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# Application of Modified Extended Tanh Technique for Solving Complex Ginzburg–Landau Equation Considering Kerr Law Nonlinearity

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**Abstract:** The purpose of this work is to find new soliton solutions of the complex Ginzburg–Landau equation (GLE) with Kerr law non-linearity. The considered equation is an imperative nonlinear partial differential equation (PDE) in the field of physics. The applications of complex GLE can be found in optics, plasma and other related fields. The modified extended tanh technique with Riccati equation is applied to solve the Complex GLE. The results are presented under a suitable choice for the values of parameters. Figures are shown using the three and two-dimensional plots to represent the shape of the solution in real, and imaginary parts in order to discuss the similarities and difference between them. The graphical representation of the results depicts the typical behavior of soliton solutions. The obtained soliton solutions are of different forms, such as, hyperbolic and trigonometric functions. The results presented in this paper are novel and reported first time in the literature. Simulation results establish the validity and applicability of the suggested technique for the complex GLE. The suggested method with symbolic computational software such as, Mathematica and Maple, is proven as an effective way to acquire the soliton solutions of nonlinear partial differential equations (PDEs) as well as complex PDEs.

**Keywords:** Modified extended tanh technique; soliton solution; complex Ginzburg–Landau equation; Riccati equation

## 1 Introduction

Exact traveling wave solutions of nonlinear PDEs has become imperative in the study physical phenomena. NPDEs are used to express different real-world phenomena of applied sciences, such as,

fluid and solid mechanics, quantum theory, shallow water waves, plasma physics, and chemical reaction diffusion models etc.

Many semi-analytical and analytical techniques have been studied to solve nonlinear PDEs, for example the auxiliary equation technique [1], the expansion (G′/G) technique [2], the exponential function technique [3,4], the generalized Kudryashov technique [5], the first integral technique [6,7], the Jacobi elliptic technique [8], the tan($\phi/2$)–expansion technique [9], the Bernoulli sub-equation technique [10], the sine-Gordon technique [11,12], the sub-equation technique [13], the Liu group technique [14] and the new extended direct algebraic technique [15–17], etc.

The complex GLE is an imperative PDE in the field of physics. The applications of complex GLE can be found in optics, plasma and other related fields. Different techniques have been suggested to acquire the solutions of NPDEs. These techniques are the first integral and (G′/G)-expansion [18], the new extended direct algebraic [19], the generalized exponential function [20] and the ansatz functions [21], etc.

The focus of this study is to acquire the soliton solutions of complex GLE with Kerr law nonlinearity employing the modified extended tanh technique with Riccati equation. This paper is presented in the following manner: In Section 2, the proposed technique is described. In Section 3, the solutions of complex GLE are presented. In Section 4, the conclusions and future recommendations are discussed.

## 2 Analysis of the Method

The NPDE is generally defined as follows:

$$H(g, D_t g, D_x g, D_{tt} g, D_{xx} g, D_t D_x g, \ldots) = 0. \tag{1}$$

Implementing the transformation:

$$g(x,t) = G(\xi), \qquad \xi = \lambda(x - vt)$$

where $\lambda$ and v are nonzero constants. Applying the above transformation, we convert Eq. (1) into a nonlinear ordinary differential equation (ODE) as follows:

$$U(G, G', G'', \ldots) = 0, \tag{2}$$

where the derivatives are with respect to $\xi$. The solutions of Eq. (2) is presented as follows

$$G(\xi) = a_0 + \sum_{k=1}^{K} \left(a_k \varphi^k(\xi) + b_k \varphi^{-k}(\xi)\right), \tag{3}$$

where $a_k, b_k$, $k = 1, 2, \ldots\ldots, N$ denote the constants that are required to be calculated.

Moreover, $\varphi(\xi)$ satisfies the following Riccati equation

$$\varphi' = b + \varphi^2, \tag{4}$$

where $b$ is a constant, Eq. (4) has the following solutions:

(i) For $b < 0$, we get

$\varphi = -\sqrt{-b}\tanh\left(\sqrt{-b}\xi\right)$, or $\varphi = -\sqrt{-b}\coth\left(\sqrt{-b}\xi\right)$.

(ii) For $b > 0$, we get

$\varphi = \sqrt{b}\tan\left(\sqrt{b}\xi\right)$, or $\varphi = -\sqrt{b}\cot\left(\sqrt{b}\xi\right)$.

