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Computational solutions of conformable space-time derivatives dynamical wave equations: Analytical mathematical techniques

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ABSTRACT

In this article, the instigator sets up the profuse traveling wave solutions four types of fractional nonlinear equations in the sense of conformable derivatives by using the novel form of modified mathematical technique. The constructed traveling wave solutions are articulated in terms of trigonometric, hyperbolic and exponential functions. The derived results are fruitful for the physical demonstrations of problems in mathematical physics and engineering.

Introduction

Oldham and Spanier are first researchers who have taken the fractional differential equations (FDEs) into consideration [1]. Exact solutions of FDEs plays an imperative role in understanding the features of numerous physical phenomena, that described by these equations. For illustration, the nonlinear oscillation of an earthquake can be modeled due to derivatives of fractional order. In fact, the physical phenomena may not depend only on the time moment but also on the former time history, which can be fruitfully, modeled utilizing the theory of fractional integrals and derivatives [2–4]. Fractional evolution equations play a important role in diverse fields like engineering, biology, physics, fluid flow, finance, electrochemistry and many more [5–17]. Several competent methods have lately been developed to get analytical solutions for FDEs. For example, the generalized tanh-coth method[18], the auxiliary equation method [19], improved F-expansion method [20], the exponential rational function method[21–23], the simplest equation method [24], the modified simple equation method, the first integral method [25], the Kudryashov method [26–31], fractional calculus and many more methods [32–44].

This research work aims is to implement the improved simple equation method to construct the novel exact solutions of concern nonlinear fractional wave models. The derived results have valuable positions in applied sciences. The remaining part of this article is arranged as; the description of conformable derivative is illustrated in Section “Approach for fractional derivatives”. The proposed method is mentioned in Section “Description of proposed method”. The Section “Applications”, application of the method. Results discussions and conclusion are provided in “Results and discussion” and “Conclusion”.

Approach for fractional derivatives

The captivating definition of fractional derivative in term of conformable derivative illustrated [45]. Liebniz and chain rule together observed via that derivative.

Conformable derivative

Definition: Suppose $h_2 : \mathbb{R}^+ \rightarrow \mathbb{R}$ is a function, for $t > 0$,

$$D_t^{\beta}(h_2(t)) = \lim_{\epsilon \to 0} \frac{h_2(t + \epsilon t^{1-\beta}) - h_2(t)}{\epsilon} \quad (1)$$

is called conformable derivative of h_2 with order $\beta, 0 < \beta \leq 1$.

$$D_t^{\beta}(ah_2 + bg) = aD_t^{\beta}(h_2) + bD_t^{\beta}(g), \ a, b \epsilon \mathbb{R} \quad (2)$$

$$D_t^{\beta}(h_2 g) = h_2 D_t^{\beta}(g) + gD_t^{\beta}(h_2) \quad (3)$$

Suppose $h_2 : \mathbb{R}^+ \rightarrow \mathbb{R}$ be a differentiable, β and g are differential

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functions, in the range of h_2.

$$D_t^{\beta}(h_2\circ(g)) = t^{1-\beta}g'(t)h_2'(t) \tag{4}$$

Following rules hold.

$$D_t^{\beta}(t^p) = pt^{p-\beta},\ p\varepsilon\mathbb{R} \tag{5}$$

$$D_t^{\beta}(\lambda_2) = 0, \tag{6}$$

where λ_2 is called a constant.

$$D_t^{\beta}\left(\frac{h_2}{g}\right) = \frac{gD_t^{\beta}(h_2) - h_2D_t^{\beta}(g)}{g^2} \tag{7}$$

h_2 is differentiable, then

$$D_t^{\beta}(h_2(t)) = t^{1-\beta}\frac{dh_2(t)}{dt}. \tag{8}$$

Description of proposed method

Consider

$$R_1\left(U, D_t^{\beta}U, D_xU, D_t^{2\beta}U, D_{xx}U, \ldots\right) = 0 \tag{9}$$

Let

$$U = U(\xi), \quad \xi = kx - l\frac{t^{\beta}}{\beta}, \tag{10}$$

Put (10) in (9),

$$R_2(U, U', U'', \ldots) = 0. \tag{11}$$

Let solution of(11) is,

$$U(\xi) = \sum^{m} i = -mA_i\Psi^i(\xi) \tag{12}$$

$$\Psi'(\xi) = C_0 + C_1\Psi + C_2\Psi^2 + C_3\Psi^3 \tag{13}$$

The general solutions of new simple ansatz Eq. (13) are as following:

$$\Psi(\xi) = -\frac{C_1 - \sqrt{4C_0C_2 - C_1^2}\tan\left(\frac{\sqrt{4C_0C_2-C_1^2}}{2}(\xi+\xi_0)\right)}{2C_2}, \quad 4C_0C_2 > C_1^2, \quad C_3 = 0. \tag{14}$$

