# DESIGNING AN ANNUAL LEAVE SCHEDULING POLICY: CASE OF A FINANCIAL CENTER 

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#### Abstract

Providing annual leave entitlements for employees can help alleviate burnout since paid-time off work directly affects the health and productivity of workers as well as the quality of the service provided. In this paper, we develop realistic vacation scheduling policies and investigate how they compare from both the employer and the employees' perspectives. Among those policies, we consider one that is used in practice, another that we propose as a compromise which performs very well in most cases, and one that is similar to machine scheduling for benchmarking. Integer programming models are formulated and solved under various settings for workload distribution over time, substitution and unit of time for vacations. We use three performance measures for comparisons: penalty cost of unused vacation days, percent vacation granted and level of employee satisfaction. We provide a real-life case study at a bank's financial center. Numerical results suggest that an all-or-nothing type of vacation policy performs economically worse than the others. Attractive annual leave scheduling policies can be designed by administering vacation schedules daily rather than weekly, ensuring full cover for off-duty employees, and offering employees some degree of choice over vacation schedules.


1. Introduction. Burnout has been recently recognized by The World Health Organization (WHO) in its International Classification of Diseases [59] as a syndrome resulting from chronic workplace stress that has not been successfully managed. WHO states that the syndrome is characterized by " 1 . feelings of energy depletion or exhaustion; 2. increased mental distance from one's job, or feelings of negativism or cynicism related to one's job; and 3. reduced professional efficacy." Consequently, employers and policymakers will have to (re)consider approaches to alleviate or prevent burnout at workplaces. Burnout syndrome has been elaborately studied in [30] among others and recently reviewed in [34]. Workplace stress is indicated to be a non-trivial contributor to health problems and costs in the United States (U.S.). The authors in [32] estimate, through statistical estimation models and optimization, that more than 120,000 deaths and $5 \%-8 \%$ of healthcare costs each year are associated with stressor related management decisions in the workplace. One of the ten workplace stressors covered in the paper is related to decisions about work

[^0]hours and shift work which have significant health consequences and we refer the reader to the references listed in the paper for several relevant studies. A notable factor in this context is that the employer is frequently the main decision maker for scheduling work hours and durations with little or no choice for the employees on the matter. Along with health and cost consequences, employee burnout has other implications. Although it is not our intention to cover possible negative impacts of employee burnout, we note that related research has started gaining more importance in the recent years (see, e.g., [51] for negative effects of burnout on customers' perception of employee service and overall organizations' service perceptions, and [50] for physical, psychological and occupational consequences) and its multidimensional effects are yet to be investigated.

An approach targeting burnout involves reducing daily working hours (see, e.g., $[57,33]$ ) or work days. Perpetual Guardian, an insurance company in New Zealand, reports a $20 \%$ increase in productivity, fall in stress and higher staff engagement after a test trial of reduced work-week duration (see [22]). Although it may require longer testing periods and more time to investigate its results and socioeconomic implications for the society, it is clear that having time-off from work is considered as a viable approach to reduce the workplace stress on employees. A multi-objective interval programming model is provided in [35] to investigate trade-offs among different aspects of job satisfaction. The authors indicate that longer working hours may increase employee earnings and belief in job security but at the cost of reduced satisfaction.

Another approach is to enforce workplaces to offer annual leave entitlement (that is, a paid vacation period consisting of consecutive days or weeks off work, usually earned based on the length of the time an employee has worked for the workplace) via labor laws. An exception is observed in U.S. where there is no statutory minimum paid vacation and it is left to the employer to offer paid vacation. For example, in California, U.S., if the employer offers paid vacation, then the earned vacation time is considered wages, and vacation pay accrues as it is earned. However, many other states and workplaces in the U.S. enforce use-it-or-lose-it policy requiring employees to use vacation prior a set date or lose it, i.e., without any sort of compensation. Even though it may not be currently mandatory everywhere (see, e.g., [40, 31, 44]), many workplaces have started to offer paid-time off work and even more generous annual leave rights to their employees [55], since paid annual leave is proven to offer many health and motivational benefits for the employees, and consequently benefits for the employers (see, e.g., [41] and references listed therein). Companies in European Union countries, for example, are required to offer at least a four-week annual paid leave to their workers as mandated in the European Working Time Directive [29]. This right cannot be forgone or forfeited. However, accruals may (inadvertently) occur bringing possible ramifications for the employer aside from the health and other negative implications for the employees. For example, when an employee is dismissed or retired, equivalent monetary amount of all accruals should be compensated for by the employer based on the employee's most recent salary, which is an additional cost. Accruals not only increase costs but also make it harder to generate feasible vacation schedules. To prevent such cases, many workplaces request employees' vacation plans by a certain date rather than treating vacation planning as a dynamic problem in which employees may request vacations at different times.

Driven by workloads and due dates, employers prioritize planning operations and scheduling their employees accordingly. In other words, they focus on getting the job done in a cost-effective manner, i.e., optimizing the number or cost of workers to meet workload demands by creating hourly, daily or weekly work schedules. Relatedly, employers usually have the right to impose restrictions on when a paid vacation can be taken and the amount of vacation that can be taken at any particular time [44]. While these rights allow the employers to exercise some control over a desired productivity level and ensure a balanced workload, employees may be forced to select their vacation days from a predetermined set of periods which may not coincide with their actual wishes or needs. Offering employees some appropriate degree of choice over their work and vacation can provide the flexibility to recuperate, rejuvenate, and have better access to the time and resources they need. Authors in [58] investigate several key interventions for burnout in hospital doctors through a survey and find that analysis of staffing levels and ensuring full cover during statutory leave are stated to be of utmost importance to doctors in order to alleviate their challenges in having access to statutory leave, advance information on when leave can be taken, and knowledge that an appropriate (paid) locum will be available. Thus, along with other approaches that consider short-term solutions to decrease fatigue and increase performance (see, e.g., [42, 31]), there is a need to consider longer-period breaks to alleviate the burnout.

In this paper, we address the problem of scheduling annual leave entitlements for employees. This research is motivated by the challenges faced by the management in scheduling paid vacations at a bank, where both the financial aspect and the lengthy working hours add to the workplace stress. Summer is the vacation season of choice for majority of the bank employees. Overlaps in vacation-period choices create difficulties for the management in trying to satisfy employee preferences on vacation periods while ensuring completion of tasks. Making a trade-off between the two is not simple. Despite the management is aware of the benefits of granting vacation to employees when they want it and for the duration they request, relevant decision guides in company policies do not allow too much room for employee satisfaction. Hiring and firing are stringent and lengthy processes in the banking industry and are not typically viable options as most workplaces in the service industry require highly skilled workers [23]. We note that these problems are not specific to the banking industry. Hospitals, academic institutions, government offices, and several other workplaces have similar difficulties in scheduling paid vacations. The relevant decision-making process should embody a multitude of factors and be able to suggest policies that consider costs and benefits to both the employer and the employees. To the best of our knowledge, there is no study that proposes different annual leave scheduling policies and provides a comparison of those to measure their impacts.

We formulate integer programming models for three vacation scheduling policies. The first model is closely related to the well known use-it-or-loose-it vacation policy that is used in practice. In particular, any vacation period from a set of employee preferences are either granted or denied in full. The second model can be used to devise a compromise between employee wishes, work demands and employer costs. This model relaxes the requirement to fully accept or fully reject any set of employee preferences by allowing shortening any vacation duration within the preferred period in accordance with the workload. The third model is a benchmark against which we compare the first two models. This model disregards employee preferences altogether and grants vacations based solely on the forecasted workload.

In this paper, we take into account several factors including workload distribution over time and the hierarchy among the employees who are eligible to perform different tasks, policies regarding preferences of employees, entitlement durations and how vacation periods are administered (i.e., in weeks or days), whether any portion of the entitlement can be deferred and corresponding caps on vacation accruals. We evaluate and compare the cross effects of these factors on three basic performance measures: monetary penalty cost paid for the unused portions of vacations, percent vacation granted, and the level of satisfaction measured by the percentage of matching granted and requested vacation periods. We further give an extension that improves the satisfaction level provided by the third model significantly, which otherwise fails to attain neither satisfactory nor acceptable levels of employee satisfaction in most of the cases. Particularly, the objective function is modified to include a weighted combination of the penalty cost saved by granting vacation days and a reward amount for every vacation day granted from the preferences of employees. A real-life case study at a bank's financial center is used to demonstrate policy performances. The numerical results show that the second model, which provides an intermediate policy, performs quite well with a small increase in the overall penalty cost paid by the employer while ensuring a high satisfaction level in employee preferences. In addition, substitution, i.e., full cover for employees on vacation, is indispensable for creating slack and flexibility for granting vacations.