(iii) For $b = 0$, we get

$$\varphi = \frac{-1}{\xi}.$$

The value of K can be computed to balance the nonlinear and linear terms of highest orders (c.f. Eq. (2)). Then, substitute Eq. (3) with its derivatives into Eq. (2) yields:

$$R(\varphi(\xi)) = 0, \tag{5}$$

where $R(\varphi(\xi))$ denotes a polynomial in $\varphi(\xi)$. Afterwards, we equate the coefficients of each power of $\varphi(\xi)$ in Eq. (5) to zero, and a system of algebraic equations is acquired. Thus, the solution of obtained algebraic equations will provide the exact solutions of Eq. (1).

## 3 Application of the Technique on CGLE

This study comprises of several solutions of the CGLE using the proposed method (c.f. Section 2).

Consider the CGLE with Kerr law nonlinearity [22,23]:

$$i\frac{\partial g}{\partial t} + a\frac{\partial g}{\partial x^2} + c|g|^2 g = \frac{\beta}{|g|^2 g}\left[2|g|^2\left(|g|^2\right)_{xx} - \left\{\left(|g|^2\right)_x\right\}^2\right] + \gamma g, \tag{6}$$

where x is the distance along the fibers and t is the dimensionless time. Moreover, $g = g(x, t)$ is a complex function which represents the wave profile occur in many phenomena such as, plasma physics and nonlinear optics. The real valued parameters a and c relates to the velocity dispersion and the nonlinearity coefficient, respectively. Furthermore, $\beta$ is the perturbation parameter and $\gamma$ represents the detuning effects. In order to find the solution (c.f. Eq. (6)), we employ the wave transformation as given below:

$$g(x,t) = G(\xi)e^{i\mu(x,t)}, \xi = \lambda(x - vt), \qquad \mu = -kx + \omega t, \tag{7}$$

where $\lambda$ and $v$. are the constants. Moreover, $k$ describes the wave frequency and the $\omega$ expresses the wave number.

From (7), we obtain:

$$g_t = i\omega G e^{i\mu} - \lambda v G' e^{i\mu}, u_x = -ikGe^{i\mu} + \lambda v G' e^{i\mu}$$

$$g_{xx} = -k^2 G e^{i\mu} - ik\lambda G' e^{i\mu} + \lambda^2 G'' e^{i\mu} - ikG' e^{i\mu},$$

$$|g|^2 = G^2, \qquad \left(|g|^2\right)_x = 2\lambda G G', \tag{8}$$

$$\left(|g|^2\right)_{xx} = 2\lambda^2(GG'' + (G')^2$$

Now, substitute Eq. (8) and Eq. (7) into Eq. (6), we get:

$$i(v + 2ak)G' = 0, \tag{9}$$

$$\lambda^2(a - 4\beta)G'' - \left(\omega + ak^2 + \gamma\right)G + cG^3 = 0. \tag{10}$$

From Eq. (9), we obtain:

$$v = -2ak$$

Balancing $G''$ and $G^3$ in Eq. (10) results $K + 2 = 3K$, and so $K = 1$. Thus, we acquire the form as given below:

$$G(\xi) = a_0 + a_1\phi(\xi) + b_1\phi^{-1}(\xi) \tag{11}$$

Now, substitute Eq. (11) into Eq. (10). Thus, the following system is acquired by equating the coefficients of each power of $\phi(\xi)$ to zero:

$$-(\omega + ak^2 + \gamma)a_0 + ca_0^3 + 6a_0a_1b_1c = 0,$$
$$2\lambda^2(a - 4\beta)a_1b - (\omega + ak^2 + \gamma)a_1 + 3ca_0^2a_1 + 3ca_1^2b_1 = 0,$$
$$3ca_0a_1^2 = 0,$$
$$2\lambda^2(a - 4\beta)a_1 + ca_1{}^3 = 0,$$
$$2\lambda^2(a - 4\beta)b_1b - (\omega + ak^2 + \gamma)b_1 + 3ca_0^2b_1 + 3ca_1b_1^2 = 0,$$
$$3ca_0b_1^2 = 0,$$
$$2\lambda^2(a - 4\beta)b_1b^2 + cb_1{}^3 = 0.$$

After solving the above system, we acquire the following solutions.

