Results in Physics 19 (2020) 103559

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Analysis of the fractional tumour-immune-vitamins model with Mittag–Leffler kernel

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ARTICLE INFO

Keywords:
Hyres-Ulam stability
Mittag–Leffler derivative
Fixed point theorems
Adams–Bashforth method

ABSTRACT

Recently, Atangana-Baleanu fractional derivative has got much attention of the researchers due to its non-locality and non-singularity. This operator contains an accurate kernel that describes the better dynamics of systems with a memory effect. In this paper, we investigate the fractional-order tumour-immune-vitamin model (TIVM) under Mittag–Leffler derivative. The existence of at least one solution and a unique solution has discussed through fixed point results. We established the Hyres-Ulam stability of the proposed model under the Mittag–Leffler derivative. The fractional Adams–Bashforth method has used to achieve numerical results. Finally, we simulate the obtained numerical results for different fractional orders to show the effect of vitamin intervention on decreased tumour cell growth and cancer risk. At the end of the paper, the conclusion has provided.

Introduction

Cancer is the uncontrolled development of abnormal cells (known as cancer cells, malignant cells, or tumour cells) anywhere in the body. Therefore, epidemiological studies used mostly to calculate the frequency of the disease and to indicate potential causes of the disease. Several epidemiological studies have shown that improved mortality rates for multiple cancers are related to diet, lifestyle, climate, and other alterations [1–3]. One of the pioneers in genomic researcher Venter pointed out that with the presence of the hundreds of independent factors, the anatomy of humans is much complicated than we imagined. Everyone talks about the genes that they got some characteristics from their parents, although, genes have little effect on the outcome of life. Genes offer valuable information about the increased risk of a disease, but they do not determine the precise cause of the disease or the actual incidence of it in most cases. The dynamic interaction between all proteins and cells that operate with environmental variables, not directly determined by the genetic code, causes most biological problems [4]. Recently, the relation between nutrition (like vitamins A, B-group, C, D, and E) and immunity got significant attention. The vitamins play a vital role in regulating the immune system to protect tissues from injury [5–7]. To describe the behaviour of the disease and to enhanced the methods of treatment, various mathematical models have been used. Since 1994, researchers have started to study cancer behaviour mathematically. Michaelis–Menten function used to study tumour-immune interaction [8,9]. In formulating a simple mathematical model, Mayer and others used ODEs [10] to explain the response of the immune system when pathogens attack the body. To formulate models of cancer and investigate the effect of tumour growth mostly on dynamics of other cells, the researchers used ODEs, PDEs, and delay differential equations (DDEs) [11–17]. Numerous models have developed to determine the primary risk factors. Mufudza proposed a mathematical model that uses DEs to illustrate the impact of estrogen on breast cancer dynamics [11]. In the development of an obesity-cancer model [18], in tumour response to chemotherapy, Roberto and others have used ODEs and discussed the connection between obesity and cancer [19]. A model that follows a balanced diet was introduced by Alharbi and Rambely in 2019. In the model, they used normal cells, and immune cells aim to describe how the immune system functions when abnormal cells appear in a tissue [20]. They also investigate the effects of the involvement of vitamins on strengthening the immune system and regulating the spread of tumour cells. They develop an ODE-governed tumour-immune-vitamin (TIVM) model consisting of two classes, namely tumour cells, and immune cells. They provided that the immune system is strengthened by a daily intake

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https://doi.org/10.1016/j.rinp.2020.103559
Received 25 September 2020; Received in revised form 9 October 2020; Accepted 12 October 2020
Available online 12 November 2020

of 55% of vitamins per day to prevent tumour cell growth [21]. The "tumour-immune-unhealthy diet model" (TIUNHDM) and TIVM dynamics lead to the study of the effect on immune and tumour cell dynamics of changes in the rate of vitamin intake. In 2018, the third WCRF and AICR [22] report proposed a dietary source of vitamins from healthy foods and beverages. However, because of their unexpected side effects, especially in cancer patients, high doses of diet supplements are not recommended. Recently, S. A. Alharbi and A. S. Rambely [23] proposed (TIVM) as

$$\begin{cases} \frac{d\mathbb{T}}{dt} &= \sigma_1\mathbb{T}[1-\sigma_2\mathbb{T}] - \sigma_3\mathbb{T}\mathbb{I} - \delta_1\mathbb{T}\mathbb{V}, \\ \frac{d\mathbb{I}}{dt} &= \omega - \eta\mathbb{I} + \frac{\rho\mathbb{I}\mathbb{T}}{f+\mathbb{T}} - \mu\mathbb{I}\mathbb{T} + \delta_2\mathbb{I}\mathbb{V}, \\ \frac{d\mathbb{V}}{dt} &= \kappa_1 - \kappa_2\mathbb{V}, \end{cases} \tag{1}$$

The description of the parameters are given below:

- The growth limit of tumour cancer cells is represented by σ_1
- σ_2 represents the tumour reduction due to the deformed tumour from the body during dietary metabolisation
- σ_3 is the rate of elimination or suppression of tumour cells due to the immune cell response
- ω denotes a constant source of immune cells that are produced daily in the body
- The natural death rate of immune cell is denoted by η
- ρ describes the rate of the presence of tumour cells incites the response of the immune system
- f is the threshold rate of the immune system
- μ represents rate of suppression of the immune cells
- δ_1 denotes the rate of the effect of vitamins on tumour cells
- δ_2 is the rate of the effect of vitamins on immune cells
- κ_1 is a regular rate of vitamins from natural sources of food and beverages
- κ_2 describes the rate of vitamins which are attracted by cells

Fractional calculus is one of the potential areas in which different properties of various materials hold more accurately than integer-order. Many researchers, extended classical calculus to fractional calculus and introduced different fractional order mathematical models [24–26]. Fractional calculus has been investigated qualitatively and numerically in [27–30]. The researcher implemented numerous effective methods to figure out the solution of linear and nonlinear FDEs, some of which are numerical FDEs [35,36] and some are analytical [31–34,37,38]. Recently, different types of fractional-order or nonlocal derivatives have been introduced by Riemann–Liouville based on the power-law. After that, Caputo-Fabrizio proposed a new fractional derivative using the exponential kernel [39–42], which faces problem to the locality of the kernel. To handle this problem, Atangana and Baleanu (AB) [43] proposed Mittag–Leffler function (MLF) of the nonsingular and nonlocal kernel. For more details see [44–47].

