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Effects of hybrid nanofluid on novel fractional model of heat transfer flow between two parallel plates

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Abstract In this paper, it has been discussed the fractional model of Brinkman type fluid (BTF) holding hybrid nanoparticles. Titanium dioxide (T_iO_2) and silver (Ag) nanoparticles were liquefied in water (H_2O) (base fluid) to make a hybrid nanofluid. The magnetohydrodynamic (MHD) free convection flow of the nanofluid ($Ag - T_iO_2 - H_2O$) was measured in a bounded microchannel. The BTF model was generalized using constant proportional Caputo fractional operator (CPC) with effective thermophysical properties. By introducing dimensionless variables, the governing equations of the model were solved by Laplace transform method. The testified outcomes are stated as M-function. The impact of associated parameters were measured graphically using Mathcad and offered a comparison with the existing results from the literature. The effect of related parameters was physically discussed. It was concluded that constant proportional Caputo fractional operator (CPC) showed better memory effect than Caputo-Fabrizio fractional operator (CF) (Saqib et al., 2020).

1. Introduction

By adding nanometer-sized particles in different base fluids, thermophysical properties can be upgraded in thermal transport systems. This procedure hints to a development in the thermal conductivity of the base fluids, creating it further consistent and continuing. The important fluids states to nanoflu-

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Nomenclature

CPC	constant proportional Caputo	J	current density
B	magnetic flux intensity	μ_0	magnetic permeability
σ_0	electrical conductivity	E	electric field intensity
V	velocity field (ms^{-1})	u	fluid velocity
ρ_{hnf}	density of hybrid nanofluid (kg m^{-3})	β_b^*	Brinkmann parameter
F_{em}	electromagnetic force	σ_{hnf}	electrical conductivity
μ_{hnf}	dynamic viscosity (kg m^{-1} s^{-1})	B_0	magnetic field
β_{hnf}	thermal expansion	T	temperature (K)
$(C_p)_{hnf}$	specific heat (jkg^{-1} K^{-1})	*Pr*	Prandtl number
K_{hnf}	thermal conductivity (Wm^{-2}–K^{-1})	α	Fractional parameter
Gr	Thermal Grashof number	t	Time (s)
Q	heat generation parameter	M	Magnetic parameter
T_w	Fluid temperature at the plate (K)		
CF	Caputo-Fabrizio		

ids with a wide series of submissions in various branches of science and engineering, with atomic devices, solar plates, heat exchangers, biotic and organic instruments and vehicle heaters [1–6]. First of all, Choi and Eastman offered the concept of nanofluids in 1995 [7]. Wong et al. [8] discussed many applications involving nanofluids. Mohain et al. [9] offered the essential thought and deliberated new novelties so as to simplify understanding nanofluids. They fixated new developments in the works, with a complete clarification of the thermophysical possessions and imitation of heat transfer in nanofluids flow. Mohain et al. [10] highlighted 2D and 3D modeling in a extensive variety of geometries using several numerical methods. Lastly, he delivered a comprehensive submissions for nanofluids in various branches of science with valuable recommendations.

Nanofluids have attained a big consideration from scientists because of their enriched heat transfer properties. Thickness shows a vigorous part in the effectiveness of nanofluids in the convection progressions as well. Lugo et al. [11] planned the rheological performance of a graphene nanofluid via rotating rheometer. Vallejo et al. [12] also examined the rheological possessions of 6 carbon-based nanofluids. Nowadays, nanofluids are categorized as hybrid nanofluids in various modules [13]. Hybrid nanofluids become by combining two distinct nanoparticles in base fluid. The key motivation of it is to moreover advance thermal features of nanofluids. Izadi et al. [14] considered adjustable heat transmission of hybrid nanofluid under the influence of outer magnetic field. Shahsavar et al. [15] investigated the innovative heat transfer in non-Newtonian hybrid nanofluid collected with entropy group. Moreover, Entropy group in the flow of hybrid nanofluid over nonlinear enlarging sheet was examined by Farooq et al. [16]. Fractional calculus has got significant attention of scientists since last few decades.

Firstly, Caputo advanced the fractional operator through Laplace convolutions of fractional derivatives and power-law functions. This fractional operator stabled the problem of the Riemann-Liouville fractional operator. CF and RL fractional operators are generally castoff in several branches of science [17,18]. Caputo and Fabrizio offered a new fractional operator with a nonsingular kernel (CF) [19]. CF has been positively engaged in a variation of real circumstances. After that in 2016, Atangana-Baleanu introduced a derivative which is better than Caputo-Fabrizio [20]. Jarad et al. [21] discussed the properties of generalized fractional derivatives of some functions with Caputo modification. Jarad et al. [22] deliberated the iteration methods on conformable derivatives by presenting new fractional derivatives and related theorems. Among them, Imran et al. [23] presented a complete explosion on convective flow of MHD viscous fluid with (ABC) and (CF) fractional derivatives. Atangana et al. [24] discussed application of freedman model and nonlinear baggs with new fractional operator. Dokuyucu et al. [25] used (ABC) to analyze the Keller'Segel model. Abro et al. [26] applied (ABC) and (CF) operators to find the effects of carbon nanotubes on MHD flow of methanol based nanofluids. Singh et al. [27] attained the solution of Fisher-Kolmogorov equation using (ABC) fractional approach. Dubey et al. [28] debated Caputo fractional derivative to find fractional power series solutions of nonlinear partial differential equations. Ikram et al. [29] described heat transference of viscous nanofluid on moving exponential perpendicular plate via (ABC) operator. Jarad et al. [30] offered new properties of fractional proportional derivatives.

Recently, Baleanu presented a new combined proportional caputo hybrid fractional operator, which gives best results in describing the memory effect of velocity and temperature fields than all other fractional operators [31]. Imran et al. [32] used (CPC) fractional approach to analyze of heat transfer flow of clay water based nanofluids. Newly, Saqib et al. [33] inspected fractional model of Brinkman type fluid with hybrid nanostructure via (CF) fractional operator. Imran et al. [34] deliberated the effect of hybrid nanofluids on heat transfer movement of a viscous fluid due to pressure gradient with (CPC) fractional derivative. Imran et al. [35] explained the application of novel way of modeling of heat and mass transfer flow of hybrid nanofluid for different base fluid water and engine oil via (C) fractional approach. Goufo used (CF) operator to analyze Korteweg-de Vries-Burgers equation [36]. Ahmad et al. [37] explored a fractional model of unsteady and an incompressible MHD viscous fluid with heat transfer via (CPC) fractional approach. Imran considered fundamental problem of fluid dynamics with (CPC) fractional derivative [38]. Imran et al. [39] used (CPC) in Stoke's first problem with

MHD effect and porosity. Furthermore, literature on hybrid nanofluids can be seen in the references [40–88].

In the past, no one has used (CPC) operator to solve (BTF) fractional model in prose. So, our inspiration is to improve (BTF) model with hybrid nanofluids and comparison the consequences, attained by Saqib et al. [33]. The Laplace transform technique to attain the logical solutions for temperature and velocity fields and showed in the form of M-function. The effect of parameters on temperature and velocity graphically.

2. Mathematical formulation

Consider MHD natural convection flow happens in the microchannel of a generalized, electrically conductive $(Ag - TiO_2 - H_2O)$ hybrid nanofluid.

The suppositions are the followings.

(a) Microchannel is considered of length infinite with width L. (i.e. L is distance between parallel plates.)
(b) The channel is along x-axis and normal to y-axis.
(c) At $t \leq 0$, temperature of the system is T_0.
(d) After $t = 0^+$, the temperature increases from T_0 to T_W.
(e) Fluid accelerates in the x-direction.
(f) Magnetic field of strength B_0 works transversely to the flow direction.

The flow of the electrically conductive $Ag - TiO_2 - H_2O$ hybrid nanofluid suffers electromotive force, which yields current. The induced magnetic field is ignored because of the hypothesis of a very small Reynold number. The electromagnetic force pivots on the intensity of electric flux [78].

The problem is governed by equations as under [33],

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (1)$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

Maxwell equation and generalized Ohm's law are related as [33],

$$\mathbf{J} = \sigma_0[\mathbf{E} + V \times \mathbf{B}]$$

The electromagnetic force is defined as [33],

$$\mathbf{F}_{em} = \mathbf{J} \times \mathbf{B} = \sigma_0[\mathbf{E} + V \times \mathbf{B}] \times \mathbf{B} = -\sigma B_0^2 u(y,t)\hat{i} \quad (3)$$

where $\hat{i}$ is along $x - axis$ and $u(y,t)$ is velocity of hybrid nanofluid. $\mathbf{F}_{em}$ is merged in velocity equation of the natural convection flow of $Ag - TiO_2 - H_2O$ hybrid nanofluid without pressure slope and a transverse magnetic field is engaged [79].