If $C_0 = 0, C_3 = 0$, then simple ansatz Eq. (13) reduces to Bernoulli equation;

$$\Psi = \frac{C_1e^{C_1(\xi+\xi_0)}}{1 - C_2e^{C_1(\xi+\xi_0)}}, \quad C_1 > 0, \tag{15}$$

$$\Psi = \frac{-C_1e^{C_1(\xi+\xi_0)}}{1 + C_2e^{C_1(\xi+\xi_0)}}, \quad C_1 < 0. \tag{16}$$

If $C_1 = 0, C_3 = 0$, then the ansatz (13) reduces to Riccati equation with following solutions:

$$\Psi(\xi) = \frac{\sqrt{C_0C_2}}{C_2}\tan(\sqrt{C_0C_2}(\xi+\xi_0)), \quad C_0C_2 > 0, \tag{17}$$

$$\Psi(\xi) = -\frac{\sqrt{-C_0C_2}}{C_2}\tanh(\sqrt{-C_0C_2}(\xi+\xi_0)), \quad C_0C_2 < 0. \tag{18}$$

Put (12) and (13) in (11). Solving the obtained systems of equations for the required values parameters. Putting all parameters values and Ψ in (11), achieved the solution of (9).

Applications

The conformable space-time EW equation

Let EW equation in [46];

$$D_t^{\beta}U(x,t) + aD_x^{\beta}U^2(x,t) - cD_{xxt}^{3\beta}U(x,t) = 0. \tag{19}$$

$$U = U(\xi), \ \xi = k\frac{x^{\beta}}{\beta} - l\frac{t^{\beta}}{\beta} \tag{20}$$

Put (20) in (19), after this integrating by taking integration of constant equal to zero

$$akU^2 + ck^2lU'' - lU = 0 \tag{21}$$

Let solution of (21) is

$$U = A_2\psi^2 + A_1\psi + \frac{A_{-2}}{\psi^2} + \frac{A_{-1}}{\psi} + A_0 \tag{22}$$

Put (22) in (21) with (13),

Case I: $C_3 = 0$,

Family- I:

$$A_1 = \frac{6cC_1C_2l}{a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}, A_2 = \frac{6cC_2^2l}{a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}, A_{-2} = 0, A_{-1} = 0,$$

$$A_0 = \frac{l\left(-\sqrt{c^2(C_1^2-4C_0C_2)^2} + cC_1^2 + 8cC_0C_2\right)}{2a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}, k = -\frac{1}{\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}} \tag{23}$$

Substitute (23) in (22),

$$U_1 = \frac{6cC_2^2l}{a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}\left(\frac{C_1 - \sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^2 + \left(\frac{6cC_1C_2l}{a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}\right)\left(\frac{C_1 - \sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right) + \frac{l\left(-\sqrt{c^2(C_1^2-4C_0C_2)^2} + cC_1^2 + 8cC_0C_2\right)}{2a\sqrt[4]{c^2(C_1^2-4C_0C_2)^2}}, \quad 4C_0C_2 > C_1^2 \tag{24}$$

Family- II:

Substitute (25) in (22),

$$A_0=\frac{l\left(-\sqrt{c^2\left(C_1^2-4C_0C_2\right)^2}+cC_1^2+8cC_0C_2\right)}{2a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}},\ A_{-2}=\frac{6cC_0^2l}{a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}},\ A_2=0$$
$$A_1=0\ ,A_{-1}=\frac{6cC_0C_1l}{a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}},\ k=-\frac{1}{\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}} \tag{25}$$

Case II: $C_0=C_3=0$,

$$U_2=\frac{6cC_0^2l}{a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}}\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^{-2}+\left(\frac{6cC_0C_1l}{a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}}\right)$$
$$\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^{-1}+\frac{l\left(-\sqrt{c^2\left(C_1^2-4C_0C_2\right)^2}+cC_1^2+8cC_0C_2\right)}{2a\sqrt[4]{c^2\left(C_1^2-4C_0C_2\right)^2}},$$
$$4C_0C_2>C_1^2 \tag{26}$$

$$A_0=0,\ A_{-2}=0,\ A_{-1}=0,\ A_2=-\frac{6\sqrt{c}C_2^2l}{aC_1},\ A_1=-\frac{6\sqrt{c}C_2l}{a},\ k=\frac{1}{\sqrt{c}C_1} \tag{27}$$

Put (27) in (22),

$$U_3=-\frac{6\sqrt{c}C_2l}{a}\left(\frac{C_1\exp(C_1(\xi+\xi_0))}{1-C_2\exp(C_1(\xi+\xi_0))}\right)-\frac{6\sqrt{c}C_2^2l}{aC_1}\left(\frac{C_1\exp(C_1(\xi+\xi_0))}{1-C_2\exp(C_1(\xi+\xi_0))}\right)^2,C_1>0. \tag{28}$$

$$U_4=-\frac{6\sqrt{c}C_2l}{a}\left(\frac{-C_1\exp(C_1(\xi+\xi_0))}{1+C_2\exp(C_1(\xi+\xi_0))}\right)-\frac{6\sqrt{c}C_2^2l}{aC_1}\left(\frac{-C_1\exp(C_1(\xi+\xi_0))}{1+C_2\exp(C_1(\xi+\xi_0))}\right)^2,C_1<0. \tag{29}$$

