xi_n = n\hbar$ in (14), one achieves

$$\mathscr{Y}(\xi_n) = \mathscr{Y}(0) + \frac{1-\omega}{B(\omega)} \mathscr{S}(\xi_n, \mathscr{Y}(\xi_n)) + \frac{\omega}{\Gamma(\omega)B(\omega)} \sum_{i=0}^{n-1} \times \int_{\xi_i}^{\xi_{i+1}} \mathscr{S}(\tau, \mathscr{Y}(\tau))(\xi_n - \tau)^{\omega - 1} d\tau. \tag{15}$$

Now, with the help of linear interpolation of $\mathscr{S}(\xi, \mathscr{Y}(\xi))$, one gets

$$\mathscr{S}(\xi, \mathscr{Y}(\xi)) \approx \mathscr{S}(\xi_{i+1}, \mathscr{Y}_{i+1}) + \frac{\xi - \xi_{i+1}}{\hbar} (\mathscr{S}(\xi_{i+1}, \mathscr{Y}_{i+1})) - \mathscr{S}(\xi_i, \mathscr{Y}_i)), \quad \xi \in [\xi_i, \xi_{i+1}], \tag{16}$$

where the notation of $\mathscr{K}_i = \mathscr{K}(t_i)$ is used.

Substituting Eq. (16) in Eq. (15), the approximate solution of the problem will obtain as in [15],