In this paper, we extend the model (1) by $\mathscr{ABC}$ derivative because of the performance of this operator in modelling infectious diseases and inspired by the above useful applications of some fractional operators in the epidemic model. Under $\mathscr{ABC}$ fractional derivative of order ϖ, we consider the model as:

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0{}^{\varpi}\mathbb{T}(t) &= \sigma_1\mathbb{T}[1-\sigma_2\mathbb{T}] - \sigma_3\mathbb{T}\mathbb{I} - \delta_1\mathbb{T}\mathbb{V}, \\ {}^{\mathscr{ABC}}\mathscr{D}_0{}^{\varpi}\mathbb{I}(t) &= \omega - \eta\mathbb{I} + \frac{\rho\mathbb{I}\mathbb{T}}{f+\mathbb{T}} - \mu\mathbb{I}\mathbb{T} + \delta_2\mathbb{I}\mathbb{V}, \\ {}^{\mathscr{ABC}}\mathscr{D}_0{}^{\varpi}\mathbb{V}(t) &= \kappa_1 - \kappa_2\mathbb{V}, \end{cases} \tag{2}$$

along with initial conditions:

$$\mathbb{T}(0) = T_0;\ \mathbb{I}(0) = I_0;\ \mathbb{V}(0) = V_0. \tag{3}$$

The manuscript is structured as: the introduction and motivation part of the manuscript is given in Section 'Introduction'. Section 'Preliminaries' is devoted to basic definitions of fractional calculus. Section 'Main work' provides the existence, uniqueness and HU-stability of the model (2) and gives the numerical results via fractional Adams–Bashforth method. Section 'Numerical simulations' presents the simulations of the numrical results for various fractional orders. The conclusion of the article is presented in Section 'Conclusion'.

Preliminaries

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Definition 0.1. [43] For fractional order $0 < \varpi \le 1$ and $\Theta \in \mathbb{H}^1(0,T)$, the left-sided $\mathscr{ABC}$ fractional derivative is defined as:

$${}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\Theta\Big(t\Big) = \frac{\mathfrak{N}(\varpi)}{(1-\varpi)}\int_0^t \mathbb{E}_{\varpi}\Big(\frac{-\varpi}{\varpi-1}(t-\Theta)^{\varpi}\Big)\Theta'\Big(\Theta\Big)d\Theta, \quad t>0.$$

where $\mathfrak{N}\Big(\varpi\Big) = \frac{\varpi}{2-\varpi}$ denotes normalization function with the property $\mathfrak{N}(0) = \mathfrak{N}(1) = 1$ and $\mathbb{E}_{\varpi}$ represents Mittag–Leffler function given below:

$$\mathbb{E}_{\varpi}\Big(\zeta\Big) = \sum_{k=0}^{\infty}\frac{\zeta^k}{\Gamma(\varpi k+1)},$$

where $Re(\varpi) > 0$.

Definition 0.2. [43] For fractional order $0 < \varpi \le 1$ and $\Theta \in \mathbb{H}^1(0,T)$, the left-sided $\mathscr{ABC}$ fractional integral is defined as:

$${}^{\mathscr{ABC}}\mathscr{I}_0^{\varpi}\Theta\Big(t\Big) = \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\Theta\Big(t\Big) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\Theta\Big(\Theta\Big)d\Theta, \quad t > 0.$$

Lemma 0.3. [43] *The solution to $\mathscr{ABC}$ FDE given by*

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\Theta(t) = \mathfrak{P}(t), \\ \Theta(0) = \Theta_0, \end{cases} \tag{4}$$

is equivalent the integral equation as follows:

$$\Theta\Big(t\Big) = \mathfrak{f}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{P}\Big(t\Big) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{P}\Big(\Theta\Big)d\Theta.$$

Main work

Existence and uniqueness theory

In this section, we will derive the existence and uniqueness results of the system (2). We can write the proposed model (2) as

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{S}(t) &= \mathscr{U}_1(t,\mathbb{T},\mathbb{I},\mathbb{V}), \\ {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{M}(\mathfrak{t}) &= \mathscr{U}_2(\mathfrak{t},\mathbb{T},\mathbb{I},\mathbb{V}), \\ {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{V}(\mathfrak{t}) &= \mathscr{U}_3(\mathfrak{t},\mathbb{T},\mathbb{I},\mathbb{V}), \end{cases} \tag{5}$$

where

$$\begin{cases} \mathscr{U}_1(t,\mathbb{T},\mathbb{I},\mathbb{V}) &= \sigma_1\mathbb{T}[1-\sigma_2\mathbb{T}] - \sigma_3\mathbb{T}\mathbb{I} - \delta_1\mathbb{T}\mathbb{V}, \\ \mathscr{U}_2(\mathfrak{t},\mathbb{T},\mathbb{I},\mathbb{V}) &= \omega - \eta\mathbb{I} + \frac{\rho\mathbb{I}\mathbb{T}}{\mathfrak{f}+\mathbb{T}} - \mu\mathbb{I}\mathbb{T} + \delta_2\mathbb{I}\mathbb{V}, \\ \mathscr{U}_3(\mathfrak{t},\mathbb{T},\mathbb{I},\mathbb{V}) &= \kappa_1 - \kappa_2\mathbb{V}. \end{cases} \tag{6}$$