Case III: $C_1=C_3=0$,
Family-I

$$A_0=\frac{l\sqrt{cC_0C_2}}{a},\ A_{-2}=0,\ A_{-1}=0,\ A_2=\frac{3\sqrt{c}C_2^{3/2}l}{a\sqrt{C_0}},\ A_1=0,\ k=-\frac{1}{2\sqrt{cC_0C_2}} \tag{30}$$

Put (30) in (22),

$$U_5=\frac{l\sqrt{cC_0C_2}}{a}+\frac{3\sqrt{c}C_2^{3/2}l}{a\sqrt{C_0}}\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2,\ C_2C_0>0. \tag{31}$$

$$U_6=\frac{l\sqrt{cC_0C_2}}{a}+\frac{3\sqrt{c}C_2^{3/2}l}{a\sqrt{C_0}}\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2,\ C_2C_0<0. \tag{32}$$

Family-II

$$A_0=\frac{l\sqrt{cC_0C_2}}{a},\ A_{-2}=\frac{3\sqrt{c}C_0^{3/2}l}{a\sqrt{C_2}},\ A_{-1}=0,\ A_2=0,\ A_1=0,\ k=-\frac{1}{2\sqrt{cC_0C_2}} \tag{33}$$

Put (33) in (22),

$$U_7=\frac{l\sqrt{cC_0C_2}}{a}+\frac{3\sqrt{c}C_0^{3/2}l}{a\sqrt{C_2}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}\right)^2,\ C_2C_0>0. \tag{34}$$

$$U_8=\frac{l\sqrt{cC_0C_2}}{a}+\frac{3\sqrt{c}C_0^{3/2}l}{a\sqrt{C_2}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}\right)^2,\ C_2C_0<0. \tag{35}$$

Family-III

$$A_0=\frac{l\sqrt{cC_0C_2}}{a},\ A_{-2}=-\frac{3\sqrt{c}C_0^{3/2}l}{2a\sqrt{C_2}},\ A_{-1}=0,\ A_2=-\frac{3\sqrt{c}C_2^{3/2}l}{2a\sqrt{C_0}},\ A_1=0,\ k=\frac{1}{4\sqrt{cC_0C_2}} \tag{36}$$

Put (36) in (22),

$$U_9 = \frac{l\sqrt{cC_0C_2}}{a} - \frac{3\sqrt{c}C_0^{3/2}l}{2a\sqrt{C_2}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}\right)^2 - \left(\frac{3\sqrt{c}\,C_2^{3/2}l}{2a\sqrt{C_0}}\right)\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2,\ C_2C_0>0. \quad (37)$$

(3 + 1)-dimensional conformable time-fractional KdV-ZK equation

$$U_{10} = \frac{l\sqrt{cC_0C_2}}{a} - \frac{3\sqrt{c}C_0^{3/2}l}{2a\sqrt{C_2}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}\right)^2 - \left(\frac{3\sqrt{c}\,C_2^{3/2}l}{2a\sqrt{C_0}}\right)\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}{C_2}\right),\ C_2C_0<0. \quad (38)$$

Consider KdV-ZK equation in[47],

$$D_t^\beta U + aUU_x + U_{xxx} + c(U_{yyx}+U_{zzx}) = 0,\ t>0,\ 0<\beta<1. \quad (39)$$

$$U = U(\xi),\ \xi = x+y+z-l\frac{t^\beta}{\beta} \quad (40)$$

Put (40) in (39), yields following ODE;

$$\frac{aU^2}{2} + (2c+1)U'' - lU = 0 \quad (41)$$

Let Eq. (22) is the solution of (41), Put (22) into (41) with (13) after solving we have the following cases;

Case I: $C_3 = 0$,

Family- I:

$$A_2 = -\frac{12(2c+1)C_2^2}{a},\ A_1 = -\frac{12(2c+1)C_1C_2}{a},\ A_0 = -\frac{12(2c+1)C_0C_2}{a},$$
$$l = (2c+1)\left(C_1^2-4C_0C_2\right),\ A_{-2}=0,\ A_{-1}=0. \quad (42)$$

Substitute (42) in (22),

Family- II:

$$U_{11} = -\frac{12(2c+1)C_2^2}{a}\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^2 - \left(\frac{12(2c+1)C_1C_2}{a}\right)\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right) - \frac{12(2c+1)C_0C_2}{a},\quad 4C_0C_2>C_1^2 \quad (43)$$

$$A_{-2} = -\frac{12(2c+1)C_0^2}{a},\ A_{-1} = -\frac{12(2c+1)C_0C_1}{a},\ A_2=0,\ A_1=0,$$
$$A_0 = -\frac{12(2c+1)C_0C_2}{a}, l = (2c+1)\left(C_1^2-4C_0C_2\right) \quad (44)$$