The rest of the paper is organized as follows. Section 2 discusses relevant background and positions our work in literature. Section 3 defines our problem and different approaches for taking employee vacation preferences into account, and presents mathematical models under these approaches subject to several other vacation scheduling policy considerations. Details of the data set used in the case study are explained in Section 4. Numerical results are discussed in Section 5 and managerial insights are highlighted in Section 6. Concluding remarks are outlined and future research directions are presented in Section 7.
2. Literature review. Annual leave consideration has been gaining significant importance in recent years with several reports of its importance and benefits to employee health and motivation as well as social implications through field study and surveys (see, e.g., $[16,24,26,55,52,60]$ ). The literature on vacation scheduling is quite narrow. In fact, we are not aware of any study that proposes and compares different approaches for scheduling annual leave entitlements. A two-stage heuristic in [25] is developed to evaluate daily absenteeism requests considering legal constraints, social criteria (e.g., married employee couples, school-age children, etc.), capacity and qualification restrictions and preferences of drivers in a transportation company. The article's aim is to develop a sector-specific decision support system to quickly generate feasible solutions. A rule-based heuristic with a conflict-resolving algorithm is used for overlapping employee requests. This is the only article, to our knowledge, with its main focus on assigning off-duty periods to requesting employees, albeit in a suboptimal fashion and not entirely for the purpose of annual leave. Apart from this study that considers daily absenteeism requests of employees, the existing research focuses mostly on task and workforce scheduling and overlooks scheduling vacations. Particularly, major emphasis is on days-off, shift or annualized hours scheduling which are classified as sub-problems of workforce scheduling in [8]. A classification of workforce scheduling problems based on daily or weekly work schedules is given in [45]. We refer the interested reader to studies in [1] and [54] for compilations of workforce scheduling literature.

Days-off scheduling problem arises in workplaces operating seven days a week with varying daily labor requirements. Since workers must be given weekly breaks, they are assigned to different work patterns with days-off shifts. Shift scheduling problem arises in systems that operate on a daily multiple-shift basis in which a labor-cost minimizing assignment of employees to shifts is sought. Shift and daysoff scheduling problems are equivalent when there is a single shift on a work day. Consequently, related problems are classified under single-shift scheduling (see, e.g., $[13,27,5,15,28,37,9])$ and multiple-shift scheduling (see, e.g., [56, 47, 14, 39, 38]) of a single category or multiple categories of employees. There are other studies which consider employee preferences and hierarchy. Both [11] and [10] consider a nurse scheduling problem and point to importance of employee preferences on the quality of service along with other benefits. Authors in [2] and [4] assign employees to work locations and shifts during a week with two off days. They develop a two-stage formulation in the former and a column generation-based heuristic in the latter. Although they consider preferences with respect to locations and off days, their focus is on weekly workforce scheduling and annual leave consideration is out of scope. An extension of their work with a heuristic for the same problem under hierarchy is given in [3] (see also [48, 49, 53, 12, 46] for related studies that also consider hierarchy).

In annualized hours scheduling, a job contract is based on a certain number of hours that an employee must work annually. The employer needs to determine the number of employees to hire and their assignments to tasks. Scheduling is done usually on a weekly basis because the workload varies during the hours of a day, between days and across weeks. Hiring temporary or new workers, using multiple shifts, varying daily and weekly working hours are typical options for scheduling the workforce in such an environment. Consequently, many service workers (e.g., drivers, airline and healthcare personnel, etc.) are on an irregular roster with several days or weeks with a heavy work schedule followed by an off period. A mixed integer linear programming formulation of a multiple-shift workforce planning problem is solved in [36] to balance the workload and minimize the workforce size. Although decision variables include weekly vacation periods, employee preferences are not taken into account which results in irregular work hours and vacation periods. Irregular work hours and vacation periods, little or no consideration given to employee preferences may contribute to workplace stress, negatively impact employee motivation, performance, retention rates, and even lead to vacation accruals. In order to minimize such negative consequences, annualized work hours must be negotiated before signing a job contract and may have to be accompanied by some sort of compensation such as reduction in working hours, additional holidays, salary increase, etc. Hence, the annualized hours scheduling literature assumes that employees have a contract that specifies the annual work hours, holiday weeks and whether employees can take off days simultaneously. Whereas [49], [20], [18] and [17] consider vacation periods as part of the planning horizon, $[21,19]$ simply exclude them from consideration.

A three-step approach is proposed in [7] for solving a single-shift scheduling problem under annualized hours. Vacation periods are set beforehand by the company. In the first step, a minimal workforce is determined. Next, overtime hours are calculated to ensure a required number of holiday weeks for each employee, and possible holiday weeks are assigned considering employee preferences as much as possible. Finally, weekly work days are assigned to available employees. This study
is extended in [6] with two mixed integer programming formulations to determine a balanced workload and minimize overtime. Vacation weeks are fixed and workers are entitled to a minimum four-week vacation out of which two should be consecutive. In the first two models, each worker chooses vacation weeks from the given set, i.e., actual worker preferences are not taken into account. The first of these models minimizes the total workload deviation and overtime, and the second one minimizes the maximum workload deviation and overtime. The main focus is on scheduling the workers in the remaining work weeks. The other two models assign off weeks from a given set without considering employee preferences. Since vacations are in weeks, they cannot be divided into several consecutive days.

Authors in [43] schedule working hours and vacation weeks of cross-trained workers with different efficiencies under annualized hours. Workers take two holiday periods: two consecutive weeks in winter and four consecutive weeks in summer. The study optimizes start days of vacation periods and weekly working hours for each employee and decisions related to overtime and temporary workers in the first model. In the second model, vacation periods are randomly generated and fixed beforehand from which an assignment is made for each employee. The third and fourth models take optimal cost solutions of the first and the second models, respectively, as additional constraints to smooth out the working hours. The authors do not consider employee vacation preferences and they allow neither dividing vacation periods into several parts nor different durations.

In this study, on the contrary to the main focus of the existing workforce scheduling literature that deals with assigning employees to various tasks and generating employee-specific schedules, we address the often overlooked problem of scheduling vacations considering employee preferences. There are few studies in the literature that consider employee preferences. However, in these studies, tasks are scheduled first, and then, the remaining days are designated as possible vacation periods. Therefore, vacation periods are fixed prior to contract signing, and consequently, employees are forced to select their vacation days from a predetermined set with irregular durations which may not coincide with their actual wishes or needs. We note that our problem is vastly different than the traditional workforce scheduling problem with priorities for off days and weekends since those models are built on work-oriented policies that use the minimum legal requirements for off-duty or rest periods and schedule employees ignoring their needs in a manner that is similar to the way machines are scheduled. Our objective is also different than those considered in the literature in that we evaluate the performances of the policies from two different perspectives, in terms of the employers' and employees' points of views, by minimizing the monetary implications as well as maximizing the employee satisfaction. Our experiments show that there are significant performance differences between the implemented policies that depend on the factors mentioned previously. Therefore, our work points to the importance of designing vacation policies and schedules and selecting the most appropriate one for a particular workplace that will benefit both the employer and employees.
3. Problem definition and mathematical models. We consider a workplace where the management is concerned with scheduling annual leave entitlements of employees during a pre-specified vacation horizon while ensuring timely completion of forecasted tasks. The models we present in this section can be applied and solved for individual departments at a workplace as long as each employee belongs to one department. We based our models on a financial center that enables us to cover a
variety of realistic constraints and practices applicable to several other institutions.
An entitlement may be split into periods, each of which can be several consecutive days or weeks. The employer may or may not consider employee preferences in scheduling entitlements. In the former case employees need to indicate their preferences considering the timing and duration of vacation periods. The employer, however, may impose a restriction on whether vacation periods should be in full weeks (i.e., Monday through Sunday) or if they can be in some number of days instead of full weeks depending on the nature of the work, simplicity and amount of effort needed for the scheduling activities. Both of these situations are observed in practice. In this paper, we consider these two as weekly and daily vacation schedules, respectively. It can be noticed that an employee's preferences may not always coincide with those possible under the imposed restriction. We refer to an employee's vacation preferences without any restriction on the timing and duration imposed by the employer as actual preferences. An employee's vacation preferences indicated under the employer's policy will be referred to as daily preferences and weekly preferences depending on the duration requirement.