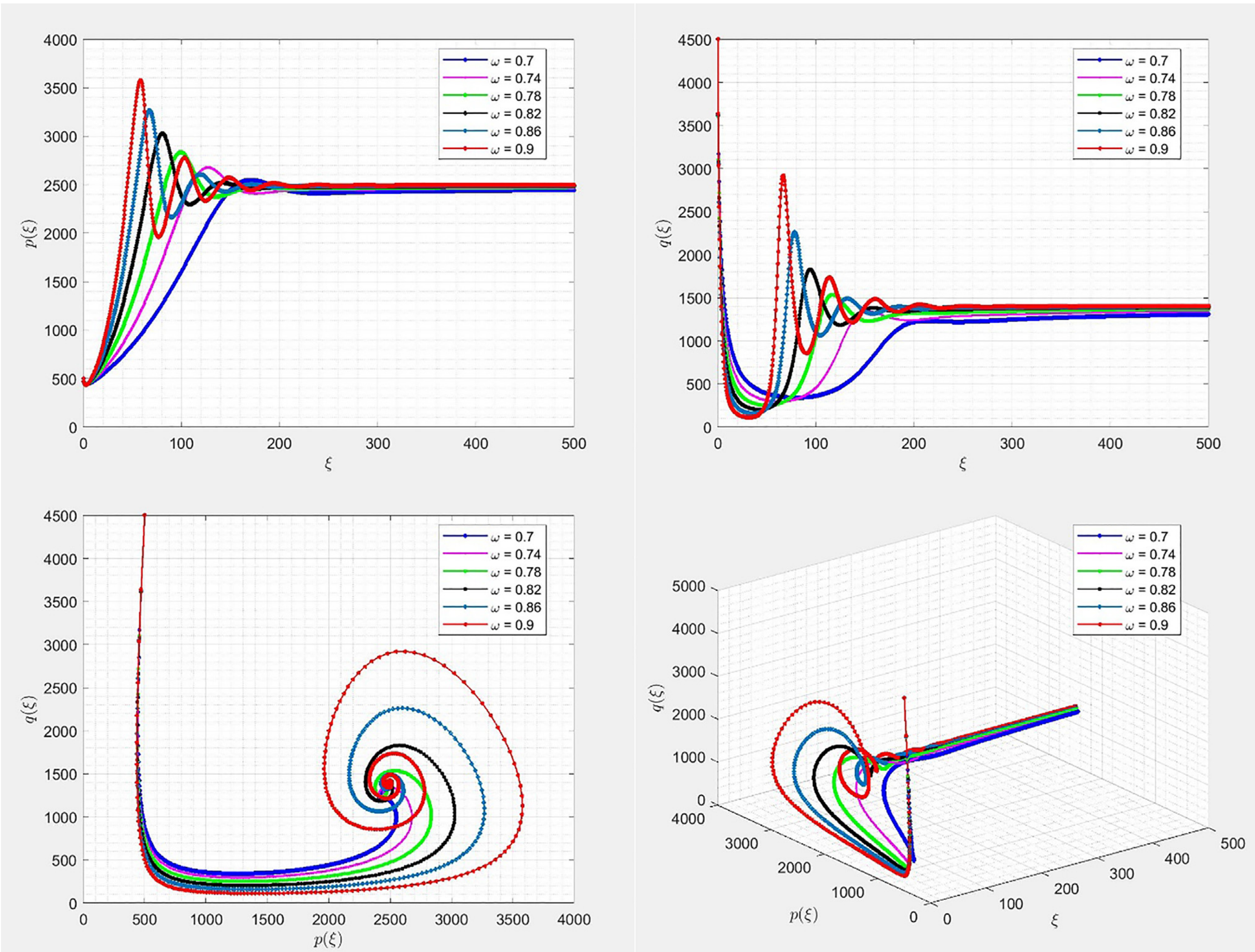

Fig. 5. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 4500$, $q_0 = 500$.

$$\mathscr{Y}_n = \mathscr{Y}_0 + \frac{\omega \hbar^{\omega}}{B(\omega)}\left(\eta_n \mathscr{S}(\xi_0, \mathscr{Y}_0) + \sum_{i=1}^{n} \theta_{n-i} \mathscr{S}(\xi_i, \mathscr{Y}_i)\right) \tag{17}$$

where

$$\eta_n = \frac{(n-1)^{\omega+1} - n^{\omega}(n-\omega-1)}{\Gamma(\omega+2)},$$
$$\theta_j = \begin{cases} \frac{1}{\Gamma(\omega+2)} + \frac{1-\omega}{\omega \hbar^{\omega}}, & j = 0 \\ \frac{(j-1)^{\omega+1} - 2j^{\omega+1} + (j+1)^{\omega+1}}{\Gamma(\omega+2)}, & j = 1, 2, \dots, n-1 \end{cases} \tag{18}$$

Using the numerical method Eq. (17) presented above, the approximate solution of the problem (3) will be achieved recursively as

$$p_n = p_0 + \frac{\omega \hbar^{\omega}}{B(\omega)}\left(\eta_n(\alpha p_0 - u p_0^2 - v p_0 q_0) + \sum_{i=1}^{n} \theta_{n-i}(\alpha p_i - u p_i^2 - v p_i q_i)\right), \tag{19}$$

$$q_n = q_0 + \frac{\omega \hbar^{\omega}}{B(\omega)}\left(\eta_n(-l q_0 + w p_0 q_0) + \sum_{i=1}^{n} \theta_{n-i}(-l q_i + w p_i q_i)\right). \tag{20}$$

Approximate solutions to the fractional guava fruit model can be computed using the numerical method described above.

Numerical experiments

In this part, we will utilize the outlined approximate scheme in Eq. (19) and Eq. (20) to obtain numerical simulations for solving the fractional-order mathematical model of (3).

In what follows we present some graphs that describe the behavior of the solutions for different values of some parameters. Numerical solutions to the fractional order model are shown in the Figs. 1–6 for different initial values and the fractional orders ω. One can observe that the numerical results obtained are fully according to the theoretical results stated in the preceding sections.