Substitute (44) in (22),

Case II: $C_0 = C_3 = 0$,

$$U_{12} = -\frac{12(2c+1)C_0^2}{a}\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^{-2} - \left(\frac{12(2c+1)C_0C_1}{a}\right)\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}(\xi+\xi_0)\right)}{2C_2}\right)^{-1} + (2c+1)\left(C_1^2-4C_0C_2\right),\quad 4C_0C_2>C_1^2 \quad (45)$$

$$A_{-1}=0,\ A_2=-\frac{12(2c+1)C_2^2}{a},\ A_1=-\frac{12(2c+1)C_1C_2}{a},\ l=(2c+1)C_1^2,\ A_0=0,\ A_{-2}=0. \tag{46}$$

Put (46) in (22),

$$U_{13}=-\frac{12(2c+1)C_1C_2}{a}\left(\frac{C_1\exp(C_1(\xi+\xi_0))}{1-C_2\exp(C_1(\xi+\xi_0))}\right)-\frac{12(2c+1)C_2^2}{a}\left(\frac{C_1\exp(C_1(\xi+\xi_0))}{1-C_2\exp(C_1(\xi+\xi_0))}\right)^2, C_1>0. \tag{47}$$

$$U_{14}=-\frac{12(2c+1)C_1C_2}{a}\left(\frac{-C_1\exp(C_1(\xi+\xi_0))}{1+C_2\exp(C_1(\xi+\xi_0))}\right)-\frac{12(2c+1)C_2^2}{a}\left(\frac{-C_1\exp(C_1(\xi+\xi_0))}{1+C_2\exp(C_1(\xi+\xi_0))}\right)^2, C_1<0. \tag{48}$$

Case III: $C_1=C_3=0$,

Family-I

$$A_0=-\frac{4(2c+1)C_0C_2}{a}, A_{-2}=0, A_{-1}=0, A_2=-\frac{12(2c+1)C_2^2}{a}, A_1=0, l=4(2c+1)C_0C_2 \tag{49}$$

Put (49) in (22),

$$U_{15}=-\frac{4(2c+1)C_0C_2}{a}-\frac{12(2c+1)C_2^2}{a}\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2, C_2C_0>0. \tag{50}$$

$$U_{16}=-\frac{4(2c+1)C_0C_2}{a}-\frac{12(2c+1)C_2^2}{a}\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2, C_2C_0<0. \tag{51}$$

Family-II

$$A_0=-\frac{12(2c+1)C_0C_2}{a}, A_{-2}=-\frac{12(2c+1)C_0^2}{a}, A_{-1}=0, A_2=0, A_1=0, l=-4(2c+1)C_0C_2 \tag{52}$$

Put (52) in (22),

$$U_{17}=-\frac{12(2c+1)C_0C_2}{a}-\frac{12(2c+1)C_0^2}{a}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}\right)^2, C_2C_0>0. \tag{53}$$

$$U_{18}=-\frac{12(2c+1)C_0C_2}{a}-\frac{12(2c+1)C_0^2}{a}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi+\xi_0)\right)}\right)^2, C_2C_0<0. \tag{54}$$

Family-III

$$A_0=\frac{8(2c+1)C_0C_2}{a},\ A_{-2}=-\frac{12(2c+1)C_0^2}{a},\ A_{-1}=0,\ A_2=-\frac{12(2c+1)C_2^2}{a},\ A_1=0,\ l=16(2c+1)C_0C_2 \tag{55}$$

Put (55) in (22),

$$U_{29}=\frac{8(2c+1)C_0C_2}{a}-\frac{12(2c+1)C_0^2}{a}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}\right)^2-\left(\frac{12(2c+1)C_2^2}{a}\right)\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi+\xi_0)\right)}{C_2}\right)^2,\ C_2C_0>0. \tag{56}$$

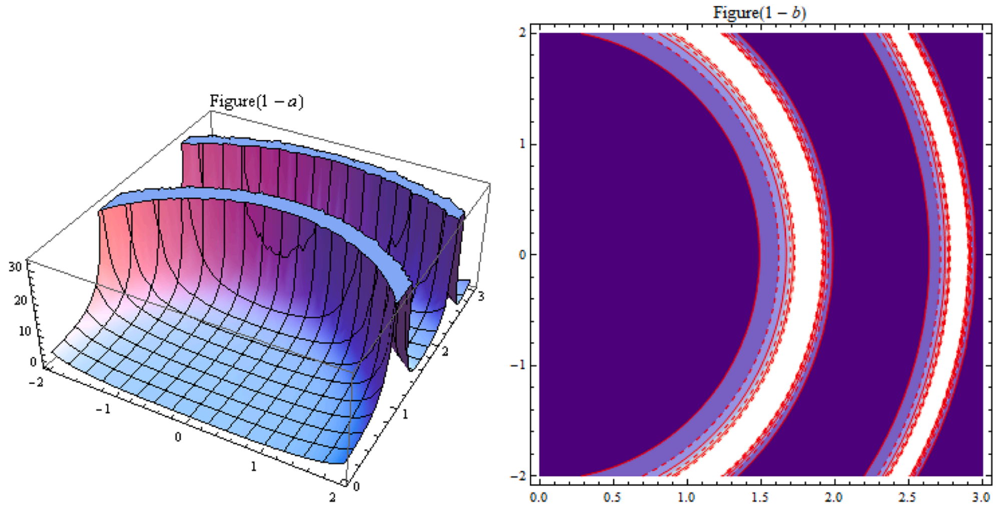

Fig. 1. Exact traveling waves solution (24).