The governing equations are [33],

$$\rho_{hnf}\left(\frac{\partial u(y,t)}{\partial t} + \beta_{b^*} u(y,t)\right) = \mu_{hnf}\frac{\partial^2 u(y,t)}{\partial y^2} - \sigma_{hnf} B_0^2 u(y,t) + g(\rho\beta_T)_{hnf}(T - T_0), \quad (4)$$

$$(\rho C_p)_{hnf}\frac{\partial T}{\partial t} = k_{hnf}\frac{\partial^2 T}{\partial y^2} + Q_0(T - T_0), \quad (5)$$

subject to the constraints

$$u(y,0) = 0, \qquad T(y,0) = T_0, \qquad y \geqslant 0, \quad (6)$$

$$u(0,t) = 0, \qquad T(0,t) = T_0, \qquad t > 0, \quad (7)$$

$$u(L,t) = 0, \qquad T(L,t) = T_w, \qquad t > 0, \quad (8)$$

By introducing dimensionless variables into Eqs. (4)–(8),

$$\tau = \frac{v}{L^2}t, \qquad V = \frac{L}{v}u, \qquad Y = \frac{y}{L}, \qquad \theta = \frac{T - T_0}{T_w - T_0}, \quad (9)$$

We have the resulting dimensionless problem

$$A_0\frac{\partial V}{\partial \tau} + \beta_b V(Y,\tau) = A_1\frac{\partial^2 V}{\partial Y^2} - A_2 M V(Y,\tau) + A_3 Gr\theta(Y,\tau), \quad (10)$$

$$A_4 Pr\frac{\partial \theta}{\partial \tau} = \lambda_{hnf}\frac{\partial^2 \theta}{\partial Y^2} + Q\theta(Y,\tau), \quad (11)$$

subject to the constraints

$$V(Y,0) = 0, \qquad \theta(Y,0) = 0, \qquad Y \geqslant 0, \quad (12)$$

$$V(0,\tau) = 0, \qquad \theta(0,\tau) = 0, \qquad \tau > 0, \quad (13)$$

$$V(L,\tau) = 0, \qquad \theta(L,\tau) = 1, \qquad \tau > 0, \quad (14)$$

where

$$A_0 = 1 - \phi_{hnf} + \frac{\phi_{Ag}\rho_{Ag} + \phi_{TiO_2}\rho_{TiO_2}}{\rho_f},$$

$$A_1 = \frac{1}{\left[1 - (\phi_{Ag} + \phi_{TiO_2})\right]^{2.5}}, \qquad A_2 = \frac{\sigma_{hnf}}{\sigma_f},$$

$$A_3 = 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho\beta_T)_{Ag} + \phi_{TiO_2}(\rho\beta_T)_{TiO_2}}{(\rho\beta_T)_f},$$

$$A_4 = 1 - \phi_{hnf} + \frac{\phi_{Ag}(\rho C_p)_{Ag} + \phi_{TiO_2}(\rho C_p)_{TiO_2}}{(\rho C_p)_f},$$

$$\beta_b = \frac{L^2\beta_{b^*}\rho_{hnf}}{\mu}, \qquad \mathrm{M} = \frac{L^2\sigma_f B_0^2}{\mu},$$

$$\mathrm{Gr} = \frac{L^3 g(\beta_T)_f(T_w - T_0)}{v^2},$$

$$Pr = \frac{(\mu C_p)}{k_f}, \qquad Q = \frac{Q_0 L^2}{k_f}, \qquad \lambda_{hnf} = \frac{k_{hnf}}{k_f}. \quad (15)$$

3. Solution of the problem

The (CPC) fractional model of the problem is as follows from Eqs. (10)–(11),

$$\frac{\partial^2 V(Y,\tau)}{\partial Y^2} - B_1\frac{\partial^\alpha V(Y,\tau)}{\partial \tau^\alpha} - B_2 V(Y,\tau) + B_3 Gr\theta(Y,\tau) = 0, \quad (16)$$

$$\frac{\partial^2 \theta(Y,\tau)}{\partial Y^2} - B_4\frac{\partial^\alpha \theta(Y,\tau)}{\partial \tau^\alpha} + B_5\theta(Y,\tau) = 0, \quad (17)$$

where

$$B_1 = \frac{A_0}{A_1}, \qquad B_2 = \frac{A_2\mathrm{M} + \beta_b}{A_1}, \qquad B_3 = \frac{A_3\mathrm{Gr}}{A_1},$$
$$B_4 = \frac{A_4\mathrm{Pr}}{\lambda_{hnf}}, \qquad B_5 = \frac{Q}{\lambda_{hnf}},$$

The (CPC) fractional derivative of order α is defined as [31].

$$ {}^{CPC}D_t^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t [K_1(\alpha)f(x) + K_0(\alpha)f\prime(x)](t-x)^{-\alpha}dx. $$

with Laplace transform of (CPC) fractional derivative is

$$ L\left[{}^{CPC}D_t^{\alpha}f(t)\right] = \left[\frac{K_1(\alpha)}{s} + K_0(\alpha)\right]s^{\alpha}L[f(t)] - K_0(\alpha)s^{\alpha-1}f(0) $$

3.1. Solution of temperature field

By applying the Laplace transform to Eq. (17) with constraints $(13)_2, (14)_2$ and using (CPC) fractional derivative, we obtain,

$$ \left[\frac{\partial^2}{\partial Y^2} - B_4\left(\frac{k_1(\alpha)}{s} + k_0(\alpha)\right)s^{\alpha} + B_5\right]\bar{\theta}(Y,s) = 0, \quad (18) $$

satisfies

$$ \bar{\theta}(0,s) = 0, \qquad \bar{\theta}(1,s) = \frac{1}{s}, \qquad \tau > 0. \quad (19) $$

Using Eq. (19), we obtain the following Laplace transform of temperature profile

$$ \bar{\theta}(Y,s) = \frac{1}{s}\left[\frac{sinh(Y\sqrt{B_4\left(\frac{k_1(\alpha)}{s} + k_0(\alpha)\right)s^{\alpha} - B_5})}{sinh(\sqrt{B_4\left(\frac{k_1(\alpha)}{s} + k_0(\alpha)\right)s^{\alpha} - B_5})}\right] \quad (20) $$

Suitably written in equivalent form

$$ \bar{\theta}(Y,s) = \frac{1}{s}\sum_{\delta=0}^{\infty}\left[e^{-(2\delta+1-Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}} - e^{-(2\delta+1+Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}\right]. \quad (21) $$

Eq. (21) can also be expressed in series form so that we can find Laplace inverse transform analytically.

$$ \bar{\theta}(Y,s) = \frac{1}{s} + \sum_{\delta=0}^{\infty}\left[\sum_{\delta_1=1}^{\infty}\sum_{\delta_2=1}^{\infty}\sum_{\delta_3=1}^{\infty}\frac{(Y-2\delta-1)^{\delta_1}(-1)^{\frac{\delta_1}{2}+\delta_2}(B_4)^{\delta_2}(k_1(\alpha))^{\delta_3}}{\delta_1!\delta_2!\delta_3!(B_5)^{\delta_2-\frac{\delta_1}{2}}(k_0(\alpha))^{\delta_3-\delta_2}s^{\delta_3-\alpha\delta_2+1}}\frac{\Gamma(\frac{\delta_1}{2}+1)\Gamma(\delta_2+1)}{\Gamma(\frac{\delta_1}{2}+1-\delta_2)\Gamma(\delta_2+1-\delta_3)}\right] $$
$$ -\sum_{\delta=0}^{\infty}\left[\sum_{\delta_4=0}^{\infty}\sum_{\delta_5=0}^{\infty}\sum_{\delta_6=0}^{\infty}\frac{(-(2\delta+1+Y))^{\delta_4}(-1)^{\frac{\delta_4}{2}+\delta_5}(B_4)^{\delta_5}(k_1(\alpha))^{\delta_6}}{\delta_4!\delta_5!\delta_6!(B_5)^{\delta_5-\frac{\delta_4}{2}}(k_0(\alpha))^{\delta_6-\delta_5}s^{\delta_6-\alpha\delta_5+1}}\frac{\Gamma(\frac{\delta_4}{2}+1)\Gamma(\delta_5+1)}{\Gamma(\frac{\delta_4}{2}+1-\delta_5)\Gamma(\delta_5+1-\delta_6)}\right]. \quad (22) $$