In the numerical experiments, we have used the time step size of $\hbar = 1.0 \times 10^{-3}$. In Figs. 1–6, we consider distinct initial conditions i.e. the values of guava borers (p_0) as well as natural enemies (q_0) at the initial time for different fractional orders. The numerical simulation of the fractional guava fruit model is demonstrated and the stability is also discussed. The time interval from 0 to 500 days is taken for numerical simulation of the fractional model. From the Figs. 1–6, we see that at initial time there are frequent changes in the values of natural enemies and guava borers for different order of AB fractional derivative (ω), but after 120 to140 days they are got stable. As 120–140 days are needed to mature the guava fruits from their flowers. In Figs. 1–6, we observe from phase prostrates that the nature of all parameters is oscillatory between 0 and 140 days. It is noticed that the solutions of the fractional guava

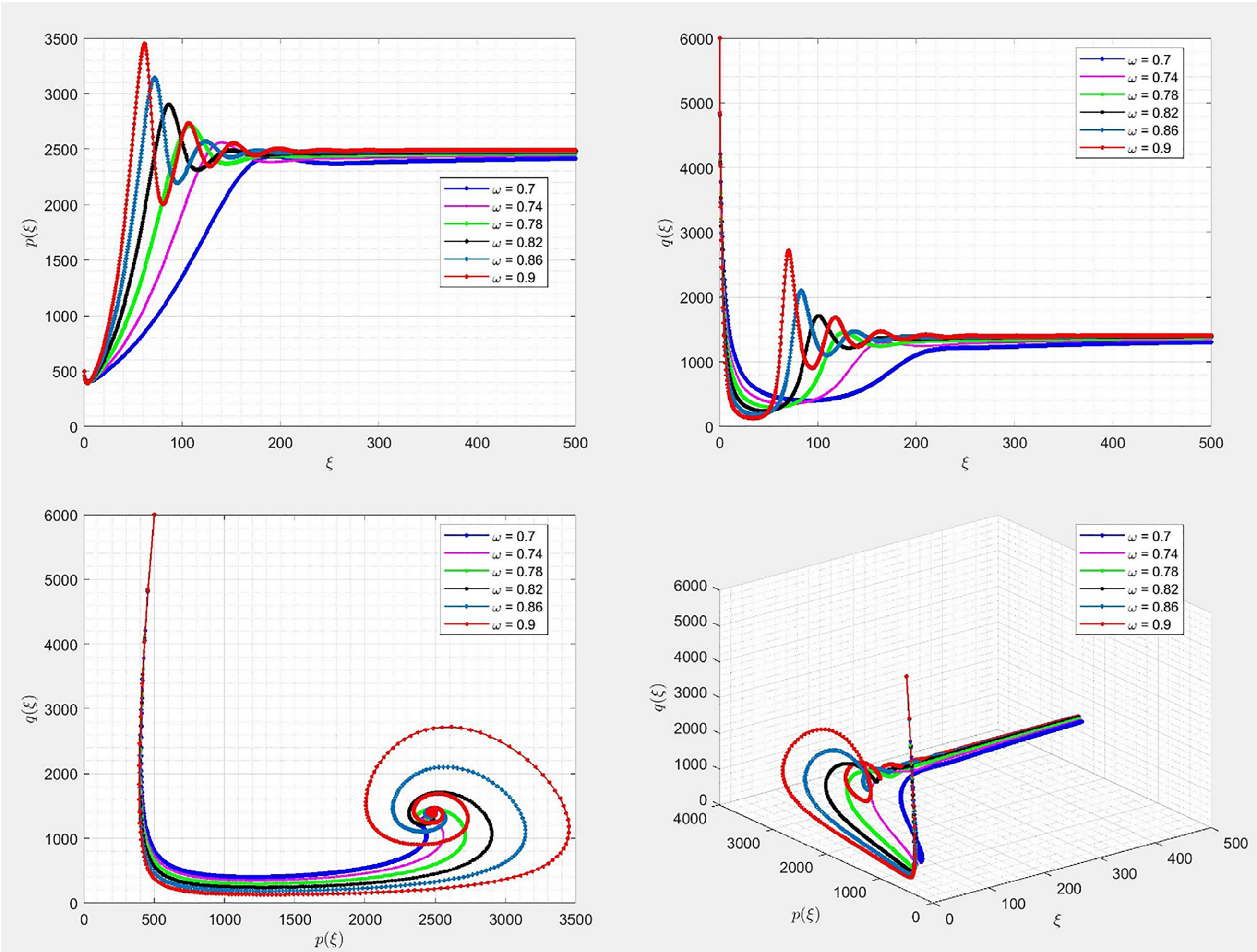

Fig. 6. Influence of different values of ω on response behavior of solutions for the values of $\alpha = 0.07$, $u = 0.0000020$, $v = 0.000046$, $w = 0.0002$, $l = 0.5$, and $p_0 = 500$, $q_0 = 6000$.