Our aim is to investigate the effects of some annual leave policies on vacation granted, level of satisfaction with respect to employee vacation preferences and the monetary implications of unused vacation days. To this end, we formulate three models (labeled as M1, M2, and M3) corresponding to different vacation policies depending on whether the actual employee preferences are taken into account and the manner in which these preferences are utilized in scheduling vacations. Among those policies, we consider one that is used in practice, another that we propose as a compromise, and one that is similar to machine scheduling for benchmarking purpose. The first is the commonly-used use-it-or-loose-it vacation policy that was explained in Section 1. The second policy is one that we propose to make up for the poor performance of the first. This policy relaxes the requirement for any preferred vacation period to be used in full or not at all, and instead allows for taking partial vacations within the preferred periods. The third policy is based on machine scheduling which does not consider any employee preferences. This policy is used for benchmarking purposes. Models M1, M2, and M3 correspond to these policies, respectively, and are explained briefly below. We then discuss variants of these models. All models are solved using a real-life case study whose details are presented in Section 4.

M1. All-or-nothing with preferences. This model is akin to the common use-it-or-loose-it vacation policy implemented at several workplaces, except that we include a penalty cost to measure implications for both the employer and the employees. Each employee must submit a set of vacation period preferences under the employer's vacation policy indicating the start and end days (or weeks) and whose durations sum up to the employee's annual leave entitlement. Any period from the set of preferences will be either granted or denied in full.
M2. Partially-satisfied preferences. We propose this model as a compromise between employee wishes, work demands and employer costs. We relax the requirement in model M1 for an employee to use a preferred vacation period in full or not use it at all. Model M2 determines a start and an end day (or week) through each of the preferred vacation periods based on workload requirements and the employer's vacation policy. That is, any preferred vacation period may be partially granted.

M3. No preference. This model can be considered as a machine-scheduling treatment to our problem and will be used as a benchmark. Vacation periods with start and end days (or weeks) for each employee are determined by the employer based on the forecasted workload and the vacation policy. This model yields an optimal vacation schedule based on the workload, but does not take employee preferences into account.

We append the letter $S$ to a model label if the company requires substitution for off-duty employees, and the letter $N$ indicating substitution is not possible. Before going into further details of the problem setting and mathematical models, we list the assumptions used by the financial center.

A1. Current workforce is sufficient to meet the daily varying workload demand.
A2. Labor law or company policy requires that at least one vacation period lasts a predetermined minimum number of days or weeks. Also, there may be an upper bound on the number of periods that an entitlement can be split into.
A3. There must be at least one work day (or week) between two consecutive vacation periods depending on the vacation policy.

Assumption 1 is reasonable for workplaces such as banks and universities in which workforce requirements must be met. In both banking industry and higher education, a minimum number of tenured and non-tenured workforce requirements must be met and maintained. If assumption 1 is not reasonable for a workplace, hiring and dismissal decisions can be incorporated into the models to ensure sufficient workforce. The first part of assumption 2 is enforced in different lengths in some workplaces and countries (e.g., Andorra, Brazil, Columbia, Hong Kong, Principality of Monaco, Turkey to name a few) or may be required by the employer. The second part of this assumption is optional. It may prevent an employee from prolonging off days simply by dividing total entitlement into many parts and adjoining them with weekends and other holidays. Finally, assumption 3 is a common practice that requires an employee to sign documents on a workday to verify returning to work after vacation. We note that these are not strong assumptions and variants can be observed at different workplaces.

Vacation horizon (can be the whole year or some designated period) consists of $T$ days if a daily vacation schedule is used or $W$ weeks $(7 W=T)$ if a weekly vacation schedule is employed. There is a set of $J$ task types with $N_{j t}$ type $j$ tasks to be performed on day $t . I$ and $I_{j}$ correspond to sets of all employees and those eligible to perform a task type $j$, respectively. Duration of work hours in a day is scaled to unity (e.g., an eight-hour work day is taken to be as one day) and employee $i$ needs $r_{i j}$ days to complete a type $j$ task where $r_{i j} \in(0,1]$. Depending on the vacation policy, each employee $i$ is entitled to $A_{i}$ days or $A_{i}^{\prime}$ weeks of annual leave. Any unused vacation day has to be compensated for by the employer at a daily penalty cost of $c_{i}$. We note that the penalty cost is differentiated with respect to the employee. If vacation preferences are taken into account to some degree, each employee requests a total of $P_{i}$ preferred vacation periods $\left(p=1,2, \ldots, P_{i}\right)$, and an employee may split the entitlement into at most $m$ periods. A starting day (week) number $s_{i p}\left(s_{i p}^{\prime}\right)$, and duration in days (weeks) $d_{i p}\left(d_{i p}^{\prime}\right)$, of each vacation period $p$ for employee $i$ are indicated on the request. If required (e.g., through labor law or by the employer), at least one of the vacation periods should be at least $f\left(f^{\prime}\right)$ days (weeks) long. Table 1 lists sets, parameters, and decision variables.

TABLE 1. Sets, parameters and decision variables used in models.

| Sets: |  |
| :--- | :--- |
| $T$ | Vacation horizon, in days, indexed by $t, \tau, h$ or $\ell$ |
| $W$ | Vacation horizon, in weeks, indexed by $k$ or $w$ |
| $J$ | Set of tasks, indexed by $j$ |
| $I$ | Set of employees, indexed by $i$ |
| $I_{j}$ | Set of employees eligible to perform task type $j$ |
| Parameters: |  |
| $N_{j t}$ | Number of type $j$ tasks on day $t$ |
| $r_{i j}$ | Time, in fraction of days, required for employee $i$ to complete a type $j$ |
| $A_{i}\left(A_{i}^{\prime}\right)$ | task, $r_{i j} \in(0,1]$ |
| $P_{i}$ | Numbal leave entitlement, in days (weeks), of employee $i$ |
| $s_{i p}\left(s_{i p}^{\prime}\right)$ | Starting day (week) of the $p$ th vacation-period preference of employee $i$ |
| $d_{i p}\left(d_{i p}^{\prime}\right)$ | Duration, in days (weeks), of the $p$ th vacation-period preference of em- <br>  <br> $m$ |
| ployee $i$ |  |
| Upper bound on the number of times an entitlement can be split |  |
| $c_{i}$ | Minimum length, in days (weeks), for at least one part of the entitlement |
| $\alpha$ | Penalty cost per day for employee $i$ |

In this setting, the employer's objective (1) is to maximize the total weighted sum of penalty cost saved by granting vacation days and a reward amount due to granting vacation days coinciding with the actual employee preferences. We note that the penalty cost paid can be simply calculated by $\sum_{i \in I} c_{i}\left(A_{i}-\sum_{t=1}^{T} y_{i t}\right)$. We maximize the penalty cost saved rather than paid since they are equivalent and because it is more sensible to combine this objective with a maximization of a reward for granting vacations from employee preferences. The second term allows the employer to place some desired level of emphasis on employee preferences relative to the penalty cost saved given in the first term. When the multiplier $\alpha$ equals to zero, the employer does not put any emphasis on the actual employee preferences. As the value of the multiplier $\alpha$ increases to positive values, so does the employer's emphasis on the
employee preferences, proportional to the penalty cost relevant for the employee.

$$
\begin{equation*}
\max \sum_{i \in I} \sum_{t=1}^{T} c_{i} \cdot y_{i t}+\sum_{i \in I} \sum_{p=1}^{P_{i}} \sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} \alpha \cdot c_{i} \cdot y_{i t} \tag{1}
\end{equation*}
$$

Note that a vacation policy is set by the employer and modeled using both constraints and the objective function. All models have the same objective function with various constraints considering employee preferences as outlined by the corresponding policy. An objective that prioritizes employee satisfaction (i.e., amount of vacation granted from the actual employee preferences) without considering other implications (i.e., cost of unused vacations) neglects the authority of the employer. Therefore, using a weighted objective function allows the employer to solve the same model by varying the weights placed on the employer's and the employees' perspectives. Since the actual monetary implications are always considered, its weight is fixed at one. We use multiplier values ranging from zero to 1.5 as a relative weight for the employee satisfaction which is expressed as a monetary reward. Given a policy, the decision maker can solve the corresponding same model under different multiplier values to prioritize the employee perspective. On the other hand, given a fixed multiplier value, all models will have the same objective function. Thus, their solutions will help to compare the performance of each corresponding policy.

We note that the first two models (M1 and M2) use employee preferences as hard constraints, whereas the last model (M3) does not use preferences at all. Since model M1 grants or denies any preferred vacation period in full, the second term merely increases values of monetary coefficients in the first term proportional to their values, and hence, will not affect the optimal solution that would be obtained in its absence. Optimal solution under model M2 may be affected since, by definition, this model allows for granting partial vacations within periods requested by the employees. Finally, using such an objective function in model M3 can provide the employer with an intermediary when a positive weight is used. However, since no actual preferences are used in the constraints of model M3, it is still possible to grant vacation outside preferences. We note that multiplier values can be customized for each employee although we used a common value for all employees in our numerical experiments to better assess its effect and not to confound the results.