We can write simply the proposed model as:

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathfrak{H}(t) = \mathfrak{E}(t,\mathfrak{H}(t)), \\ \mathfrak{H}(0) = \mathfrak{H}_0 \geqslant 0, \end{cases} \tag{7}$$

where

$$\begin{cases} \mathfrak{H}(t) & = (\mathbb{T},\mathbb{I},\mathbb{V})^{\mathbb{T}}, \\ \mathfrak{H}_0 & = (\mathbb{T}_0,\mathbb{I}_0,\mathbb{V}_0)^{\mathbb{T}}, \\ \mathfrak{E}(\mathfrak{t},\mathfrak{H}(\mathfrak{t})) & = (\mathscr{U}_1,\mathscr{U}_2,\mathscr{U}_3)^{\mathbb{T}}. \end{cases} \tag{8}$$

Applying $\mathscr{ABC}$ fractional integral to (7) and using initial conditions, we obtain the equivalent form of (7) as

$$\mathfrak{H}\left(t\right) = \mathfrak{H}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{E}(t,\mathfrak{H}(t)) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))d\Theta. \tag{9}$$

Now define a Banach space $\mathscr{B} = \mathfrak{C}(\mathbb{X},R^3)$ on $\mathbb{X} = [0,\mathscr{T}]$ with the following norm

$$\|\mathfrak{H}\| = \sup_{t\in X}\{\mathfrak{H}(t) : \mathfrak{H}\in\mathscr{B}\}.$$

Suppose that for each $\mathfrak{H}\in\mathscr{B}$ and $t\in[0,\mathscr{T}]$, the function $\mathfrak{E}(t,\mathfrak{H}(t))$ fulfill the conditions given below.

- $\exists$ two constants $\lambda_{\mathfrak{E}}$ and $\rho_{\mathfrak{E}}$ such that

$$|\mathfrak{E}(t,\mathfrak{H}(t))| \leqslant \lambda_{\mathfrak{E}}|\mathfrak{H}| + \rho_{\mathfrak{E}}. \tag{10}$$

- $\exists$ a constant $L_{\mathfrak{E}} > 0$ such that

$$|\mathfrak{E}(t,\mathfrak{H}_1(t)) - \mathfrak{E}(t,\mathfrak{H}_2(t))| \leqslant L_{\mathfrak{E}}|\mathfrak{H}_1 - \mathfrak{H}_2|. \tag{11}$$

Now we define the operators $\mathfrak{P}_1$ and $\mathfrak{P}_2$ such that:

$$\mathfrak{P}_1\mathfrak{H}\left(t\right) = \mathfrak{H}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{E}(t,\mathfrak{H}(t)),$$

$$\mathfrak{P}_2\mathfrak{H}\left(t\right) = \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))d\Theta,$$

where $\mathfrak{P}_1 + \mathfrak{P}_2 = \mathscr{B}$.

Theorem 0.4. *If the following conditions along with (10) and (11) hold.*

(i) $\frac{(1-\varpi)}{\mathfrak{N}(\varpi)}L_{\mathfrak{E}} < 1$.

(ii) $\nabla_1 = \left[\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right]\rho_{\mathfrak{E}} < 1$.

(iii) $\nabla_2 = \left\{\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right\}\lambda_{\mathfrak{E}} < 1$.

Then the system (2) posses at least one solution.

Proof. Let us define a closed and convex set as $\mathscr{B}_\tau = \{\mathfrak{H}\in\mathscr{B} : \|\mathfrak{H}\| \leqslant \tau\}$. First we have to prove that $\mathfrak{P}_1\mathfrak{H}_1 + \mathfrak{P}_2\mathfrak{H}_2 \in \mathscr{B}_\tau$, for $\mathfrak{H}_1,\mathfrak{H}_2\in\mathscr{B}_\tau$. For this use Eq. (10), we have

$$\begin{aligned} &\|\mathfrak{P}_1\mathfrak{H}_1 + \mathfrak{P}_2\mathfrak{H}_2\| \leqslant \\ &\leqslant \sup_{t\in[0,\mathscr{T}]}\left\{|\mathfrak{H}_0| + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}|\mathfrak{E}(t,\mathfrak{H}(t))| + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}|\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))|d\Theta\right\} \\ &\leqslant \left\{|\mathfrak{H}_0| + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}(\lambda_{\mathfrak{E}}\|\mathfrak{H}\| + \rho_{\mathfrak{E}}) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}(\lambda_{\mathfrak{E}}\|\mathfrak{H}\| + \rho_{\mathfrak{E}})d\Theta\right\} \\ &= |\mathfrak{H}_0| + \left\{\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right\}\rho_{\mathfrak{E}} + \left\{\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right\}\lambda_{\mathfrak{E}}\tau \\ &= \nabla_1 + \nabla_2\tau \leqslant \tau. \end{aligned}$$

This shows that $\mathfrak{P}_1\mathfrak{H}_1 + \mathfrak{P}_2\mathfrak{H}_2 \in \mathscr{B}_\tau$. Next we show that $\mathfrak{P}_1$ is contraction. For any $\mathfrak{H}_1,\mathfrak{H}_2\in\mathscr{B}_\tau$ and using Lipschitz condition, we have:

$$\begin{aligned} \|\mathfrak{P}_1\mathfrak{H}_1 - \mathfrak{P}_1\mathfrak{H}_2\| &= \sup_{t\in[0,\mathscr{T}]}\frac{(1-\varpi)}{\mathfrak{N}(\varpi)}|\mathfrak{E}(t,\mathfrak{H}_1(t)) - \mathfrak{E}(t,\mathfrak{H}_2(t))| \\ &\leqslant \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}L_{\mathfrak{E}}\sup_{t\in[0,\mathscr{T}]}|\mathfrak{H}_1(t) - \mathfrak{H}_2(t)| \\ &\leqslant \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}L_{\mathfrak{E}}\|\mathfrak{H}_1 - \mathfrak{H}_2\|. \end{aligned}$$