Taking inverse Laplace transform on Eq. (22) and showing in M-function mentioned in [26], we get,

$$ \theta(Y,\tau) = 1 + \sum_{\delta=0}^{\infty}\sum_{\delta_1=1}^{\infty}\sum_{\delta_2=1}^{\infty}\frac{(Y-2\delta-1)^{\delta_1}(-1)^{\frac{\delta_1}{2}+\delta_2}(B_4)^{\delta_2}(k_0(\alpha))^{\delta_2}}{\delta_1!\delta_2!(B_5)^{\delta_2-\frac{\delta_1}{2}}} $$
$$ M_3^2\left[\frac{k_1(\alpha)}{k_0(\alpha)}\tau\Big|^{(\frac{\delta_1}{2}+1,0),(\delta_2+1,0)}_{(\frac{\delta_1}{2}+1-\delta_2,0),(\delta_2+1,-1),(1-\alpha\delta_2,1)}\right] \quad (23) $$
$$ -\sum_{\delta=0}^{\infty}\sum_{\delta_4=0}^{\infty}\sum_{\delta_5=0}^{\infty}\frac{(-(2\delta+1+Y))^{\delta_4}(-1)^{\frac{\delta_4}{2}+\delta_5}(B_4)^{\delta_5}(k_0(\alpha))^{\delta_5}}{\delta_4!\delta_5!(B_5)^{\delta_5-\frac{\delta_4}{2}}}M_3^2\left[\frac{k_1(\alpha)}{k_0(\alpha)}\tau\Big|^{(\frac{\delta_4}{2}+1,0),(\delta_5+1,0)}_{(\frac{\delta_4}{2}+1-\delta_5,0),(\delta_5+1,-1),(1-\alpha\delta_5,1)}\right]. $$

3.2. Nusselt number

We have computed the heat transfer rate in terms of Nusselt number through the following relation and presented in Table 1.

$$ Nu = -\frac{\partial\theta(Y,\tau)}{\partial Y}|_{Y=0} $$

3.3. Solution of Velocity Field

By applying the Laplace transform to Eq. (16) with constraints $(13)_1, (14)_1$ and using (CPC) fractional derivatives, we obtain

$$ \left[\frac{\partial^2}{\partial Y^2} - B_1\left(\frac{k_1(\alpha)}{s} + k_0(\alpha)\right)s^{\alpha} - B_2\right]\bar{V}(Y,s) = -B_3\bar{\theta}(Y,s), \quad (25) $$

satisfies

$$ \bar{V}(0,s) = 0, \qquad \bar{V}(1,s) = 0. \quad (26) $$

Using Eq. (26), we obtain the following Laplace transform of velocity profile

$$ \bar{V}(Y,s) = \frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-2\delta\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}} - e^{-(2\delta+2)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right]\times $$
$$ \sum_{\omega=0}^{\infty}\left[e^{-(2\omega+1-Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}} - e^{-(2\omega+1+Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}}\right] \quad (27) $$
$$ -\frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-(2\delta+1-Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}} - e^{-(2\delta+1+Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] $$

where

$$ B_6 = B_4 - B_1, \qquad B_7 = B_2 + B_5 $$

Eq. (27) can also be written as,

$$ \bar{V}(Y,s) = \bar{V}_1(Y,s) + \bar{V}_2(Y,s) + \bar{V}_3(Y,s) + \bar{V}_4(Y,s) + \bar{V}_5(Y,s) + \bar{V}_6(Y,s) \quad (28) $$

where

$$ \bar{V}_1(Y,s) = \frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-2\delta\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \times \sum_{\omega=0}^{\infty}\left[e^{-(2\omega+1-Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}}\right] \quad (29) $$

Table 1 Statistically analysis of Nusselt number for the effect of fractional parameter α.

α	Nu t = 2	Nu t = 3	Nu t = 4
0.1	1.764	1.713	1.664
0.2	1.769	1.726	1.687
0.3	1.775	1.740	1.708
0.4	1.781	1.753	1.729
0.5	1.788	1.766	1.748
0.6	1.795	1.779	1.766
0.7	1.802	1.791	1.783
0.8	1.810	1.803	1.798
0.9	1.817	1.815	1.813

$$\bar{V}_2(Y,s) = -\frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-(2\delta+2)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \times \sum_{\omega=0}^{\infty}\left[e^{-(2\omega+1-Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}}\right] \tag{30}$$

$$\bar{V}_3(Y,s) = -\frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-2\delta\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \times \sum_{\omega=0}^{\infty}\left[e^{-(2\omega+1+Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}}\right] \tag{31}$$

$$\bar{V}_4(Y,s) = \frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-(2\delta+2)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \times \sum_{\omega=0}^{\infty}\left[e^{-(2\omega+1+Y)\sqrt{B_1\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}+B_2}}\right] \tag{32}$$

$$\bar{V}_5(Y,s) = -\frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-(2\delta+1-Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \tag{33}$$

$$\bar{V}_6(Y,s) = \frac{B_3}{s}\sum_{\delta=0}^{\infty}\left[\frac{e^{-(2\delta+1+Y)\sqrt{B_4\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_5}}}{B_6\left(\frac{k_1(\alpha)}{s}+k_0(\alpha)\right)s^{\alpha}-B_7}\right] \tag{34}$$

It is difficult to find inverse Laplace transform of Eqs. (29)–(34), so we can write in suitably series form

$$\bar{V}_1(Y,s) = B_3\sum_{\omega=0}^{\infty}\sum_{\omega_1=0}^{\infty}\sum_{\omega_2=0}^{\infty}\sum_{\omega_3=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(Y-2\omega-1)^{\omega_1}(B_1)^{\omega_2}(B_4)^{i_2}}{\omega_1!\omega_2!\omega_3!i_1!i_2!i_3!j_2!(B_2)^{\omega_2-\frac{\omega_1}{2}}(B_5)^{i_2-\frac{i_1}{2}}}\times \frac{(B_6)^{j_1}(-2\delta)^{i_1}(-1)^{\frac{i_1}{2}+i_2+1}(k_1(\alpha))^{\omega_3+i_3+j_2}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_3-\omega_2+i_3-i_2+j_2-j_1}s^{\omega_3-\alpha\omega_2+i_3-\alpha i_2+j_2-\alpha j_1+1}}\times \frac{\Gamma(\frac{\omega_1}{2}+1)\Gamma(\omega_2+1)\Gamma(\frac{i_1}{2}+1)\Gamma(i_2+1)\Gamma(j_1+1)}{\Gamma(\frac{\omega_1}{2}+1-\omega_2)\Gamma(\omega_2+1-\omega_3)\Gamma(\frac{i_1}{2}+1-i_2)\Gamma(i_2+1-i_3)\Gamma(j_1+1-j_2)} \tag{35}$$

$$\bar{V}_2(Y,s) = -B_3\sum_{\omega=0}^{\infty}\sum_{\omega_1=0}^{\infty}\sum_{\omega_2=0}^{\infty}\sum_{\omega_3=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty}\sum_{i_6=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(Y-2\omega-1)^{\omega_1}(B_1)^{\omega_2}(B_4)^{i_5}}{\omega_1!\omega_2!\omega_3!i_4!i_5!i_6!j_2!(B_2)^{\omega_2-\frac{\omega_1}{2}}(B_5)^{i_5-\frac{i_4}{2}}}\times \frac{(B_6)^{j_1}(-2\delta-2)^{i_4}(-1)^{\frac{i_4}{2}+i_5+1}(k_1(\alpha))^{\omega_3+i_6+j_2}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_3-\omega_2+i_6-i_5+j_2-j_1}s^{\omega_3-\alpha\omega_2+i_6-\alpha i_5+j_2-\alpha j_1+1}}\times \frac{\Gamma(\frac{\omega_1}{2}+1)\Gamma(\omega_2+1)\Gamma(\frac{i_4}{2}+1)\Gamma(i_5+1)\Gamma(j_1+1)}{\Gamma(\frac{\omega_1}{2}+1-\omega_2)\Gamma(\omega_2+1-\omega_3)\Gamma(\frac{i_4}{2}+1-i_5)\Gamma(i_5+1-i_6)\Gamma(j_1+1-j_2)} \tag{36}$$