In what follows, we provide the constraints for each model under daily preferences considering substitution in Subsections 3.1-3.3. Models under weekly preferences follow a similar structure in which parameters and decision variables are replaced with those defined considering weekly vacation schedules (see Table 1) and have the additional constraint set (2) below that links the vacation days and weeks.

$$
\begin{equation*}
z_{i k}=y_{i t}, \quad \forall k, t=\lceil k / 7\rceil \tag{2}
\end{equation*}
$$

We note that all models can easily be modified when substitution is not allowed by restricting the indices in constraint sets (3)-(6) related to performing tasks by eligible employees.
3.1. M1. All-or-nothing with preferences under daily vacation schedules. Any period in an employee's set of vacation preferences is either granted or denied in full. Constraints for M1 are explained and formulated in (3)-(12) below.

Time spent on a day's tasks cannot exceed work hours equivalent of one day.

$$
\begin{equation*}
\sum_{j \in J} r_{i j} \cdot x_{i j t} \leq 1, \quad \forall i, t \tag{3}
\end{equation*}
$$

All tasks must be completed each day.

$$
\begin{equation*}
\sum_{i \in I_{j}} \frac{x_{i j t}}{r_{i j}} \geq N_{j t}, \quad \forall j, t \tag{4}
\end{equation*}
$$

At least one eligible employee must be present everyday for each task type.

$$
\begin{equation*}
\sum_{i \in I_{j}}\left(1-y_{i t}\right) \geq 1, \quad \forall j, t \tag{5}
\end{equation*}
$$

An employee cannot be present at work if $\mathrm{s} / \mathrm{he}$ is on vacation on that day.

$$
\begin{equation*}
x_{i j t}+y_{i t} \leq 1, \quad \forall i, j, t \tag{6}
\end{equation*}
$$

Days off during a vacation period equal to the length of the requested period if granted.

$$
\begin{equation*}
\sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} y_{i t}=d_{i p} \cdot u_{i p}, \quad \forall i, p \tag{7}
\end{equation*}
$$

An employee cannot take vacation outside his/her preferences.

$$
\begin{equation*}
y_{i t}=0, \quad \forall i, t=T /\left\{s_{i p}, s_{i p}+1, \ldots, s_{i p}+d_{i p}-1\right\}_{p=1}^{P_{i}} \tag{8}
\end{equation*}
$$

Number of times an employee takes a vacation is the number of periods granted.

$$
\begin{equation*}
\sum_{p=1}^{P_{i}} u_{i p}=n_{i}, \quad \forall i \tag{9}
\end{equation*}
$$

Disjunctive constraints enforce at least one vacation period to be minimum $f$ days.

$$
\begin{align*}
& d_{i p} \cdot u_{i p} \geq f \cdot q_{i p}, \quad \forall i, p  \tag{10}\\
& \sum_{p=1}^{P_{i}} q_{i p} \geq n_{i}, \quad \forall i \tag{11}
\end{align*}
$$

Annual leave entitlement can be split into at most a predetermined number of parts (optional constraint).

$$
n_{i} \leq m
$$

3.2. M2. Partially-satisfied preferences under daily vacation schedules. Employee preferences are flexible such that any period in the set of preferences may be partially granted. Constraint sets (3)-(6), (8), and (12) are also part of model M2 but will not be repeated in order to save space. Other constraints are given in (13)-(19).

Days off during a requested period equal to the number of days granted.

$$
\begin{equation*}
\sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} y_{i t}=\sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} \sum_{h=1}^{\min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}} h \cdot u_{i p t h}, \quad \forall i, p \tag{13}
\end{equation*}
$$

Number of times an employee takes a vacation is the number of times a vacation starts.

$$
\begin{equation*}
\sum_{p=1}^{P_{i}} \sum_{t=s_{i p}}^{s_{i p}+d_{i_{p}}-1} \sum_{h=1}^{\min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}} u_{i p t h}=n_{i}, \quad \forall i \tag{14}
\end{equation*}
$$

Each day during a period can be either a starting day for a vacation or not.

$$
\begin{equation*}
\sum_{h=0}^{\min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}} u_{i p t h}=1, \quad \forall i, p, t=s_{i p}, s_{i p}+1, \ldots, s_{i p}+d_{i p}-1 \tag{15}
\end{equation*}
$$

A vacation can start at most once during a period.

$$
\begin{equation*}
\sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} \sum_{h=1}^{\min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}} u_{i p t h} \leq 1, \quad \forall i, p \tag{16}
\end{equation*}
$$

Days off resume from the day an employee begins a vacation until the end of the vacation granted within a period.

$$
\begin{array}{ll}
y_{i \tau} \geq u_{i p t h}, & \forall i, p, t=s_{i p}, s_{i p}+1, \ldots, s_{i p}+d_{i p}-1 \\
& h=1,2, \ldots, \min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\} \\
& \tau=t, t+1, \ldots, t+h-1 \tag{17}
\end{array}
$$

Disjunctive constraints enforce at least one vacation period to be minimum $f$ days.

$$
\begin{gather*}
h \cdot u_{i p t h} \geq f \cdot q_{i p t h}, \quad \forall i, p, t=s_{i p}, s_{i p}+1, \ldots, s_{i p}+d_{i p}-1, \\
h=1,2, \ldots, \min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}  \tag{18}\\
\sum_{p=1}^{P_{i}} \sum_{t=s_{i p}}^{s_{i p}+d_{i p}-1} \sum_{h=1}^{\min \left\{d_{i p}, s_{i p}+d_{i p}-t\right\}} q_{i p t h} \geq n_{i}, \quad \forall i \tag{19}
\end{gather*}
$$

3.3. M3. No preference under daily vacation schedules. Model M3 schedules annual leave entitlements based solely on the forecasted workload and does not consider employee vacation preferences. It is used as a benchmark to compare with other models. Constraint sets (3)-(6), and (12) from model M1 are also part of model M3 but will not be repeated in order to save space. We explain and formulate additional constraints in (20)-(27) below.

Days off for an employee cannot exceed the duration of the annual leave entitlement.

$$
\begin{equation*}
\sum_{t=1}^{T} y_{i t} \leq A_{i}, \quad \forall i \tag{20}
\end{equation*}
$$

Days off equal to the sum of days granted through all vacation periods.

$$
\begin{equation*}
\sum_{t=1}^{T} y_{i t}=\sum_{t=1}^{T} \sum_{h=0}^{\min \left\{A_{i}, T-t+1\right\}} h \cdot u_{i t h}, \quad \forall i \tag{21}
\end{equation*}
$$

Number of times an employee takes a vacation is the number of times a vacation starts.

$$
\begin{equation*}
\sum_{t=1}^{T} \sum_{h=1}^{\min \left\{A_{i}, T-t+1\right\}} u_{i t h}=n_{i}, \quad \forall i \tag{22}
\end{equation*}
$$

Each day during a period can be either a starting day for a vacation or not.

$$
\begin{equation*}
\sum_{h=0}^{\min \left\{A_{i}, T-t+1\right\}} u_{i t h}=1, \quad \forall i, t \tag{23}
\end{equation*}
$$

An employee needs to be off for the period starting on his/her first of vacation.

$$
\begin{equation*}
y_{i \tau} \geq u_{i t h}, \quad \forall i, t, h=1, \ldots, \min \left\{A_{i}, T-t+1\right\}, \tau=t, t+1, \ldots, t+h-1 \tag{24}
\end{equation*}
$$

An employee cannot start another vacation until one day after the current one ends.

$$
\begin{array}{ll}
u_{i t h}+u_{i \tau \ell} \leq 1, & t=1,2, \ldots, T-1, h=1,2, \ldots, \min \left\{A_{i}, T-t+1\right\} \\
& \tau=t+1, t+2, \ldots, \min \{t+h, T\} \\
& \ell=1,2, \ldots, \min \left\{A_{i}, T-\tau+1\right\} \tag{25}
\end{array}
$$

Disjunctive constraints enforce at least one vacation period to be minimum $f$ days.

$$
\begin{align*}
& h \cdot u_{i t h} \geq f \cdot q_{i t h}, \quad \forall i, t, h  \tag{26}\\
& \sum_{t=1}^{T} \sum_{h=1}^{\min \left\{A_{i}, T-t+1\right\}} q_{i t h} \geq n_{i}, \quad \forall i \tag{27}
\end{align*}
$$

4. Case study. We solved all of the models for a medium-sized financial center with 30 employees. The current paid-vacation scheduling practice at the bank resembles all-or-nothing type of policy that corresponds to model M1. Summer is the preferred vacation season for most of the employees in the financial center. Therefore, we consider a 91-day (13-week) vacation horizon. Note that a longer vacation horizon, e.g., a year, can also be used, but having a smaller interval increases the likelihood of overlapping vacation preferences and allows us to observe how the models perform in such cases. We obtained real-life data for some of the parameters from the bank and needed to generate others either because they are confidential or they exhibit variations. These will be explained in the remainder of this section.