fruit model are periodic, thus the solutions of the fractional model are orbitally stable. It is also noticeable that there is significant impact of the order of fractional operator on the values of guava borers and their enemies.

Conclusions and outcomes of the paper

At the last by observing all above discussed important aspects via all the sections further based on useful and interesting theoretical results with well supported numerical outcomes, we set the concluding remarks in the form of three crucial parts as

(1) Mathematical modeling of realistic problems by applying the logic of fractional operators for the system of differential equations have been a subject of research activities since few past years. The AB derivative has few crucial features i.e. it contains strong memory effect of the system. Therefore, a study of guava model associated with a newly AB fractional derivative which is founded on nonlocal and non-singular kernel is discussed in the present paper.

(2) We study a fractional guava model pertaining to Atangan-Baleanu (AB) derivative for the interaction into natural enemies and guava pests. The present system is discussed as a Lotka-Volterra kind. The positivity analysis and equilibrium analysis for the fractional guava model is analyzed in a effective way. By utilizing the Picard Lindelof scheme the EU analysis of the solution the fractional guava fruit model is verified effectively.

(3) By utilizing a numerical technique the approximate numerical solution of the fractional guava fruit model is obtained. The numerical outcomes which are also perform as a control, as the parameters are represented to verify our theoretical outcomes. The graphical depictions of solution of the guava model with AB derivative at the various fractional order values, parameters displays new vista and interesting nature of the model.

Compliance with ethics requirements

This work does not contain any studies with human or animal subjects.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Lotka AI. Fluctuation in the abundance of the species considered mathematically (with comment by Voltra, V). Nature 1927;119:12–3.