$$\bar{V}_3(Y,s) = -B_3\sum_{\omega=0}^{\infty}\sum_{\omega_4=0}^{\infty}\sum_{\omega_5=0}^{\infty}\sum_{\omega_6=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(-(2\omega+1+Y))^{\omega_4}(B_1)^{\omega_5}(B_4)^{i_2}}{\omega_4!\omega_5!\omega_6!i_1!i_2!i_3!j_2!(B_2)^{\omega_5-\frac{\omega_4}{2}}(B_5)^{i_2-\frac{i_1}{2}}}\times \frac{(B_6)^{j_1}(-2\delta)^{i_1}(-1)^{\frac{i_1}{2}+i_2+1}(k_1(\alpha))^{\omega_6+i_3+j_2}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_6-\omega_5+i_3-i_2+j_2-j_1}s^{\omega_6-\alpha\omega_5+i_3-\alpha i_2+j_2-\alpha j_1+1}}\times \frac{\Gamma(\frac{\omega_4}{2}+1)\Gamma(\omega_5+1)\Gamma(\frac{i_1}{2}+1)\Gamma(i_2+1)\Gamma(j_1+1)}{\Gamma(\frac{\omega_4}{2}+1-\omega_5)\Gamma(\omega_5+1-\omega_6)\Gamma(\frac{i_1}{2}+1-i_2)\Gamma(i_2+1-i_3)\Gamma(j_1+1-j_2)} \tag{37}$$

$$\bar{V}_4(Y,s) = B_3\sum_{\omega=0}^{\infty}\sum_{\omega_4=0}^{\infty}\sum_{\omega_5=0}^{\infty}\sum_{\omega_6=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty}\sum_{i_6=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(-(2\omega+1+Y))^{\omega_4}(B_1)^{\omega_5}(B_4)^{i_5}}{\omega_4!\omega_5!\omega_6!i_4!i_5!i_6!j_2!(B_2)^{\omega_5-\frac{\omega_4}{2}}(B_5)^{i_5-\frac{i_4}{2}}}\times \frac{(B_6)^{j_1}(-2\delta-2)^{i_4}(-1)^{\frac{i_4}{2}+i_5+1}(k_1(\alpha))^{\omega_6+i_6+j_2}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_6-\omega_5+i_6-i_5+j_2-j_1}s^{\omega_6-\alpha\omega_5+i_6-\alpha i_5+j_2-\alpha j_1+1}}\times \frac{\Gamma(\frac{\omega_4}{2}+1)\Gamma(\omega_5+1)\Gamma(\frac{i_4}{2}+1)\Gamma(i_5+1)\Gamma(j_1+1)}{\Gamma(\frac{\omega_4}{2}+1-\omega_5)\Gamma(\omega_5+1-\omega_6)\Gamma(\frac{i_4}{2}+1-i_5)\Gamma(i_5+1-i_6)\Gamma(j_1+1-j_2)} \tag{38}$$

$$\bar{V}_5(Y,s) = -B_3\sum_{\delta=0}^{\infty}\sum_{\delta_1=0}^{\infty}\sum_{\delta_2=0}^{\infty}\sum_{\delta_3=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(Y-2\delta-1)^{\delta_1}(-1)^{\frac{\delta_1}{2}+\delta_2+1}(B_4)^{\delta_2}(k_1(\alpha))^{\delta_3+j_2}}{\delta_1!\delta_2!\delta_3!j_2!(B_5)^{\delta_2-\frac{\delta_1}{2}}(k_0(\alpha))^{\delta_3-\alpha\delta_2+j_2-j_1}s^{\delta_3-\alpha\delta_2+j_2-\alpha j_1+1}}\times \frac{(B_6)^{j_1}\Gamma(\frac{\delta_1}{2}+1)\Gamma(\delta_2+1)\Gamma(j_1+1)}{(B_7)^{j_1+1}\Gamma(\frac{\delta_1}{2}+1-\delta_2)\Gamma(\delta_2+1-\delta_3)\Gamma(j_1+1-j_2)} \tag{39}$$

$$\bar{V}_6(Y,s) = B_3\sum_{\delta=0}^{\infty}\sum_{\delta_4=0}^{\infty}\sum_{\delta_5=0}^{\infty}\sum_{\delta_6=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\frac{(-(2\delta+1+Y))^{\delta_4}(-1)^{\frac{\delta_4}{2}+\delta_5+1}(B_4)^{\delta_5}(k_1(\alpha))^{\delta_6+j_2}}{\delta_4!\delta_5!\delta_6!j_2!(B_5)^{\delta_5-\frac{\delta_4}{2}}(k_0(\alpha))^{\delta_6-\alpha\delta_5+j_2-j_1}s^{\delta_6-\alpha\delta_5+j_2-\alpha j_1+1}}\times \frac{(B_6)^{j_1}\Gamma(\frac{\delta_4}{2}+1)\Gamma(\delta_5+1)\Gamma(j_1+1)}{(B_7)^{j_1+1}\Gamma(\frac{\delta_4}{2}+1-\delta_5)\Gamma(\delta_5+1-\delta_6)\Gamma(j_1+1-j_2)} \tag{40}$$

Taking inverse Laplace transform on Eq. (28) and showing in M-function mentioned in [26], we have,

$$V(Y,\tau) = V_1(Y,\tau) + V_2(Y,\tau) + V_3(Y,\tau) + V_4(Y,\tau) + V_5(Y,\tau) + V_6(Y,\tau) \tag{41}$$

where

$$V_1(Y,\tau) = B_3\sum_{\omega=0}^{\infty}\sum_{\omega_1=0}^{\infty}\sum_{\omega_2=0}^{\infty}\sum_{\omega_3=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{j_1=0}^{\infty}\frac{(Y-2\omega-1)^{\omega_1}(B_1)^{\omega_2}(B_4)^{i_2}}{\omega_1!\omega_2!\omega_3!i_1!i_2!i_3!(B_2)^{\omega_2-\frac{\omega_1}{2}}(B_5)^{i_2-\frac{i_1}{2}}}\times \frac{(B_6)^{j_1}(-2\delta)^{i_1}(-1)^{\frac{i_1}{2}+i_2+1}(k_1(\alpha))^{\omega_3+i_3}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_3-\omega_2+i_3-i_2-j_1}}\times M_6^5\left[\left.\frac{k_1(\alpha)}{k_0(\alpha)}\tau\right|^{(\frac{\omega_1}{2}+1,0),(\omega_2+1,0),(\frac{i_1}{2}+1,0),(i_2+1,0),(j_1+1,0)}_{(\frac{\omega_1}{2}+1-\omega_2,0),(\omega_2+1-\omega_3,0),(\frac{i_1}{2}+1-i_2,0),(i_2+1-i_3,0),(j_1+1,-1),(\omega_3-\alpha\omega_2+i_3-\alpha i_2-\alpha j_1+1,1)}\right] \tag{42}$$

$$V_2(Y,\tau) = -B_3\sum_{\omega=0}^{\infty}\sum_{\omega_1=0}^{\infty}\sum_{\omega_2=0}^{\infty}\sum_{\omega_3=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty}\sum_{i_6=0}^{\infty}\sum_{j_1=0}^{\infty}\frac{(Y-2\omega-1)^{\omega_1}(B_1)^{\omega_2}(B_4)^{i_5}}{\omega_1!\omega_2!\omega_3!i_4!i_5!i_6!(B_2)^{\omega_2-\frac{\omega_1}{2}}(B_5)^{i_5-\frac{i_4}{2}}}\times \frac{(B_6)^{j_1}(-2\delta-2)^{i_4}(-1)^{\frac{i_4}{2}+i_5+1}(k_1(\alpha))^{\omega_3+i_6}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_3-\omega_2+i_6-i_5-j_1}}\times M_6^5\left[\left.\frac{k_1(\alpha)}{k_0(\alpha)}\tau\right|^{(\frac{\omega_1}{2}+1,0),(\omega_2+1,0),(\frac{i_4}{2}+1,0),(i_5+1,0),(j_1+1,0)}_{(\frac{\omega_1}{2}+1-\omega_2,0),(\omega_2+1-\omega_3,0),(\frac{i_4}{2}+1-i_5,0),(i_5+1-i_6,0),(j_1+1,-1),(\omega_3-\alpha\omega_2+i_6-\alpha i_5-\alpha j_1+1,1)}\right] \tag{43}$$