$$U_{30} = \frac{8(2c+1)C_0C_2}{a} - \frac{12(2c+1)C_0^2}{a}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}\right)^2 - \left(\frac{12(2c+1)C_2^2}{a}\right)\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}{C_2}\right), \; C_2C_0<0. \tag{57}$$

(2 + 1)-dimensional conformable time-fractional Zoomeron equation

Let fractional Zoomeron equation [46]

$$D_{tt}^{2\beta}\left(\frac{U_{xy}}{U}\right) - \left(\frac{U_{xy}}{U}\right)_{xx} + 2D_t^{\beta}(U^2)_x = 0, \; 0<\beta\leq 1. \tag{58}$$

Let

$$U = U(\xi), \; \xi = k\frac{x^{\beta}}{\beta} + h\frac{y^{\beta}}{\beta} - l\frac{t^{\beta}}{\beta} \tag{59}$$

Put (59) in (58), after this integrating twice by taking integration of constant equal to zero;

$$-\mathrm{a}U + \mathrm{kh}\left(l^2-k^2\right)U'' - 2\mathrm{kl}U^3 = 0 \tag{60}$$

Let solution of (60) is

$$U = \frac{A_{-1}}{\psi} + A_1\psi + A_0 \tag{61}$$

Put (61) in (60) with (13), after solving following possible solutions casess.

Case I: $C_3 = 0$,

Family- I:

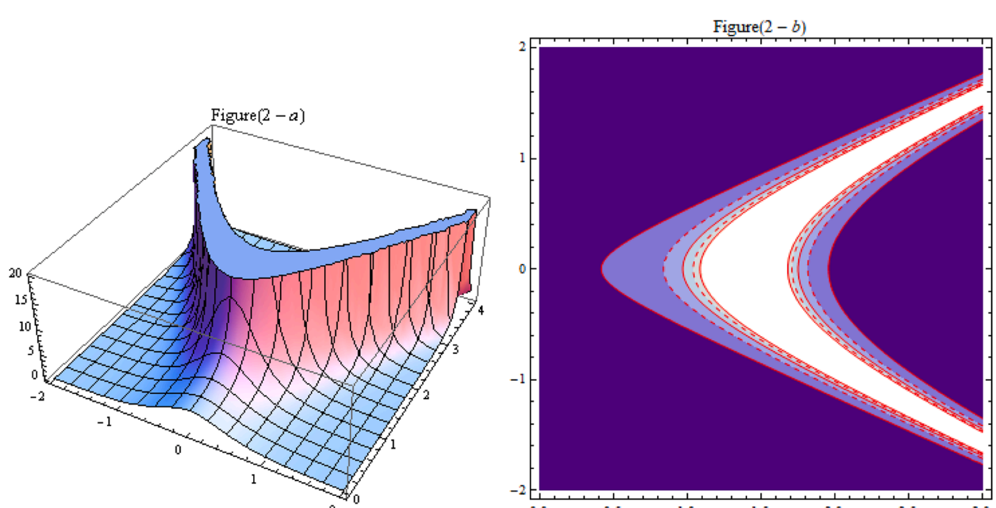

Fig. 2. Exact traveling waves solution (28).

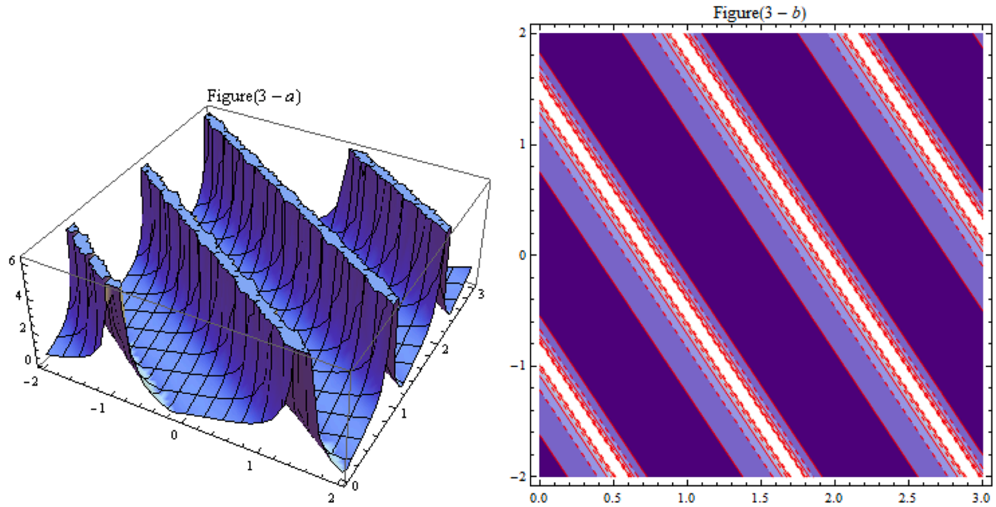

Fig. 3. Exact traveling waves solution (63).

