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Erratum to 'Meir-Keeler α-contractive fixed and common fixed point theorems'

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Abstract
In this note we correct some errors that appeared in the article (Abdeljawad in Fixed Point Theory Appl. 2013:19, 2013) by modifying some conditions in the main theorems and by giving an example to support.
MSC: 47H10; 54H25

After examining the calculations in the proof of the uniqueness part in Theorem 8 in [1] and Steps 3 and 4 of Theorem 16, we found that they do not lead to strict inequalities, and hence the proofs failed. In this note, we slightly modify some of the used conditions to achieve our claim.

The following theorem is a modification to Theorem 8 in [1]. The proof is the same as in [1] except the uniqueness part will be proved by using the new modified condition (H) in the statement of the theorem.

Theorem 1 *Let* (X,d) *be an* (f,g)*-orbitally complete metric space, where* f, g *are self-mappings of* X. *Also, let* $\alpha : X \times X \to [0,\infty)$ *be a mapping. Assume the following*:

1. (f,g) *is* α*-admissible and there exists an* $x_0 \in X$ *such that* $\alpha(x_0, fx_0) \geq 1$;
2. *the pair* (f,g) *is generalized Meir-Keeler* α*-contractive*.

   Then the sequence $d_n = d(x_n, x_{n+1})$ *is monotone decreasing. If, moreover, we assume that*
3. *on the* (f,g)*-orbit of* x_0, *we have* $\alpha(x_n, x_j) \geq 1$ *for all* n *even and* $j > n$ *odd and that* f *and* g *are continuous on the* (f,g)*-orbit of* x_0.

Then either (1) f *or* g *has a fixed point in the* (f,g)*-orbit* $\{x_n\}$ *of* x_0 *or* (2) f *and* g *have a common fixed point* p *and* $\lim x_n = p$. *If, moreover, we assume that the following condition* (H) *holds*:

(H) *If for all fixed points* x *and* y *of* (f,g), $\alpha(x,y) \geq 1$,

then the uniqueness of the fixed point is obtained.

Proof To prove uniqueness, assume p is the common fixed point obtained as $x_n \to p$ and q is another common fixed point. Then, Eq. (5) in [1] and the condition (H) yield

$$\begin{aligned} d(p,q) &= d(fp,gq) \\ &\leq \alpha(p,q) d(fp,gq) \\ &< \max\left\{ d(p,q), d(p,fp), d(q,gq), \frac{d(p,gq)+d(q,fp)}{2} \right\} = d(p,q). \end{aligned}$$

Thus we reach $d(p,q) < d(p,q)$, and hence a contradiction, which implies that $p = q$. □

Using the new modified condition (H) for the pair (f,f), we modify the uniqueness part of Corollary 9 in [1].

The following example shows that we lose the uniqueness if our modified (H) condition is not satisfied.

Example 2 Let $X = [0,2]$ with the absolute value metric $d(x,y) = |x-y|$. Define $f : X \to X$ by

$$f(x) = \begin{cases} 0, & x \in \{0, \frac{1}{4}\}, \\ 1, & x \in (0, \frac{1}{2}) - \{\frac{1}{4}\}, \\ \frac{3}{2}, & x \in [\frac{1}{2}, 2]. \end{cases}$$

Also, define

$$\alpha(x,y) = \begin{cases} 1, & x,y \in [\frac{1}{2}, 2], \\ 0, & \text{otherwise}. \end{cases}$$

Notice that f has two common fixed points $x = 0$ and $x = \frac{3}{2}$. This is because f satisfies all the hypotheses of the corollary (which is Corollary 9 in [1]) except the condition (H), *i.e.*, $\alpha(0, \frac{3}{2}) = 0 < 1$.

The following theorem is a modification of Theorem 16 in [1]. The proofs of Step 3 and Step 4 are given only according to the new modified conditions (H) and (f-H).

Theorem 3 *Let f, g be continuous self-maps of a metric space (X,d) such that $g \in C_f$. Assume that $\alpha(x_n, x_m) \geq 1$ for all $m > n$. If g is a generalized Meir-Keeler α-f-contractive map such that α satisfies the condition* (f-H): *If $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_m) \geq 1$ for all $m > n$ and $fx_n \to z$, then $\alpha(z, fz) \geq 1$. Also assume that the condition* (H) *is satisfied. Then f and g have a unique common fixed point.*

Proof

- Step 3. We show that $\eta = fz = gz$ is a common fixed point for f and g. Assume that $f\eta \neq \eta$. Then $f^2z \neq fz$, and by the help of the (f-H) condition, we have

$$\begin{aligned} d(\eta, f\eta) &= d(gz, fgz) = d(gz, gfz) \\ &\leq \alpha(z, fz) d(gz, gfz) \\ &< \max\left\{d(fz, ffz), d(fz, gz), d(ffz, gfz), \frac{d(fz, gfz) + d(ffz, gz)}{2}\right\} \\ &= \max\left\{d(\eta, f\eta), 0, 0, d(\eta, f\eta)\right\}. \end{aligned}$$

Thus we have $d(\eta, f\eta) < d(\eta, f\eta)$, which gives a contradiction and therefore $f\eta = \eta$. Moreover, $g\eta = gfz = f\eta = \eta$.

- Step 4. The uniqueness of the common fixed point. Assume that $\eta = fz = gz$ is our common fixed point for f and g, where $fx_n \to z$, and ω is another common fixed point. Then, by the (H) condition, we have

$$\begin{aligned} d(\eta,\omega) &= d(g\eta,g\omega) \\ &\le \alpha(\eta,\omega)d(g\eta,g\omega) \\ &< \max\left\{d(f\eta,f\omega),d(f\eta,g\eta),d(f\omega,g\omega),\frac{d(f\eta,g\omega)+d(f\omega,g\eta)}{2}\right\}, \end{aligned}$$

which gives $d(\eta,\omega) < d(\eta,\omega)$, a contradiction, and hence $\eta = \omega$. □

Instead of the modified condition (f-H) above, the following condition can be used (s-f-H). If $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_m) \ge 1$ for all $m > n$ and $fx_n \to z$, then $\alpha(fx_n,fz) \ge k$ for all n, where $k > 1$, and hence Step 3 will be proved as follows.

We show that $\eta = fz = gz$ is a common fixed point for f and g. Assume that $f\eta \ne \eta$. Then $f^2z \ne fz$, and by the help of the (s-f-H) condition, we have

$$d(\eta,f\eta) = d(gz,fgz) = d(gz,gfz) = \lim_{n\to\infty} d(gfx_n,gfz)$$

and

$$\begin{aligned} d(gfx_n,gfz) &\le k^{-1}\alpha(fx_n,fz)d(gfx_n,gfz) \\ &\le k^{-1}\max\left\{d(ffx_n,ffz),d(ffx_n,gfx_n),d(ffz,gfz),\frac{d(ffx_n,gfz)+d(ffz,gfx_n)}{2}\right\}. \end{aligned}$$

If we let $n \to \infty$ above and use the continuity and commutativity of f and g, then we reach $d(\eta,f\eta) \le k^{-1}d(\eta,f\eta) < d(\eta,f\eta)$, and hence $f\eta = \eta$. Moreover, $g\eta = gfz = f\eta = \eta$.

Finally, according to the modifications above, the (H) condition only in Theorem 18 of [1] is needed to be modified.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the manuscript.

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