$$V_3(Y,\tau) = -B_3\sum_{\omega=0}^{\infty}\sum_{\omega_4=0}^{\infty}\sum_{\omega_5=0}^{\infty}\sum_{\omega_6=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\sum_{j_1=0}^{\infty}\frac{(-(2\omega+1+Y))^{\omega_4}(B_1)^{\omega_5}(B_4)^{i_2}}{\omega_4!\omega_5!\omega_6!i_1!i_2!i_3!(B_2)^{\omega_5-\frac{\omega_4}{2}}(B_5)^{i_2-\frac{i_1}{2}}}\times \frac{(B_6)^{j_1}(-2\delta)^{i_1}(-1)^{\frac{i_1}{2}+i_2+1}(k_1(\alpha))^{\omega_6+i_3}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_6-\omega_5+i_3-i_2-j_1}}\times M_6^5\left[\left.\frac{k_1(\alpha)}{k_0(\alpha)}\tau\right|^{(\frac{\omega_4}{2}+1,0),(\omega_5+1,0),(\frac{i_1}{2}+1,0),(i_2+1,0),(j_1+1,0)}_{(\frac{\omega_4}{2}+1-\omega_5,0),(\omega_5+1-\omega_6,0),(\frac{i_1}{2}+1-i_2,0),(i_2+1-i_3,0),(j_1+1,-1),(\omega_6-\alpha\omega_5+i_3-\alpha i_2-\alpha j_1+1,1)}\right] \tag{44}$$

$$V_4(Y,\tau) = B_3\sum_{\omega=0}^{\infty}\sum_{\omega_4=0}^{\infty}\sum_{\omega_5=0}^{\infty}\sum_{\omega_6=0}^{\infty}\sum_{\delta=0}^{\infty}\sum_{i_4=0}^{\infty}\sum_{i_5=0}^{\infty}\sum_{i_6=0}^{\infty}\sum_{j_1=0}^{\infty}\frac{(-(2\omega+1+Y))^{\omega_4}(B_1)^{\omega_5}(B_4)^{i_5}}{\omega_4!\omega_5!\omega_6!i_4!i_5!i_6!(B_2)^{\omega_5-\frac{\omega_4}{2}}(B_5)^{i_5-\frac{i_4}{2}}}\times \frac{(B_6)^{j_1}(-2\delta-2)^{i_4}(-1)^{\frac{i_4}{2}+i_5+1}(k_1(\alpha))^{\omega_6+i_6}}{(B_7)^{j_1+1}(k_0(\alpha))^{\omega_6-\omega_5+i_6-i_5-j_1}}\times M_6^5\left[\left.\frac{k_1(\alpha)}{k_0(\alpha)}\tau\right|^{(\frac{\omega_4}{2}+1,0),(\omega_5+1,0),(\frac{i_4}{2}+1,0),(i_5+1,0),(j_1+1,0)}_{(\frac{\omega_4}{2}+1-\omega_5,0),(\omega_5+1-\omega_6,0),(\frac{i_4}{2}+1-i_5,0),(i_5+1-i_6,0),(j_1+1,-1),(\omega_6-\alpha\omega_5+i_6-\alpha i_5-\alpha j_1+1,1)}\right] \tag{45}$$

$$V_5(Y,\tau) = -B_3\sum_{\delta=0}^{\infty}\sum_{\delta_1=0}^{\infty}\sum_{\delta_2=0}^{\infty}\sum_{\delta_3=0}^{\infty}\sum_{j_1=0}^{\infty}\frac{(Y-2\delta-1)^{\delta_1}(-1)^{\frac{\delta_1}{2}+\delta_2+1}(B_4)^{\delta_2}(k_1(\alpha))^{\delta_3}(B_6)^{j_1}}{\delta_1!\delta_2!\delta_3!(B_5)^{\delta_2-\frac{\delta_1}{2}}(k_0(\alpha))^{\delta_3-\alpha\delta_2-j_1}(B_7)^{j_1+1}}\times M_4^3\left[\left.\frac{k_1(\alpha)}{k_0(\alpha)}\tau\right|^{(\frac{\delta_1}{2}+1,0),(\delta_2+1,0),(j_1+1,0)}_{(\frac{\delta_1}{2}+1-\delta_2,0),(\delta_2+1-\delta_3,0),(j_1+1,-1),(\delta_3-\alpha\delta_2-\alpha j_1+1,1)}\right] \tag{46}$$

$$V_6(Y,\tau) = B_3 \sum_{\delta=0}^{\infty} \sum_{\delta_4=0}^{\infty} \sum_{\delta_5=0}^{\infty} \sum_{\delta_6=0}^{\infty} \sum_{j_1=0}^{\infty} \frac{(-(2\delta+1+Y))^{\delta_4} (-1)^{\frac{\delta_4}{2}+\delta_5+1} (B_4)^{\delta_5} (k_1(\alpha))^{\delta_6} (B_6)^{j_1}}{\delta_4!\delta_5!\delta_6!(B_5)^{\delta_5-\frac{\delta_4}{2}} (k_0(\alpha))^{\delta_6-\alpha\delta_5-j_1} (B_7)^{j_1+1}} \times M_4^3 \left[\frac{k_1(\alpha)}{k_0(\alpha)} \tau \Big|^{(\frac{\delta_4}{2}+1,0),(\delta_5+1,0),(j_1+1,0)}_{(\frac{\delta_4}{2}+1-\delta_5,0),(\delta_5+1-\delta_6,0),(j_1+1,-1),(\delta_6-\alpha\delta_5-\alpha j_1+1,1)} \right]. \tag{47}$$

4. Graphical outcomes and arguments

The silver-titanium dioxide-water $(Ag - TiO_2 - H_2O)$ hybrid nanofluid was considered with fractional approach. The (BTF) model with resistance and under the effect of magnetic field was measured with partial differential equations. The constant proportional Caputo (CPC) fractional operator was used to examine the behavior of hybrid nanofluid with Laplace transform technique. The temperature and velocity fields are stated in forms of M-function. The impacts of parameters M, α , Q, ϕ_{hnf}, Gr and β_b on velocity and temperature fields are too measured and presented graphically with their physical significance (see Fig. 1).

Figs. 2–5 are designed to discribe the impact of α on temperature and velocity fields by comparing (CPC) operator with (CF) operator which is discussed in [33]. By increasing values of α the temperature and velocity are reduced because of loss in momentum, thermal and boundary sheets. It is also concluded that (CPC) operator gives better memory effect than (CF) operator. Figs. 6 and 7 represent the comparison of temperature and velocity fields for nanofluid particles $(Ag - H_2O)$ and $(TiO_2 - H_2O)$. Since Ag is good conductor and TiO_2 is semiconductor, so $(Ag - H_2O)$ had high temperature profile than $(TiO_2 - H_2O)$. While the nanofluid density has significant importance in the velocity field. Due to nanoparticles mixing with base fluid, the subsequent hybrid nanofluids develop much thicker which reduce the speed. $(Ag - H_2O)$ reduces velocity more than $(TiO_2 - H_2O)$ comparatively.

Figs. 8 and 9 are planned to learn the effect of Q which is very conventional. The temperature and velocity fields are increasing functions of Q. This is because of heat generating by system via increasing Q. Figs. 10 and 11 show the impact

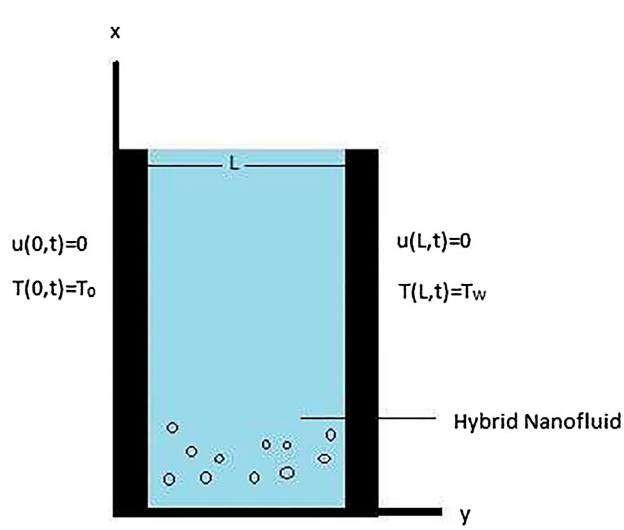

Fig. 1 Configuration of microchannel and coordinate system.