where $\frac{(1-\varpi)}{\mathfrak{N}(\varpi)}L_{\mathfrak{E}} < 1$. Hence, we proved that $\mathfrak{P}_1$ is a contraction. Next, we need to prove that $\mathfrak{P}_2$ is relatively compact. Let $\mathfrak{H}\in\mathscr{B}_\tau$, consider

$$\begin{aligned} \|\mathfrak{P}_2\mathfrak{H}\| &\leqslant \sup_{t\in[0,\mathscr{T}]}\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}|\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))|d\Theta \\ &\leqslant \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\sup_{t\in[0,\mathscr{T}]}[\lambda_{\mathfrak{E}}|\mathfrak{H}| + \rho_{\mathfrak{E}}]d\Theta \\ &\leqslant \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}[\lambda_{\mathfrak{E}}\|\mathfrak{H}\| + \rho_{\mathfrak{E}}]d\Theta \\ &\leqslant \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}[\lambda_{\mathfrak{E}}\tau + \rho_{\mathfrak{E}}]. \end{aligned}$$

Hence, $\mathfrak{P}_2$ is uniformly bounded on $\mathscr{B}_\tau$. Lastly, we prove that $\mathfrak{P}_2$ is equicontinuous. For this let $\mathfrak{H}\in\mathscr{B}_\tau$ and $t_1,t_2\in[0,\mathscr{T}]$ such that $t_1 < t_2$. Then

$$\begin{aligned} \|\mathfrak{P}_2\mathfrak{H}(t_1) - \mathfrak{P}_2\mathfrak{H}(t_2)\| &\leqslant \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_{t_1}^{t_2}(t_2-\Theta)^{\varpi-1}|\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))|d\Theta \\ &+ \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^{t_2}\left[(t_1-\Theta)^{\varpi-1} - (t_2-\Theta)^{\varpi-1}\right] \\ &|\mathfrak{E}(\Theta,\mathfrak{H}(\Theta))|d\Theta \\ &\leqslant \frac{2[\lambda_{\mathfrak{E}}\tau + \rho_{\mathfrak{E}}]}{\mathfrak{N}(\varpi)\Gamma(\varpi)}[(t_2-t_1)^{\varpi}]. \end{aligned}$$

Thus, $\|\mathfrak{P}_2\mathfrak{H}(t_1) - \mathfrak{P}_2\mathfrak{H}(t_2)\| \to 0$ as $t_2\to t_1$. Hence the operator $\mathfrak{P}_2$ is relatively compact by Arzela-Ascoli theorem, so $\mathfrak{P}_2$ is equicontinuous. Thus the integral Eq. (9) has at least one solution. Therefore the consequent model (14) has at least one solution □

Theorem 0.5. *Suppose that the condition (11) holds. Then the Eq. (9) posses at most one solution if*

$$\left\{\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right\}L_{\mathfrak{E}} < 1.$$

Proof. For $t\in[0,\mathscr{T}]$, and $\mathfrak{H},\mathfrak{H}^*\in\mathscr{B}$ we have

$$\begin{aligned} \|\mathfrak{P}\mathfrak{H}(t) - \mathfrak{P}\mathfrak{H}^*(t)\| &\leqslant \max_{t\in[0,\mathscr{T}]}\frac{(1-\varpi)}{\mathfrak{N}(\varpi)}|\mathfrak{E}(t,\mathfrak{H}(t)) - \mathfrak{E}(t,\mathfrak{H}^*(t))| \\ &+ \max_{t\in[0,\mathscr{T}]}\frac{\varpi}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}|\mathfrak{E}(t,\mathfrak{H}(t)) \\ &- \mathfrak{E}(t,\mathfrak{H}^*(t))|d\Theta \leqslant \left(\frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\right. \\ &\left. + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right)L_{\mathfrak{E}}\|\mathfrak{H} - \mathfrak{H}^*\| \end{aligned}$$

Since $\left\{\frac{(1-\varpi)}{\mathfrak{N}(\varpi)} + \frac{T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right\}L_{\mathfrak{E}} < 1$, thus the operator $\mathfrak{P}$ satisfies the contraction condition. By Banach fixed point theorem the integral Eq. (9) posses at most one solution. Hence the model (2) has at most one solution. □

HU-stability

In this section, we will explore the HU-stability of (7).

Definition 0.6. The nodel (2) is HU-stable. If for any $\zeta > 0$ and $\widetilde{\mathfrak{H}} \in \mathscr{B}$, we have

$$\left| {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\widetilde{\mathfrak{H}}\left(t\right) - \mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right)\right| \leqslant \zeta, \tag{12}$$

then $\exists \mathfrak{H} \in \mathscr{B}$ satisfying system (2) with initial conditions $\mathfrak{H}(0) = \widetilde{\mathfrak{H}}(0) = \widetilde{\mathfrak{H}}_0$, such that $\left\|\widetilde{\mathfrak{H}} - \mathfrak{H}\right\| \leqslant \mu^*\zeta$, for $\mu^* > 0$. where

$$\begin{cases} \widetilde{\mathfrak{H}}\left(t\right) = \left(\widetilde{\mathbb{T}}, \widetilde{\mathbb{I}}, \widetilde{\mathbb{V}}\right)^T, \\ \widetilde{\mathfrak{H}}_0 = \left(\widetilde{T_0}, \widetilde{I_0}, \widetilde{V_0}\right), \\ \mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) = \left(\widetilde{\mathscr{U}_1}, \widetilde{\mathscr{U}_2}, \widetilde{\mathscr{U}_3}\right)^T, \\ \zeta = \max(\zeta_i)^T, \ i = 1, 2, \cdots, 5, \\ \mu^* = \max(\mu_i^*)^T, \ i = 1, 2, \cdots, 5. \end{cases} \tag{13}$$

Remark 0.7. Let us take a small perturbation $\Lambda \in C[0, \mathscr{T}]$ such that $\Lambda(0) = 0$ along with following properties:

(i) $|\Lambda(t)| \leqslant \zeta$, for $t \in [0, \mathscr{T}]$ and $\zeta > 0$,
(ii) For $t \in [0, \mathscr{T}]$, we have

$${}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\widetilde{\mathfrak{H}}\left(t\right) = \mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) + \Lambda\left(t\right),$$

where $\Lambda(t) = (\Lambda_1(t), \Lambda_2(t), \Lambda_3(t))^T$.