The employees perform eight types of tasks during an eight-hour shift every day. Employees and their tasks are ranked by hierarchy (management and corresponding highest-level task at the top). The financial center provided an average number of these tasks performed over the planning horizon, estimates of minimum and maximum number of daily tasks but also stated that the daily task load distribution may exhibit various patterns in practice. As a result, we generated plausible task load distributions by fixing the total number of tasks over the horizon for each level. Task loads are such that there should be at least one employee for each task
type every day, and hence, scheduling is non-trivial. The bank requires cover for each off-duty employee. An employee assigned to a particular task may substitute for another responsible for the next lower-level task, albeit requiring roughly $20 \%$ more time to complete the task. Note that this is often the case, i.e., skill level and experience of an employee affect the completion time of a task, especially if it is a manual one [61]. The two employees in the top management do not have any substitute other than each other. The penalty cost for an unused vacation day is not disclosed. Therefore, we generated the relevant costs according to the hierarchy, i.e., more for the top management and decreasing with the levels considering the same behavior in their salaries. Table 2 shows the corresponding parameter values for employee group (EG), number of employees (NE), daily penalty cost per employee (DC), unit task duration in minutes (TD), minimum and maximum daily number of tasks per employee (NT), and total number of tasks over the horizon (TT).

Table 2. Parameter settings for employees and tasks.

| EG | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NE | 2 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| DC | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| TD | 60 | 40 | 24 | 20 | 16 | 12 | 8 | 6 |
| NT | $[5,13]$ | $[9,21]$ | $[17,57]$ | $[21,69]$ | $[29,117]$ | $[37,197]$ | $[61,297]$ | $[80,477]$ |
| TT | 845 | 1,380 | 3,342 | 4,193 | 6,791 | 10,440 | 16,083 | 25,449 |

Figure 1 shows the four task load distributions covering a variety of practical situations. Uniform, cyclic and random task loads are generated as implied by their names. The spread of task types and loads differentiates the two cyclic distributions. Daily tasks are generated randomly and then adjusted to ensure equal load for each employee group over the vacation horizon across all distributions, which lets us compare the task load distributions' effects on model performances.


Figure 1. Daily and average task loads over the vacation horizon under different task load distributions.

The financial center schedules vacations on a daily basis. However, we consider both daily and weekly vacation schedules to provide insight into their effects. Furthermore, entitlement information is company confidential, so we had to generate an entitlement for each of the 30 employees, and consequently, the distribution of
entitlements. We note that paid vacation days vary across countries that mandate annual leave. Values between 20 and 30 days are reported in [44] and [31] for most of the countries belonging to the Organisation for Economic Co-operation and Development. Therefore, we set annual leave entitlements to 21 and 28 days. These numbers correspond to three and four weeks, respectively, which also enables us to make a fair comparison between daily and weekly vacation scheduling policies. We vary the percentages of employees entitled to 21 or 28 days in two entitlement distributions which we label as C1 and C2. In C1, $50 \%$ of the employees are entitled to 21 days and the rest are entitled to 28 days. In $\mathrm{C} 2,70 \%$ of the employees are entitled to 21 days and the rest are entitled to 28 days. We include the optional constraint in (12) since the management does not allow an entitlement to be divided into more than three parts. Furthermore, at least one of the vacation periods should last a minimum of $f=7$ days (i.e., one week).

Some employees may not be able to use their entitlements in full due to recurring busy periods, follow-ups required, contractual restrictions on vacation periods or simply because the workforce may be insufficient. An employer may allow deferring any remaining vacation to the next year. Vacation accruals, however, may be a burden on other employees if they are too long or taken during busy periods, and may negatively affect the functioning of a workplace. In our numerical study, we generate three accrual scenarios labeled as S1, S2, S3. Scenario S1 is the base case with no accrual, scenarios S2 and S3 correspond to 7 and 14 days of accrual, respectively. In terms of weekly vacation schedules, these accruals amount to one week and two weeks, respectively.

We randomly generated daily vacation preferences of each employee, i.e., how many parts an entitlement is split into, start day and duration of each vacation period. We further note that the daily vacation preferences are considered as the actual preferences. Therefore, we tried to cover as many days as possible from these actual preferences when we generated the corresponding weekly preferences. Multiplier values used in the objective function of the models for rewarding days granted from employee preferences are varied between 0 and 1.5 with 0.5 increments.

For each model considered (M1, M2, M3), we have a total of 192 problem instances under each of the daily and weekly vacation schedules. CPLEX (version 12.9) is used for the numerical study on a computer equipped with Intel Core i51.3 GHz dual-core processor and 8GB RAM. We set an upper bound of 14,400 seconds for the run time. All instances are solved within the allotted time, in $0.02-$ 1.31 seconds for models M1 and M2, 11.02-13,377.42 seconds for M3 under daily vacation schedules; in 0.02-12.92 seconds considering all models under weekly vacation schedules. We report only a subset of our numerical study results due to limited space. However, a supplement reporting all experiments is available from the authors upon request.
5. Discussions of the case study results. Analyses of our case study show that different parameter settings in the workplace may have various effects on each model's performance. We evaluate and compare models based on three performance measures:

- Penalty cost paid is the monetary amount that has to be compensated for by the employer due to unused vacation days. We include penalty cost saved in the objective function (see Section 3) and calculate the penalty cost by subtracting the saved amount from the total monetary equivalent of entitlements.
- Percent vacation granted is the percentage of entitlements granted.
- Satisfaction level is measured as a percentage of the entitlements granted from the actual preferences of employees.
In the following subsections, we first investigate individual and cross effects of: (1) substitution, (2) duration of vacation period (i.e., daily vs. weekly vacation schedules), (3) taking employee preferences into account and the manner in which preferences are handled, and (4) accruals and entitlement on penalty cost paid, percent vacation granted and percent satisfaction under different task load distributions. Then, we provide a discussion about the amount of vacation granted and satisfaction level for individual employee groups in the hierarchy, discuss which factors are important and provide some possible extensions. The discussions presented in this section may help an employer identify an appropriate vacation scheduling policy based on particular workplace settings.
5.1. Effects of substitution. Substitution is one of the most influential factors on the resulting penalty costs, vacation granted and percent satisfaction. Therefore, we will further discuss its cross effects in the following subsections. Prominent effects of substitution are summarized below and demonstrated in Tables 3-6 for 50\%-50\% entitlement distribution and no accrual.
- Not allowing substitution severely impairs model performances. Enforcing substitution results in significantly less penalty costs, more vacation days and higher satisfaction levels compared to not allowing it.
- Fluctuation in task load distribution (e.g., cyclic or random) severely impairs model performances unless substitution is allowed or required. Substitution lessens the negative impacts of fluctuations in the workload.


### 5.2. Effects of vacation period duration: Daily versus weekly vacation

 schedules. Values of percent vacation granted under both daily and weekly vacation schedules are very close under a given task load distribution, but vary across different task load distributions (in particular under models M1 and M3) and depending on if substitution is allowed or not. In addition, the accrual amount and the emphasis placed on preferences also affect model M3's performance.On the other hand, percent satisfaction is significantly affected by the unit of measure for vacations (days or weeks). Additionally, percent vacation granted and percent satisfaction are, in general, higher when vacations are administered daily rather than weekly. Pertinent results are summarized in Tables 3-6.
5.2.1. Observations under Model M1. Under Uniform Task Load Distribution: Under the financial center's current policy (corresponding to model M1 under substitution), percent vacation granted is very high (above $78.2 \%$ considering both daily and weekly vacation schedules). With substitution, the employer's choice of daily vacation schedules rather than weekly ones makes sense since a significant increase in employee satisfaction level can be obtained by increasing the penalty cost paid relatively a small amount. On the other hand, without substitution, values of the percent vacation granted are relatively lower (below $68.7 \%$ considering both daily and weekly vacation schedules). Without substitution, weekly vacation schedules grant longer durations than daily vacation schedules, although the opposite holds for satisfaction levels. Therefore, although higher satisfaction levels can be achieved under daily vacation schedules, the employer may prefer the weekly format to grant relatively more days at lower penalty costs if substitution is not possible.