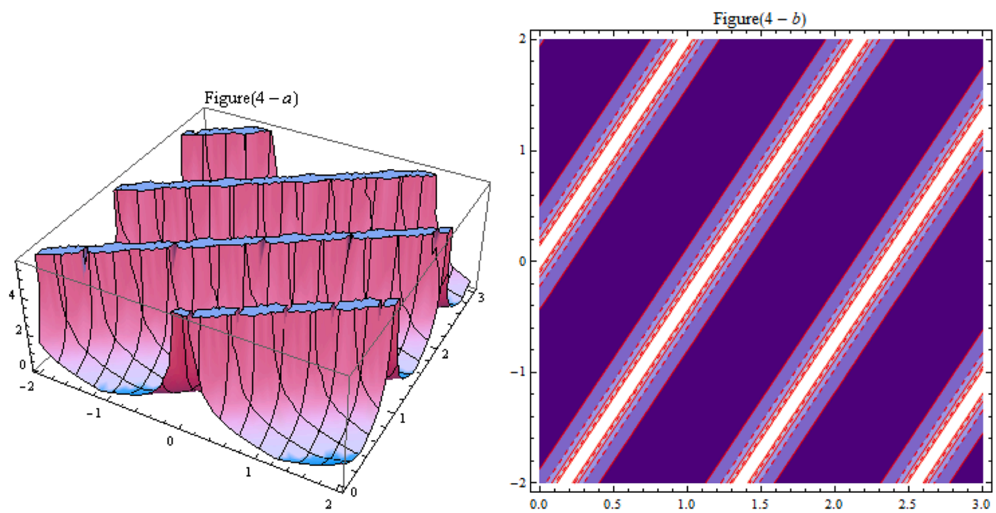

Fig. 4. Exact traveling waves solution (65).

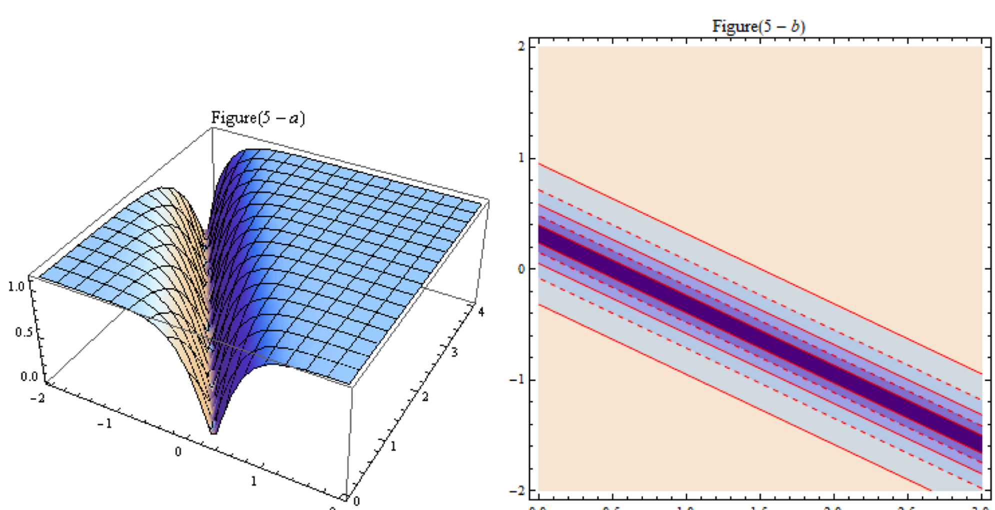

Fig. 5. Exact traveling waves solution (83).

$$A_1 = \frac{\sqrt{2a}C_2}{\sqrt{4C_0C_2\text{kl} - C_1^2\text{kl}}},\ A_0 = \frac{\sqrt{a}C_1}{\sqrt{2}\sqrt{(C_1^2 - 4C_0C_2)(-\text{kl})}},\ A_{-1} = 0,\quad h = -\frac{2a}{(C_1^2 - 4C_0C_2)k(l^2 - k^2)} \tag{62}$$

Substitute (62) in (61),

$$U_{31} = \frac{\sqrt{2a}C_2}{\sqrt{4C_0C_2\text{kl} - C_1^2\text{kl}}}\left(\frac{C_1 - \sqrt{4C_0C_2 - C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2 - C_1^2}(\xi + \xi_0)\right)}{2C_2}\right) + \frac{\sqrt{a}C_1}{\sqrt{2}\sqrt{(C_1^2 - 4C_0C_2)(-\text{kl})}},\ 4C_0C_2 > C_1^2 \tag{63}$$

Family- II:

$$A_1 = 0,\ A_0 = -\frac{\sqrt{a}C_1}{\sqrt{2}\sqrt{(C_1^2 - 4C_0C_2)(-\text{kl})}},\ A_{-1} = -\frac{\sqrt{2a}C_0}{\sqrt{4C_0C_2\text{kl} - C_1^2\text{kl}}},\quad h = -\frac{2a}{(C_1^2 - 4C_0C_2)k(l^2 - k^2)} \tag{64}$$