[2] Anisiu MC. Lotka, volterra and their model. Didactica Mathematica 2014;32:9–17.
[3] NIPHM, Aisa based IPM package. Department of agriculture and Cooperation, Ministry of Agriculture, Government of India; 2014.
[4] Rafikov M. Optimization of biological pest control of sugarcane borer. In: 18th IEEE International Conference on Control Applications, Part of 2009 IEEE Multi-conference on Systems and Control, IEEE, Saint Petersburg, Russia, July 8-10, 2009, http://dx.doi.org/10.1109/CCA.2009.5280989.
[5] Rafikov M, Balthazar JM, Bremen HFV. Mathematical modeling and control of population systems: Application in biological pest control. Appl Math Comput 2007;200:557–73.
[6] Rafikov M, Balthazar JM. Optimal pest control problem in population dynamics. Comput Appl Math 2005;24(1):65–81.
[7] Karmaker S, Ruhi FY, Mallick UK. Mathematical analysis of a model on guava for biological pest control. Math Modell Eng Problems 2018;5(4):427–40.
[8] Kumar D, Singh J. Fractional calculus in medical and health science. London, UK: CRC Press; 2020.
[9] Atangana A, Alkahtani BT. Analysis of non- homogenous heat model with new trend of derivative with fractional order. Chaos Soliton Fract 2016;89:566–71.
[10] Singh J, Kumar D, Hammouch Z, Atangana A. A fractional epidemiological model for computer viruses pertaining to a new fractional derivative. Appl Math Comput 2018;316:504–15.
[11] Atangana A, Baleanu D. New fractional derivative with nonlocal and non-singular kernel, Theory and application to heat transfer model. Therm Sci 2016;20(2):763–9.
[12] Singh J, Kumar D, Baleanu D. A new analysis of fractional fish farm model associated with Mittag-Leffler type kernel. Int J Biomath 2020;13(2). 2050010.
[13] Kumar D, Singh J, Baleanu D. On the analysis of vibration equation involving a fractional derivative with Mittag-Leffler law. Math Methods Appl Sci 2020;43 (1):443–57.
[14] Singh J, Jassim HK, Kumar D. An efficient computational technique for local fractional Fokker Planck equation 124525. Phys A 2020.
[15] Ghanbari B, Kumar D. Numerical solution of predator-prey model with Beddington-DeAngelis functional response and fractional derivatives with Mittag-Leffler kernel. Chaos 2019;29. 063103.
[16] Das M, Maiti A, Samanta GP. Stability analysis of a prey-predator fractional order model incorporating prey refuge". Ecol Genet Genom 2018;7–8:33–46.
[17] Das M, Samanta GP. A delayed fractional order food chain model with fear effect and prey refuge". Math Comput Simul 2020;178:218–45.
[18] Das M, Samanta GP. A prey-predator fractional order model with fear effect and group defense". Int J Dyn Control 2020. doi: https://doi.org/10.1007/s40435-020-00626-x.
[19] De la Sen M. About robust stability of caputo linear fractional dynamic systems with time delays through fixed point theory 867932. Fixed Point Theory Appl 2011;2011. doi: https://doi.org/10.1155/2011/867932.
[20] Srivastava HM, Dubey VP, Kumar R, Singh J, Kumar D, Baleanu D. An efficient computational approach for a fractional-order biological population model with carrying capacity. Chaos Solitons Fractals 2020;138. 109880.
[21] Dubey VP, Kumar R, Kumar D. Approximate analytical solution of fractional order biochemical reaction model and its stability analysis. Int J Biomath 2019;12(5). 1950059.
[22] Yang XJ, Machado JAT. A new fractal nonlinear Burgers' equation arising in the acoustic signals propagation. Math Meth Appl Sci 2020;42(18):7539–44.
[23] Cădariu L, Radu V. Fixed point methods for the generalized stability of functional equations in a single variable, Fixed Point Theory and Applications, 2008 (2008), article number: 749392, doi: 10.1155/2008/749392.

Prof. Jagdev Singh is a Professor in the Department of Mathematics, JECRC University, Jaipur-303905, Rajasthan, India. He did his Master of Science (M.Sc.) in Mathematics and Ph.D. in Mathematics from University of Rajasthan, India. He primarily teaches the subjects like mathematical modeling, real analysis, functional analysis, integral equations and special functions in post-graduate level course in mathematics. His area of interest is Mathematical Modelling, Mathematical Biology, Fluid Dynamics, Special Functions, Fractional Calculus, Applied Functional Analysis, Nonlinear Dynamics, Analytical and Numerical Methods. He has published three books: Advance Engineering Mathematics (2007), Engineering Mathematics-I (2008), Engineering Mathematics-II (2013). His works have been published in the Nonlinear Dynamics, Chaos Solitions & Fractals, Physica A, Journal of Computational and Nonlinear Dynamics, Applied Mathematical Modelling, Entropy, Advances in Nonlinear Analysis, Romanian Reports in Physics, Applied Mathematics and Computation, Chaos and several other peer-reviewed international journals. His 194 research papers have been published in various Journals of repute with h-index of 44 He has attained a number of National and International Conferences and presented several research papers. He has also attended Summer Courses, Short Terms Programs and Workshops. He is member of Editorial Board of various Journals of Mathematics. He is reviewer of various Journals.

Dr. Behzad Ganbari is an Assistant Professor in the Department of Engineering science, Kermanshah University of Technology, Kermanshah, Iran. He did his Master of Science (M.Sc.) and Ph.D in Applied Mathematics in University of Guilan, Iran. His area of interest is Mathematical Modelling, Mathematical Biology, Fluid Dynamics, Special Functions, Fractional Calculus, Applied Functional Analysis, Nonlinear Dynamics, Analytical and Numerical Methods. His works have been published in the Nonlinear Dynamics, Chaos Solitions & Fractals, Physica A, Applied Mathematics and Computation, Chaos and several other peer-reviewed international journals. He is reviewer of various Journals.