Under Cyclic Task Load Distributions: Under the financial center's current policy (corresponding to model M1 under substitution), values of the percent vacation granted may be considered acceptable (above $67.1 \%$ considering both daily and weekly vacation schedules). With substitution, the employer may prefer to use daily vacation schedules rather than weekly ones since a significant increase in employee satisfaction level can be obtained by increasing the penalty cost relatively a small amount. On the other hand, if substitution is not possible, values of the percent vacation granted are below $31.8 \%$ (considering both daily and weekly vacation schedules), which may be considered unacceptable. In addition, we cannot definitely say that the employer should choose daily or weekly vacation schedules over the other. We observed cases where weekly vacation schedules are costlier than daily ones with more total vacation days at higher satisfaction levels (e.g., under Cyclic-2 task load distribution, without accrual and entitlement distribution (70\%, $30 \%)$ ), and other cases resulting in the opposite situation.

Under Random Task Load Distribution: Under the financial center's current policy (corresponding to model M1 under substitution), values of the percent vacation granted may be considered acceptable (above $63.7 \%$ considering both daily and weekly vacation schedules). With substitution, the employer may prefer to use daily vacation schedules rather than weekly ones since a significant increase in employee satisfaction level can be obtained by increasing the penalty cost relatively a small amount. On the other hand, if substitution is not possible, values of the percent vacation granted are below $4 \%$ (considering both daily and weekly vacation schedules) and neither daily nor weekly vacation schedules yield satisfactory levels of performance measures. Therefore, under random task loads, the employer should enforce substitution.
5.2.2. Observations under Model M2. Daily vacation schedules are less costly and result in more total vacation days with higher satisfaction levels compared to weekly vacation schedules considering all task load distributions except for the uniform distribution. With a uniform task load distribution, both daily and weekly vacation schedules yield similar total vacation days, whereas the former results in significantly higher satisfaction levels albeit at higher penalty cost. In other words, the employer may prefer daily vacation schedules to ensure higher satisfaction levels by increasing the penalty cost a relatively small amount.
5.2.3. Observations under Model M3. Under Uniform Task Load Distribution: Daily vacation schedules yield higher satisfaction levels compared to the weekly ones if employee preferences are taken into account in the objective function (i.e., $\alpha>0$ ) and if the accrual is very high (i.e., two weeks).

Under Cyclic and Random Task Load Distributions: The employer may prefer daily rather than weekly vacation schedules since, in general, the daily schedules result in more vacation days at higher satisfaction levels and lower penalty cost. We note that in a few cases we observed lower satisfaction levels under daily vacation schedules compared to those under weekly ones but the differences were not significant.
5.3. Effects of employee preferences. Recall that the first two models (M1 and M2) take employee preferences into account at least partially whereas the last model (M3) disregards any employee preference unless the objective function is modified to include preferences. In what follows, we compare the models by investigating the effects of taking employee preferences into account and the manner in which
preferences are handled on the total penalty cost, percent vacation granted and percent satisfaction (see Tables 3-6).

TABLE 3. Comparison of models under uniform task load distribution.

| Vacation |  | With Substitution |  |  |  | No Substitution |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Schedules | Model | $\alpha=0$ | $=0.5$ | $\alpha=1$ | $\alpha=1.5$ | $\alpha=0$ | $=0.5$ | $\alpha=1$ | $\alpha=1.5$ |
| Penalty Cost Paid |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 680 | 680 | 680 | 680 | 1404 | 1404 | 1404 | 1404 |
|  | M2 | 475 | 475 | 475 | 475 | 1151 | 1151 | 1151 | 1151 |
|  | M3 | 336 | 336 | 336 | 336 | 630 | 630 | 630 | 630 |
| Weekly | M1 | 427 | 427 | 427 | 427 | 1120 | 1120 | 1120 | 1120 |
|  | M2 | 427 | 427 | 434 | 434 | 1120 | 1120 | 1120 | 1120 |
|  | M3 | 336 | 336 | 336 | 336 | 630 | 630 | 630 | 630 |
| \% Vacation Granted |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 85.9 | 85.9 | 85.9 | 85.9 | 58.8 | 58.8 | 58.8 | 58.8 |
|  | M2 | 90.5 | 90.5 | 90.5 | 90.5 | 68.0 | 68.0 | 68.0 | 68.0 |
|  | M3 | 94.3 | 94.3 | 94.3 | 94.3 | 88.6 | 88.6 | 88.6 | 88.6 |
| Weekly | M1 | 91.4 | 91.4 | 91.4 | 91.4 | 67.6 | 67.6 | 67.6 | 67.6 |
|  | M2 | 91.4 | 91.4 | 91.4 | 91.4 | 67.6 | 67.6 | 67.6 | 67.6 |
|  | M3 | 94.3 | 94.3 | 94.3 | 94.3 | 88.6 | 88.6 | 88.6 | 88.6 |
| \% Satisfaction |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 85.9 | 85.9 | 85.9 | 85.9 | 58.8 | 58.8 | 58.8 | 58.8 |
|  | M2 | 90.5 | 90.5 | 90.5 | 90.5 | 68.0 | 68.0 | 68.0 | 68.0 |
|  | M3 | 27.6 | 59.7 | 59.7 | 59.7 | 22.6 | 50.7 | 53.1 | 54.4 |
| Weekly | M1 | 55.4 | 55.9 | 55.9 | 55.9 | 42.4 | 45.4 | 45.4 | 45.4 |
|  | M2 | 54.0 | 55.9 | 56.2 | 56.2 | 38.4 | 46.0 | 46.0 | 46.0 |
|  | M3 | 32.0 | 62.0 | 62.0 | 62.0 | 27.8 | 51.2 | 51.2 | 51.2 |

TABLE 4. Comparison of models under cyclic-1 task load distribution.

5.3.1. Effects of Employee Preferences on Penalty Cost. The least amount of penalty is paid under M3, followed by M2 closely, then by M1 (see Figure 2 below). This is expected since model M3 is flexible in choosing vacation days. Similarly, model M2 outperforms model M1 since the former allows partial vacations whereas the latter

Table 5. Comparison of models under cyclic-2 task load distribution.

| Vacation |  | With Substitution |  |  |  | No Substitution |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Schedules | Model | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1$ | $\alpha=1.5$ | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1$ | $\alpha=1.5$ |
| Penalty Cost Paid |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 843 | 843 | 843 | 843 | 2079 | 2079 | 2079 | 2079 |
|  | M2 | 549 | 549 | 549 | 549 | 1635 | 1635 | 1635 | 1635 |
|  | M3 | 112 | 112 | 112 | 112 | 643 | 647 | 650 | 654 |
| Weekly | M1 | 602 | 602 | 602 | 602 | 2142 | 2142 | 2142 | 2142 |
|  | M2 | 588 | 602 | 602 | 602 | 1946 | 1946 | 1946 | 1946 |
|  | M3 | 336 | 336 | 336 | 336 | 1435 | 1435 | 1435 | 1435 |
| \% Vacation Granted |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 78.6 | 78.6 | 78.6 | 78.6 | 21.0 | 21.0 | 21.0 | 21.0 |
|  | M2 | 87.6 | 87.6 | 87.6 | 87.6 | 37.6 | 37.6 | 37.6 | 37.6 |
|  | M3 | 98.1 | 98.1 | 98.1 | 98.1 | 81.9 | 81.8 | 81.4 | 81.2 |
| Weekly | M1 | 84.8 | 84.8 | 84.8 | 84.8 | 20.0 | 20.0 | 20.0 | 20.0 |
|  | M2 | 84.8 | 84.8 | 84.8 | 84.8 | 26.7 | 26.7 | 26.7 | 26.7 |
|  | M3 | 94.3 | 94.3 | 94.3 | 94.3 | 47.6 | 47.6 | 47.6 | 47.6 |
| \% Satisfaction |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 78.6 | 78.6 | 78.6 | 78.6 | 21.0 | 21.0 | 21.0 | 21.0 |
|  | M2 | 87.6 | 87.6 | 87.6 | 87.6 | 37.6 | 37.6 | 37.6 | 37.6 |
|  | M3 | 27.9 | 58.0 | 58.2 | 58.2 | 21.0 | 36.5 | 37.3 | 38.1 |
| Weekly | M1 | 51.2 | 52.8 | 52.8 | 52.8 | 7.9 | 11.3 | 11.3 | 11.3 |
|  | M2 | 49.4 | 53.2 | 53.2 | 53.2 | 13.7 | 17.0 | 17.0 | 17.0 |
|  | M3 | 26.3 | 62.4 | 62.4 | 62.4 | 14.0 | 23.3 | 23.3 | 23.3 |

TABLE 6. Comparison of models under random task load distribution.