**Case 1.**

$$a_0 = a_1 = 0, b = \frac{w + ak^2 + \gamma}{2\lambda^2(a - 4\beta)}, b_1 = -\frac{w + ak^2 + \gamma}{\lambda\sqrt{2c(4\beta - a)}},$$

hence, the solution is formed as:

If $\dfrac{w + ak^2 + \gamma}{a - 4\beta} > 0$, then

$$g_1(x,t) = -\sqrt{\frac{w + ak^2 + g}{c}}\cot\left(\sqrt{\frac{w + ak^2 + g}{2(a - 4b)}}(x - vt)\right)e^{i(-kx+wt)},$$

$$g_{2(x,t)} = \sqrt{\frac{w + ak^2 + g}{c}}\tan\left(\sqrt{\frac{w + ak^2 + g}{2(a - 4b)}}(x - vt)\right)e^{i(-kx+wt)}.$$

If $\dfrac{w + ak^2 + \gamma}{a - 4\beta} < 0$, then

$$g_3(x,t) = \sqrt{-\frac{w + ak^2 + g}{c}}\tanh\left(\sqrt{-\frac{w + ak^2 + g}{2l^2(a - 4b)}}(x - vt)\right)e^{i(-kx+wt)},$$

$$g_4(x,t) = \sqrt{-\frac{w + ak^2 + g}{c}}\coth\left(\sqrt{-\frac{w + ak^2 + g}{2(a - 4b)}}(x - vt)\right)e^{i(-kx+wt)}.$$

**Case 2.**

$$a_0 = a_1 = 0, b = \frac{w + ak^2 + \gamma}{2\lambda^2(a - 4\beta)}, b_1 = \frac{w + ak^2 + \gamma}{\lambda\sqrt{2c(4\beta - a)}},$$

hence, the solution is formed as:

If $\dfrac{\omega + ak^2 + \gamma}{a - 4\beta} > 0$, then

$$g_5(x,t)=\sqrt{\frac{w+ak^2+\gamma}{c}}\cot\left(\sqrt{\frac{w+ak^2+\gamma}{2(a-4b)}}(x-vt)\right)\exp[i(-kx+wt)],$$

$$g_6(x,t)=\sqrt{\frac{w+ak^2+\gamma}{c}}\tan\left(\sqrt{\frac{w+ak^2+\gamma}{2(a-4b)}}(x-vt)\right)\exp[i(-kx+wt)].$$

If $\dfrac{\omega+ak^2+\gamma}{a-4\beta}<0$, then

$$g_7(x,t)=-\sqrt{-\frac{w+ak^2+\gamma}{c}}\tan h\left(\sqrt{-\frac{w+ak^2+\gamma}{2\lambda^2(a-4b)}}(x-vt)\right)\exp[i(-kx+wt)],$$

$$g_7(x,t)=-\sqrt{-\frac{w+ak^2+\gamma}{c}}\cot h\left(\sqrt{-\frac{w+ak^2+\gamma}{2(a-4b)}}(x-vt)\right)\exp[i(-kx+wt)].$$

**Case 3.**

$$a_0=a_1=0, b=\frac{w+ak^2+\gamma}{2\lambda^2(a-4\beta)}, b_1=\frac{w+ak^2+\gamma}{\lambda\sqrt{2c(4\beta-a)}},$$

hence, the solution is formed as:

If $\dfrac{w+ak^2+\gamma}{a-4\beta}>0$, then

$$g_5(x,t)=\sqrt{\frac{w+ak^2+g}{c}}\cot\left(\sqrt{\frac{w+ak^2+g}{2(a-4b)}}(x-vt)\right)e^{i(-kx+wt)},$$

$$g_6(x,t)=-\sqrt{\frac{w+ak^2+g}{c}}\tan\left(\sqrt{\frac{w+ak^2+g}{2(a-4b)}}(x-vt)\right)e^{i(-kx+wt)}.$$

If $\dfrac{w+ak^2+\gamma}{a-4\beta}<0$, then

$$g_7(x,t)=-\sqrt{-\frac{w+ak^2+g}{c}}\tanh\left(\sqrt{-\frac{w+ak^2+g}{2l^2(a-4b)}}(x-vt)\right)e^{i(-kx+wt)},$$

$$g_8(x,t)=-\sqrt{-\frac{w+ak^2+\gamma}{c}}\coth\left(\sqrt{-\frac{w+ak^2+\gamma}{2(a-4b)}}(x-vt)\right)e^{i(-kx+wt)}.$$