Lemma 0.8. *The solution $\widetilde{\mathfrak{H}}_h(t)$ to the following perturbed system*

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\widetilde{\mathfrak{H}}\left(t\right) = \mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) + \Lambda\left(t\right), \\ \mathfrak{H}\left(0\right) = \widetilde{\mathfrak{H}}_0, \end{cases} \tag{14}$$

fulfills the following relation

$$\left|\widetilde{\mathfrak{H}}_h\left(t\right) - \widetilde{\mathfrak{H}}\left(t\right)\right| \leqslant \left[\frac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right]\zeta. \tag{15}$$

Proof. Upon using fractional integral Eq. (14) gets the form

$$\widetilde{\mathfrak{H}}_h\left(t\right) = \left\{\widetilde{\mathfrak{H}}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) + \frac{\varpi}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{E}\left(\Theta, \widetilde{\mathfrak{H}}\left(\Theta\right)\right)d\Theta + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\Lambda\left(t\right) + \frac{\varpi}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\Lambda\left(\Theta\right)d\Theta,\right. \tag{16}$$

also

$$\widetilde{\mathfrak{H}}\left(t\right) = \widetilde{\mathfrak{H}}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) + \frac{\varpi}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{E}\left(\Theta, \widetilde{\mathfrak{H}}\left(\Theta\right)\right)d\Theta \tag{17}$$

Using remark (i), we reach

$$\begin{aligned} \left|\widetilde{\mathfrak{H}}_h\left(t\right) - \widetilde{\mathfrak{H}}\left(t\right)\right| \leqslant & \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}|\Lambda(t)| + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}|\Lambda(\Theta)|d\Theta \\ \leqslant & \left[\frac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right]\zeta. \end{aligned}$$

Hence $\widetilde{\mathfrak{H}}_h(t)$ fulfill the condition (15). □

Theorem 0.9. *If presumptions of theorem (3.1) hold, then system (2) is HU-stable.*

Proof. Suppose that $\widetilde{\mathfrak{H}} \in \mathscr{B}$ be another solution of (12) and $\mathfrak{H} \in \mathscr{B}$ be at most one solution of model (2) with initial condition $\mathfrak{H}(0) = \widetilde{\mathfrak{H}}_0$.

$$\mathfrak{H}\left(t\right) = \widetilde{\mathfrak{H}}_0 + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{E}(t, \mathfrak{H}(t)) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\mathfrak{E}(\Theta, \mathfrak{H}(\Theta))d\Theta.$$

Using Lemma (3.5), we reach

$$\begin{aligned} \left|\widetilde{\mathfrak{H}}\left(t\right) - \mathfrak{H}\left(t\right)\right| \leqslant & \left|\widetilde{\mathfrak{H}}_h\left(t\right) - \widetilde{\mathfrak{H}}\left(t\right)\right| + \left|\widetilde{\mathfrak{H}}_h\left(t\right) - \mathfrak{H}\left(t\right)\right| \\ \leqslant & 2\left[\frac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right]\zeta + \frac{1-\varpi}{\mathfrak{N}(\varpi)}\left|\mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) - \mathfrak{E}(t, \mathfrak{H}(t))\right| \\ & + \frac{\varpi}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\int_0^t (t-\Theta)^{\varpi-1}\left|\mathfrak{E}\left(t, \widetilde{\mathfrak{H}}\left(t\right)\right) - \mathfrak{E}(t, \mathfrak{H}(t))\right|d\Theta \\ \leqslant & 2\left[\frac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right]\zeta + \left(\frac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right)L_{\mathfrak{E}}\left\|\widetilde{\mathfrak{H}} - \mathfrak{H}\right\| \end{aligned}$$

which means that

$$\left\|\widetilde{\mathfrak{H}} - \mathfrak{H}\right\| \leqslant \frac{2\varsigma\zeta}{1-\Psi}, \tag{18}$$

where

$$\begin{cases} \varsigma = \dfrac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}, \\ \Psi = \left(\dfrac{(1-\varpi)\Gamma(\varpi) + T^{\varpi}}{\mathfrak{N}(\varpi)\Gamma(\varpi)}\right)L_{\mathfrak{E}}. \end{cases} \tag{19}$$

Eq. (18) becomes $\left\|\widetilde{\mathfrak{H}} - \mathfrak{H}\right\| \leqslant \mu^*\zeta$ for $\mu^* = \frac{2\varsigma}{1-\Psi}$. Hence, our proposed model (2) is HU-stable. □

Numerical scheme

Here, the numerical results of the system (2) are obtained through the fractional Adams–Bashforth method. Consider the model (2) as

$$\begin{cases} {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{T}\left(t\right) & = \mathfrak{F}_1(t, \mathbb{T}(t)), \\ {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{I}\left(t\right) & = \mathfrak{F}_2(t, \mathbb{I}(t)), \\ {}^{\mathscr{ABC}}\mathscr{D}_0^{\varpi}\mathbb{V}\left(t\right) & = \mathfrak{F}_3(t, \mathbb{V}(t)), \end{cases} \tag{20}$$

where

$$\begin{cases} \mathfrak{F}_1(t, \mathbb{T}(t)) & = \sigma_1\mathbb{T}[1 - \sigma_2\mathbb{T}] - \sigma_3\mathbb{T}\mathbb{I} - \delta_1\mathbb{T}\mathbb{V}, \\ \mathfrak{F}_2(t, \mathbb{I}(t)) & = \omega - \eta\mathbb{I} + \dfrac{\rho\mathbb{I}\mathbb{T}}{f + \mathbb{T}} - \mu\mathbb{I}\mathbb{T} + \delta_2\mathbb{I}\mathbb{V}, \\ \mathfrak{F}_3(t, \mathbb{V}(t)) & = \kappa_1 - \kappa_2\mathbb{V}. \end{cases}$$