| Vacation |  | With Substitution |  |  |  | No Substitution |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Schedules | Model | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1$ | $\alpha=1.5$ | $\alpha=0$ | $\alpha=0.5$ | $\alpha=1$ | $\alpha=1.5$ |
| Penalty Cost Paid |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 884 | 884 | 884 | 884 | 2647 | 2647 | 2647 | 2647 |
|  | M2 | 561 | 561 | 561 | 561 | 2491 | 2491 | 2491 | 2491 |
|  | M3 | 336 | 336 | 336 | 336 | 2214 | 2218 | 2221 | 2221 |
| Weekly | M1 | 609 | 609 | 616 | 616 | 2597 | 2597 | 2597 | 2597 |
|  | M2 | 609 | 609 | 630 | 630 | 2597 | 2597 | 2597 | 2597 |
|  | M3 | 336 | 336 | 336 | 336 | 2583 | 2583 | 2583 | 2583 |
| \% Vacation Granted |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 77.3 | 77.3 | 77.3 | 77.3 | 1.4 | 1.4 | 1.4 | 1.4 |
|  | M2 | 87.5 | 87.5 | 87.5 | 87.5 | 9.3 | 9.3 | 9.3 | 9.3 |
|  | M3 | 94.3 | 94.3 | 94.3 | 94.3 | 32.9 | 32.7 | 32.2 | 32.2 |
| Weekly | M1 | 83.8 | 83.8 | 83.8 | 83.8 | 3.8 | 3.8 | 3.8 | 3.8 |
|  | M2 | 83.8 | 83.8 | 83.8 | 83.8 | 3.8 | 3.8 | 3.8 | 3.8 |
|  | M3 | 94.3 | 94.3 | 94.3 | 94.3 | 4.8 | 4.8 | 4.8 | 4.8 |
| \% Satisfaction |  |  |  |  |  |  |  |  |  |
| Daily | M1 | 77.3 | 77.3 | 77.3 | 77.3 | 1.4 | 1.4 | 1.4 | 1.4 |
|  | M2 | 87.5 | 87.5 | 87.5 | 87.5 | 9.3 | 9.3 | 9.3 | 9.3 |
|  | M3 | 26.8 | 59.3 | 59.3 | 59.3 | 8.2 | 15.6 | 16.1 | 15.6 |
| Weekly | M1 | 51.2 | 52.5 | 52.8 | 52.8 | 1.6 | 2.3 | 2.3 | 2.3 |
|  | M2 | 49.0 | 52.5 | 53.6 | 53.6 | 2.2 | 3.3 | 3.3 | 3.3 |
|  | M3 | 26.9 | 58.6 | 58.6 | 58.6 | 1.8 | 3.0 | 3.0 | 3.0 |

either grants or denies a vacation period request in full. Including the employee preferences in the objective function does not significantly affect the penalty cost (mostly stays the same or slightly increases). Penalty costs are close to each other under weekly schedules, whereas differences are larger under the daily ones.

We next discuss the cross effects of employee preferences, substitution and task load distribution. Even the worst-performing one among those models that require substitution (M1S) results in more penalty costs than the best-performing model among those without substitution (M3N). This is observed in almost all problem


Figure 2. Penalty cost paid in all problem instances.
instances except for a few cases. Particularly, the penalty cost under M3N may be slightly less than that under M1S with daily schedules, no accrual and non-uniform task load distributions. We may naively say that ignoring employee preferences offsets the negative effects of a fluctuating task load. However, this is not the case if the entitlements are too long. These few exceptions are not observed under weekly schedules because, vacations need to be in blocks of days (i.e., weeks). Penalty cost is the minimum under uniform, maximum under random, and varies between these values under cyclic task load distributions.
5.3.2. Effects of Employee Preferences on Percent Vacation Granted and Satisfaction. Percent vacation granted behaves in the same manner as the penalty cost, i.e., it is the largest under M3, followed by M2, and then M1 (see Figure 3 below). However, the differences are not significant under weekly schedules. Larger differences exist under daily schedules, especially if substitution is not allowed or under non-uniform task loads (see Figures 4 and 5). In addition, without substitution and under non-uniform task load distributions, daily vacation schedules results in higher percent vacation granted than the weekly ones.

Even though the percent vacation granted by model M3 is the largest, the satisfaction level may be very low. The discrepancy between the two may be undesirable. In fact, percent satisfaction may be as low as $25 \%$ under model M3 even with substitution (see Figure 6). Modifying the objective function to include employee


Figure 3. Percent vacation granted in all problem instances.


Figure 4. Percent vacation granted in all problem instances (with substitution).


Figure 5. Percent vacation granted in all problem instances (without substitution).
preferences $(\alpha>0)$ may improve this poor behavior. Model performances vary depending on the cross effects of several factors. We discuss them below, where we also present cross effects of these factors on percent vacation granted.

- Percent vacation granted and percent satisfaction are the minimum under random, maximum under uniform, and varies between these values under cyclic task load distributions.
- With substitution, the satisfaction level depends on whether schedules are daily or weekly and accrual. With daily schedules, percent satisfaction is the highest under M2S, followed by M1S in majority of the instances and then M3S. Furthermore, M3 performs significantly worse than M2, but its performance increases with accrual, and approaches to that of model M2. With weekly schedules, satisfaction levels under M2S and M1S are very close to each other and M3S is significantly worse when employee preferences are disregarded. However, if the preferences are taken into account, M3S yields slightly higher satisfaction levels than the other two models (see Figure 7).
- Without substitution, satisfaction levels are not as high as those with substitution (under $75 \%$ with daily and $60 \%$ with weekly vacation schedules, see Figure 8). With daily vacation schedules, percent satisfaction is the highest under M2N, followed closely by M3N in majority of the instances and then M1N. With weekly vacation schedules, satisfaction levels under all models are very close to each other. If employee preferences are considered in the objective function $(\alpha>0)$, M3S catches up and yields slightly higher satisfaction levels than the other two models.
5.4. Effects of accrual and entitlement distribution. The penalty cost and the amount of vacation granted increase with accrual, whereas the percent vacation granted stays relatively constant if substitution is required. Without substitution,


Figure 6. Percent vacation granted from actual preferences in all problem instances.


Figure 7. Effect of $\alpha$ with substitution.


Figure 8. Effect of $\alpha$ without substitution.
the penalty cost increases and the percent vacation granted generally decreases with accrual. We note, however, that the vacation granted may increase or decrease with accrual depending on the model, task load distribution, and how large the accrual is. In particular, if the task load distribution is uniform, vacation days granted increase with accrual under all models. However, under cyclic or random task load distributions, vacation days granted may decrease with accrual. In particular, it becomes more difficult to grant vacations under a fluctuating task load without substitution under model M1. In terms of satisfaction levels, accruals result in different performances across models depending on if the substitution is required or not and whether daily or weekly vacation schedules are used. Particularly, if substitution is required, satisfaction increases with accrual under M3 both under daily and weekly vacation schedules. Under other models (M1 and M2) that include employee preferences as hard constraints, the satisfaction level may decrease (increase) if daily (weekly) vacation schedules are used. Without substitution, all models behave poorly, and we cannot say definitely if satisfaction levels increase or decrease with accrual.

Entitlement distribution's effect is overshadowed by those of substitution and task load distribution. In addition, when the number of employees requesting 21 or 28 days vary, comparing models M1 and M2 (which consider employee vacation preferences) is not meaningful. Therefore, we focus on model M3's performance to measure the effects of entitlement distribution on the penalty cost paid, percent vacation granted and satisfaction level. All measures considered are quite insensitive to the entitlement distribution if substitution is required. The penalty cost is smaller and the percent vacation granted is higher under $\mathrm{C} 2(70 \%, 30 \%)$ than those respective values under $\mathrm{C} 1(50 \%, 50 \%)$. This is expected since the majority of the employees are entitled to shorter vacation days. Satisfaction levels in general increase but may also decrease depending on the accrual and task load distribution.
5.5. Analysis of vacation granted to different employee groups in hierarchy. We now examine the vacation days granted, percent vacation granted and satisfaction levels for employee groups in the hierarchy and discuss how models compare with respect to these measures. Results of our case study show that the outcomes vary considerably across the employee groups, and we summarize our observations below.