**Case 4.**

$$a_0=0, a_1=\sqrt{\frac{2\lambda^2(4\beta-a)}{c}},\ b_1=\frac{\sqrt{2}(\omega+ak^2+\gamma)}{8\lambda\sqrt{c(a-4\beta)}}, b=\frac{\omega+ak^2+\gamma}{8\lambda^2(4\beta-a)},$$

therefore, the solution is formed as:

If $\frac{\omega + ak^2 + \gamma}{4\beta - a} > 0$, then

$$g_9(x,t) = \sqrt{\frac{w + ak^2 + g}{4c}}\left(\tan\left(\sqrt{\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right) + \cot\left(\sqrt{\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right)\right)e^{i(-kx+wt)},$$

$$g_{10}(x,t) = -\sqrt{\frac{w + ak^2 + g}{4c}}\left(\tan\left(\sqrt{\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right) + \cot\left(\sqrt{\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right)\right)e^{i(-kx+wt)}$$

If $\frac{w + ak^2 + \gamma}{4\beta - a} < 0$, then

$$g_{11}(x,t) = \sqrt{-\frac{w + ak^2 + g}{4c}}\left(\tanh\left(\sqrt{-\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right) + \coth\left(\sqrt{-\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right)\right)e^{i(-kx+wt)},$$

$$g_{12}(x,t) = -\sqrt{-\frac{w + ak^2 + g}{4c}}\left(\tanh\left(\sqrt{-\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right) + \coth\left(\sqrt{-\frac{w + ak^2 + g}{8(4b - a)}}(x - vt)\right)\right)e^{i(-kx+wt)}$$

## 4 Representation of Obtained Solutions

This part studies the physical interpretation of obtained solutions under a suitable choice for the values of parameters. It represents each one of the selected solutions by three and two-dimensional plots, we represent the shape of the solution in real, and imaginary plots to show the similarities and difference between them in these cases. We have plotted each of $g_3(x,t)$ in Figs. 1–3 considering the following conditions:

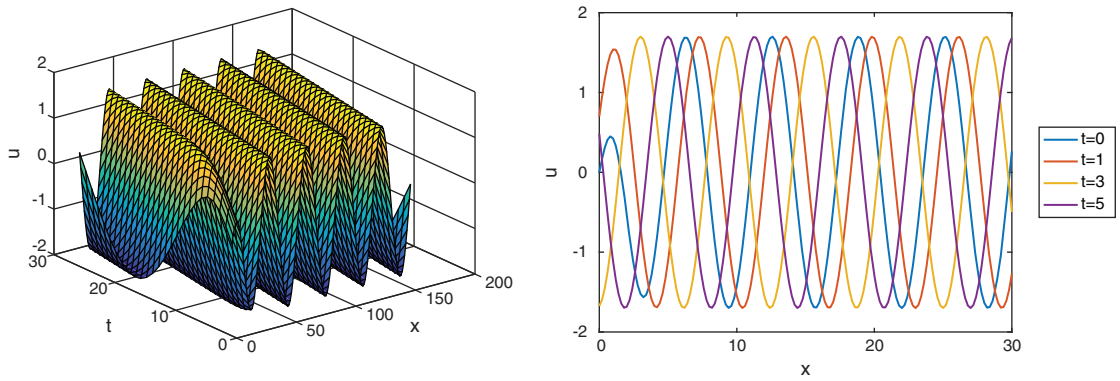


**Figure 1:** Plotting the real part of $g_3(x,t)$ with $0 \leq x, \leq 30, 0 \leq t \leq 5$

Wave solution of $g_3(x,t)$ in three and two dimensional when, $a = c = \omega = k = \gamma = \lambda = \beta = 1$, in the next interval $0 \leq x, \leq 30, 0 \leq t \leq 5$.

Furthermore, we have plotted each of $g_{11}(x,t)$ in Figs. 4–6 considering the following conditions: Wave solution of $g_{11}(x,t)$ in three and two dimensional when, $a = 5, c = \omega = k = \gamma = \lambda = \beta = 1$, in the next interval $0 \leq x, \leq 30, 0 \leq t \leq 5$.