The equivalent form of (20) is given by

$$\begin{cases} \mathbb{T}(t) &= \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t,\mathbb{T}(t)) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t(t-\Theta)^{\varpi-1}\mathfrak{F}_1(\Theta,\mathbb{T}(\Theta))d\Theta, \\ \mathbb{I}(t) &= \mathbb{I}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_2(t,\mathbb{I}(t)) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t(t-\Theta)^{\varpi-1}\mathfrak{F}_2(\Theta,\mathbb{I}(\Theta))d\Theta, \\ \mathbb{V}(t) &= \mathbb{V}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_3(t,\mathbb{V}(t)) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^t(t-\Theta)^{\varpi-1}\mathfrak{F}_3(\Theta,\mathbb{V}(\Theta))d\Theta, \end{cases} \tag{21}$$

We take the first equation of (21) to deduce an iterative scheme, and for the remaining equations of (21) we will write only the main results

$$\begin{cases} \mathbb{T}(t) &= \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t,\mathbb{T}(\mathfrak{t})) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^{\mathfrak{t}}(\mathfrak{t}-\Theta)^{\varpi-1}\mathfrak{F}_1(\Theta,\mathbb{T}(\Theta))\mathfrak{d}\Theta, \end{cases} \tag{22}$$

At $t = t_{b+1}$, for $b = 0,1,2,\cdots$, Eq. (22) becomes

$$\mathbb{T}(t_{b+1}) = \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t_b,\mathbb{T}(t_b)) + \frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\int_0^{t_{b+1}}(t_{b+1}-\Theta)^{\varpi-1}\mathfrak{F}_1(\Theta,\mathbb{T}(\Theta))d\Theta,$$

$$\begin{cases} \mathbb{T}\left(t_{b+1}\right) = \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t_b,\mathbb{T}(t_b)) \\ +\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\sum_{z=0}^{b}\int_{t_z}^{t_{z+1}}(t_{b+1}-\Theta)^{\varpi-1}\mathfrak{F}_1(\Theta,T(\Theta))d\Theta. \end{cases} \tag{23}$$

Now, we use the following interpolation polynomial to approximate the function $\mathfrak{F}_1(\Theta,\mathbb{T}(\Theta))$ on the interval $[t_z, t_{z+1}]$.

$$\mathfrak{F}_1(\Theta,\mathbb{T}(\Theta)) \cong \frac{\mathfrak{F}_1(t_z,\mathbb{T}(t_z))}{h}(t-t_{z-1}) + \frac{\mathfrak{F}_1(t_{z-1},\mathbb{T}(t_{z-1}))}{h}(t-t_z)$$

So Eq. (23) gets the form

$$\begin{cases} \mathbb{T}(t_{b+1}) &= \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t_b,\mathbb{T}(t_b)) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\frac{1}{\Gamma(\varpi)}\sum_{z=0}^{q}\left[\frac{\mathfrak{F}_1(t_z,\mathbb{T}(t_z))}{h}\int_{t_z}^{t_{z+1}}(t-t_{z-1})(t_{b+1}-t)^{\varpi-1}dt\right. \\ &\left.-\frac{\mathfrak{F}_1(t_{z-1},\mathbb{T}(t_{z-1}))}{h}\int_{t_z}^{t_{z+1}}(t-t_z)(t_{b+1}-t)^{\varpi-1}dt\right]. \end{cases} \tag{24}$$

Without the loss of generality, let

$$\begin{cases} I_{z-1,\varpi} = \int_{t_z}^{t_{z+1}}(t-t_{z-1})(t_{b+1}-t)^{\varpi-1}dt, \\ I_{z,\varpi} = \int_{t_z}^{t_{z+1}}(t-t_z)(t_{b+1}-t)^{\varpi-1}dt. \end{cases} \tag{25}$$

Eq. (24) becomes

$$\begin{cases} \mathbb{T}\left(t_{b+1}\right) = \mathbb{T}\left(0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t_b,\mathbb{T}(t_b)) \\ \frac{\varpi}{\Gamma(\varpi)\mathfrak{N}(\varpi)}\sum_{m=0}^{q}\left[\frac{\mathfrak{F}_1(t_z,\mathbb{T}(t_z))}{h}I_{z-1,\varpi} - \frac{\mathfrak{F}_1(t_{z-1},\mathbb{T}(t_{z-1}))}{h}I_{z,\varpi}\right]. \end{cases} \tag{26}$$

Next, we evaluate the integrals $I_{z-1,\varpi}$ and $I_{z,\varpi}$ as follows

$$\begin{aligned} I_{z-1,\varpi} &= \int_{t_z}^{t_{z+1}}(t-t_{z-1})(t_{b+1}-t)^{\varpi-1}dt \\ &= -\frac{1}{\varpi}\left[(t_{z+1}-t_{z-1})(t_{b+1}-t_{z+1})^{\varpi} - (t_z-t_{z-1})(t_{b+1}-t_z)^{\varpi}\right] \\ &\quad -\frac{1}{\varpi(\varpi+1)}\left[(t_{b+1}-t_{z+1})^{\varpi+1} - (t_{b+1}-t_z)^{\varpi+1}\right]. \end{aligned}$$