Dr. Devendra Kumar is an Assistant Professor in the Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India. He did his Master of Science (M.Sc.) in Mathematics and Ph.D. in Mathematics from University of Rajasthan, India. He primarily teaches the subjects like real and complex analysis, functional analysis, integral equations and special functions in post-graduate level course in mathematics. His area of interest is Mathematical Modelling, Special Functions, Fractional Calculus, Applied Functional Analysis, Nonlinear Dynamics, Analytical and Numerical Methods. He has published two books: Engineering Mathematics-I (2008), Engineering Mathematics-II (2013). His works have been published in the Nonlinear Dynamics, Chaos Solitions & Fractals, Physica A, Journal of Computational and Nonlinear Dynamics, Applied Mathematical Modelling, Entropy, Advances in Nonlinear Analysis, Romanian Reports in Physics, Applied Mathematics and Computation, Chaos and several other peer-reviewed international journals. His 210 research papers have been published in various Journals of repute with h-index of 44. He has attained a number of National and International Conferences and presented several research papers. He has also attended Summer Courses, Short Terms Programs and Workshops. He is member of Editorial Board of various Journals of Mathematics. He is reviewer of various Journals.

Prof. Dumitru Baleanu research interests include fractional dynamics and its applications, fractional differential equations, discrete mathematics, dynamic systems on time scales, the wavelet method and its applications, quantization of the systems with constraints, Hamilton-Jacobi formalism, geometries admitting generic and non-generic symmetries. He has published more than 600 papers indexed in SCI. He is one of the editors of 5 books published by Springer, one published by AIP Conference Proceedings and one of the co-authors of the monograph book titled "Fractional Calculus: Models and Numerical Methods", published in 2012 by World Scientific Publishing. Dumitru is an editorial board member of the following journals indexed in SCI: Abstract and Applied Analysis, Central European Journal of Physics, Advances in Difference Equations, Scientific Research and Essays (SRE) and Fractional Calculus and Applied Analysis. He is also an editorial board member of 12 different journals which are not indexed in SCI. Dumitru is a member of the advisory board of the "Mathematical Methods and Modeling for Complex Phenomena" book collection, published jointly by Higher Education Press and Springer. Also he sits as a scientific board member for the Chemistry and Physics of InTech Scientific Board for 2011/2012. He was also the thesis supervisor of 4 master students. Throughout his career, Dumitru was a visiting scientist of prestigious institutions such as: ICTP - Trieste, Italy, Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, Russia, Middle East Technical University, Ankara, Turkey, Doppler Institute and Technical University, Praque, Czech Republic, University of Olomouc, Czech Republic, Southern Illinois University, Carbondale, USA, University of Illinois at Chicago, USA, ISEP Porto, Portugal, Departamento de Matematica Aplicada Universidad Complutense de Madrid, Madrid, Spain and Yeshiva University, New York, USA. Dumitru was awarded the 2nd IFAC Workshop on Fractional Differentiation and its Applications Prize (19 - 21 July, Porto, Portugal, 2006) and Revista de Chimie Award (SCI indexed journal) in 2010. He received certificates of appreciation of ASME, Design Engineering Division, Technical Committee on Multibody Systems and Nonlinear Dynamics, 2009 and 2011 as an organizer of the 4th and 5th Symposia on Fractional Derivatives and Their Applications. Dumitru was an invited lecturer to prestigious international conferences and was the national organizer of 3rd IFAC Workshop on Fractional Differentiation and its Applications, Ankara, Turkey, 05 - 07 November, 2008. He is the chair of the program committee of the 5th Fractional Differentiation and Its Applications, Hohai University, China, May 14-17, 2012. Dumitru is a referee of more than 70 journals indexed in SCI, has received more than 4000 citations (excluded from citation overview: Self citations of all authors) in journals covered by SCI and his Hirsch index is 37. He was on the Thompson Reuter list of high cited researchers in 2015, 2016, 2017 and 2018.