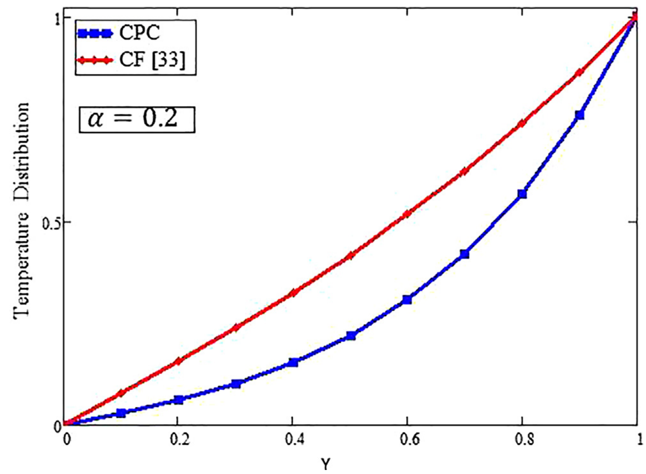

Fig. 2 Comparison between the temperatures with CPC and CF Saqib et al. [33], while t = 6, $Pr = 6.2$, Q = 0.3 and $\phi_{hnf} = 0.2$.

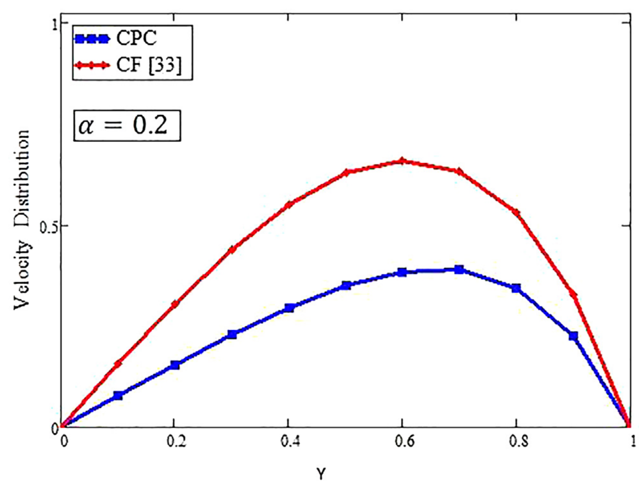

Fig. 3 Comparison between the velocities with CPC and CF Saqib et al. [33], while t = 3, $Pr = 6.2$, Q = 0.5, M = 0.2, Gr = 20, $\beta = 0.8$ and $\phi_{hnf} = 0.08$.

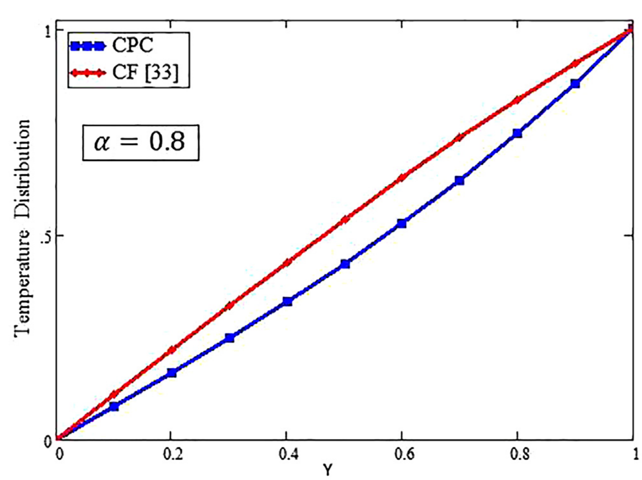

Fig. 4 Comparison between the temperatures with CPC and CF Saqib et al. [33], while t = 6, $Pr = 6.2$, Q = 0.6 and $\phi_{hnf} = 0.04$.

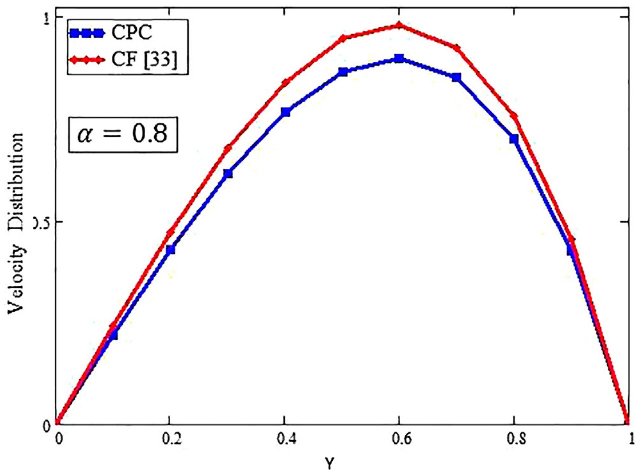

Fig. 5 Comparison between the velocities with CPC and CF Saqib et al. [33], while t = 2, $Pr = 6.2$, Q = 0.5, M = 0.2, Gr = 23, $\beta = 0.8$ and $\phi_{hnf} = 0.08$.

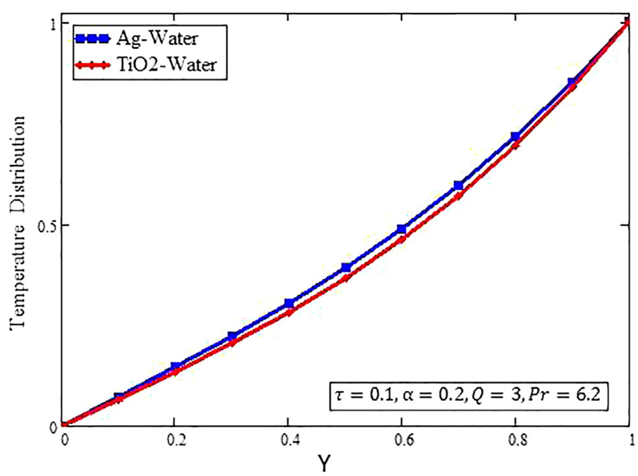

Fig. 6 Comparison between nanofluids for temperature, while t = 0.1, $Pr = 6.2$, Q = 3 and $\alpha = 0.2$.

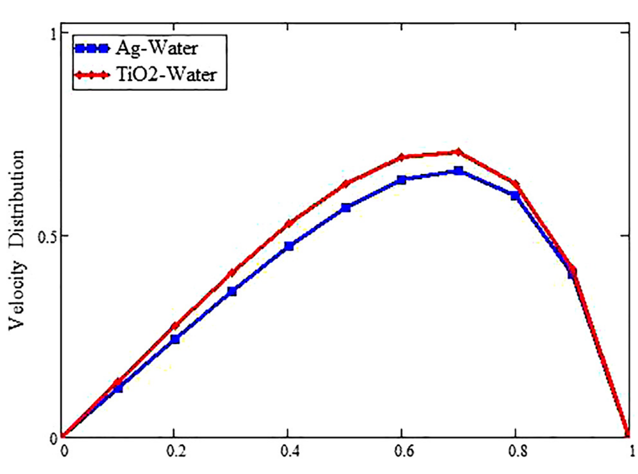

Fig. 7 Comparison between nanofluids for velocity, while t = 0.4, $Pr = 6.2$, Q = 0.5, M = 10, Gr = 40, $\beta = 2$ and $\alpha = 0.2$.

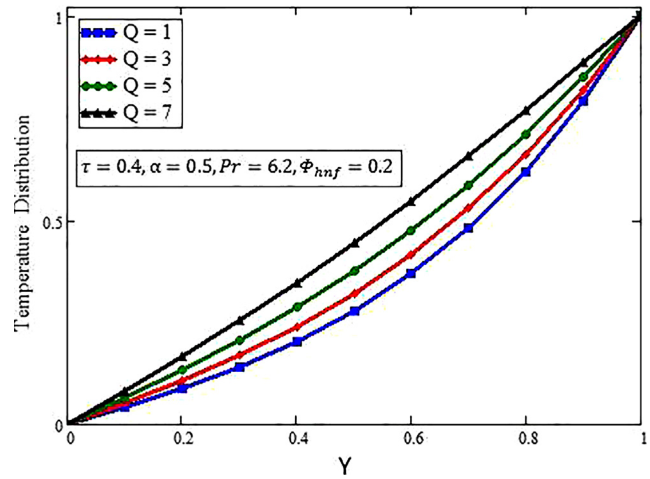

Fig. 8 Effect of Heat source Q on Temperature while t = 0.4, $Pr = 6.2$, $\phi_{hnf} = 0.2$ and $\alpha = 0.5$.