Put $t_z = zh$,we get

$$\begin{aligned} I_{z-1,\varpi} &= -\frac{h^{\varpi+1}}{\varpi}[(z+1-z+1)(b+1-z-1)^{\varpi} \\ &\quad -(z-z+1)(b+1-z)^{\varpi}] \\ &\quad -\frac{h^{\varpi+1}}{\varpi(\varpi+1)}\left[(b+1-z-1)^{\varpi+1} - (b+1-z)^{\varpi+1}\right] \\ &= \frac{h^{\varpi+1}}{\varpi(\varpi+1)}[(b-z)^{\varpi}(-2(\varpi+1)-(b-z)) \\ &\quad +(b+1-z)^{\varpi}(\varpi+1+b+1-z)], \end{aligned}$$

thus, we obtain

$$\begin{cases} I_{z-1,\varpi} &= \frac{h^{\varpi+1}}{\varpi(\varpi+1)}[(b-z+1)^{\varpi}(b-z+2+\varpi) \\ &-(b-z)^{\varpi}(b-z+2+2\varpi)] \end{cases} \tag{27}$$

and

$$\begin{aligned} I_{z,\varpi} &= \int_{t_z}^{t_{z+1}}(t-t_z)(t_{b+1}-t)^{\varpi-1}dt \\ &= -\frac{1}{\varpi}\left[(t_{z+1}-t_z)(t_{b+1}-t_{z+1})^{\varpi}\right] \\ &\quad -\frac{1}{\varpi(\varpi+1)}\left[(t_{b+1}-t_{z+1})^{\varpi+1} - (t_{b+1}-t_z)^{\varpi+1}\right] \end{aligned}$$

Put $t_z = zh$, we get

$$\begin{aligned} I_{z,\varpi} &= -\frac{h^{\varpi+1}}{\varpi}\left[(z+1-z)(b+1-z-1)^{\varpi}\right] \\ &\quad -\frac{h^{\varpi+1}}{\varpi(\varpi+1)}\left[(b+1-z-1)^{\varpi+1} - (b+1-z)^{\varpi+1}\right], \\ &= \frac{h^{\varpi+1}}{\varpi(\varpi+1)}\left[(b-z)^{\varpi}(-\varpi-1-b+z) + (b+1-z)^{\varpi+1}\right]. \end{aligned}$$

Thus, we get

$$\begin{cases} I_{z,\varpi} &= \frac{h^{\varpi+1}}{\varpi(\varpi+1)}\left[(b+1-z)^{\varpi+1} - (b-z)^{\varpi}(b-z+1+\varpi)\right]. \end{cases} \tag{28}$$

Upon substituting Eqs. (27) and (28) into the Eq. (26), we get

$$\begin{cases} \mathbb{T}(t_{b+1}) &= \mathbb{T}\left(t_0\right) + \frac{(1-\varpi)}{\mathfrak{N}(\varpi)}\mathfrak{F}_1(t_b,\mathbb{T}(t_b)) \\ &+\frac{\varpi}{\mathfrak{N}(\varpi)}\sum_{z=0}^{b}\left\{\frac{\mathfrak{F}_1(t_z,\mathbb{T}(t_z))}{\Gamma(\varpi+2)}h^{\varpi}[(b+1-z)^{\varpi}(b-z+2+\varpi)\right. \\ &(b-z)^{\varpi}(b-z+2+2\varpi)] \\ &-\frac{\mathfrak{F}_1(t_{z-1},\mathbb{T}(\mathfrak{t}_{z-1}))}{\Gamma(\varpi+2)}h^{\varpi}\left[(b+1-z)^{\varpi+1}\right. \\ &\left.\left.-(b-z)^{\varpi}(b-z+1+\varpi)\right]\right\}. \end{cases} \tag{29}$$

Similarly for the remaining equations, we have

$$\begin{cases} \mathbb{I}(t_{b+1}) &= \mathbb{I}\left(t_0\right) + \dfrac{(1-\varpi)}{\Re(\varpi)}\mathfrak{F}_2(t_b, \mathbb{I}(t_b)) \\ & +\dfrac{\varpi}{\Re(\varpi)}\displaystyle\sum_{z=0}^{b}\left\{\dfrac{\mathfrak{F}_2(t_z, \mathbb{I}(t_z))}{\Gamma(\varpi+2)}h^{\varpi}[(b+1-z)^{\varpi}(b-z+2+\varpi)\right. \\ & (b-z)^{\varpi}(b-z+2+2\varpi)] \\ & -\dfrac{\mathfrak{F}_2(t_{z-1}, \mathbb{I}(t_{z-1}))}{\Gamma(\varpi+2)}h^{\varpi}\left[(b+1-z)^{\varpi+1}\right. \\ & \left.\left. -(b-z)^{\varpi}(b-z+1+\varpi)\right]\right\}. \end{cases} \tag{30}$$

$$\begin{cases} \mathbb{V}(t_{b+1}) &= \mathbb{V}\left(t_0\right) + \dfrac{(1-\varpi)}{\Re(\varpi)}\mathfrak{F}_3(t_b, \mathbb{V}(t_b)) \\ & +\dfrac{\varpi}{\Re(\varpi)}\displaystyle\sum_{z=0}^{b}\left\{\dfrac{\mathfrak{F}_3(t_z, \mathbb{V}(t_z))}{\Gamma(\varpi+2)}h^{\varpi}[(b+1-z)^{\varpi}(b-z+2+\varpi)\right. \\ & (b-z)^{\varpi}(b-z+2+2\varpi)] \\ & -\dfrac{\mathfrak{F}_3(t_{z-1}, \mathbb{V}(t_{z-1}))}{\Gamma(\varpi+2)}h^{\varpi}\left[(b+1-z)^{\varpi+1}\right. \\ & \left.\left. -(b-z)^{\varpi}(b-z+1+\varpi)\right]\right\}. \end{cases} \tag{31}$$