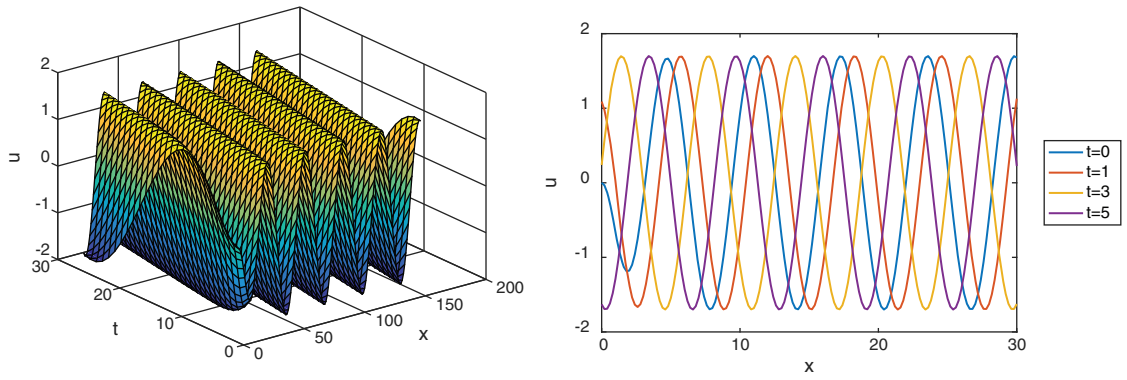


**Figure 2:** Plotting the imaginary part of $g_3(x,t)$ with $0 \leq x, \leq 30, 0 \leq t \leq 5$

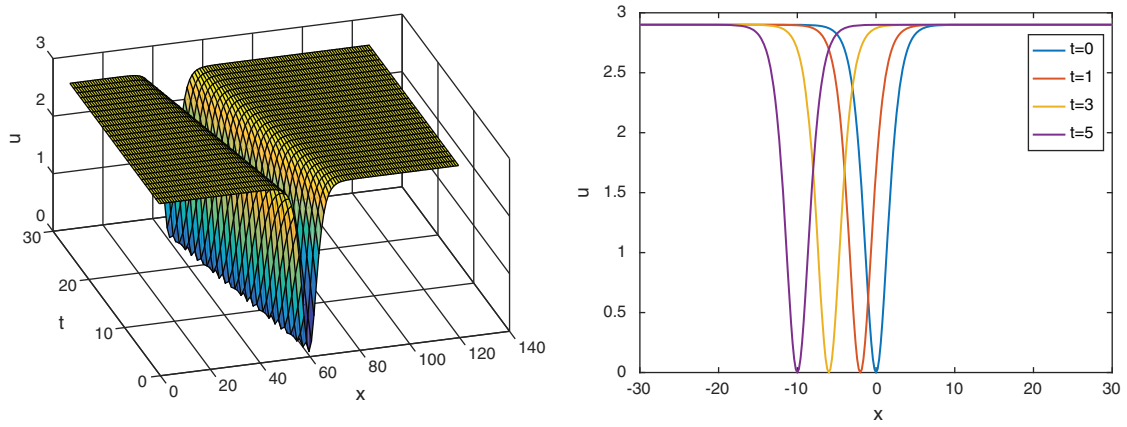


**Figure 3:** Plotting of $|g_3(x,t)|^2$ with $-30 \leq x, \leq 30, t = 0, 1, 3, 5$

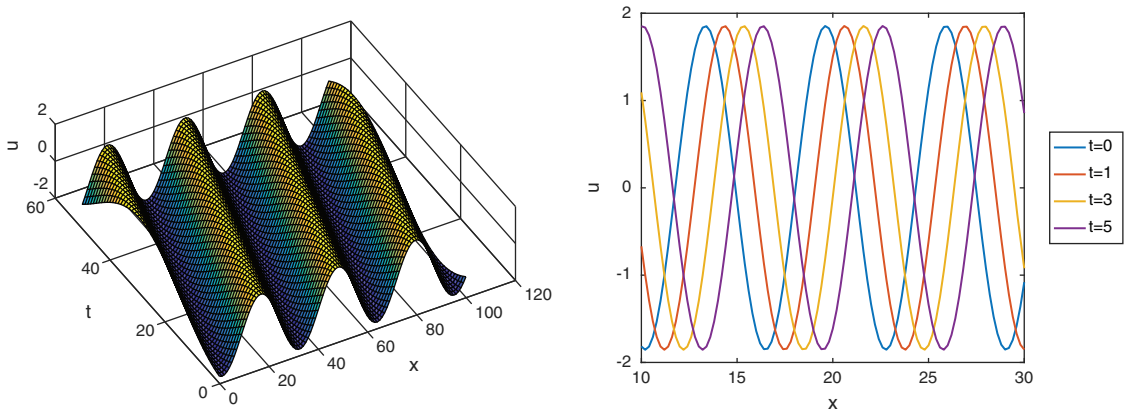


**Figure 4:** Plotting the real part of $g_{11}(x,t)$ with $10 \leq x, \leq 30, 0 \leq t \leq 5$

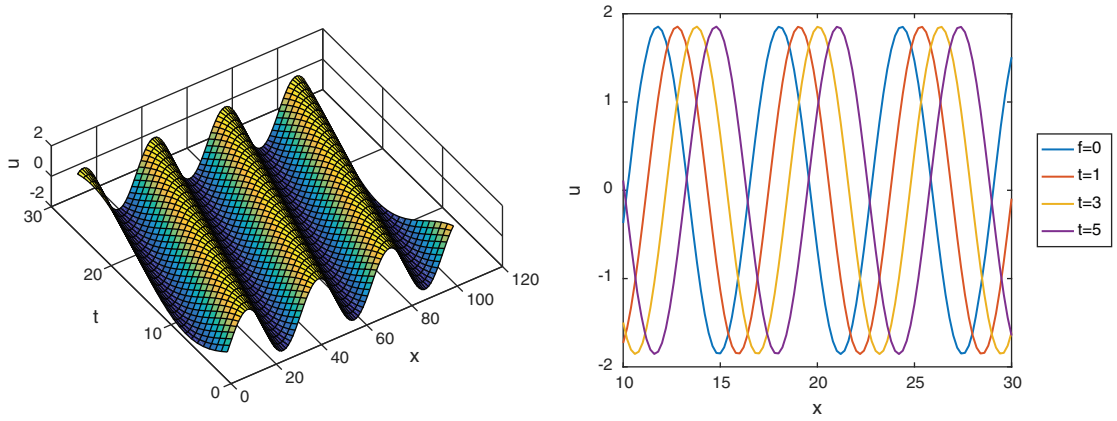


**Figure 5:** Plotting the imaginary part of $g_{11}(x,t)$ with $10 \leq x, \leq 30, 0 \leq t \leq 5$

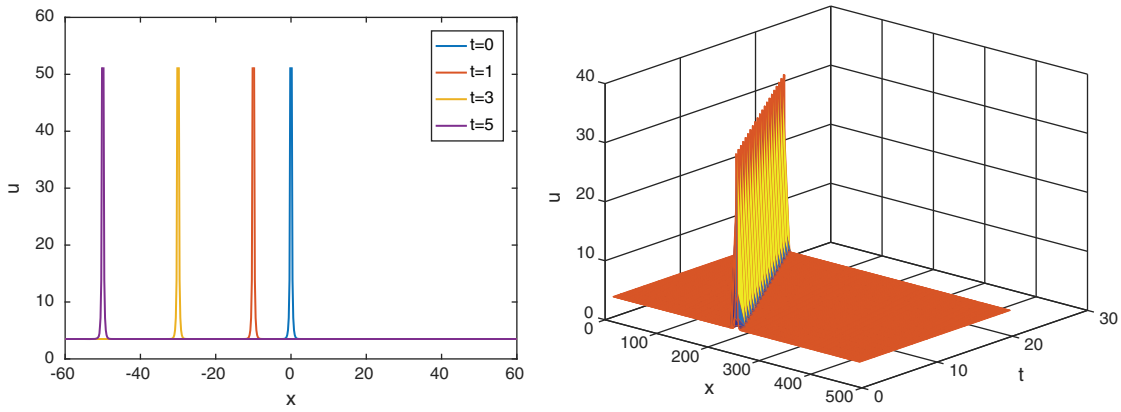


**Figure 6:** Plotting of $|g_{11}(x,t)|^2$ with $-60 \leq x, \leq 60, t = 0, 1, 3, 5$

## 5 Conclusions

In this work, new generalized travelling wave solutions of the Complex GLE with Kerr law non-linearity were obtained. The modified extended tanh method with Riccati equation was implemented. The acquired results are represented by trigonometric and hyperbolic functions. The results depict the typical soliton behavior of the solution. The suggested technique is a powerful and efficient way to discuss complex NPDEs. The results presented in this paper are novel and reported first time in the literature. Simulation results establish the validity and applicability of the suggested technique.

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