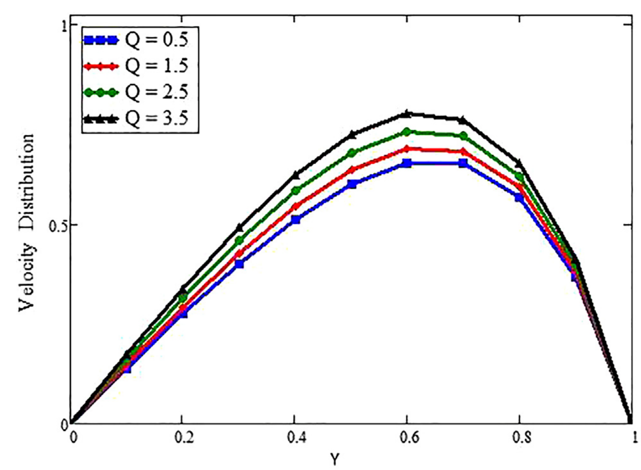

Fig. 9 Effect of Heat source Q on Velocity while t = 4, $Pr = 6.2$, $\phi_{hnf} = 0.08$, M = 2, Gr = 30, $\beta = 0.2$ and $\alpha = 0.5$.

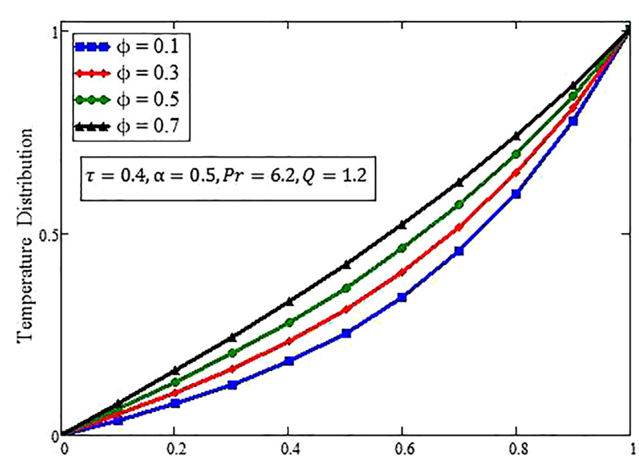

Fig. 10 Effect of ϕ_{hnf} on Temperature while t = 0.4, $Pr = 6.2$, Q = 1.2 and $\alpha = 0.5$.

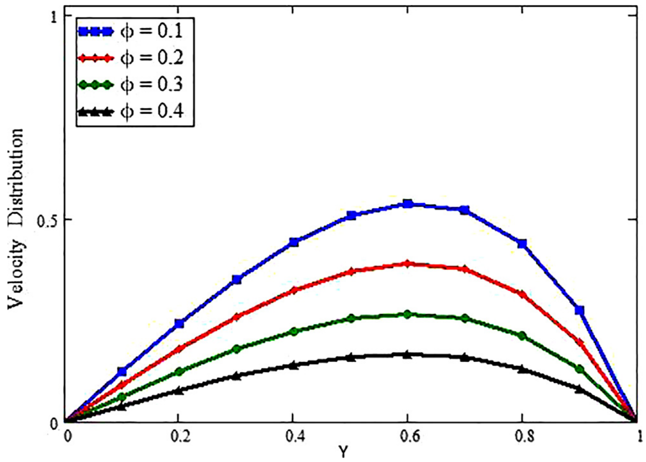

Fig. 11 Effect of ϕ_{hnf} on velocity while t = 2, $Pr = 6.2$, Q = 0.5, M = 0.3, Gr = 20, $\beta = 0.2$ and $\alpha = 0.5$.

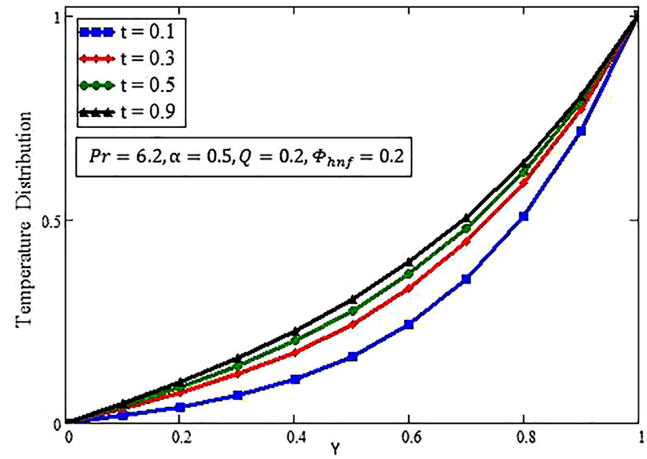

Fig. 12 Effect of time t on temperature while $Pr = 6.2$, Q = 0.2, $\phi_{hnf} = 0.2$ and $\alpha = 0.5$.

Table 2 Thermophysical possessions of nanoparticles & base fluid.

Material	Base Fluid H_2O	Nanoparticles Ag	Nanoparticles TiO_2
ρ	997.1	10500	425
C_p	4179	235	6862
k	0.613	429	8.9538
$\beta_T \times 10^{-5}$	21	1.89	0.9
σ	0.05	3.6×10^7	1×10^{-12}
Pr	6.2	–	–

of ϕ_{hnf} on the temperature and velocity fields. The temperature field represents as increasing function of ϕ_{hnf}. Table 2 shows that greater values of ϕ_{hnf} improve the ability of hybrid nanofluid to grip more heat. Though, the impact of ϕ_{hnf} on the velocity field is the conflicting. From Table 2, it can be seen that by increasing ϕ_{hnf} increment in thickness and viscidness, and a reduction in velocity. The same tendency for ϕ_{hnf} was resulted by Saqib et al. [33] (see Table 3).

The consequence of time t on temperature and velocity fields are discussed in Figs. 12 and 13 and concluded that by the passage of time temperature increased where as velocity decreases. It is because of viscosity of hybrid nanofluids. Figs. 14 and 15 are designed to see the comparison of different base fluids (water, kerosene oil, engine oil) for temperature and velocity fields. It is concluded that water with hybrid nanoparticles has great temperature because of great thermal conduc-

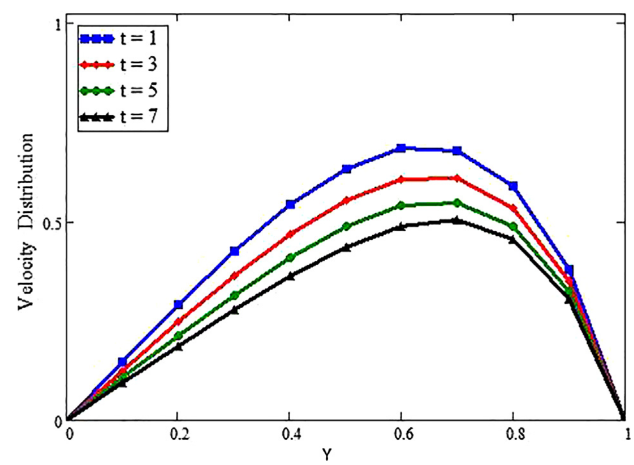

Fig. 13 Effect of t on velocity while $Pr = 6.2$, Q = 0.5, M = 4, Gr = 30, $\beta = 0.2$ $\phi_{hnf} = 0.08$ and $\alpha = 0.5$.

Table 3 Thermophysical Properties of nanofluid & hybrid nanofluid.

NanoFluid	Hybrid Nanofluid
$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hnf} = (1-\phi_{hnf})\rho_f + \phi_{Ag}\rho_{Ag} + \phi_{TiO_2}\rho_{TiO_2}$
$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$	$\mu_{hnf} = \frac{\mu_f}{[1-(\phi_{Ag}+\phi_{TiO_2})]^{2.5}}$
$(\rho\beta_T)_{nf} = (1-\phi)(\rho\beta_T)_f + \phi(\rho\beta_T)_s$	$(\rho\beta_T)_{hnf} = (1-\phi_{hnf})(\rho\beta_T)_f + \phi_{Ag}(\rho\beta_T)_{Ag} + \phi_{TiO_2}(\rho\beta_T)_{TiO_2}$
$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$	$(\rho C_p)_{hnf} = (1-\phi_{hnf})(\rho C_p)_f + \phi_{Ag}(\rho C_p)_{Ag} + \phi_{TiO_2}(\rho C_p)_{TiO_2}$
$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}$	$\frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3\left(\frac{\phi_{Ag}\sigma_{Ag}+\phi_{TiO_2}\sigma_{TiO_2}}{\sigma_f}-\phi_{hnf}\right)}{\left(\frac{\phi_{Ag}\sigma_{Ag}+\phi_{TiO_2}\sigma_{TiO_2}}{\phi_{hnf}\sigma_f}+2\right)-\left(\frac{\phi_{Ag}\sigma_{Ag}+\phi_{TiO_2}\sigma_{TiO_2}}{\sigma_f}-\phi_{hnf}\right)}$
$\frac{K_{nf}}{K_f} = \frac{k_s+2k_f-2\phi(k_s-k_f)}{k_s+2k_f+\phi(k_s-k_f)}$	$\frac{K_{hnf}}{K_f} = \frac{\frac{\phi_{Ag}k_{Ag}+\phi_{TiO_2}k_{TiO_2}}{\phi_{hnf}}+2k_f+2(\phi_{Ag}k_{Ag}+\phi_{TiO_2}k_{TiO_2})-2\phi_{hnf}k_f}{\frac{\phi_{Ag}k_{Ag}+\phi_{TiO_2}k_{TiO_2}}{\phi_{hnf}}+2k_f+(\phi_{Ag}k_{Ag}+\phi_{TiO_2}k_{TiO_2})-\phi_{hnf}k_f}$