Numerical simulations

Here we present the simulation of the numerical results for different fractional-order via Matlab. We take the initial conditions from ([23]) as $\mathbb{T}(0) = 1$, $\mathbb{I}(0) = 1.22$, $\mathbb{V}(0) = 5$. The parameter values of the Table 1 are used for the simulations of the numerical results. Since the fractional differential derivative has a great deal of freedom that offers a full spectrum of geometry, thus a few fractional orders have been taken to interpret the model's complex behaviors under consideration. From the figures, we have observed that strengthening immune cells through vitamin intervention can tend to slow tumour cells growth and division. The tumor cells and vitamin intervention is decreasing while the immune cells are increasing as shown in the figures. It is noticed that the smaller the fractional-order, the faster the process of decay or growth; thus, stability on smaller fractional orders occurs quickly. Also, we have analyzed that, as the fractional order approaches 1, then the fractional-order solution tends to the integer-order solution. Thus, the arbitrary order model of the TIVM model generalizes the integer-order model and provide global dynamics of the opposite relation between immune cells and the intervention of vitamins and tumour cells growth and division. see Fig. 1–3.

Table 1
Values of the parameters for simulation.

Name	Parameters values
σ_1	0.4426
σ_2	0.4
σ_3	0.5140
ω	0.7
η	0.57
ρ	0.7829
f	0.8620
μ	0.1859
δ_1	0.6142
δ_2	0.3628
κ_1	0.5463
κ_2	0.9757

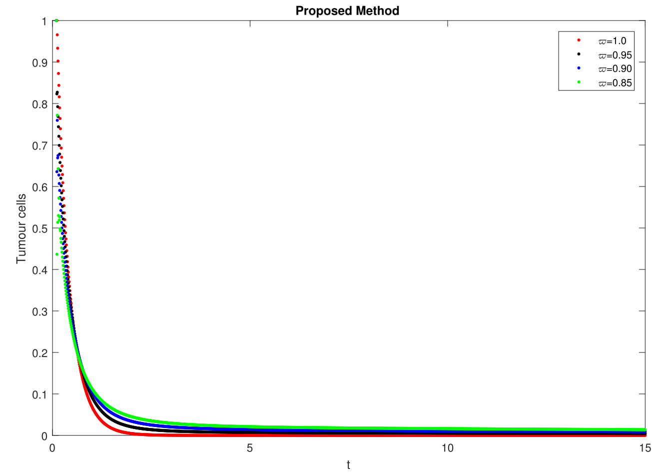

Fig. 1. Dynamical behavior of tumour cells at $\varpi = 1.0, 0.95, 0.90, 0.85$.

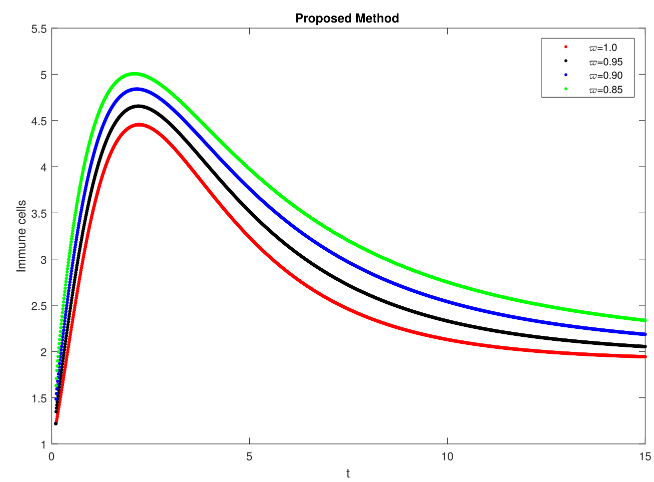

Fig. 2. Dynamical behavior of immune cells at $\varpi = 1.0, 0.95, 0.90, 0.85$.

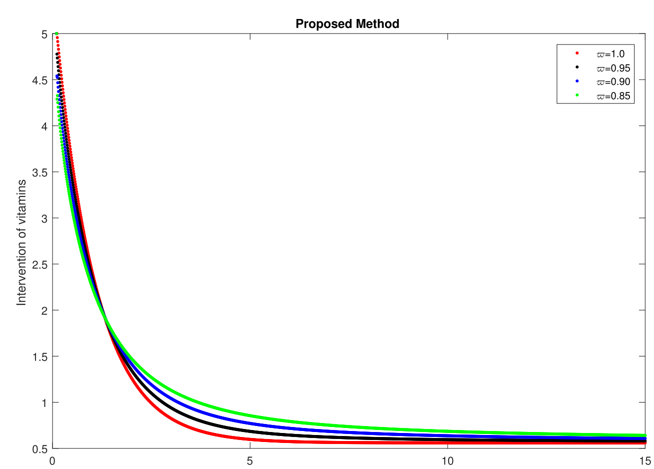

Fig. 3. Dynamical behavior of intervention of vitamin at $\varpi = 1.0, 0.95, 0.90, 0.85$.

Conclusion

In this paper, the tumour-immune-vitamins model is generalized by Atangana-Baleanu fractional derivative. The proposed model has studied with two aspects: qualitatively and quantitatively. The existence of at least one and a unique solution has explored by the fixed point theory. The stability of the solution has carried out through HU-stability. Numerical results have obtained for the proposed model through the Admas-Bashforth method. Numerical simulations have provided for the numerical results: showing the global dynamics of the TIV model. Also, the simulations showed that the fractional-order curves approach an integer-order when $\varpi\rightarrow 1$. Thus, the fractional-order TIV model provides better results than the integer-order model. In future, we will study the considered model under fractal-fractional operators.

Conflict of interest

There exist no conflict of interest regarding this research work.

Funding

No source exist of funding this work.

We are thankful to the reviewers for their careful reading and suggestions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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