- In general, enforcing substitution allows for higher levels of vacation and satisfaction for all groups. In terms of satisfaction levels, model M2 performs the best for almost all groups, with model M1 being a close contender and model M3 being the worst. We note that if employee preferences are considered in the objective function of model M3 $(\alpha>0)$, then it may be possible to improve satisfaction levels under this model. Particularly, we observed cases with weekly vacation schedules in which satisfaction levels obtained under model M3 are slightly better than those under model M2.
- Employees at the top of the hierarchy, i.e., the managers, hardly get any vacation days. The situation is worse for these employees under weekly vacation schedules. However, we observed that they receive some vacation (under half of the total they request) under cyclic task load distributions if model M3 is used. Similarly, employees in the second group are not granted any vacation unless substitution is required. Employees in the other groups are granted relatively more vacation days and have higher satisfaction levels than those in the first group.
- Satisfaction levels under daily vacation schedules are, in general, better than those under weekly vacation schedules. We can say that in the absence of substitution, using weekly vacation schedules is inferior to using daily vacation schedules. The worst-case scenario among all considered settings is realized if substitution is not allowed, task load distribution is random and if the employer uses a weekly vacation schedule. In this particular case, almost no vacation is granted to anyone.

6. Overall evaluation of performance measures and managerial insights. In this subsection, we examine the discrepancy between the percent vacation granted and the percent satisfaction resulting from using a particular vacation policy, i.e., a daily or a weekly vacation schedule. We further report on the corresponding penalty cost incurred to identify an applicable set of models among all those we considered. Finally, we provide some insights and recommendations.

We now list major observations obtained from the results of the case study. Figure 9 and Figure 10 show all three performance measures (i.e., percent vacation granted, percent satisfaction, and penalty cost) when the entitlement distribution is $\mathrm{C} 1(50 \%, 50 \%)$ relevant to the discussion. We note that the behavior is similar when the entitlement distribution is $\mathrm{C} 2(70 \%, 30 \%)$, but the corresponding figures are omitted due to space limitation.
O1. The discrepancy between percent vacation granted and percent satisfaction is the smallest for models M1 and M2 with almost similar values in magnitude. In fact, there is no discrepancy under daily vacation schedules, whereas there is some positive discrepancy under weekly vacation schedules.
O2. Although the least amount of penalty cost paid is under model M3, the maximum discrepancy is also observed for this model under all parameter settings.

O3. The penalty cost paid under model M2 is larger than that under model M3, and the penalty cost paid under model M1 is at least as large as that under model M2.
O4. Models M1 and M2 behave almost the same only if there is no accrual, weekly vacation schedules are used, and if the entitlement distribution is C1. Under all other parameter settings, model M1 is worse than model M2 in terms of percent vacation granted, satisfaction level, and penalty cost.
O5. Penalty cost is the highest under the random task load distribution.
O6. The task load distribution (except for the random case) does not significantly affect the percent vacation granted when substitution is enforced. However, without substitution, the percent vacation granted and the satisfaction level vary across different task load distributions depending on whether the vacation schedules are daily or weekly and the accrual scenario. In general, as the accrual increases, both the percent vacation and the satisfaction level decrease under non-uniform task load distributions. Almost no vacation is granted under any model if the task load distribution is random and if weekly vacations are used. The situation is only slightly better under models M2 and M3 if daily vacation schedules are used. However, model M1 does not grant any vacation at all, even under daily vacation schedules.
O7. Under uniform task load distribution with daily vacation schedules and no substitution, M2 may be considered the most viable model if the accrual amount is not too large. Because the percent vacation granted is high and comparable to that under model M3 with a relatively small increase in the penalty cost paid. Furthermore, there is no discrepancy between the percent vacation granted and the percent satisfaction, which makes M2 a more attractive model than M3 (see Figure 10).
O8. Under random and cyclic task load distributions with daily vacation schedules and no substitution, M3 seems to be the most viable model despite the positive (and sometimes very large) discrepancy between the percent vacation granted and the percent satisfaction (see Figure 10). This is because other models (M1 and M2) cannot grant as much vacation as M3 does, and their penalty costs are higher.
Based on the models and factors we considered and the observations above, we now provide some managerial insights and recommendations. We further note that the models provided in this study can be extended with additional workplace policies and can be implemented over shorter or longer vacation horizons.

- Model M1, which is used in practice at the financial center considered in the case study (and at many other workplaces due to its simplicity in implementation), results in the highest penalty cost for the employer, especially without substitution. Additionally, the percent vacation granted is significantly lower than those achieved by models M2 and M3. Therefore, an employer may be better off avoiding using all-or-nothing types of vacation policies, also considering health and other consequences for the employees.
- The above observations suggest that model M2 can be very attractive. An employer can grant most of the vacation days of entitlements while ensuring high satisfaction levels in terms of actual employee preferences with a relatively small increase in the penalty cost by using model M2 over M3. Another advantage of using model M2 over M3 is that M2 takes employee preferences into account via hard constraints. In contrast, no such consideration is part


Figure 9. Penalty cost paid versus percent vacation granted (overall and from preferences) in all problem instances (with substitution).
of the latter model. Such positive implications of using model M2 may also help with alleviating burnout at a workplace.

- An employer may prefer using model M3 with a modified objective that considers employee preferences, which will compensate for its shortcoming in ignoring employee preferences altogether. As a result, the employer can implement an attractive intermediate policy to ensure a high satisfaction level while reducing the penalty costs and granting most of the vacation days of entitlements. These may also help with alleviating burnout. However, from a practical point of view, even the modified model M3 may result in discontent among the employees since it has no guarantees that vacations will be granted based on the actual preferences.
- Penalty costs do not vary much between the models if weekly vacation schedules are used. In contrast, they are significantly different from each other if daily vacation schedules are used (see Figures 9 and 10). As a result, an employer who uses weekly vacation schedules may overlook the penalty cost and prioritize other performance measures to select an appropriate model. On the other hand, if an employer uses daily vacation schedules, all performance


Figure 10. Penalty cost paid versus percent vacation granted (overall and from preferences) in all problem instances (without substitution).
measures need to be weighed in, and tradeoffs should be evaluated. An employer who wants to ensure a high satisfaction level is better off using daily vacation schedules (for models M1 and M2 in particular) instead of weekly ones. In general, the discrepancy between the percent vacation granted and the satisfaction level increases in the latter.
7. Conclusion and future work. This study is motivated by the challenges and consequences faced by many employees while taking annual leave and the recent recognition of burnout by WHO. Although several media sources and scholarly articles point to the importance of providing paid vacation to employees, there is no study, to the best of our knowledge, that develops any theoretical or practical approach to generate realistic vacation schedules or investigates how different vacation policies compare from both the employer and the employees' perspectives. Through a real-life case study, we were able to show that employers should look into alternative vacation policies and thoroughly investigate the effects of several factors. We hope that this study will pave the way to develop other policies and approaches to the often-neglected vacation scheduling problem.

We formulate several models corresponding to different vacation policies that aim to minimize unprecedented costs of unused vacation days and differ on the degree that employee preferences are considered in scheduling vacations. Using a case study at a financial center, we evaluate and compare the models based on three main resultant performance measures: penalty cost of unused vacation days, percent vacation granted, and satisfaction level. We further report on the effects of several policy parameters and constraints on the way vacations are administered (daily vs. weekly), vacation accruals, hierarchy, and substitution among employees. Additionally, we consider four different distributions for the task load, including uniform, two different cyclic, and random distributions, to demonstrate how each policy and model performs under these different workload situations.

Our case study results show that the third model is the best performing one in terms of percent vacation granted and penalty cost, but it ignores employee preferences altogether. The second model, which takes employee vacation preferences into account, at least partially, performs comparable, especially under substitution. It exceeds the performance of the third model in the satisfaction level as expected. The first model, which either fully grants or denies a vacation request, results in the highest penalty costs for the employer and is the worst in terms of percent vacation granted, especially without substitution. Although variants of this first model are used in practice in several workplaces, we recommend employers to exercise caution in using such all-or-nothing types of approaches for scheduling vacations.

The most influential factors on the model performances are substitution and distribution of the task load. We observed that both absence of substitution and also fluctuations in the workload severely impair model performances. In practice, a decision maker rarely has control over the task load distribution but may at the very least enforce substitution to make up for the adverse effects of fluctuations. Nevertheless, randomness in the task load significantly hinders the ability of any model considered in this study to grant vacations. Therefore, a possible future work may consider developing more sophisticated models that may yield satisfactory or acceptable vacation schedules under a random task load distribution. Several other extensions to our work are possible. One immediate extension is to include possible hiring and dismissal scenarios. Additional objectives or constraints may be included to smooth out differences in the amounts of vacation granted among the employees. Effects of using different vacation scheduling approaches or policies for employees in different levels of the hierarchy may also be investigated. Researchers, practitioners, and policymakers will need to collaborate on identifying additional objectives and constraints whose socioeconomic implications are aligned. Future studies should consider paid annual leave policies designed to offer employees some appropriate degrees of choice and flexibility over their work and vacation and developing measures for the various resultant impacts on the employer, employees, and society. Finally, we believe that developing a decision support system with such models and possible extensions can be informative and valuable to both the employees and the decision makers.

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