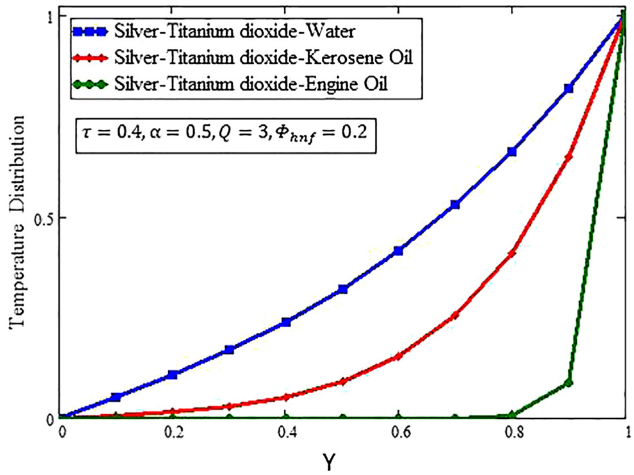

Fig. 14 Evaluation between different base fluids (Water, Kerosene Oil and Engine Oil) for Temperature while t = 0.4, $\alpha = 0.5$, Q = 3, $\phi_{hnf} = 0.2$ and $\alpha = 0.5$.

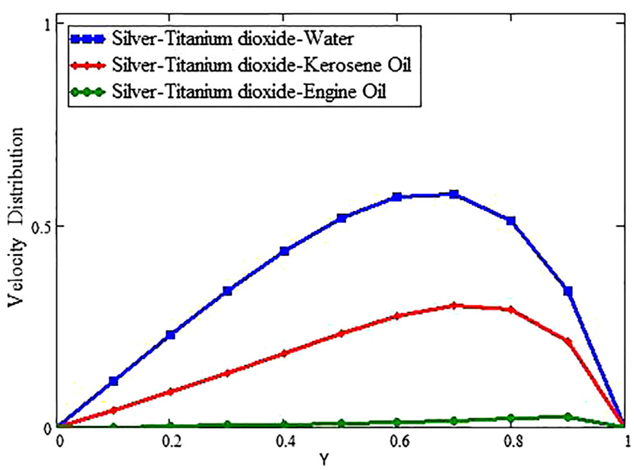

Fig. 15 Evaluation between base fluids (Water, Kerosene Oil and Engine Oil) for Velocity while t = 4, Q = 0.5, M = 4, Gr = 30, $\beta = 0.2$ $\phi_{hnf} = 0.08$ and $\alpha = 0.5$.

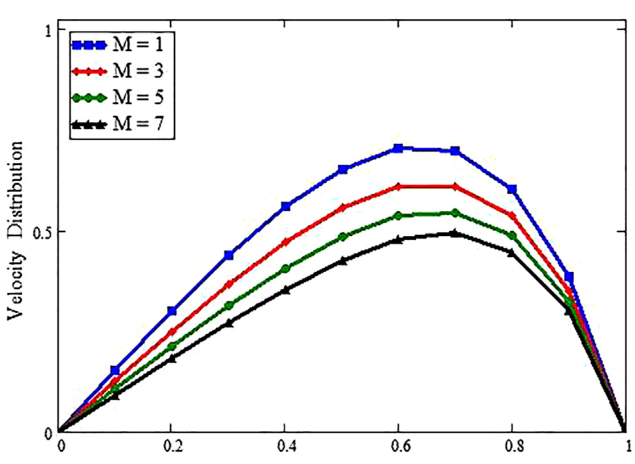

Fig. 16 Effect of Magnetic field M on velocity while t = 4, Q = 0.5, Pr = 6.2, Gr = 30, $\beta = 0.2$ $\phi_{hnf} = 0.08$ and $\alpha = 0.5$.

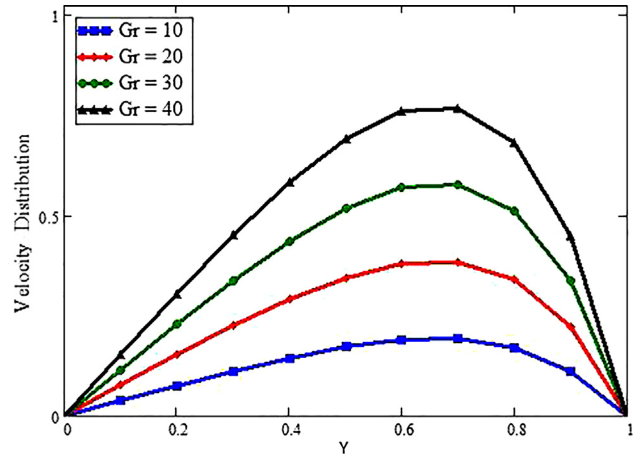

Fig. 17 Effect of Gr on velocity while t = 4, Q = 0.5, Pr = 6.2, M = 4, $\beta = 0.2$, $\phi_{hnf} = 0.08$ and $\alpha = 0.5$.

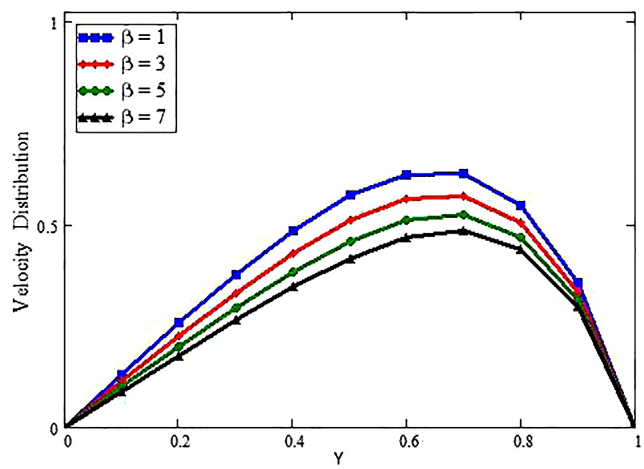

Fig. 18 Effect of Brinkman type fluid β on velocity while t = 4, Q = 0.5, Pr = 6.2, M = 2, Gr = 30, $\phi_{hnf} = 0.08$ and $\alpha = 0.5$.

tivity than other base fluids and also resulted that water-based hybrid nanoparticles has advanced velocity away from the plate while near the plate velocity of engine oil-based fluid declines. The reason of this is because of the distinctions in thermal conductivities of base fluids.

The impact of M is demonstrated in Fig. 16. It directs that the with greater value of M, velocity reduces. Since M relays to resistive type forces, known as Lorentz forces. So the velocity was decreased. Fig. 17 exposes the impact of Gr on the velocity field. It is realized that with high values of Gr, the velocity faster. Since Gr is related to buoyancy forces which rise the natural convection, so the velocity rises in speed. The influence of β_b on the velocity field is depicted in Fig. 18. By rising β_b the fluid velocity reduces due to greater values of β_b firming the drag forces, which inclines to reduce the velocity field. Same impact was discussed in [33].

5. Conclusions

In this paper, the behavior of $(Ag - TiO_2 - H_2O)$ hybrid nanofluid discussed. The movement of the hybrid nanofluid was measured in a microchannel. The heat source and MHD effects were too considered. (CPC) fractional operator used to solve problem by converting it into fractional model. The exact results were gained using the Laplace transform technique and expressed in the form of M-function. The exact solutions for the temperature and velocity fields were also discussed graphically through physical significance. The important results of this study are as follows.

- Temperature is increasing function of ϕ_{hnf} while velocity is decreasing function of ϕ_{hnf}.
- Temperature and velocity of $(Ag - TiO_2 - H_2O)$ is high in comparison with other base fluids.
- Nusselt number is increasing function of fractional parameter α
- Velocity rises with the rising values of Gr whereas it falls with rising values of M and β.
- In the comparison between (CPC) and (CF) [33], we found that (CPC) is best in describing the memory of velocity and temperature fields.
- Both velocity and temperature rise with the rising values of Q.

The thermophysical properties and values of nanoparticles as follows [33].

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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