Substitute (64) in (61),

$$U_{32} = -\frac{\sqrt{2a}C_0}{\sqrt{4C_0C_2\text{kl} - C_1^2\text{kl}}}\left(\frac{C_1 - \sqrt{4C_0C_2 - C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2 - C_1^2}(\xi + \xi_0)\right)}{2C_2}\right)^{-1} + -\frac{\sqrt{a}C_1}{\sqrt{2}\sqrt{(C_1^2 - 4C_0C_2)(-\text{kl})}},\ 4C_0C_2 > C_1^2 \tag{65}$$

Case II: $C_1 = C_3 = 0$,
Family-I

$$A_1 = \frac{\sqrt{aC_2}}{\sqrt{2\text{kl}C_0}},\ A_0 = 0,\ A_{-1} = 0,\ h = \frac{a}{2C_0C_2k(l^2 - k^2)} \tag{66}$$

Put (66) in (61),

$$U_{33} = \frac{\sqrt{aC_2}}{\sqrt{2\text{kl}C_0}}\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi + \xi_0)\right)}{C_2}\right),\ C_2C_0 > 0. \tag{67}$$

$$U_{34} = \frac{\sqrt{aC_2}}{\sqrt{2\text{kl}C_0}}\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi + \xi_0)\right)}{C_2}\right),\ C_2C_0 < 0. \tag{68}$$

Family-II

$$A_1 = 0,\ A_0 = 0,\ A_{-1} = \frac{\sqrt{aC_0}}{\sqrt{2C_2\text{kl}}},\ h = \frac{a}{2C_0C_2k(l^2 - k^2)} \tag{69}$$

Put (69) in (61),

$$U_{35} = \frac{\sqrt{aC_0}}{\sqrt{2C_2\text{kl}}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi + \xi_0)\right)}\right),\ C_2C_0 > 0. \tag{70}$$

$$U_{36} = \frac{\sqrt{aC_0}}{\sqrt{2C_2\text{kl}}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi + \xi_0)\right)}\right),\ C_2C_0 < 0. \tag{71}$$

Family-III

$$A_1 = -\frac{\sqrt{aC_2}}{2\sqrt{2C_0kl}},\ A_0 = 0,\ A_{-1} = \frac{\sqrt{aC_0}}{2\sqrt{2C_2kl}},\ h = \frac{a}{8C_0C_2k(l^2 - k^2)} \tag{72}$$

Put (72) in (61),

$$U_{37} = \frac{\sqrt{aC_0}}{2\sqrt{2C_2kl}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi + \xi_0)\right)}\right) - \left(\frac{\sqrt{aC_2}}{2\sqrt{2C_0kl}}\right)\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}(\xi + \xi_0)\right)}{C_2}\right),\ C_2C_0 > 0. \tag{73}$$

$$U_{38} = \frac{\sqrt{aC_0}}{2\sqrt{2C_2kl}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi + \xi_0)\right)}\right) - \left(\frac{\sqrt{aC_2}}{2\sqrt{2C_0kl}}\right)\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}(\xi + \xi_0)\right)}{C_2}\right),\ C_2C_0 < 0. \tag{74}$$

The conformable space-time modified EW equation:

Consider fractional MEW equation in [48],

$$D_t^\beta U(x,t) + \sigma D_x^\beta U^3(x,t) - \delta D_{xxt}^{3\beta}U(x,t) = 0. \tag{75}$$

Put transformation of (20) into (75) and after this integrating by taking integration of constant equal to zero yields;

$$-lU + k\sigma U^3 + \delta lk^2U'' = 0 \tag{76}$$

Let Eq. (61) is the solution of Eq. (76), following possible solutions are;
Case I: $C_3 = 0$,
Family- I:

$$A_1 = \frac{2^{3/4}C_2\sqrt{\delta l}}{\sqrt{\sigma}\sqrt[4]{(4C_0C_2 - C_1^2)\delta}},\ A_0 = \frac{C_1\sqrt{\delta l}}{\sqrt[4]{2}\sqrt{\sigma}\sqrt[4]{(4C_0C_2 - C_1^2)\delta}},\quad A_{-1} = 0,\ k = -\frac{\sqrt{2}}{\sqrt{4C_0C_2\delta - C_1^2\delta}} \tag{77}$$

Substitute (77) in (61),

$$U_{39}=\frac{2^{3/4}C_2\sqrt{\delta l}}{\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)\delta}}\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}\,(\xi+\xi_0)\right)}{2C_2}\right)+\left(\frac{C_1\sqrt{\delta l}}{\sqrt[4]{2}\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)\delta}}\right),\ 4C_0C_2>C_1^2 \tag{78}$$

Family- II:

$$A_1=0,\ A_0=-\frac{C_1\sqrt{\delta l}}{\sqrt[4]{2}\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)(-\delta)}},\ A_{-1}=-\frac{2^{3/4}C_0\sqrt{\delta}\sqrt{l}}{\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)(-\delta)}},\ k=-\frac{\sqrt{2}}{\sqrt{4C_0C_2\delta-C_1^2\delta}} \tag{79}$$

Substitute (79) in (61),

$$U_{40}=-\frac{2^{3/4}C_0\sqrt{\delta}\sqrt{l}}{\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)(-\delta)}}\left(\frac{C_1-\sqrt{4C_0C_2-C_1^2}\tan\left(\frac{1}{2}\sqrt{4C_0C_2-C_1^2}\,(\xi+\xi_0)\right)}{2C_2}\right)^{-1}-\left(\frac{C_1\sqrt{\delta l}}{\sqrt[4]{2}\sqrt{\sigma}\sqrt[4]{\left(4C_0C_2-C_1^2\right)(-\delta)}}\right),\ 4C_0C_2>C_1^2 \tag{80}$$

Case II: $C_1=C_3=0$,
Family-I

$$A_1=\frac{\sqrt[4]{2}C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_0}\sqrt{\sigma}},\ A_0=0,\ A_{-1}=0,\ k=-\frac{1}{\sqrt{2C_2C_0\delta}} \tag{81}$$

Put (81) in (61),

$$U_{41}=\frac{\sqrt[4]{2}C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_0}\sqrt{\sigma}}\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}\,(\xi+\xi_0)\right)}{C_2}\right),\ C_2C_0>0. \tag{82}$$

$$U_{42}=\frac{\sqrt[4]{2}C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_0}\sqrt{\sigma}}\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}{C_2}\right),\ C_2C_0<0. \tag{83}$$

Family-II

$$A_1=0,\ A_0=0,\ A_{-1}=\frac{\sqrt[4]{2}C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_2}\sqrt{\sigma}},\ k=-\frac{1}{\sqrt{2C_0\delta C_2}} \tag{84}$$

Put (84) in (61),

$$U_{43}=\frac{\sqrt[4]{2}C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_2}\sqrt{\sigma}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}\,(\xi+\xi_0)\right)}\right),\ C_2C_0>0. \tag{85}$$

$$U_{44}=\frac{\sqrt[4]{2}C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{C_2}\sqrt{\sigma}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}\right),\ C_2C_0<0. \tag{86}$$

Family-III

$$A_1=\frac{C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_0}\sqrt{\sigma}},\ A_0=0,\ A_{-1}=-\frac{C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_2}\sqrt{\sigma}},\ k=-\frac{1}{2\sqrt{2C_2C_0\delta}} \tag{87}$$

Put (87) in (61),

$$U_{45} = -\frac{C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_2}\sqrt{\sigma}}\left(\frac{C_2}{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}\,(\xi+\xi_0)\right)}\right) - \left(\frac{C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_0}\sqrt{\sigma}}\right)\left(\frac{\sqrt{C_0C_2}\tan\left(\sqrt{C_0C_2}\,(\xi+\xi_0)\right)}{C_2}\right),\ C_2C_0>0. \tag{88}$$

$$U_{46} = -\frac{C_0^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_2}\sqrt{\sigma}}\left(\frac{C_2}{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}\right) - \left(\frac{C_2^{3/4}\sqrt[4]{\delta}\sqrt{l}}{\sqrt[4]{2}\sqrt[4]{C_0}\sqrt{\sigma}}\right)\left(\frac{\sqrt{-C_0C_2}\tanh\left(\sqrt{-C_0C_2}\,(\xi+\xi_0)\right)}{C_2}\right),\ C_2C_0<0. \tag{89}$$

Results and discussion

Fractional differential equations (FDEs) are formed by modeling due to nonlinear corporal system. The study of these types physical models have a strong position in applied science. Different researchers used distinct techniques for the determination the exact results of Eqs. (19), (39), (58), (75) in [49–52]. Some of our results are likely similar with other discovered due to the followings. Our solutions in Eqs. (31) and (32) are approximately similar to the results discussed in Eqs. (4.8) and (4.9) respectively in [47]. Further our results in Eqs. (67) and (68) are likely same form as discussed solutions in $u_{7,8}(x,t)$ and $u_{11,12}(x,t)$ gradually in [48]. Moreover our constructed solution in Eqs. (80) and (83) have also some similarity with the solutions mentioned in Eqs. (15) and (27) in [46]. we have found that our modified mathematical method is for explore of solutions which more general and powerful as compared in previous research literature. Hence From this we found that our described technique provide a best plate form as a mathematical tools for solving fractional order nonlinear wave problem in applied science see Figs. 1–5.

Conclusion

Exploring the exact solutions of conformable space-time derivative of EW, KdV-ZK, $(2+1)$-Zoomeron and MEW equations via novel effective technique, called a improve simple equation method.The work emphasizes our assurance that this method is a dedicated procedure to manage such types of fractional nonlinear space-time equations. The benefit of this technique is that we apply it in a straightforward way without utilizing linearizion, discretization and restrictive suppositions. Hence it is a very simply implementable mathematical tool for solving real-life problems budding in engineering and sciences.

CRediT authorship contribution statement

Asghar Ali: Data curation, Writing - original draft, Writing - review & editing. **Aly R. Seadawy:** Conceptualization, Methodology, Software, Supervision. **Dumitru Baleanu:** Visualization, Investigation, Software, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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